This is the March 2011 version of the Grade 8 Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, and identify misconceptions and connections related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multistate initiatives to develop common content elaborations and learning expectations.

Grade 8	
Domain	Cluster
The Number System	• Know that there are numbers that are not rational, and approximate them by rational numbers.
Expressions and Equations	• Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations.
Functions	• Define, evaluate, and compare functions. • Use functions to model relationships between quantities.
Geometry	• Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Statistics and Probability	• Investigate patterns of association in bivariate data.

Grade 8

Domain	**The Number System**
Cluster	***Know that there are numbers that are not rational, and approximate them by rational numbers.***
Standards	1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. 2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

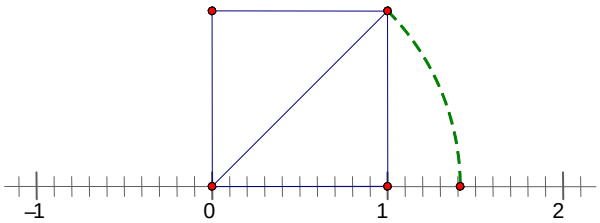
Instructional Strategies and Resources

Instructional Strategies

The distinction between rational and irrational numbers is an abstract distinction, originally based on ideal assumptions of perfect construction and measurement. In the real world, however, all measurements and constructions are approximate. Nonetheless, it is possible to see the distinction between rational and irrational numbers in their decimal representations.

A rational number is of the form a/b, where a and b are both integers, and b is not 0. In the elementary grades, students learned processes that can be used to locate any rational number on the number line: Divide the interval from 0 to 1 into b equal parts; then, beginning at 0, count out a of those parts. The surprising fact, now, is that there are numbers on the number line that cannot be expressed as a/b, with a and b both integers, and these are called irrational numbers.

Students construct a right isosceles triangle with legs of 1 unit. Using the Pythagorean theorem, they determine that the length of the hypotenuse is $\sqrt{2}$. In the figure below, they can rotate the hypotenuse back to the original number line to show that indeed $\sqrt{2}$ is a number on the number line.



In the elementary grades, students become familiar with decimal fractions, most often with decimal representations that terminate a few digits to the right of the decimal point. For example, to find the exact decimal representation of 2/7, students might use their calculator to find 2/7 = 0.2857142857…, and they might guess that the digits 285714 repeat. To show that the digits do repeat, students in Grade 7 actually carry out the long division and recognize that the remainders repeat in a predictable pattern—a pattern that creates the repetition in the decimal representation (see 7.NS.2.d).

Thinking about long division generally, ask students what will happen if the remainder is 0 at some step. They can reason that the long division is complete, and the decimal representation terminates. If the reminder is never 0, in contrast, then the remainders will repeat in a cyclical pattern because at each step with a given remainder, the process for finding the next remainder is always the same. Thus, the digits in the decimal representation also repeat. When dividing by 7, there are 6 possible nonzero remainders, and students can see that the decimal repeats with a pattern of at most 6 digits. In general, when finding the decimal representation of m/n, students can reason that the repeating portion of decimal will have at most n-1 digits. The important point here is that students can see that the pattern will repeat, so they can imagine the process continuing without actually carrying it out.

Conversely, given a repeating decimal, students learn strategies for converting the decimal to a fraction. One approach is to notice that rational numbers with denominators of 9 repeat a single digit. With a denominator of 99, two digits repeat; with a denominator of 999, three digits repeat, and so on. For example,

13/99 = 0.13131313…
74/99 = 0.74747474…
237/999 = 0.237237237…
485/999 = 0.485485485…

From this pattern, students can go in the other direction, conjecturing, for example, that the repeating decimal 0.285714285714… = 285714/999999. And then they can verify that this fraction is equivalent to 2/7.

Once students understand that (1) every rational number has a decimal representation that either terminates or repeats, and (2) every terminating or repeating decimal is a rational number, they can reason that on the number line, irrational numbers (i.e., those that are not rational) must have decimal representations that neither terminate nor repeat. And although students at this grade do not need to be able to prove that $\sqrt{2}$ is irrational, they need to know that $\sqrt{2}$ is irrational (see 8.EE.2), which means that its decimal representation neither terminates nor repeats. Nonetheless, they can approximate $\sqrt{2}$ without using the square root key on the calculator. Students can create tables like those below to approximate $\sqrt{2}$ to one, two, and then three places to the right of the decimal point.

