Mathematics Model Curriculum

Ohio | Department of Education

This is the March 2011 version of the Grade 4 Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, and identify misconceptions and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multistate initiatives to develop common content elaborations and learning expectations.

Grade 4	
Domain	**Cluster**
Operations and Algebraic Thinking	• Use the four operations with whole numbers to solve problems. • Gain familiarity with factors and multiples. • Generate and analyze patterns.
Number and Operations in Base Ten	• Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations— Fractions	• Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions, and compare decimal fractions.
Measurement and Data	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Represent and interpret data. • Geometric measurement: understand concepts of angle and measure angles.
Geometry	• Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Grade 4

Domain	Operations and Algebraic Thinking
Cluster	*Use the four operations with whole numbers to solve problems.*
Standards	1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Students need experiences that allow them to connect mathematical statements and number sentences or equations. This allows for an effective transition to formal algebraic concepts. They represent an unknown number in a word problem with a symbol. Word problems which require multiplication or division are solved by using drawings and equations.

Students need to solve word problems involving multiplicative comparison (product unknown, partition unknown) using multiplication or division as shown in Table 2 of the Common Core State Standards for Mathematics, page 89. They should use drawings or equations with a symbol for the unknown number to represent the problem. Students need to be able to distinguish whether a word problem involves multiplicative comparison or additive comparison (solved when adding and subtracting in Grades 1 and 2).

Present multistep word problems with whole numbers and whole-number answers using the four operations. Students should know which operations are needed to solve the problem. Drawing pictures or using models will help students understand what the problem is asking. They should check the reasonableness of their answer using mental computation and estimation strategies.

Examples of multistep word problems can be accessed from the released questions on the NAEP (National Assessment of Educational Progress) Assessment at http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx.

For example, a constructed response question from the 2007 Grade 4 NAEP assessment reads, "Five classes are going on a bus trip and each class has 21 students. If each bus holds only 40 students, how many buses are needed for the trip?"

Instructional Resources/Tools

Table 2. Common multiplication and division situations (Common Core State Standards for Mathematics 2010)

The National Assessment of Educational Progress (NAEP) Assessments - http://nces.ed.gov/nationsreportcard/itmrlsx/search.aspx.

Common Misconceptions

Diverse Learners

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Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #1 , **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.** More information about this critical area of focus can be found by clicking here.

Represent and solve problems involving multiplication and division (Grade 3 OA 3).

Solve problems involving the four operations, and identify and explain patterns in arithmetic (Grade 3 OA 8).

Grade 4

Domain	Operations and Algebraic Thinking
Cluster	*Gain Familiarity with factors and multiples.*
Standards	4. Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number is the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

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Instructional Strategies and Resources

Instructional Strategies

Students need to develop an understanding of the concepts of number theory such as prime numbers and composite numbers. This includes the relationship of factors and multiples. Multiplication and division are used to develop concepts of factors and multiples. Division problems resulting in remainders are used as counter-examples of factors.

Review vocabulary so that students have an understanding of terms such as factor, product, multiples, and odd and even numbers.

Students need to develop strategies for determining if a number is prime or composite, in other words, if a number has a whole number factor that is not one or itself. Starting with a number chart of 1 to 20, use multiples of prime numbers to eliminate later numbers in the chart. For example, 2 is prime but 4, 6, 8, 10, 12,… are composite. Encourage the development of rules that can be used to aid in the determination of composite numbers. For example, other than 2, if a number ends in an even number (0, 2, 4, 6 and 8), it is a composite number.

Using area models will also enable students to analyze numbers and arrive at an understanding of whether a number is prime or composite. Have students construct rectangles with an area equal to a given number. They should see an association between the number of rectangles and the given number for the area as to whether this number is a prime or composite number.

Definitions of prime and composite numbers should not be provided, but determined after many strategies have been used in finding all possible factors of a number.

Provide students with counters to find the factors of numbers. Have them find ways to separate the counters into equal subsets. For example, have them find several factors of 10, 14, 25 or 32, and write multiplication expressions for the numbers.

