This is the March 2011 version of the Grade 6 Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, and identify misconceptions and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multistate initiatives to develop common content elaborations and learning expectations.

Grade 6	
Domain	Cluster
Ratios and Proportional Relationships	• Understand ratio concepts and use ratio reasoning to solve problems.
The Number System	• Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Compute fluently with multi-digit numbers and find common factors and multiples. • Apply and extend previous understandings of numbers to the system of rational numbers.
Expressions and Equations	• Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables.
Geometry	• Solve real-world and mathematical problems involving area, surface area, and volume.
Statistics and Probability	• Develop understanding of statistical variability. • Summarize and describe distributions.

Grade 6

Domain	**Ratio and Proportional Relationships**
Cluster	***Understand ratio concepts and use ratio reasoning to solve problems***
Standards	1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* 2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* 3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative thinking. Examples with ratio and proportion must involve measurements, prices and geometric contexts, as well as rates of miles per hour or portions per person within contexts that are relevant to sixth graders. Experience with proportional and nonproportional relationships, comparing and predicting ratios, and relating unit rates to previously learned unit fractions will facilitate the development of proportional reasoning. Although algorithms provide efficient means for finding solutions, the cross-product algorithm commonly used for solving proportions will not aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Students develop the understanding that ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percents are examples of part-to-whole ratios. Fractions are written as the part being identified compared to the whole amount. A percent is the part identified compared to the whole (100). Provide students with multiple examples of ratios, fractions and percents of this type. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28.

Percents are often taught in relationship to learning fractions and decimals. This cluster indicates that percents are to be taught as a special type of rate. Provide students with opportunities to find percents in the same ways they would solve rates and proportions.

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16. This form of ratios is often used to compare the event that can happen to the event that cannot happen.

Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon and students per bus. For example, 3 cans of pudding cost $2.48 at Store A and 6 cans of the same pudding costs $4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem:

- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking ½ of $4.50.

Using ratio tables develops the concept of proportion. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

Store A	
3 cans	6 cans
$2.48	#4.96

Store B	
6 cans	3 cans
$4.50	$2.25

Students should also solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with models such as ratio tables, t-charts or double number line diagrams will help students internalize the size relationships between same system measurements and relate the process of converting to the solution of a ratio.

Multiplicative reasoning is used when finding the missing element in a proportion. For example, use 2 cups of syrup to 5 cups of water to make fruit punch. If 6 cups of syrup are used to make punch, how many cups of water are needed?

$\frac{2}{5} = \frac{6}{x}$ Recognize that the relationship between 2 and 6 is 3 times; $2 \cdot 3 = 6$

To find x, the relationship between 5 and x must also be 3 times. $3 \cdot 5 = x$, therefore, $x = 15$

$\frac{2}{5} = \frac{6}{15}$ The final proportion.

Other ways to illustrate ratios that will help students see the relationships follow. Begin written representation of ratios with the words “out of” or “to” before using the symbolic notation of the colon and then the fraction bar; for example, 3 out of 7, 3 to 5, 6:7 and then 4/5.

Use skip counting as a technique to determine if ratios are equal.

Labeling units helps students organize the quantities when writing proportions.

$$\frac{3\ \text{eggs}}{2\ \text{cups of flour}} = \frac{z\ \text{eggs}}{8\ \text{cups of flour}}$$

Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used. For example, have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

Instructional Resources/Tools

100 grids (10 x 10) for modeling percents
Ratio tables – to use for proportional reasoning
Bar Models – for example, 4 red bars to 6 blue bars as a visual representation of a ratio and then expand the number of bars to show other equivalent ratios

Ohio Resource Center
Something Fishy
ORC #257: Students will estimate the size of a large population by applying the concepts of ratio and proportion through the capture-recapture statistical procedure.

How Many Noses Are in Your Arm?
ORC # 130: Students will apply the concept of ratio and proportion to determine the length of the Statue of Liberty's torch-bearing arm.

