This is the March 2011 version of the Grade 5 Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, and identify misconceptions and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multistate initiatives to develop common content elaborations and learning expectations.

Grade 5	
Domain	Cluster
Operations and Algebraic Thinking	• Write and interpret numerical expressions. • Analyze patterns and relationships.
Number and Operations in Base Ten	• Understand the place value system. • Perform operations with multi-digit whole numbers and with decimals to hundredths.
Number and Operations—Fractions	• Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Measurement and Data	• Convert like measurement units within a given measurement system. • Represent and interpret data. • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
Geometry	• Graph points on the coordinate plane to solve real-world and mathematical problems. • Classify two-dimensional figures into categories based on their properties.

Grade 5

Domain	Operations and Algebraic Thinking
Cluster	***Write and interpret numerical expressions.***
Standards	1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. 2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7).Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Students should be given ample opportunities to explore numerical expressions with mixed operations. This is the foundation for evaluating numerical and algebraic expressions that will include whole-number exponents in Grade 6.

There are conventions (rules) determined by mathematicians that must be learned with no conceptual basis. For example, multiplication and division are always done before addition and subtraction.

Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Using the same digits, with the operations in a different order, have students evaluate the expressions and discuss why the value of the expression is different. For example, have students evaluate 5 × 3 + 6 and 5 + 3 × 6. Discuss the rules that must be followed. Have students insert parentheses around the multiplication or division part in an expression. A discussion should focus on the similarities and differences in the problems and the results. This leads to students being able to solve problem situations which require that they know the order in which operations should take place.

After students have evaluated expressions without grouping symbols, present problems with one grouping symbol, beginning with parentheses, then in combination with brackets and/or braces.

Have students write numerical expressions in words without calculating the value. This is the foundation for writing algebraic expressions. Then, have students write numerical expressions from phrases without calculating them.

Instructional Resources/Tools

Calculators (scientific or four-function)

The Ohio Resource Center
ORC # 11463 From the National Council of Teachers of Mathematics, Illuminations: Order of Operations Bingo. Instead of calling numbers to play Bingo, you call (and write) numerical expressions to be evaluated for the numbers on the Bingo cards. The operations in this lesson are addition, subtraction, multiplication, and division; the numbers are all single-digit whole numbers.

Common Misconceptions

Students may believe the order in which a problem with mixed operations is written is the order to solve the problem. Allow students to use calculators to determine the value of the expression, and then discuss the order the calculator

used to evaluate the expression. Do this with four-function and scientific calculators.

Diverse Learners

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Connections:

This cluster is connected to the Grade 5 Critical Area of Focus #2, **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.** More information about this critical area of focus can be found by clicking here.

Evaluating numerical expressions with whole-number exponents (Grade 6 OA 1).

Grade 5

Domain	Operations and Algebraic Thinking
Cluster	***Analyze patterns and relationships.***
Standards	3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Content Elaborations (in development)

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Instructional Strategies and Resources

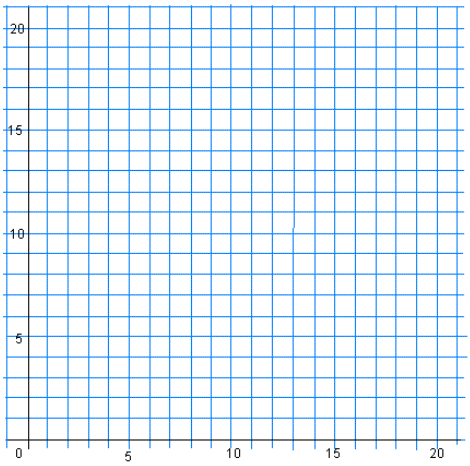
Instructional Strategies

Students should have experienced generating and analyzing numerical patterns using a given rule in Grade 4.

Given two rules with an apparent relationship, students should be able to identify the relationship between the resulting sequences of the terms in one sequence to the corresponding terms in the other sequence. For example, starting with 0, multiply by 4 and starting with 0, multiply by 8 and generate each sequence of numbers (0, 4, 8, 12, 16, …) and (0, 8, 16, 24, 32,…). Students should see that the terms in the second sequence are double the terms in the first sequence, or that the terms in the first sequence are half the terms in the second sequence.

