This is the March 2011 version of the Grade 7 Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, and identify misconceptions and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multi-state initiatives to develop common content elaborations and learning expectations.

Grade 7	
Domain	Cluster
Ratios and Proportional Relationships	• Analyze proportional relationships and use them to solve real-world and mathematical problems.
The Number System	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
Expressions and Equations	• Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
Geometry	• Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
Statistics and Probability	• Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use, and evaluate probability models.

Grade 7

Domain	Ratios and Proportional Relationships
Cluster	***Analyze proportional relationships and use them to solve real-world and mathematical problems.***
Standards	1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.* 2. Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. 3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multi-state partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Building from the development of rate and unit concepts in Grade 6, applications now need to focus on solving unit-rate problems with more sophisticated numbers: fractions per fractions.

Proportional relationships are further developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. This is the time to push for a deep understanding of what a representation of a proportional relationship looks like and what the characteristics are: a straight line through the origin on a graph, a “rule” that applies for all ordered pairs, an equivalent ratio or an expression that describes the situation, etc. This is not the time for students to learn to cross multiply to solve problems.

Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on “out of 100”; now percents above 100 are encountered.

Providing opportunities to solve problems based within contexts that are relevant to seventh graders will connect meaning to rates, ratios and proportions. Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-ended problem scenarios with and without numbers to give students the opportunity to demonstrate conceptual understanding, inviting students to create a similar problem to a given problem and explain their reasoning.

Instructional Resources/Tools

Play money - act out a problem with play money
Advertisements in newspapers
Unlimited manipulatives or tools (don't restrict the tools to one or two, give students many options)

Common Misconceptions

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development Web page. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #1, **Developing understanding of and applying proportional relationships** and Critical Area of Focus #2**, Developing understanding of operations with rational numbers and working with expressions and linear equations**. More information about this critical area of focus can be found by clicking here.

This cluster grows out of Ratio and Proportional Relationships (Grade 6) and the Number System (Grade 6), and relates to Expressions and Equations (Grade 7).
Cross Curricular connections - economics, personal finance, reading strategies.

Domain	The Number System
Cluster	***Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.***
Standards	1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. 3. Solve real-world and mathematical problems involving the four operations with rational numbers.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

This cluster builds upon the understandings of rational numbers in Grade 6:

- quantities can be shown using + or – as having opposite directions or values,
- points on a number line show distance and direction,
- opposite signs of numbers indicate locations on opposite sides of 0 on the number line,
- the opposite of an opposite is the number itself,
- the absolute value of a rational number is its distance from 0 on the number line,
- the absolute value is the magnitude for a positive or negative quantity, and
- locating and comparing locations on a coordinate grid by using negative and positive numbers.

Learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers.

Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Number lines present a visual image for students to explore and record addition and subtraction results. Two-color counters or colored chips can be used as a physical and kinesthetic model for adding and subtracting integers. With one color designated to represent positives and a second color for negatives, addition/subtraction can be represented by placing the appropriate numbers of chips for the addends and their signs on a board. Using the notion of opposites, the board is simplified by removing pairs of opposite colored chips. The answer is the total of the remaining chips with the sign representing the appropriate color. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules. Fractional rational numbers and whole numbers should be used in computations and explorations. Students should be able to give contextual examples of integer operations, write and solve equations for real-world problems and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers.

For example, beginning with known facts, students predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2 and 3 below).

Table 1	Table 2	Table 3
4 x 4 = 16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x - 4 =
4 x - 2 =	-4 x 2 =	-2 x - 4 =
4 x - 3 =	-4 x 3 =	-3 x - 4 =
4 x - 4 =	-4 x 4 =	-4 x - 4 =

Using the language of “the opposite of” helps some students understand the multiplication of negatively signed numbers (-4 x -4 = 16, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products and commutativity of multiplication. Then students should be asked to answer these questions and prove their responses.

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?
- How is the numerical value of the product of any two numbers found?

Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language $(-(p/q) = (-p)/q = p/(-q))$ is written for the teacher’s information, not as an expectation for students.)

Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals.

In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

Instructional Resources/Tools

Two-color counters
Calculators
From the National Library of Virtual Manipulatives
- **Circle 3** – A puzzle involving adding positive real numbers to sum to three.
- **Circle 21** – A puzzle involving adding positive and negative integers to sum to 21.