Another way to find the factor of a number is to use arrays from square tiles or drawn on grid papers. Have students build rectangles that have the given number of squares. For example if you have 16 squares:
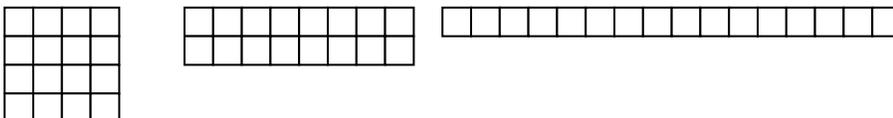
The idea that a product of any two whole numbers is a common multiple of those two numbers is a difficult concept to understand. For example. 5 x 8 is 40; the table below shows the multiples of each factor.

5	10	15	20	25	30	35	**40**	45
8	16	24	32	**40**	48	56	64	72

Ask students what they notice about the number 40 in each set of multiples; 40 is the 8th multiple of 5, and the 5th multiple of 8.

Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in Grade 6.

Writing multiplication expressions for numbers with several factors and for numbers with a few factors will help students in making conjectures about the numbers. Students need to look for commonalities among the numbers.

Instructional Resources/Tools

Calculators
Counters
Grid papers

The Ohio Resource Center

ORC # 397 From the National Council of Teachers of Mathematics, Illuminations: The Factor Game engages students in a friendly contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors. Students are then guided through an analysis of game strategies and introduced to the definitions of prime and composite numbers.

Understanding factoring through geometry - Using square unit tiles, students work with a partner to construct all rectangles whose area is equal to a given number. After several examples, students see that prime numbers are associated with exactly two rectangles, whereas composite numbers are associated with more than two rectangles.

ORC # 4209, From the National Council of Teachers of Mathematics, Illuminations, The Product Game – Classifying Numbers. Students construct Venn diagrams to show the relationships between the factors or products of two or more numbers in the Product Game.

ORC # 1161, From the National Council of Teachers of Mathematics, Illuminations, The Product Game. In the Product Game, students start with factors and multiply to find the product. In The Factor Game, students start with a number and find its factors.

ORC # 4001, From the National Council of Teachers of Mathematics, Illuminations, Multiplication: It's in the Cards – More Patterns with Products.

National Library of Virtual Manipulatives

The National Library of Virtual Manipulatives contains Java applets and activities for K-12 mathematics.

Sieve of Eratosthenes – relate number patterns with visual patterns. Click on the link for *Activities* for directions on engaging students in finding all prime numbers 1-100.

Common Misconceptions
When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.

Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.

Diverse Learners
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Specific strategies for mathematics may include:
Some students may need to start with numbers that have only one pair of factors, then those with two pairs of factors before finding factors of numbers with several factor pairs.

Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #1, **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.** More information about this critical area of focus can be found by clicking here.

Understand properties of multiplication and the relationship between multiplication and division (Grade 3 OA 5 – 6).

Geometric measurement: understand concepts of area and relate area to multiplication and to addition (Grade 3 MD 7a).

The concepts of prime, factor and multiple are important in the study of relationships found among the natural numbers. Compute fluently with multi-digit numbers and find common factors and multiples (Grade 6 NS 4).

Domain	Operations and Algebraic Thinking
Cluster	**_Generate and analyze patterns._**
Standards	5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. _For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way._

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources
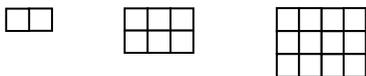
Instructional Strategies

In order for students to be successful later in the formal study of algebra, their algebraic thinking needs to be developed. Understanding patterns is fundamental to algebraic thinking. Students have experience in identifying arithmetic patterns, especially those included in addition and multiplication tables. Contexts familiar to students are helpful in developing students' algebraic thinking.

Students should generate numerical or geometric patterns that follow a given rule. They should look for relationships in the patterns and be able to describe and make generalizations.

As students generate numeric patterns for rules, they should be able to "undo" the pattern to determine if the rule works with all of the numbers generated. For example, given the rule, "Add 4" starting with the number 1, the pattern 1, 5, 9, 13, 17, … is generated. In analyzing the pattern, students need to determine how to get from one term to the next term. Teachers can ask students, "How is a number in the sequence related to the one that came before it?", and "If they started at the end of the pattern, will this relationship be the same?" Students can use this type of questioning in analyzing numbers patterns to determine the rule.

Students should also determine if there are other relationships in the patterns. In the numeric pattern generated above, students should observe that the numbers are all odd numbers.

Provide patterns that involve shapes so that students can determine the rule for the pattern. For example,

Students may state that the rule is to multiply the previous number of squares by 3.