If You Hopped Like a Frog
This book introduces the concepts of ratio and proportion by comparing what humans would be able to do if they had the capabilities of different animals.

Common Misconceptions

Fractions and ratios may represent different comparisons. Fractions always express a part-to-whole comparison, but ratios can express a part-to-whole comparison or a part-to-part comparison.

Even though ratios and fractions express a part-to-whole comparison, the addition of ratios and the addition of fractions are distinctly different procedures. When adding ratios, the parts are added, the wholes are added and then the total part is compared to the total whole. For example, (2 out of 3 parts) + (4 out of 5 parts) is equal to six parts out of 8 total parts (6 out of 8) if the parts are equal. When dealing with fractions, the procedure for addition is based on a common denominator: $(\frac{2}{3}) + (\frac{4}{5}) = (\frac{10}{15}) + (\frac{12}{15})$ which is equal to $(\frac{22}{15})$. Therefore, the addition process for ratios and for fractions is distinctly different.

Often there is a misunderstanding that a percent is always a natural number less than or equal to 100. Provide examples of percent amounts that are greater than 100%, and percent amounts that are less 1%.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections

This cluster is connected to the Grade 6 Critical Area of Focus #1, **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.** More information about this critical area of focus can be found by clicking here.

In Grade 6, students develop the foundational understanding of ratio and proportion that will be extended in Grade 7 to include scale drawings, slope and real-world percent problems.

Grade 6

Domain	The Number System
Cluster	***Apply and extend previous understandings of multiplication and division to divide fractions by fractions.***
Standards	1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $3/4$-cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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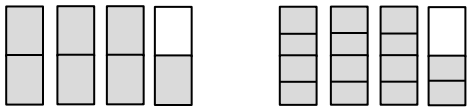
Instructional Strategies and Resources

Instructional Strategies

Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought.

For example: 12÷3 means; How many groups of three would make 12? Or how many in each of 3 groups would make 12? Thus $\frac{7}{2} \div \frac{1}{4}$ can be solved the same way. How many groups of $\frac{1}{4}$ make $\frac{7}{2}$? Or, how many objects in a group when $\frac{7}{2}$ fills one fourth?

Creating the picture that represents this problem makes seeing and proving the solutions easier.



Set the problem in context and represent the problem with a concrete or pictorial model.
$\frac{5}{4} \div \frac{1}{2}$ $\frac{5}{4}$ cups of nuts fills $\frac{1}{2}$ of a container. How many cups of nuts will fill the entire container?

Teaching "invert and multiply" without developing an understanding of why it works first leads to confusion as to when to apply the shortcut.

Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed.

Instructional Resources/Tools

Models for Multiplying and Dividing Fractions This teacher resource gives shows how the area model can be used in multiplication and division of fractions. There is also a section on the relationship to decimals.

From the National Library of Virtual Manipulatives: Fractions - Rectangle Multiplication Use this virtual manipulative to graphically demonstrate, explore, and practice multiplying fractions.

Common Misconceptions

Students may believe that dividing by $\frac{1}{2}$ is the same as dividing in half. Dividing by half means to find how many $\frac{1}{2}$ s there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts. Thus 7 divided by $\frac{1}{2}$ = 14 and 7 divided in half equals 3 $\frac{1}{2}$.

Diverse Learners

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Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #2, **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.** More information about this critical area of focus can be found by clicking here.

This cluster continues the work from Number and Operations in Base Ten and Number and Operations – Fractions. In Grade 7, this cluster will be extended in The Number System to rational numbers and in Ratios and Proportional Reasoning.

Grade 6

Domain	The Number System
Cluster	***Compute fluently with multi-digit numbers and find common factors and multiples.***
Standards	2. Fluently divide multi-digit numbers using the standard algorithm. 3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. 4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of standard algorithms. Fluency with an algorithm denotes an ability that is efficient, accurate, appropriate and flexible. Division was introduced in Grade 3 conceptually, as the inverse of multiplication. In Grade 4, division continues using place-value strategies, properties of operations, the relationship with multiplication, area models, and rectangular arrays to solve problems with one digit divisors. In Grade 6, fluency with the algorithms for division and all operations with decimals is developed.