Common Misconceptions

Diverse Learners

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Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #2, **Developing understanding of operations with rational numbers and working with expressions and linear equations**. More information about this Critical Area of Focus can be found by clicking here.

Domain	Expressions and Equations
Cluster	***Use properties to generate equivalent expressions***
Standards	1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multi-state partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Have students build on their understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions that were developed in Grade 6. Students continue to use properties that were initially used with whole numbers and now develop the understanding that properties hold for integers, rational and real numbers.

Provide opportunities to build upon this experience of writing expressions using variables to represent situations and use the properties of operations to generate equivalent expressions. These expressions may look different and use different numbers, but the values of the expressions are the same.

Provide opportunities for students to experience expressions for amounts of increase and decrease. In Standard 2, the expression is rewritten and the variable has a different coefficient. In context, the coefficient aids in the understanding of the situation. Another example is this situation which represents a 10% decrease: b - 0.10b = 1.00b - 0.10b which equals 0.90b or 90% of the amount.

One method that students can use to become convinced that expressions are equivalent is by substituting a numerical value for the variable and evaluating the expression. For example 5(3 + 2x) is equal to
5•3 + 5•2x Let x = 6 and substitute 6 for x in both equations.

5(3 + 2•6)	5•3 + 5•2•6
5(3 + 12)	15 + 60
5(15)	75
75	

Provide opportunities for students to use and understand the properties of operations. These include: the commutative, associative, identity, and inverse properties of addition and of multiplication, and the zero property of multiplication. Another method students can use to become convinced that expressions are equivalent is to justify each step of simplification of an expression with an operation property.

Instructional Resources/Tools

Online Algebra Tiles - Visualize multiplying and factoring algebraic expressions using tiles.

Common Misconceptions

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example having a student simplify an expression like 8 + 4(2x - 5) + 3x can bring to light several misconceptions. Do the students immediately add the 8 and 4 before distributing the 4? Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis? Do they collect all like terms

8 + 4 – 5, and 2x + 3x? Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations.

Diverse Learners

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Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #2, **Developing understanding of operations with rational numbers and working with expressions and linear equations**. More information about this critical area of focus can be found by clicking here.

The concepts in this cluster build from Operations and Algebriac Thinking **Write and interpret numerical expressions** 1&2 (Grade 5).

Provides foundation for beginning equation work (Grade 8).

Provides foundation for writing equivalent non-linear expressions in the High School Conceptual Category Algebra.

Domain	Expressions and Equations
Cluster	***Solve real-life and mathematical problems using numerical and algebraic expressions and equations.***
Standards	3. Solve multi-step, real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 $3/4$ inches long in the center of a door that is 27 $1/2$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* 4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

To assist students' assessment of the reasonableness of answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation. Connections between performing the inverse operation and undoing the operations are appropriate here. It is appropriate to expect students to show the steps in their work. Students should be able to explain their thinking using the correct terminology for the properties and operations.

Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also the context for students to practice using rational numbers including: integers, and positive and negative fractions and decimals. As students analyze a situation, they need to identify what operation should be completed first, then the values for that computation. Each set of the needed operation and values is determined in order. Finally an equation matching the order of operations is written. *For example, Bonnie goes out to eat and buys a meal that costs $12.50 that includes a tax of $.75. She only wants to leave a tip based on the cost of the food*. In this situation, students need to realize that the tax must be subtracted from the total cost before being multiplied by the percent of tip and then added back to obtain the final cost. C = (12.50 - .75)(1 + T) + .75 = 11.75(1 +T) + .75 where C = cost and T = tip.

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not

be appropriate for the situation, therefore limiting the solutions for that particular problem.

Instructional Resources/Tools

Solving for a Variable This activity for students uses a pan balance to model solving equations for a variable.
Solving an Inequality This activity for students illustrates the solution to inequalities modeled on a number line.

Common Misconceptions

Diverse Learners

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Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #2, **Developing understanding of operations with rational numbers and working with expressions and linear equations**, and to Critical Area of Focus #3, **Solving problems invloving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**. More information about these critical area of focus can be found by clicking here.

Grade 7

Domain	Geometry
Cluster	***Draw, construct, and describe geometrical figures and describe the relationships between them.***
Standards	1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems.

Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of time you multiple the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks (not the hexagon) provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. Questions about the relationship of the original block to the created shape should be asked and recorded. A sample of a recording sheet is shown.