Instructional Resources/Tools

From PBS Teachers: Snake Patterns –s-s-s: Students will use given rules to generate several stages of a pattern and will be able to predict the outcome for any stage.

From the National Council of Teachers of Mathematics, Illuminations: Patterns that Grow – Growing Patterns. Students use numbers to make growing patterns. They create, analyze, and describe growing patterns and then record them. They also analyze a special growing pattern called Pascal's triangle.

From the National Council of Teachers of Mathematics, Illuminations: Patterns that Grow – Exploring Other Number Patterns. Students analyze numeric patterns, including Fibonacci numbers. They also describe numeric patterns and then record them in table form.

From the National Council of Teachers of Mathematics, Illuminations: Patterns that Grow – Looking Back and Moving Forward. In this final lesson of the unit, students use logical thinking to create, identify, extend, and translate patterns. They make patterns with numbers and shapes and explore patterns in a variety of mathematical contexts.

Misconceptions

Diverse Learners

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Connections:

This cluster goes beyound the Fourth Grade Critical Areas of Focus to address **Analyzing patterns**. More information about this critical area of focus can be found by clicking here.

Solve problems involving the four operations, and identify and explain patterns in arithmetic (3.OA.4.9).

Grade 4

Domain	Number and Operations Base Ten
Cluster	*Generalize place value understanding for multi-digit whole numbers.*
Standards	1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 3. Use place value understanding to round multi-digit whole numbers to any place.

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Instructional Strategies and Resources

Instructional Strategies

Provide multiple opportunities in the classroom setting and use real-world context for students to read and write multi-digit whole numbers.

Students need to have opportunities to compare numbers with the same number of digits, e.g., compare 453, 698 and 215; numbers that have the same number in the leading digit position, e.g., compare 45, 495 and 41,223; and numbers that have different numbers of digits and different leading digits, e.g., compare 312, 95, 5245 and 10,002.

Students also need to create numbers that meet specific criteria. For example, provide students with cards numbered 0 through 9. Ask students to select 4 to 6 cards; then, using all the cards make the largest number possible with the cards, the smallest number possible and the closest number to 5000 that is greater than 5000 or less than 5000.

In Grade 4, rounding is not new, and students need to build on the Grade 3 skill of rounding to the nearest 10 or 100 to include larger numbers and place value. What is new for Grade 4 is rounding to digits other than the leading digit, e.g., round 23,960 to the nearest hundred. This requires greater sophistication than rounding to the nearest ten thousand because the digit in the hundreds place represents 900 and when rounded it becomes 1000, not just zero.

Students should also begin to develop some rules for rounding, building off the basic strategy of; "Is 48 closer to 40 or 50?" Since 48 is only 2 away from 50 and 8 away from 40, 48 would round to 50. Now students need to generalize the rule for much larger numbers and rounding to values that are not the leading digit.

Instructional Resources/Tools

Place value boxes
Place value flip charts
Number cards

Common Misconceptions

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition method.

Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole.

Students need to be aware of the greatest place value. In this example, there is one number with the lead digit in the thousands and another number with its lead digit in the hundreds.

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Specific Strategies for mathematics may include:
Development of a clear understanding of the value of the digits in a number is critical for the understanding of and using numbers in computations. Helping students build the understanding that 12345 means one ten thousand or 10,000, two thousands or 2000, three hundreds or 300, four tens or 40, and 5 ones or 5. Additionally, the answer is the sum of each of these values 10,000 + 2000 + 300 + 40 + 5.

Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #1**, Developing an understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends.** More information about this critical area of focus can be found by clicking here.

A strong foundation in whole-number place value and rounding is critical for the expansion to decimal place value and decimal rounding.

Understand place value (Grade 2 NBT 1 – 4).

Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 3 NBT 1).

Grade 4

Domain	Number and Operations in Base Ten
Cluster	***Use place value understanding and properties of operations to perform multi-digit arithmetic***
Standards	4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. 5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Content Elaborations (in development)

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Instructional Strategies and Resources

Instructional Strategies

A crucial theme in multi-digit arithmetic is encouraging students to develop *strategies* that they understand, can explain, and can think about, rather than merely follow a sequence of directions that they don't understand.