Fluency is something that develops over time; practice should be given over the course of the year as students solve problems related to other mathematical studies. Opportunities to determine when to use paper pencil algorithms, mental math or a computing tool is also a necessary skill and should be provided in problem solving situations.

Greatest common factor and *least common multiple* are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as prime and composite numbers in Grade 4. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples. For example, consider answering the question: "If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?" Being able to see and write the relationships between numbers will be beneficial as further algebraic understandings are developed. Another focus is to be able to see how the GCF is useful in expressing the numbers using the distributive property, (36 + 24) = 12(3+2), where 12 is the GCF of 36 and 24. This concept will be extended in Expressions and Equations as work progresses from understanding the number system and solving equations to simplifying and solving algebraic equations in Grade 7.

Instructional Resources/Tools

Greatest Common Factor Lesson: This lesson is a resource for teachers or for students after participating in lessons exploring GCF.

Common Misconceptions

Diverse Learners

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Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #2, **Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems**. More information about this critical area of focus can be found by clicking here.

This cluster connects to the other Grade 6 clusters within The Number System Domain. It marks the final opportunity for students to demonstrate fluency with the four operations with whole numbers and decimals. Grade 7 will extend these learnings in The Number System and in Expressions and Equations.

Grade 6

Domain	The Number System
Cluster	***Apply and extend previous understandings of numbers to the system of rational numbers.***
Standards	5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. 7. Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ^{o}C > –7 ^{o}C to express the fact that –3 ^{o}C is warmer than –7 ^{o}C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* 8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

The purpose of this cluster is to begin study of the existence of negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. Starting with examples of having/owing and above/below zero sets the stage for understanding that there is a mathematical way to describe opposites. Students should already be familiar with the counting numbers (positive whole numbers and zero), as well as with fractions and decimals (also positive). They are now ready to understand that all numbers have an opposite. These special numbers can be shown on vertical or horizontal number lines, which then can be used to solve simple problems. Demonstration of understanding of positives and negatives involves translating among words, numbers and models: given the words “7 degrees below zero,” showing it on a thermometer and writing -7; given -4 on a number line, writing a real-life example and mathematically -4. Number lines also give the opportunity to model absolute value as the distance from zero.

Simple comparisons can be made and order determined. Order can also be established and written mathematically: -3° C > -5° C or -5° C < -3° C. Finally, absolute values should be used to relate contextual problems to their meanings and solutions.

Using number lines to model negative numbers, prove the distance between opposites, and understand the meaning of absolute value easily transfers to the creation and usage of four-quadrant coordinate grids. Points can now be plotted in all four quadrants of a coordinate grid. Differences between numbers can be found by counting the distance between numbers on the grid. Actual computation with negatives and positives is handled in Grade 7.

Instructional Resources/Tools

Common Misconceptions

Diverse Learners

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Connections:

This cluster does not directly address one of the Grade 6 Critical Areas of Focus. However, it is the foundation for working with rational numbers, algebraic expressions and equations, functions, and the coordinate plane in subsequent grades. More information about this critical area of focus can be found by clicking here.

Grade 6

Domain	**Expressions and Equations**
Cluster	***Apply and extend previous understandings of arithmetic to algebraic expressions.***
Standards	1. Write and evaluate numerical expressions involving whole-number exponents. 2. Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\ s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$. 3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* 4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Content Elaborations (in development)

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Instructional Strategies and Resources

Instructional Strategies

The skills of reading, writing and evaluating expressions are essential for future work with expressions and equations, and are a Critical Area of Focus for Grade 6. In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Students should now focus on what terms are to be solved first rather than invoking the PEMDAS rule. Likewise, the division symbol (3 ÷ 5) was used and should now be replaced with a fraction bar ($\frac{3}{5}$). Less confusion will occur as students write algebraic expressions and equations if *x* represents only variables and not multiplication. The use of a dot (•) or parentheses between number terms is preferred.