Shape	Original Side Length	Created Side Length	Scale Relationship of Created to Original
Square	1 unit		
Triangle	1 unit		
Rhombus	1 unit		

This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. In other words, if the original side can be stated to represent 2.5 inches, what would be the new lengths and what would be the scale?

Shape	Original Side Length	Created Side Length	Scale
Square	2.5 inches		
Parallelogram	3.25 cms		
Trapezoid	(Actual measurements)	Length 1 Length 2	

Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

After students have explored multiple iterations with a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Starting with simple shapes and whole-number side lengths allows all students access to discover and understand the relationships. An interesting discovery is the relationship of the scale of the side lengths to the scale of the respective perimeters (same scale) and areas (scale squared). A sample recording sheet is shown.

Shape	Side Length	Scale	Original Perimeter	Scaled Perimeter	Perimeter Scale	Original Area	Scaled Area	Area Scale
Rectangle	2 x 3 in.	2	10 inches	20 inches	2	6 sq. in.	24 sq in.	4
Triangle	1.5 inches	2	4.5 inches	9 inches	2	2.25 sq. in.	9 sq in.	4

Students should move on to drawing scaled figures on grid paper with proper figure labels, scale and dimensions. Provide word problems that require finding missing side lengths, perimeters or areas. For example, if a 4 by 4.5 cm rectangle is enlarged by a scale of 3, what will be the new perimeter? What is the new area? or If the scale is 6, what will the new side length look like? or Suppose the area of one triangle is 16 sq units and the scale factor between this triangle and a new triangle is 2.5. What is the area of the new triangle?

Reading scales on maps and determining the actual distance (length) is an appropriate contextual situation.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles. Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.

This cluster is related to the following Grade 7 cluster “Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.” Further construction work can be replicated with quadrilaterals, determining the angle sum, noticing the variety of polygons that can be created with the same side lengths but different angle measures, and ultimately generalizing a method for finding the angle sums for regular polygons and the measures of individual angles. For example, subdividing a polygon into triangles using a vertex (N-2)180° or subdividing a polygons into triangles using an interior point 180°N - 360° where N = the number of sides in the polygon. An extension would be to realize that the two equations are equal.

Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found. Challenges can also be given: “See how many different two-dimensional figures can be found by slicing a cube” or “What three-dimensional figure can produce a hexagon slice?” This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices and resulting two-dimensional figures.

Instructional Resources/Tools

Straws, clay, angle rulers, protractors, rulers, grid paper
Road Maps - convert to actual miles
Dynamic computer software - Geometer's SketchPad. This cluster lends itself to using dynamic software. Students sometimes can manipulate the software more quickly than do the work manually. However, being able to use a protractor and a straight edge are desirable skills.

Common Misconceptions

Student may have misconceptions about
Correctly setting up proportions
How to read a ruler

Doubling side measures does not double perimeter

Diverse Learners

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Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #3, **Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**. More information about this critical area of focus can be found by clicking here.

Connections should be made between this cluster and the Grade 7 Geometry Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (7.G.4-6).
Grades 6 and 7: Ratios and Proportional Relationships
This cluster leads to the development of the triangle congruence criteria in Grade 8.

Domain	Geometry
Cluster	***Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.***
Standards	4. Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure. 6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Content Elaborations (in development)

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

This is the students' initial work with circles. Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, pi and area. Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius. Given multiple-size circles, students should then explore the relationship between the radius and the length measure of the circle (circumference) finding an approximation of pi and ultimately deriving a formula for circumference. String or yarn laid over the circle and compared to a ruler is an adequate estimate of the circumference. This same process can be followed in finding the relationship between the diameter and the area of a circle by using grid paper to estimate the area.

Another visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. In figuring area of a circle, the squaring of the radius can also be explained by showing a circle inside a square. Again, the formula is derived and then learned. After explorations, students should then solve problems, set in relevant contexts, using the formulas for area and circumference.