It is important for students to have seen and used a variety of strategies and materials to broaden and deepen their understanding of place value before they are required to use standard algorithms. The goal is for them to *understand* all the steps in the algorithm, and they should be able to explain the meaning of each digit. For example, a 1 can represent one, ten, one hundred, and so on. For multi-digit addition and subtraction in Grade 4, the goal is also fluency, which means students must be able to carry out the calculations efficiently and accurately.

Start with a student's understanding of a certain strategy, and then make intentional, clear-cut connections for the student to the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.

Sometimes students benefit from 'being the teacher' to an imaginary student who is having difficulties applying standard algorithms in addition and subtraction situations. To promote understanding, use examples of student work that have been done incorrectly and ask students to provide feedback about the student work.

It is very important for some students to talk through their understanding of connections between different strategies and standard addition and subtractions algorithms. Give students many opportunities to talk with classmates about how they could explain standard algorithms. Think-Pair-Share is a good protocol for all students.

When asking students to gain understanding about multiplying larger numbers, provide frequent opportunities to engage in mental math exercises. When doing mental math, it is difficult to even *attempt* to use a strategy that one does not fully understand. Also, it is a natural tendency to use numbers that are 'friendly' (multiples of 10) when doing mental math, and this promotes its understanding.

Use a variation of an area model. For example, to multiply 23 × 36, arrange the partial products as follows:

	20	+	3
	600	90	
30 + 6	120	18	

Then add the four partial products to get 828.

As students developed an understanding of multiplying a whole number up to four digits by a one-digit whole number, and multiplying two two-digit numbers through various strategies, they should do the same when finding whole-number quotients and remainders. By relating division to multiplication and repeated subtraction, students can find partial quotients. An explanation of partial quotients can be viewed at http://www.teachertube.com, search for *Outline of partial quotients*. This strategy will help them understand the division algorithm.

Students will have a better understanding of multiplication or division when problems are presented in context.

Students should be able to illustrate and explain multiplication and division calculations by using equations, rectangular arrays and the properties of operations. These strategies were used in Grade 3 as students developed an understanding of multiplication.

To give students an opportunity to communicate their understandings of various strategies, organize them into small groups and ask each group to create a poster to explain a particular strategy and then present it to the class.

Vocabulary is important. Students should have an understanding of terms such as, sum, difference, fewer, more, less, ones, tens, hundreds, thousands, digit, whole numbers, product, factors and multiples.

Instructional Resources/Tools

Place-value mats
bound place value flip books (so that the digit in a certain place can be switched)
base ten blocks
 tens frames
 hundreds flats
 Smartboard
Make literature connections by using the resource book, Read Any Good Math Lately?, to identify books related to certain math topics. Books can provide a 'hook' for learning, to activate background knowledge, and to build student interest.

Common Misconceptions

Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

Diverse Learners

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Specific strategies for mathematics may include:
If students are having difficulty with lining up similar place values in numbers as they are adding and subtracting, it is sometimes helpful to have them write their calculations on grid paper. This assists the student with lining up the

numbers more accurately.

If students are having a difficult time with a standard addition algorithm, a possible modification to the algorithm might be helpful. Instead of the 'shorthand' of 'carrying,' students could add by place value, moving left to right placing the answers down below the 'equals' line. For example:

```
  249      (start with 200 + 300 to get the 500, then 40 + 70 to get 110, and 9 + 2 for 11)
+372
  500
  110
   11
  621
```

Connections:

This Cluster is connected to the Fourth Grade Critical Areas of Focus #1 , **Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends,** and go beyond to address **Adding and subtracting multi-digit whole numbers**. More information about this critical area of focus can be found by clicking here.

Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 3 NBT 2 – 3)

Use the four operations with whole numbers to solve problems (Grade 4 OA 2 – 3).

Generalize place value understanding for multi-digit whole numbers (Grade 4 NBT 1 – 2).

Grade 4

Domain	Number and Operations – Fractions
Cluster	***Extend understanding of fractions equivalence and ordering.***
Standards	1. Explain why a fraction *a/b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Content Elaborations (in development)

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Instructional Strategies and Resources

Instructional Strategies

Students' initial experience with fractions began in Grade 3. They used models such as number lines to locate unit fractions, and fraction bars or strips, area or length models, and Venn diagrams to recognize and generate equivalent fractions and make comparisons of fractions.