Provide opportunities for students to write expressions for numerical and real-world situations. Write multiple statements that represent a given algebraic expression. For example, the expression *x* – 10 could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees,"', "I scored ten fewer points than my brother," etc. Students should also read an algebraic expression and write a statement.

Through modeling, encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc.

Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression *x* + *x* + *x* + *x* + 4•2, students could write 2*x* + 2*x* + 8 or some other equivalent expression. Make

the connection to the simplest form of this expression as $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

Provide expressions and formulas to students, along with values for the variables so students can evaluate the expression. Evaluate expressions using the order of operations with and without parentheses. Include whole-number exponents, fractions, decimals, etc. Provide a model that shows step-by-step thinking when simplifying an expression. This demonstrates how two lines of work maintain equivalent algebraic expressions and establishes the need to have a way to review and justify thinking.

Provide a variety of expressions and problem situations for students to practice and deepen their skills. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The use of negatives and positives should mirror the level of introduction in Grade 6 The Number System; students are developing the concept and not generalizing operation rules.

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow you to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

Instructional Resources/Tools

From the National Library of Virtual Manipulatives: Online algebra tiles that can be used to represent expressions and equations.

Online game **Late Delivery.** In this game, the student helps the mail carrier deliver five letters to houses with numbers such as $3(a + 2)$.

Common Misconceptions

Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like, x^3, $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $x \bullet x \bullet x$, means x times x times x, not $3x$ or 3 times x; $4x$ means 4 times x or $x+x+x+x$, not forty-something. When evaluating $4x$ when $x = 7$, substitution does not result in the expression meaning 47. Use of the "x" notation as both the variable and the operation of multiplication can complicate this understanding.

Diverse Learners

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Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations**. More information about this critical area of focus can be found by clicking here.

The learning in this cluster is foundational in the transition to algebraic representation and problem solving which is extended and formalized in Grade 7, the Number System and Expressions and Equations.

Grade 6

Domain	Expressions and Equations
Cluster	***Reason about and solve one-variable equations and inequalities.***
Standards	5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. 6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. 8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation $x + 21 = 32$ students know that $21 + 9 = 30$ therefore the solution must be 2 more than 9 or 11, so $x = 11$.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both types of equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is being introduced in Grade 6. When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. As an extension to this concept, certain situations may require a solution between two numbers. For example, a probem situatation may have a solution that requires more than 10 but not greater than 25. Therefore, the exploration with students as to what this would look like both on a number line and symbolically is a reasonable extension.

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations/inequalities for a situation. This process should go both ways; Students should be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story does not come naturally for many students. A strategy for assisting with this is to give students an equation and ask them to come up with the situation/story that the equation could be referencing.

Instructional Resources/Tools

Common Misconceptions

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with

disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org..
Connections: This cluster is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations.** More information about this critical area of focus can be found by clicking here.

Grade 6

Domain	Expressions and Equations
Cluster	***Represent and analyze quantitative relationships between dependent and independent variables.***
Standards	9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

The goal is to help students connect the pieces together. This can be done by having students use multiple representations for the mathematical relationship. Students need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs and equations. They also need to be able to start with any of the representations and develop the others.

Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components. For example, how far I travel is dependent on the time and rate that I am traveling.

Throughout the expressions and equations domain in Grade 6, students need to have an understanding of how the expressions or equations relate to situations presented, as well as the process of solving them.

The use of technology, including computer apps, CBLs, and other hand-held technology allows the collection of real-time data or the use of actual data to create tables and charts. It is valuable for students to realize that although real-world data often is not linear, a line sometimes can model the data .

Instructional Resources/Tools

Use graphic organizers as tools for connecting various representations.
Pedal Power – NCTM illuminations lesson on translating a graph to a story.