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Instructional Resources/Tools

circular objects of several different sizes
string or yarn
tape measures, rulers
grid paper
paper plates

NCTM Illuminations

Square Circles: This lesson features two creative twists on the standard lesson of having students measure several circles to discover that the ratio of the circumference to the diameter seems always to be a little more than 3. This lesson starts with squares, so students can first identify a simpler constant ratio (4) of perimeter to length of a side before moving to the more difficult case of the circle. The second idea is to measure with a variety of units, so students can more readily see that the ratio of the measurements remains constant, not only across different sizes of figures, but even for the same figure with different measurements. From these measurements, students will discover the constant ratio of 1:4 for all squares and the ratio of approximately 1:3.14 for all circles.

Apple Pi: Using estimation and measurement skills, students will determine the ratio of circumference to diameter and explore the meaning of π. Students will discover the circumference and area formulas based on their investigations.

Circle Tool: With this three-part online applet, students can explore with graphic and numeric displays how the circumference and area of a circle compare to its radius and diameter. Students can collect data points by dragging the radius to various lengths and clicking the "Add to Table" button to record the data in the table.

Geometry of Circles: Using a MIRA™ geometry tool, students determine the relationships between radius, diameter, circumference and area of a circle.

Ohio Resource Center
Circles and Their Areas: Given that units of area are squares, how can we find the area of a circle or other curved region? Imagine a waffle-like grid inside a circle and a larger grid containing the circle. The area of the circle lies between the area of the inside grid and the area of the outside grid..

Exploring $c/d = \pi$: Students measure circular objects to collect data to investigate the relationship between the circumference of a circle and its diameter. They find that, regardless of the size of the object or the size of the measuring unit, it always takes a little more than three times the length of the diameter to measure the circumference.

Parallel Lines: Students use Geometer's Sketchpad® to explore relationships among the angles formed when parallel lines are cut by a transversal. The software is integral to the lesson, and step-by-step instructions are provided.

Common Misconceptions

Students may believe:
Pi is an exact number rather than understanding that 3.14 is just an approximation of pi.

Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

Diverse Learners

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Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #3, **Solving problems invloving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**. More information about this critical area of focus can be found by clicking here.

This cluster builds from understandings of Geometry and in Measurement and Data Grades 3-6. It also utilizes the scope of the number system experienced thus far and begins the formal use of equations, formulas and variables in representing and solving mathematical situations.

Grade 7

Domain	**Statistics and Probability**
Cluster	***Use random sampling to draw inferences about a population***
Standards	1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. 2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multi-state partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

In Grade 6, students used measures of center and variability to describe data. Students continue to use this knowledge in Grade 7 as they use random samples to make predictions about an entire population and judge the possible discrepancies of the predictions. Providing opportunities for students to use real-life situations from science and social studies shows the purpose for using random sampling to make inferences about a population.

Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample. Have students compare the random sample to population, asking questions like “Are all the elements of the entire population represented in the sample?” and “Are the elements represented proportionally?” Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis.

Provide students with random samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.

Instructional Resources/Tools

Guidelines for Assessment and instruction in Statistics Education (GAISE) Report, American Statistical Association

Ohio Resource Center
Mathline Something Fishy #257: Students estimate the size of a large population by applying the concepts of ratio and proportion through the capture-recapture statistical procedure.

Random Sampling and Estimation # 8347: In this session, students estimate population quantities from a random sample.

Bias in Sampling #11062: This content resource addresses statistics topics that teachers may be uncomfortable teaching due to limited exposure to statistical content and vocabulary. This resource focuses a four-component statistical problem-solving process and the meaning of variation and bias in statistics and to investigate how data vary.

From the National Council of Teachers of Mathematics, Illuminations - Capture Recapture: In this lesson, students experience an application of proportion that scientists use to solve real-life problems. Students estimate the size of a

total population by taking samples and using proportions.

Common Misconceptions

Students may believe:

One random sample is not representative of the entire population. Many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception.

Diverse Learners

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Connections:

This cluster is connected to the Grade 7 Critical Area of Focus #4, **Drawing inferences about populations based on samples**. More information about this critical area of focus can be found by clicking here.

Initial understanding of statistics, specifically variability and the measures of center and spread begins in Grade 6.

Grade 7

Domain	Statistics and Probability
Cluster	***Draw informal comparative inferences about two populations***
Standards	3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* 4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multi-state partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

In Grade 6, students used measures of center and variability to describe sets of data. In the cluster "Use random sampling to draw inferences about a population" of Statistics and Probability in Grade 7, students learn to draw inferences about one population from a random sampling of that population. Students continue using these skills to draw informal comparative inferences about two populations.