x	x^2
1.0	1.00
1.1	1.21
1.2	1.44
1.3	1.69
1.4	1.96
1.5	2.25
1.6	2.56
1.7	2.89
1.8	3.24
1.9	3.61
2.0	4.00

x	x^2
1.40	1.9600
1.41	1.9881
1.42	2.0164
1.43	2.0449
1.44	2.0736
1.45	2.1025
1.46	2.1316
1.47	2.1609
1.48	2.1904
1.49	2.2201
1.50	2.2500

x	x^2
1.410	1.988100
1.411	1.990921
1.412	1.993744
1.413	1.996569
1.414	1.999396
1.415	2.002225
1.416	2.005056
1.417	2.007889
1.418	2.010724
1.419	2.013561
1.420	2.016400

From knowing that $1^2 = 1$ and $2^2 = 4$, or from the picture above, students can reason that there is a number between 1 and 2 whose square is 2. In the first table above, students can see that between 1.4 and 1.5, there is a number whose square is 2. Then in the second table, they locate that number between 1.41 and 1.42. And in the third table they can locate $\sqrt{2}$ between 1.414 and 1.415. Students can develop more efficient methods for this work. For example, from the picture above, they might have begun the first table with 1.4. And once they see that $1.42^2 > 2$, they do not need generate the rest of the data in the second table.

Use set diagrams to show the relationships among real, rational, irrational numbers, integers, and counting numbers. The diagram should show that the all real numbers (numbers on the number line) are either rational or irrational.

Given two distinct numbers, it is possible to find both a rational and an irrational number between them.

Instructional Resources/Tools

Graphing calculators
Dynamic geometry software

Common Misconceptions

Some students are surprised that the decimal representation of pi does not repeat. Some students believe that if only we keep looking at digits farther and farther to the right, eventually a pattern will emerge.

A few irrational numbers are given special names (pi and e), and much attention is given to sqrt(2). Because we name so few irrational numbers, students sometimes conclude that irrational numbers are unusual and rare. In fact, irrational numbers are much more plentiful than rational numbers, in the sense that they are “denser” in the real line.

Diverse Learners Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development webpage. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.
Connections: This cluster goes beyond the Grade 8 Critical Areas of Focus to address **Working with irrational numbers, interger exponents, and scientific notation**. More information about this critical area of focus can be found by clicking here. This cluster builds on previous understandings from Grades 6-7, The Number System.

Domain	Expressions and Equations
Cluster	***Work with radicals and integer exponents.***
Standards	1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. 2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. 3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* 4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Content Elaborations (in development)

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Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Although students begin using whole-number exponents in Grades 5 and 6, it is in Grade 8 when students are first expected to know and use the properties of exponents and to extend the meaning beyond counting-number exponents. It is no accident that these expectations are simultaneous, because it is the properties of counting-number exponents that provide the rationale for the properties of integer exponents. In other words, s*tudents should not be told these properties but rather should derive them through experience and reason.*

For counting-number exponents (and for nonzero bases), the following properties follow directly from the meaning of exponents.

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$

Students should have experience simplifying numerical expressions with exponents so that these properties become natural and obvious. For example,

$$2^3 \cdot 2^5 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 2^8$$

$$(5^3)^4 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) = 5^{12}$$

$$(3 \cdot 7)^4 = (3 \cdot 7) \cdot (3 \cdot 7) \cdot (3 \cdot 7) \cdot (3 \cdot 7) = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (7 \cdot 7 \cdot 7 \cdot 7) = 3^4 \cdot 7^4$$

If students reason about these examples with a sense of generality about the numbers, they begin to articulate the properties. For example, “I see that 3 twos is being multiplied by 5 twos, and the results is 8 twos being multiplied together, where the 8 is the sum of 3 and 5, the number of twos in each of the original factors. That would work for a base other than two (as long as the bases are the same).”

Note: When talking about the meaning of an exponential expression, it is easy to say (incorrectly) that “3^5 means 3

multiplied by itself 5 times." But by writing out the meaning, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$, students can see that there are only 4 multiplications. So a better description is "3^5 means 5 3s multiplied together."

Students also need to realize that these simple descriptions work only for counting-number exponents. When extending the meaning of exponents to include 0 and negative exponents, these descriptions are limiting: Is it sensible to say "3^0 means 0 3s multiplied together" or that "3^{-2} means -2 3s multiplied together"?

The motivation for the meanings of 0 and negative exponents is the following principle: *The properties of counting-number exponents should continue to work for integer exponents.*

For example, Property 1 can be used to reason what 3^0 should be. Consider the following expression and simplification: $3^0 \cdot 3^5 = 3^{0+5} = 3^5$. This computation shows that the when 3^0 is multiplied by 3^5, the result (following Property 1) should be 3^5, which implies that 3^0 must be 1. Because this reasoning holds for any base other than 0, we can reason that $a^0 = 1$ for any nonzero number a.