Common Misconceptions

Students may misunderstand what the graph represents in context. For example, that moving up or down on a graph does not necessarily mean that a person is moving up or down.

Diverse Learners

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Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #3, **Writing, interpreting and using expressions, and equations**. More information about this critical area of focus can be found by clicking here.

This cluster, Expressions and Equations, is closely tied to Ratios and Proportional Relationships, allowing the ideas in each to be connected and taught together.

Grade 6

Domain	**Geometry**
Cluster	***Solve real-world and mathematical problems involving area, surface area, and volume.***
Standards	1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. 2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. 3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. 4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

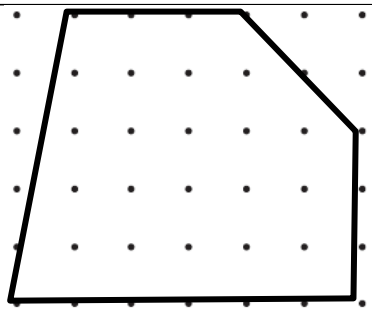
Instructional Strategies and Resources

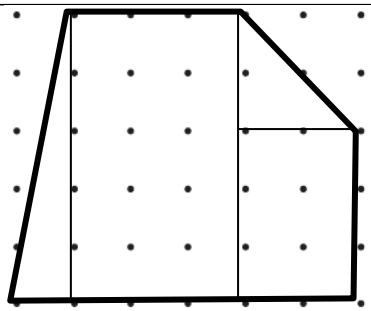
Instructional Strategies

It is very important for students to continue to physically manipulate materials and make connections to the symbolic and more abstract aspects of geometry. Exploring possible nets should be done by taking apart (unfolding) three-dimensional objects. This process is also foundational for the study of surface area of prisms. Building upon the understanding that a net is the two-dimensional representation of the object, students can apply the concept of area to find surface area. The surface area of a prism is the sum of the areas for each face.

Multiple strategies can be used to aid in the skill of determining the area of simple two-dimensional composite shapes. A beginning strategy should be to use rectangles and triangles, building upon shapes for which they can already determine area to create composite shapes. This process will reinforce the concept that composite shapes are created by joining together other shapes, and that the total area of the two-dimensional composite shape is the sum of the areas of all the parts.

A follow-up strategy is to place a composite shape on grid or dot paper. This aids in the decomposition of a shape into its foundational parts. Once the composite shape is decomposed, the area of each part can be determined and the sum of the area of each part is the total area.





Fill prisms with cubes of different edge lengths (including fractional lengths) to explore the relationship between the length of the repeated measure and the number of units needed. An essential understanding to this strategy is the volume of a rectangular prism does not change when the units used to measure the volume changes. Since focus in Grade 6 is to use fractional lengths to measure, if the same object is measured using one centimeter cubes and then measured using half centimeter cubes, the volume will appear to be eight times greater with the smaller unit. However, students need to understand that the value or the number of cubes is greater but the volume is the same.

Instructional Resources/Tools

Cubes of fractional edge length

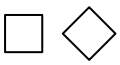
Squares that can be joined together used to develop possible nets for a cube.

Use floor plans as a real world situation for finding the area of composite shapes.

Online dot paper: http://illuminations.nctm.org/lessons/DotPaper.pdf#search=%22dot paper%22

ORC # 5279, #5280, #5281, lessons on area: http://illuminations.nctm.org/LessonDetail.aspx?ID=L580

Common Misconceptions

Students may believe that the orientation of a figure changes the figure. In Grade 6, some students still struggle with recognizing common figures in different orientations. For example, a square rotated 45° is no longer seen as a square and instead is called a diamond. 
This impacts students' ability to decompose composite figures and to appropriately apply formulas for area. Providing multiple orientations of objects within classroom examples and work is essential for students to overcome this misconception.