Provide opportunities for students to deal with small populations, determining measures of center and variability for each population. Then have students compare those measures and make inferences. The use of graphical representations of the same data (Grade 6) provides another method for making comparisons. Students begin to develop understanding of the benefits of each method by analyzing data with both methods.

When students study large populations, random sampling is used as a basis for the population inference. This build on the skill developed in the Grade 7 cluster "Use random sampling to draw inferences about a population" of Statistics and Probability. Measures of center and variability are used to make inferences on each of the general populations. Then the students have make comparisons for the two populations based on those inferences.

This is a great opportunity to have students examine how different inferences can be made based on the same two sets of data. Have students investigate how advertising agencies uses data to persuade customers to use their products. Additionally, provide students with two populations and have them use the data to persuade both sides of an argument.

Instructional Resources/Tools

Ohio Resource Center
Baseball Stats ORC #1494: In this lesson students explore and compare data sets and statistics in baseball.

Representation of Data—Cholera and War ORC #9740: The object of this activity is to study excellent examples of the presentation of data. Students analyze (1) a map of cholera cases plotted against the location of water wells in London in 1854 and (2) a map of Napoleon's march on Moscow in 1812-1813 to see what inferences they can draw from the data displays.

Representation of Data—The U. S. Census ORC # 9741: The object of this activity is to study an excellent example of

the presentation of data. Students analyze an illustration of the 1930 U.S. census compared to the 1960 census to see what inferences they can draw from the data displays.

Common Misconceptions

Diverse Learners

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Connections:

This Cluster is connected to the Grade 7 Critical Area of Focus #4**, Drawing inferences about populations based on samples**. More information about this critical area of focus can be found by clicking here.

Measures of center and variability are developed in Statistics and Probability Grade 6. This cluster expands standards 1 and 2 in Grade 7 to make inferences between populations.

Domain	Statistics and Probability
Cluster	***Investigate chance processes and develop, use, and evaluate probability models.***
Standards	5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* 7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* 8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multi-state partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Grade 7 is the introduction to the formal study of probability. Through multiple experiences, students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions.

Help students understand the probability of chance is using the benchmarks of probability: 0, 1 and ½. Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as 1/2. Then advance to situations in which the probability is somewhere between any two of these benchmark values. This builds to the concept of expressing the probability as a number between 0 and 1. Use this understaind to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen. Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that

the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data; i.e., a sample space of the compound event. From each sample space, students determine the probability or fraction of each possible outcome. Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability.

Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities.

Instructional Resources/Tools

From the National Council of Teachers of Mathematics, Illuminations:
Boxing Up: In this lesson students explore the relationship between theoretical and experimental probabilities.
Capture-Recapture: In this lesson students estimate the size of a total population by taking samples and using proportions to estimate the entire population.

Ohio Resource Center
Probability Basics ORC #24 Probability Basics This is a 7+ minute video that explores theoretical and experimental probability with tree diagrams and the fundamental counting principle.
Probability Using Dice ORC #9737 This activity explores the probabilities of rolling various sums with two dice. Extensions of the problem and a complete discussion of the underlying mathematical ideas are included.
How to Fix and Unfair Game ORC #9718 This activity explores a fair game and "How to Fix an Unfair Game."

Remove One ORC # 253 A game is analyzed and the concepts of probability and sample space are discussed. In addition to the lesson plan, the site includes ideas for teacher discussion, extensions of the lesson, additional resources (including a video of the lesson procedures) and a discussion of the mathematical content.

Dart Throwing ORC #10131 The object of this activity is to study an excellent example of the presentation of data. Students analyze an illustration of the 1930 U.S. census compared to the 1960 census to see what inferences they can draw from the data displays.

Common Misconceptions

Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same.

Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but

not necessarily. Theoretical probability does use all possibilities. Note examples in simulations when some possibilities are not shown.

Diverse Learners

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Connections:

This cluster goes beyond the Grade 7 Critical Areas of Focus to address Investigating chance. More information about this critical area of focus can be found by clicking here.

Ratio and Proportional Relationships in Grade 6 is the development of fractions as ratios and percents as ratios. In Grade 7, students write the same number represented as a fraction, decimal or percent.

Random sampling and simulation are closely connected in Grade 7.SP. Random sampling and simulation are used to determine the experimental probability of event occurring in a population or to describe a population.