Students extend their understanding of unit fractions to compare two fractions with different numerators and different denominators.

Students should use models to compare two fractions with different denominators by creating common denominators or numerators. The models should be the same (both fractions shown using fraction bars or both fractions using circular models) so that the models represent the same whole. The models should be represented in drawings. Students should also use benchmark fractions such as $\frac{1}{2}$ to compare two fractions. The result of the comparisons should be recorded using >, < and = symbols.

$\frac{3}{4}$

$\frac{1}{2}$

$\frac{1}{3}$
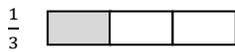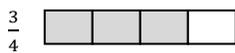
Instructional Resources/Tools

Pattern blocks
Fraction bars or strips

Misconceptions

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $\frac{1}{2}$ to sixths. They would multiply the denominator by 3 to get $\frac{1}{6}$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction.

Students need to use a fraction in the form of one such as $\frac{3}{3}$ so that the numerator and denominator do not contain the original numerator or denominator.
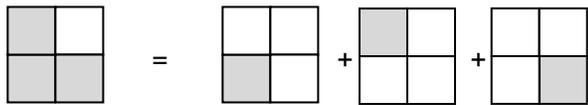
Diverse Learners

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Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #2, **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.** More information about this critical area of focus can be found by clicking here.

Develop understanding of fractions as numbers (Grade 3 NF 3).

Domain	Number and Operations - Fractions
Cluster	***Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.***

Standards	3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.
	    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
	    b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
	    Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
	    c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
	    d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
	4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
	    a. Understand a fraction *a*/*b* as a multiple of 1/*b*.
	    For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
	    b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number.
	    For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)
	    c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
	    For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

In Grade 3, students added unit fractions with the same denominator. Now, they begin to represent a fraction by decomposing the fraction as the sum of unit fraction and justify with a fraction model. For example, $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

Students also represented whole numbers as fractions. They use this knowledge to add and subtract mixed numbers with like denominators using properties of number and appropriate fraction models. It is important to stress that whichever model is used, it should be the same for the same whole. For example, a circular model and a rectangular model should not be used in the same problem.

Understanding of multiplication of whole numbers is extended to multiplying a fraction by a whole number. Allow students to use fraction models and drawing to show their understanding.

Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.

Instructional Resources/Tools

Fraction tiles/bars

Circular fraction models

Rulers with markings of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$

Number lines

Common Misconceptions

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #2 , **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.** More information about this critical area of focus can be found by clicking here.

Represent and interpret data (Grade 4 MD 4).

Grade 4

Domain	Number and Operations - Fractions
Cluster	*Understand decimal notations for fractions, and compare decimal fractions.*

Standards	5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.
	  For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.
	6. Use decimal notation for fractions with denominators 10 or 100.
	  For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
	7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

The place value system developed for whole numbers extends to fractional parts represented as decimals. This is a connection to the metric system. Decimals are another way to write fractions. The place-value system developed for whole numbers extends to decimals. The concept of one whole used in fractions is extended to models of decimals.

Students can use base-ten blocks to represent decimals. A 10 x 10 block can be assigned the value of one whole to allow other blocks to represent tenths and hundredths. They can show a decimal representation from the base-ten blocks by shading on a 10 x 10 grid.

Students need to make connections between fractions and decimals. They should be able to write decimals for fractions with denominators of 10 or 100. Have students say the fraction with denominators of 10 and 100 aloud. For example $\frac{4}{10}$ would "four tenths" or $\frac{27}{100}$ would be "twenty-seven hundredths." Also, have students represent decimals in word form with digits and the decimal place value, such as $\frac{4}{10}$ would be 4 tenths.

Students should be able to express decimals to the hundredths as the sum of two decimals or fractions. This is based on understanding of decimal place value. For example 0.32 would be the sum of 3 tenths and 2 hundredths. Using this understanding students can write 0.32 as the sum of two fractions $\left(\frac{3}{10} + \frac{2}{100}\right)$.

Students' understanding of decimals to hundredths is important in preparation for performing operations with decimals to hundredths in Grade 5.