Have students form ordered pairs and graph them on a coordinate plane. Patterns can be also discerned in graphs.

Graphing ordered pairs on a coordinate plane is introduced to students in the Geometry domain where students solve real-world and mathematical problems. For the purpose of this cluster, only use the first quadrant of the coordinate plane, which contains positive numbers only. Provide coordinate grids for the students, but also have them make coordinate grids. In Grade 6, students will position pairs of integers on a coordinate plane.



The graph of both sequences of numbers is a visual representation that will show the relationship between the two sequences of numbers.

Encourage students to represent the sequences in T-charts so that they can see a connection between the graph and

the sequences.

0	0
1	4
2	8
3	12
4	16

0	0
1	8
2	16
3	24
4	32

Instructional Resources/Tools

Grid paper

Common Misconceptions

Students reverse the points when plotting them on a coordinate plane. They count up first on the *y*-axis and then count over on the *x*-axis. The location of every point in the plane has a specific place. Have students plot points where the numbers are reversed such as (4, 5) and (5, 4). Begin with students providing a verbal description of how to plot each point. Then, have them follow the verbal description and plot each point.

Diverse Learners

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Connections:

This Cluster goes beyond the Grade 5 Critical Area of Focus to address the concepts of **Modeling numerical relationships with the coordinate plane.** More information about this critical area of focus can be found by clicking here.

Generate and analyze patterns (Grade 4 OA 3).

Graphing points in the first quadrant of a coordinate plane (Grade 5 G 1-2).

Grade 5

Domain	Number and Operations in Base Ten
Cluster	***Understand the place value system.***
Standards	1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. 3. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, *e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).* b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 4. Use place value understanding to round decimals to any place.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

In Grade 5, the concept of place value is extended to include decimal values to thousandths. The strategies for Grades 3 and 4 should be drawn upon and extended for whole numbers and decimal numbers. For example, students need to continue to represent, write and state the value of numbers including decimal numbers. For students who are not able to read, write and represent multi-digit numbers, working with decimals will be challenging.

Money is a good medium to compare decimals. Present contextual situations that require the comparison of the cost of two items to determine the lower or higher priced item. Students should also be able to identify how many pennies, dimes, dollars and ten dollars, etc., are in a given value. Help students make connections between the number of each type of coin and the value of each coin, and the expanded form of the number. Build on the understanding that it always takes ten of the number to the right to make the number to the left.

Number cards, number cubes, spinners and other manipulatives can be used to generate decimal numbers. For example, have students roll three number cubes, then create the largest and small number to the thousandths place. Ask students to represent the number with numerals and words.

Instructional Resources/Tools

Nation Library of Virtual Manipulatives; Base Block Decimals, Student use a Ten Frames to demonstrate decimal relationships.

Common Misconceptions

A common misconception that students have when trying to extend their understanding of whole number place value to decimal place value is that as you move to the left of the decimal point, the number increases in value. Reinforcing the concept of powers of ten is essential for addressing this issue.

A second misconception that is directly related to comparing whole numbers is the idea that the longer the number the greater the number. With whole numbers, a 5-digit number is always greater that a 1-, 2-, 3-, or 4-digit number. However, with decimals a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12, 0.009 or 0.499. One method for comparing decimals it to make all numbers have the same number of digits to the right of the decimal point by adding zeros to the number, such as 0.500, 0.120, 0.009 and 0.499. A second method is to use a place-value chart to place the numerals for comparison.

Diverse Learners

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Connections:

This cluster is connected to the Grade 5 Critical Area of Focus #2, **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations**. More information about this critical area of focus can be found by clicking here.

Understand decimal notation for fractions, and compare decimal fractions (Grade 4 NF 7).

Students need to have a firm grasp of place value for future work with computing with numbers, exponents and scientific notation.

Grade 5

Domain	Number and Operations – Base Ten
Cluster	***Perform operations with multi-digit whole numbers and with decimals to hundredths.***
Standards	5. Fluently multiply multi-digit whole numbers using the standard algorithm. 6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. 7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and their strategies.