Diverse Learners

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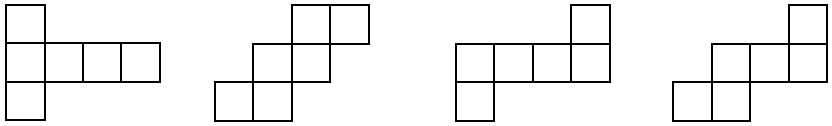
Specific strategies for mathematics may include:
Understanding that there are multiple nets for the same object may be difficult for some to visualize, provide concrete examples of nets for the object. Both the composition and decomposition of rectangular prisms should be explored. The understanding that there may be multiple nets that create a cube may be challenging. For example the following are a few of the possible nets that will create a cube.



Connections

This cluster does not direct relate to one of the Grade 6 Critical Areas of Focus. This cluster focuses on additional content for development. Students in Grade 6 build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume.

An understanding of how to find the area, surface area and volume of an object is developed in Grade 5 and should be built upon in Grade 6 to facilitate understanding of the formulas found in Measurement and Data and when to use the appropriate formula.

The use of floor plans and composite shapes on dot paper is a foundational concept for scale drawing and determining the actual area based on a scale drawing Grade 7 (Geometry and Ratio and Proportional Relationships).

Grade 6

Domain	Statistics and Probability
Cluster	***Develop understanding of statistical variability.***
Standards	1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Grade 6 is the introduction to the formal study of statistics for students. Students need multiple opportunities to look at data to determine and word statistical questions. Data should be analyzed from many sources, such as organized lists, box-plots, bar graphs and stem-and-leaf plots. This will help students begin to understand that responses to a statistical question will vary, and that this variability is described in terms of spread and overall shape. At the same time, students should begin to relate their informal knowledge of mean, mode and median to understand that data can also be described by single numbers. The single value for each of the measures of center (mean, median or mode) and measures of spread (range, interquartile range, mean absolute deviation) is used to summarize the data. Given measures of center for a set of data, students should use the value to describe the data in words. The important purpose of the number is not the value itself, but the interpretation it provides for the variation of the data. Interpreting different measures of center for the same data develops the understanding of how each measure sheds a different light on the data. The use of a similarity and difference matrix to compare mean, median, mode and range may facilitate understanding the distinctions of purpose between and among the measures of center and spread.

Include activities that require students to match graphs and explanations, or measures of center and explanations prior to interpreting graphs based upon the computation measures of center or spread. The determination of the measures of center and the process for developing graphical representation is the focus of the cluster "Summarize and describe distributions" in the Statistics and Probability domain for Grade 6. Classroom instruction should integrate the two clusters.

Instructional Resources/Tools

Newspaper and magazine graphs for analysis of the spread, shape and variation of data

From the National Council of Teachers of Mathematics, Illuminations: Numerical and Categorical Data.
In this unit of three lessons, students formulate and refine questions, and collect, display and analyze data.
ORC # 391, 392, 393

Data Analysis and Probability Virtual Manipulatives Grades 6-8
#5048 Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without requiring students to spend time hand-drawing the display. Classroom time can then be spent discussing the patterns and variability of the data.

Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, American Statistics Association

Common Misconceptions

Students may believe all graphical displays are symmetrical. Exposing students to graphs of various shapes will show this to be false.

The value of a measure of center describes the data, rather than a value used to interpret and describe the data.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #4, **Developing Understanding of statistical thinking**. More information about this critical area of focus can be found by clicking here.

Measures of center and measures of variability are used to draw informal comparative inferences about two populations in 7.SP.4.

Grade 6

Domain	Statistics and Probability
Cluster	***Summarize and describe distributions.***
Standards	4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. 5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

This cluster builds on the understandings developed in the Grade 6 cluster "Develop understanding of statistical variability." Students have analyzed data displayed in various ways to see how data can be described in terms of variability. Additionally, in Grades 3-5 students have created scaled picture and bar graphs, as well as line plots. Now students learn to organize data in appropriate representations such as box plots (box-and-whisker plots), dot plots, and stem-and-leaf plots. Students need to display the same data using different representations. By comparing the different graphs of the same data, students develop understanding of the benefits of each type of representation.