In decimal numbers, the value of each place is 10 times the value of the place to its immediate right. Students need an understanding of decimal notations before they try to do conversions in the metric system. Understanding of the decimal place value system is important prior to the generalization of moving the decimal point when performing operations involving decimals.
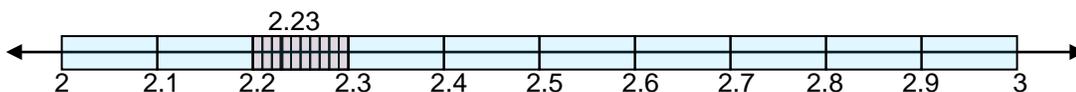
Students extend fraction equivalence from Grade 3 with denominators of 2, 3 4, 6 and 8 to fractions with a denominator of 10. Provide fraction models of tenths and hundredths so that students can express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100.

When comparing two decimals, remind students that as in comparing two fractions, the decimals need to refer to the same whole. Allow students to use visual models to compare two decimals. They can shade in a representation of each decimal on a 10 x 10 grid. The 10 x 10 grid is defined as one whole. The decimal must relate to the whole.

| 0.3 | 0.03 |
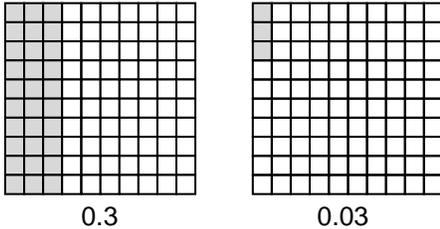
Flexibility with converting fractions to decimals and decimals to fractions provides efficiency in solving problems involving all four operations in later grades.

Instructional Resources/Tools

Length or area models
10 x 10 square on a grid
Decimal place-value mats
Base-ten blocks
Number lines

From the National Council of Teachers of Mathematics, Illuminations: A Meter of Candy – In this series of three hands-on activities, students develop and reinforce their understanding of hundredths as fractions, decimals and percentages. Students explore with candy pieces as they physically make and connect a set and linear model (meter) to produce area models (grids and pie graphs). At this time, students are not to do percents. The relationships among fractions, decimals and percents are developed in Grade 6.

Common Misconceptions

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that).03 is greater than 0.3.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #2, **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.** More information about this critical area of focus can be found by clicking here.

Connect with understanding and generating equivalent fractions (Grade 4 NF 1 – 2).

Students will perform operations with decimals to hundredths in Grade 5 (Grade 5 NBT 5 -7).

Grade 4

Domain	Measurement and Data
Cluster	***Solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit.***
Standards	1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* 2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. 3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

In order for students to have a better understanding of the relationships between units, they need to use measuring devices in class. The number of units needs to relate to the size of the unit. They need to discover that there are 12 inches in 1 foot and 3 feet in 1 yard. Allow students to use rulers and yardsticks to discover these relationships among these units of measurements. Using 12-inch rulers and yardstick, students can see that three of the 12-inch rulers, which is the same as 3 feet since each ruler is 1 foot in length, are equivalent to one yardstick. Have students record the relationships in a two column table or t-charts. A similar strategy can be used with rulers marked with centimeters and a meter stick to discover the relationships between centimeters and meters.

Present word problems as a source of students' understanding of the relationships among inches, feet and yards.

Students are to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

Present problems that involve multiplication of a fraction by a whole number (denominators are 2, 3, 4, 5 6, 8, 10, 12 and 100). Problems involving addition and subtraction of fractions should have the same denominators. Allow students to use strategies learned with these concepts.

Students used models to find area and perimeter in Grade 3. They need to relate discoveries from the use of models to develop an understanding of the area and perimeter formulas to solve real-world and mathematical problems.

Instructional Resources/Tools

Yardsticks(meter sticks) and rulers (marked with customary and metric units)
Teaspoons and tablespoons
Graduated measuring cups (marked with customary and metric units)

Common Misconceptions

Students believe that larger units will give the larger measure. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room

with one-inch tiles, with one-foot rulers, and with yard sticks. Students should notice that it takes fewer yard sticks to measure the room than rulers or tiles.

Diverse Learners
Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Fourth Grade Critical Areas of Focus #1 , **Developing understanding and fluency with multi-digit multiplication , and developing understanding of dividiing to find quotients involving multi-digit dividends**, and #2, **Developing an understanding of fractions equivalence, addition and subtraction of fractions with like denominators, and multipication of fractions by whole numbers**. More information about these critical areas of focus can be found by clicking here.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures (Grade 3 MD 8).