Connections between the algorithm for multiplying multi-digit whole numbers and strategies such as partial products or lattice multiplication are necessary for students' understanding.

You can multiply by listing all the partial products. For example:

234	
× 8	
32	Multiply the ones (8 × 4 ones = 32 ones)
240	Multiply the tens (8 × 3 tens = 24 tens or 240
1600	Multiply the hundreds (8 × 2 hundreds = 16 hundreds or 1600)
1872	Add the partial products

The multiplication can also be done without listing the partial products by multiplying the value of each digit from one factor by the value of each digit from the other factor. Understanding of place value is vital in using the standard algorithm.

In using the standard algorithm for multiplication, when multiplying the ones, 32 ones is 3 tens and 2 ones. The 2 is written in the ones place. When multiplying the tens, the 24 tens is 2 hundreds and 4 tens. But, the 3 tens from the 32 ones need to be added to these 4 tens, for 7 tens. Multiplying the hundreds, the 16 hundreds is 1 thousand and 6 hundreds. But, the 2 hundreds from the 24 tens need to be added to these 6 hundreds, for 8 hundreds.

234
× 8
1872

As students developed efficient strategies to do whole number operations, they should also develop efficient strategies with decimal operations.

Students should learn to estimate decimal computations before they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations, not on how many decimal places are involved. For example, to estimate the product of 32.84 × 4.6, the estimate would be more than 120, closer to 150. Students

should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84 × 4.6 should be close to 150.

Have students use estimation to find the product by using exactly the same digits in one of the factors with the decimal point in a different position each time. For example, have students estimate the product of 275 × 3.8; 27.5 × 3.8 and 2.75 × 3.8, and discuss why the estimates should or should not be the same.

Instructional Resources/Tools

Decimal place-value chart

From the National Library of Virtual Manipulatives: Base Blocks Decimals– Add and subtract decimal values using base blocks. (Note: make sure the Base equals 10).

Common Misconceptions

Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

$$\begin{array}{r} 15.34 \\ +\ \ 12.9 \\ \hline 16.63 \end{array}$$

To help students add and subtract decimals correctly, have them first estimate the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Diverse Learners

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Connections:

This cluster is connected to the Grade 5 Critical Area of Focus #2, **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.** More information about this critical area of focus can be found by clicking here.

Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 4 NBT 5 and 6).

Grade 5

Domain	Number and Operations - Fractions
Cluster	***Use equivalent fractions as a strategy to add and subtract fractions.***
Standards	1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* 2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Content Elaborations (in development)

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Instructional Strategies and Resources

Instructional Strategies

To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them.

Students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions.

As with solving word problems with whole number operations, regularly present word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems. Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable.

Estimation is about getting useful answers, it is not about getting the right answer. It is important for students to learn which strategy to use for estimation. Students need to think about what might be a close answer.

Resources/Tools

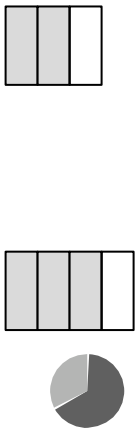
From the National Library of Virtual Manipulatives - Fraction Bars – Learn about fractions using fraction bars.

From the National Library of Virtual Manipulatives - Fractions - Adding – Illustrates what it means to find a common denominator and combine.

From the National Library of Virtual Manipulatives - Number Line Bars – Use bars to show addition, subtraction, multiplication, and division on a number line.

Misconceptions

Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.



These models of fractions are difficult to compare because the size of the whole is not the same for all representations

These models of fractions use the same size rectangle to represent the whole unit and are therefore much easier to compare fractions.

Diverse Learners

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Connections:

This Cluster is connected to the Grade 5 Critical Area of Focus #1, **Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)**. More information about this critical area of focus can be found by clicking here.

Develop an understanding of fractions as numbers (Grade 3 NF 3 a – c).

Grade 5

Domain	Number and Operations - Fractions
Cluster	***Apply and extend previous understandings of multiplication and division to multiply and divide fractions.***
Standards	3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* 4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. 5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. 6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. 7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Content Elaborations (in development)

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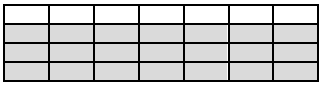
Instructional Strategies and Resources

Instructional Strategies

Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division.