Further interpretation of the variability comes from the range and center-of-measure numbers. Prior to learning the computation procedures for finding mean and median, students will benefit from concrete experiences.

To find the median visually and kinesthetically, students should reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch. This number is the median.

The concept of mean (concept of fair shares) can be demonstrated visually and kinesthetically by using stacks of linking cubes. The blocks are redistributed among the towers so that all towers have the same number of blocks. Students should not only determine the range and centers of measure, but also use these numbers to describe the variation of the data collected from the statistical question asked. The data should be described in terms of its shape, center, spread (range) and interquartile range or mean absolute deviation (the absolute value of each data point from the mean of the data set). Providing activities that require students to sketch a representation based upon given measures of center and spread and a context will help create connections between the measures and real-life situations.

Continue to have students connect contextual situations to data to describe the data set in words prior to computation. Therefore, determining the measures of spread and measures of center mathematically need to follow the development of the conceptual understanding. Students should experience data which reveals both different and identical values for each of the measures. Students need opportunities to explore how changing a part of the data may change the measures of center and measure of spread. Also, by discussing their findings, students will solidify understanding of the meanings of the measures of center and measures of variability, what each of the measures do and do not tell about a set of data, all leading to a better understanding of their usage.

Using graphing calculators to explore bos plots (box-and-whisker plots) removes the time intensity from their creation

and permits more time to be spent on the meaning. It is important to use the interquartile range in box plots when describing the variation of the data. The mean absolute deviation describes the distance each point is from the mean of that data set. Patterns in the graphical displays should be observed, as should any outliers in the data set. Students should identify the attributes of the data and know the appropriate use of the attributes when describing the data. Pairing contextual situations with data and its box-and-whisker plot is essential.

Instructional Resources/Tools
Graphing calculators may also be used for creating lists and displaying the data.
Guidelines for Assessment and Instruction in Statistics Education (GAISE) report. American Statistical Association

Ohio Resource Center
Hollywood Box Office #10112. This rich problem focuses on measures of center and graphical displays.

Wet Heads #275. In this lesson, students create stem-and-leaf plots and back-to-back stem-and-leaf plots to display data collected from an investigative activity.

Stella's Stumpers Basketball Team Weight #13966. This problem situation uses the mean to determine a missing data element.

Learning Conductor Lessons. Use the interactive applets in these standards-based lessons to improve understanding of mathematical concepts. Scroll down to the **statistics** section for your specific need.

From the National Council of Teachers of Mathematics, Illuminations: Height of Students in our Class. This lesson has students creating box-and-whisker plots with an extension of finding measures of center and creating a stem-and-leaf plot.

National Library of Virtual Manipulatives. Students can use the appropriate applet from this page of virtual manipulatives to create graphical displays of the data set. This provides an important visual display of the data without the tediousness of the student hand drawing the display.

Common Misconceptions

Students often use words to help them recall how to determine the measures of center. However, student's lack of understanding of what the measures of center actually represent tends to confuse them. Median is the number in the middle, but that middle number can only be determined after the data entries are arranged in ascending or descending order. Mode is remembered as the "most," and often students think this means the largest value, not the "most frequent" entry in the set. Vocabulary is important in mathematics, but conceptual understanding is equally as important. Usually the mean, mode, or median have different values, but sometimes those values are the same.

Diverse Learners
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Specific strategies for mathematics may include:
The use of a similarities and differences matrix to compare mean, median, mode, range, interquartile range, and mean absolute deviation will facilitate student understanding of the uniqueness of these values.

Connections:

This cluster is connected to the Grade 6 Critical Area of Focus #4, **Developing Understanding of statistical thinking**. More information about this critical area of focus can be found by clicking here.

Measures of center and measures of variability are used to draw informal comparative inferences about two populations in Grade 7 Statistics and Probability.