Geometric measurement; understand concepts of area and relate area to multiplication and to addition.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (Grade 4 NF 3 – 4).

Domain	Measurement and Data
Cluster	*Represent and interpret data.*
Standards	4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

**Expectations for Learning (in development)**

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Data has been measured and represented on line plots in units of whole numbers, halves or quarters. Students have also represented fractions on number lines. Now students are using line plots to display measurement data in fraction units and using the data to solve problems involving addition or subtraction of fractions.

Have students create line plots with fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$ and plot data showing multiple data points for each fraction.
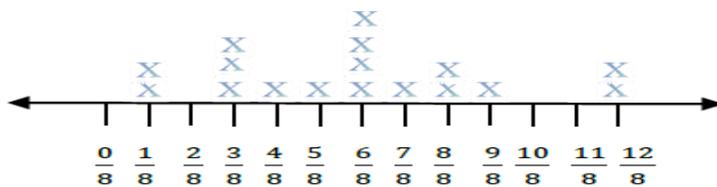
Pose questions that students may answer, such as

- "How many one-eighths are shown on the line plot?" Expect "two one-eighths" as the answer. Then ask, "What is the total of these two one-eighths?" Encourage students to count the fractional numbers as they would with whole-number counting, but using the fraction name.
- "What is the total number of inches for insects measuring $\frac{3}{8}$ inches?" Students can use skip counting with fraction names to find the total, such as, "three-eighths, six-eighths, nine-eighths. The last fraction names the total. Students should notice that the denominator did not change when they were saying the fraction name. have them make a statement about the result of adding fractions with the same denominator.
- "What is the total number of insects measuring $\frac{1}{8}$ inch or $\frac{5}{8}$ inches?" Have students write number sentences to represent the problem and solution such as, $\frac{1}{8} + \frac{1}{8} + \frac{5}{8} = \frac{7}{8}$ inches.

Use visual fraction strips and fraction bars to represent problems to solve problems involving addition and subtraction of fractions.

Instructional Resources/Tools

Fraction bars or strips

Common Misconceptions

Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.

Diverse Learners
Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Specific strategies for mathematics may include:
Create number lines with the same denominator without using the equivalent form of a fraction. For example, on a number line using eighths, use $\frac{4}{8}$ instead of $\frac{1}{2}$. This will help students later when they are adding or subtracting fractions with unlike denominators. When representations have unlike denominators, students ignore the denominators and add the numerators only.

Have students create stories to solve addition or subtraction problems with fractions to use with student created fraction bars/strips.

Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #2 , **Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.** More information about this critical area of focus can be found by clicking here.

Understand a fraction as a number on the number line; represent fractions on a number line diagram (Grade 3 NF 2).

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size (Grade 3 NF 3).

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters (Grade 3 MD 4).

Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem (Grade 4 NF 3d).

Grade 4

Domain	Measurement and Data
Cluster	*Geometric measurement: understand concepts of angle and measure angles.*
Standards	5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. 6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. 7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

**Expectations for Learning (in development)**

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Angles are geometric shapes composed of two rays that are infinite in length. Students can understand this concept by using two rulers held together near the ends. The rulers can represent the rays of an angle. As one ruler is rotated, the size of the angle is seen to get larger. Ask questions about the types of angles created. Responses may be in terms of the relationship to right angles. Introduce angles as acute (less than the measure of a right angle) and obtuse (greater than the measure of a right angle). Have students draw representations of each type of angle. They also need to be able to identify angles in two-dimensional figures.

Students can also create an angle explorer (two strips of cardboard attached with a brass fastener) to learn about angles.
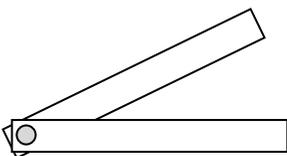
They can use the angle explorer to get a feel of the relative size of angles as they rotate the cardboard strips around.

Students can compare angles to determine whether an angle is acute or obtuse. This will allow them to have a benchmark reference for what an angle measure should be when using a tool such as a protractor or an angle ruler. Provide students with four pieces of straw, two pieces of the same length to make one angle and another two pieces of the same length to make an angle with longer rays.

Another way to compare angles is to place one angle over the other angle. Provide students with a transparency to compare two angles to help them conceptualize the spread of the rays of an angle. Students can make this comparison by tracing one angle and placing it over another angle. The side lengths of the angles to be compared need to be different.