Ask questions such as, "What does 2 × 3 mean?" and "What does 12 ÷ 3 mean?" Then, follow with questions for multiplication with fractions, such as, "What does $\frac{3}{4} \times \frac{1}{3}$ mean?" "What does $\frac{3}{4} \times 7$ mean?" (7 sets of $\frac{3}{4}$) and What does $7 \times \frac{3}{4}$ mean?" ($\frac{3}{4}$ of a set of 7)

The meaning of $4 \div \frac{1}{2}$ (how many $\frac{1}{2}$ are in 4) and $\frac{1}{2} \div 4$ (how many groups of 4 are in $\frac{1}{2}$) also should be illustrated with models or drawings like:

$\frac{3}{4}$ 7 groups of $\frac{3}{4}$  or $\frac{21}{4}$

Encourage students to use models or drawings to multiply or divide with fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

Models to consider when multiplying or dividing fractions include, but are not limited to: area models using rectangles or squares, fraction strips/bars and sets of counters.

Use calculators or models to explain what happens to the result of multiplying a whole number by a fraction ($3 \times \frac{1}{2}$, $4 \times \frac{1}{2}$, $5 \times \frac{1}{2}$ …and $4 \times \frac{1}{2}$, $4 \times \frac{1}{3}$, $4 \times \frac{1}{4}$,…) and when multiplying a fraction by a number greater than 1.

Use calculators or models to explain what happens to the result when dividing a unit fraction by a non-zero whole number ($\frac{1}{8} \div 4$, $\frac{1}{8} \div 8$, $\frac{1}{8} \div 16$,…) and what happens to the result when dividing a whole number by a unit fraction ($4 \div \frac{1}{4}$, $8 \div \frac{1}{4}$ $12 \div \frac{1}{4}$,…).

Present problem situations and have students use models and equations to solve the problem. It is important for students to develop understanding of multiplication and division of fractions through contextual situations.

Instructional Resources/Tools

The National Library of Virtual Manipulatives: contains Java applets and activities for K-12 mathematics.
Fractions - Rectangle Multiplication – students can visualize and practice multiplying fractions using an area representation.
Number Line Bars– Fractions: students can divide fractions using number line bars.

ORC # 5812 Divide and Conquer - Students can better understand the algorithm for dividing fractions if they analyze division through a sequence of problems starting with division of whole numbers, followed by division of a whole number by a unit fraction, division of a whole number by a non-unit fraction, and finally division of a fraction by a fraction (addressed in Grade 6).

Common Misconceptions

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that the results will be smaller.

Additionally, students may believe that division always results in a smaller number. Using models when dividing with fractions will enable students to see that the results will be larger.

Diverse Learners

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Some students may require experience with fractional numbers that result in whole-number products and quotients.

Specific strategies for mathematics may include:

The use of models and drawings will eliminate the confusion that eight divided by one-half ($8 \div \frac{1}{2}$) and eight divided in half ($8 \div 2$) produce the same results.

Connections

This cluster is connected to the Grade 5 Critical Area of Focus #1, **Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)**. More information about this critical area of focus can be found by clicking here.

Foundation for Learning in Grade 6: The Number System, Ratios and Proportional Relationships (Grade 6 NF1).

Grade 5

Domain	Measurement and Data
Cluster	***Convert like measurement units within a given measurement system.***
Standards	1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Content Elaborations (in development)

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Instructional Strategies and Resources

Instructional Strategies

Students should gain ease in converting units of measures in equivalent forms within the same system. To convert from one unit to another unit, the relationship between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit. For example, students have discovered that there are 12 inches in 1 foot and 3 feet in 1 yard. This understanding is needed to convert inches to yards. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers are equivalent to one yardstick (3 × 12 inches = 36 inches; 36 inches = 1 yard). Using this knowledge, students can decide whether to multiply or divide when making conversions.