To make a judgment about the meaning of 3^{-4}, the approach is similar: $3^{-4} \cdot 3^4 = 3^{-4+4} = 3^0 = 1$. This computation shows that 3^{-4} should be the reciprocal of 3^4, because their product is 1. And again, this reasoning holds for any nonzero base. Thus, we can reason that $a^{-n} = 1/a^n$.

Putting all of these results together, we now have the properties of integer exponents, shown in the above chart. For mathematical completeness, one might prove that properties 1-3 continue to hold for integer exponents, but that is not necessary at this point.

A supplemental strategy for developing meaning for integer exponents is to make use of patterns, as shown in the chart to the right.

Properties of Integer Exponents
For any nonzero real numbers a and b and integers n and m:
1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

Patterns in Exponents

⋮	⋮
5^4	625
5^3	125
5^2	25
5^1	5
5^0	1
5^{-1}	1/5
5^{-2}	1/25
5^{-3}	1/125
⋮	⋮

As the exponent decreases by 1, the value of the expression is divided by 5, which is the base. Continue that pattern to 0 and negative exponents.

The meanings of 0 and negative-integer exponents can be further explored in a place-value chart:

thousands	*hundreds*	*tens*	*ones*		*tenths*	*hundredths*	*thousandths*
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
3	2	4	7	.	5	6	8

Thus, integer exponents support writing any decimal in expanded form like the following:

$$3247.568 = 3 \cdot 10^3 + 2 \cdot 10^2 + 4 \cdot 10^1 + 7 \cdot 10^0 + 5 \cdot 10^{-1} + 6 \cdot 10^{-2} + 8 \cdot 10^{-3}.$$

Expanded form and the connection to place value is important for helping students make sense of scientific notation, which allows very large and very small numbers to be written concisely, enabling easy comparison. To develop familiarity, go back and forth between standard notation and scientific notation for numbers near, for example, 10^{12} or 10^{-9}. Compare numbers, where one is given in scientific notation and the other is given in standard notation. Real-world problems can help students compare quantities and make sense about their relationship.

Provide practical opportunities for students to flexibly move between forms of squared and cubed numbers. For example, If $3^2 = 9$ then $\sqrt{9} = 3$. This flexibility should be experienced symbolically and verbally.

Opportunities for conceptually understanding irrational numbers should be developed. One way is for students to draw a square that is one unit by one unit and find the diagonal using the Pythagorean Theorem. The diagonal drawn has an

irrational length of $\sqrt{2}$. Other irrational lengths can be found using the same strategy by finding diagonal lengths of rectangles with various side lengths.

Instructional Resources/Tools

Square tiles and cubes to develop understanding of squared and cubed numbers
Calculators to verify and explore patterns
Webquests using data mined from sites like the U.S. Census Bureau, scientific data (planetary distances)
Place value charts to connect the digit value to the exponent (negative and positive)
Powers of 10 online video.

Common Misconceptions

Students may mix up the product of powers property and the power of a power property. Is $x^2 \cdot x^3$ equivalent to x^5 or x^6? Students may make the relationship that in scientific notation, when a number contains one nonzero digit and a positive exponent, that the number of zeros equals the exponent. This pattern may incorrectly be applied to scientific notation values with negative values or with more than one nonzero digit.

Diverse Learners

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Connections:

This cluster goes beyond the Grade 8 Critical Areas of Focus to address **Working with irrational numbers, interger exponents, and Scientific notation.** More information about this critical area of focus can be found by clicking here.

This cluster connects to previous understandings of place value, very large and very small numbers.

Grade 8

Domain	Expressions and Equations
Cluster	***Understand the connections between proportional relationships, lines, and linear equations.***
Standards	5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. 6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

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Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

This cluster focuses on extending the understanding of ratios and proportions. Unit rates have been explored in Grade 6 as the comparison of two different quantities with the second unit a unit of one, (unit rate). In seventh grade unit rates were expanded to complex fractions and percents through solving multistep problems such as: discounts, interest, taxes, tips, and percent of increase or decrease. Proportional relationships were applied in scale drawings, and students should have developed an informal understanding that the steepness of the graph is the slope or unit rate. Now unit rates are addressed formally in graphical representations, algebraic equations, and geometry through similar triangles.