Students are ready to use a tool to measure angles once they understand the difference between an acute angle and an obtuse angle. Angles are measured in degrees. There is a relationship between the number of degrees in an angle

and circle which has a measure of 360 degrees. Students are to use a protractor to measure angles in whole-number degrees. They can determine if the measure of the angle is reasonable based on the relationship of the angle to a right angle. They also make sketches of angles of specified measure.

Instructional Resources/Tools

Cardboard cut in strips to make an angle explorer
Brass fasteners
Protractor
Angle ruler
Straws
Transparencies
Angle explorers

Ohio Resource Center
Sir Cumference and the Great Knight of Angleland: In this story, young Radius, son of Sir Cumference and Lady Di of Ameter, undertakes a quest, the successful completion of which will earn him his knighthood. With the help of a family heirloom that functions much like a protractor, he is able to locate the elusive King Lell and restore him to the throne of Angleland. In gratitude, King Lell bestows knighthood on Sir Radius. This book is an entry on the Ohio Resource Center *Mathematics Bookshelf*

From the National Council of Teachers of Mathematics, Figure This: What's My Angle? math Challenge # 10 - Students can estimate the measures of the angles between their fingers when they spread out their hand.

From PBS Teachers: 3rd Grade Measuring Game, Identify acute, obtuse and right angles in this online interactive game
From PBS Teachers: Star Gazing, Determine the correct angle at which to place a telescope in order to see as many stars as possible in this online interactive game.

Common Misconceptions

Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.

Diverse Learners

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Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #3, **Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.** More information about this critical area of focus can be found by clicking here.

Connect measuring angle to the Geometry domain in which students draw and identify angles as right, acute and obtuse (Grade 4 G 1).

Grade 4

Domain	Geometry
Cluster	***Draw and identify lines and angles, and classify shapes by properties of their lines and angles.***
Standards	1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Angles
Students can and should make geometric distinctions about angles without measuring or mentioning degrees. Angles should be classified in comparison to right angles, such as larger than, smaller than or the same size as a right angle.

Students can use the corner of a sheet of paper as a benchmark for a right angle. They can use a right angle to determine relationships of other angles.

Symmetry
When introducing line of symmetry, provide examples of geometric shapes with and without lines of symmetry. Shapes can be classified by the existence of lines of symmetry in sorting activities. This can be done informally by folding paper, tracing, creating designs with tiles or investigating reflections in mirrors.

With the use of a dynamic geometric program, students can easily construct points, lines and geometric figures. They can also draw lines perpendicular or parallel to other line segments.

Two-dimensional shapes
Two-dimensional shapes are classified based on relationships by the angles and sides. Students can determine if the sides are parallel or perpendicular, and classify accordingly. Characteristics of rectangles (including squares) are used to develop the concept of parallel and perpendicular lines. The characteristics and understanding of parallel and perpendicular lines are used to draw rectangles. Repeated experiences in comparing and contrasting shapes enable students to gain a deeper understanding about shapes and their properties.

Informal understanding of the characteristics of triangles is developed through angle measures and side length relationships. Triangles are named according to their angle measures (right, acute or obtuse) and side lengths (scalene, isosceles or equilateral). These characteristics are used to draw triangles.

Instructional Resources/Tools

Mirrors
Geoboards
GeoGebra (a free software for learning and teaching); http://www.geogebra.com.

Common Misconceptions

Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.
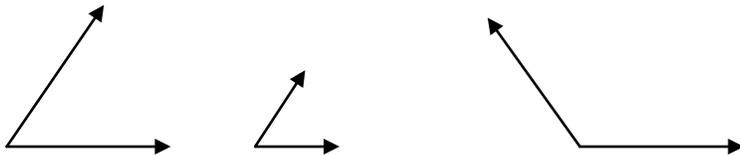
Diverse Learners

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Specific strategies for mathematics may include:
Have students draw or trace a shape, then fold in different ways to find all lines of symmetry.

Connections:

This cluster is connected to the Fourth Grade Critical Area of Focus #3, **Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, particular angle measures, and symmetry.** More information about this critical area of focus can be found by clicking here.

Geometric measurement: understand concepts of angles and measure angles (Grade 4 MD 3).

Symetry can be related to experiences in art.