Once students have an understanding of the relationships between units and how to do conversions, they are ready to solve multi-step problems that require conversions within the same system. Allow students to discuss methods used in solving the problems. Begin with problems that allow for renaming the units to represent the solution before using problems that require renaming to find the solution.

Instructional Resources/Tools

Yardsticks(meter sticks) and rulers (marked with customary and metric units)
Teaspoons and tablespoons
Graduated measuring cups (marked with customary and metric units)

From the National Council of Teachers of Mathematics, Illuminations: - Discovering Gallon Man. Students experiment with units of liquid measure used in the customary system of measurement. They practice making volume conversions in the customary system.

From the National Council of Teachers of Mathematics, Illuminations: – Do You Measure Up? Students learn the basics of the metric system. They identify which units of measurement are used to measure specific objects, and they learn to convert between units within the same system.

Common Misconceptions

When solving problems that require renaming units, students use their knowledge of renaming the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the renaming and the number to use. The same procedures used in renaming whole numbers should not be taught when solving problems involving measurement conversions. For example, when subtracting 5 inches from 2 feet, students may take one foot from the 2 feet and use it as 10 inches. Since there were no inches with the 2 feet, they put 1 with 0 inches and make it 10 inches.

	2 feet	is thought of as	2 feet	0 inches	becomes	1 foot	10 inches
-	5 inches		-	5 inches		-	5 inches
						1 foot	5 inches

Diverse Learners

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Connections:

This cluster is connected to the Grade 5 Critical Area of Focus #2, **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.** More information about this critical area of focus can be found by clicking here.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit (Grade 4 MD 1).

Grade 5

Domain	Measurement and Data
Cluster	***Represent and interpret data.***
Standards	2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Using a line plot to solve problems involving operations with unit fractions now includes multiplication and division. Revisit using a number line to solve multiplication and division problems with whole numbers. In addition to knowing how to use a number line to solve problems, students also need to know which operation to use to solve problems.

Use the tables for common addition and subtraction, and multiplication and division situations (Table 1 and Table 2 in the Common Core State Standards for Mathematics) as a guide to the types of problems students need to solve without specifying the type of problem. Allow students to share methods used to solve the problems. Also have students create problems to show their understanding of the meaning of each operation.

Resources/Tools

From the National Council of Teachers of Mathematics, Illuminations: Fractions in Every Day Life - This activity enables students to apply their knowledge about fractions to a real-life situation. It also provides a good way for teachers to assess students' working knowledge of fraction multiplication and division. Students should have prior knowledge of adding, subtracting, multiplying, and dividing fractions before participating in this activity. This will help students to think about how they use fractions in their lives, sometimes without even realizing it. The basic idea behind this activity is to use a recipe and alter it to serve larger or smaller portions.

Misconceptions

Diverse Learners

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Connections:

This cluster is connected to the Grade 5 Critical Area of Focus #1, **Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)**. More information about this critical area of focus can be found by clicking here.

Use equivalent fractions as a strategy to add and subtract fractions (Grade 5 NF 1 and 2).

Apply and extend previous understandings of multiplication and division to multiply and dvide fractions (Grade 5 NF 4 and 7).

Grade 5

Domain	Measurement and Data
Cluster	***Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.***
Standards	3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. 4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. 5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

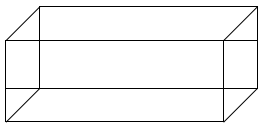
Instructional Strategies

Volume refers to the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters.

Students need to experience finding the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is presented. Provide multiple opportunities for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a I- or 2- cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box. Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the same size.

By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how addition plays a part in finding volume. This will lead to an understanding of the formula for the volume of a right rectangular prism, b x h, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Ask what strategies can be used to determine the volume of the prism based on the number of cubes in the bottom layer. Expect responses such as “adding the same number of cubes in each layer as were on the bottom layer” or multiply the number of cubes in one layer times the number of layers.

Instructional Resources/Tools

Cubes
Rulers (marked in standard or metric units)
Grid paper

http://illuminations.nctm.org/ActivityDetail.aspx?ID=6 - Determining the Volume of a Box by Filling It with Cubes, Rows of Cubes, or Layers of Cubes

Misconceptions

Diverse Learners

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Connections:

This cluster is connected to the Grade 5 Critical Area of Focus #3, **Developing understanding of volume.** More information about this critical area of focus can be found by clicking here.

Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 4 NBT 5).

Grade 5

Domain	Geometry
Cluster	***Graph points on the coordinate plane to solve real-world and mathematical problems.***
Standards	1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). 2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Students need to understand the underlying structure of the coordinate system and see how axes make it possible to locate points anywhere on a coordinate plane. This is the first time students are working with coordinate planes, and only in the first quadrant. It is important that students create the coordinate grid themselves. This can be related to two number lines and reliance on previous experiences with moving along a number line.

Multiple experiences with plotting points are needed. Provide points plotted on a grid and have students name and write the ordered pair. Have students describe how to get to the location. Encourage students to articulate directions as they plot points.

Present real-world and mathematical problems and have students graph points in the first quadrant of the coordinate plane. Gathering and graphing data is a valuable experience for students. It helps them to develop an understanding of coordinates and what the overall graph represents. Students also need to analyze the graph by interpreting the coordinate values in the context of the situation.

Instructional Resources/Tools

Grid/graph paper

From the National Council of Teachers of Mathematics, Illuminations: Finding Your Way Around - Students explore two-dimensional space via an activity in which they navigate the coordinate plane.

From the National Council of Teachers of Mathematics, Illuminations: Describe the Way – In this lesson, students will review plotting points and labeling axes. Students generate a set of random points all located in the first quadrant.

Common Misconceptions

When playing games with coordinates or looking at maps, students may think the order in plotting a coordinate point is not important. Have students plot points so that the position of the coordinates is switched. For example, have students plot (3, 4) and (4, 3) and discuss the order used to plot the points. Have students create directions for others to follow so that they become aware of the importance of direction and distance.

Diverse Learners

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Connections:

This cluster goes beyond the Grade 5 Critical Areas of Focus to address **Modeling numerical relationships with the coordinate plane.** More information about this critical area of focus can be found by clicking here.

Grade 5

Domain	Geometry
Cluster	***Classify two-dimensional figures into categories based on their properties.***
Standards	3. Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* 4. Classify two-dimensional figures in a hierarchy based on properties.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

This cluster builds from Grade 3 when students described, analyzed and compared properties of two-dimensional shapes. They compared and classified shapes by their sides and angles, and connected these with definitions of shapes. In Grade 4 students built, drew and analyzed two-dimensional shapes to deepen their understanding of the properties of two-dimensional shapes. They looked at the presence or absence of parallel and perpendicular lines or the presence or absence of angles of a specified size to classify two-dimensional shapes. Now, students classify two-dimensional shapes in a hierarchy based on properties. Details learned in earlier grades need to be used in the descriptions of the attributes of shapes. The more ways that students can classify and discriminate shapes, the better they can understand them. The shapes are not limited to quadrilaterals.

Students can use graphic organizers such as flow charts or T-charts to compare and contrast the attributes of geometric figures. Have students create a T-chart with a shape on each side. Have them list attributes of the shapes, such as number of side, number of angles, types of lines, etc. they need to determine what's alike or different about the two shapes to get a larger classification for the shapes.

Pose questions such as, "Why is a square always a rectangle?" and "Why is a rectangle not always a square?"

Resources/Tools

Rectangles and Parallelograms: Students use dynamic software to examine the properties of rectangles and parallelograms, and identify what distinguishes a rectangle from a more general parallelogram. Using spatial relationships, they will examine the properties of two-and three-dimensional shapes.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L270: In this lesson, students classify polygons according to more than one property at a time. In the context of a game, students move from a simple description of shapes to an analysis of how properties are related.

Misconceptions

Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.

Diverse Learners

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Connections:

This cluster is connected to the Grade 5 Critical Area of Focus #3, **Developing understanding of volume**. More information about this critical area of focus can be found by clicking here.

Reason with shapes and their attributes (Grade 3 G 1).

Draw and identify lines and anQies, and classify shapes by properties of their lines and anQies (Grade 4 G 1 – 2).