Distance time problems are notorious in mathematics. In this cluster, they serve the purpose of illustrating how the rates of two objects can be represented, analyzed and described in different ways: graphically and algebraically. Emphasize the creation of representative graphs and the meaning of various points. Then compare the same information when represented in an equation.

By using coordinate grids and various sets of three similar triangles, students can prove that the slopes of the corresponding sides are equal, thus making the unit rate of change equal. After proving with multiple sets of triangles, students can be led to generalize the slope to y = mx for a line through the origin and y = mx + b for a line through the vertical axis at b.

Instructional Resources/Tools

Carnegie Math™
graphing calculators
SMART™ technology with software emulator
National Library of Virtual Manipulatives (NLVM)©,
The National Council of Teachers of Mathematics, Illuminations
Annenberg™ video tutorials, www.nsdl.org to access applets
Texas Instruments® website (www.ticares.com)

Common Misconceptions

Diverse Learners

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Specific strategies for mathematics may include:
Use the Frayer Model© to help students show the connections between a table, graph, proportional relationships, the relationship described in words and with an equation.

Connections:

This cluster is connected to the Grade 8 Critical Area of Focus #1, **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equaitons** and Critical Area of Focus #3, **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem**. More information about this critical area of focus can be found by clicking here.

Domain	Expressions and Equations
Cluster	***Analyze and solve linear equations and pairs of simultaneous linear equations.***
Standards	7. Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. 8. Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

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Instructional Strategies and Resources

Instructional Strategies

In Grade 6, students applied the properties of operations to generate equivalent expressions, and identified when two expressions are equivalent. This cluster extends understanding to the process of solving equations and to their solutions, building on the fact that solutions maintain equality, and that equations may have only one solution, many solutions, or no solution at all. Equations with many solutions may be as simple as $3x = 3x$, $3x + 5 = x + 2 + x + x + 3$, or $6x + 4x = x(6 + 4)$, where both sides of the equation are equivalent once each side is simplified.

Table 3 on page 90 of CCSS generalizes the properties of operations and serves as a reminder for teachers of what these properties are. Eighth graders should be able to describe these relationships with real numbers and justify their reasoning using words and not necessarily with the algebraic language of Table 3. In other words, students should be able to state that $3(-5) = (-5)3$ because multiplication is commutative and it can be performed in any order (it is commutative), or that $9(8) + 9(2) = 9(8 + 2)$ because the distributive property allows us to distribute multiplication over addition, or determine products and add them. Grade 8 is the beginning of using the generalized properties of operations, but this is not something on which students should be assessed.

Pairing contextual situations with equation solving allows students to connect mathematical analysis with real-life events. Students should experience analyzing and representing contextual situations with equations, identify whether there is one, none, or many solutions, and then solve to prove conjectures about the solutions. Through multiple opportunities to analyze and solve equations, students should be able to estimate the number of solutions and possible values(s) of solutions prior to solving. Rich problems, such as computing the number of tiles needed to put a border around a rectangular space or solving proportional problems as in doubling recipes, help ground the abstract symbolism to life.

Experiences should move through the stages of concrete, conceptual and algebraic/abstract. Utilize experiences with the pan balance model as a visual tool for maintaining equality (balance) first with simple numbers, then with pictures symbolizing relationships, and finally with rational numbers allows understanding to develop as the complexity of the

problems increases. Equation-solving in Grade 8 should involve multistep problems that require the use of the distributive property, collecting like terms, and variables on both sides of the equation.

This cluster builds on the informal understanding of slope from graphing unit rates in Grade 6 and graphing proportional relationships in Grade 7 with a stronger, more formal understanding of slope. It extends solving equations to understanding solving systems of equations, or a set of two or more linear equations that contain one or both of the same two variables. Once again the focus is on a solution to the system. Most student experiences should be with numerical and graphical representations of solutions. Beginning work should involve systems of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection, simplify the computation and hone in on finding a solution. More complex sytems can be investigated and solve by using graphing technology.

Contextual situations relevant to eighth graders will add meaning to the solution to a system of equations. Students should explore many problems for which they must write and graph pairs of equations leading to the generalization that finding one point of intersection is the single solution to the system of equations. Provide opportunities for students to connect the solutions to an equation of a line, or solution to a system of equations, by graphing, using a table and writing an equation. Students should receive opportunities to compare equations and systems of equations, investigate using graphing calculators or graphing utilities, explain differences verbally and in writing, and use models such as equation balances.

Problems such as, “Determine the number of movies downloaded in a month that would make the costs for two sites the same, when Site A charges $6 per month and $1.25 for each movie and Site B charges $2 for each movie and no monthly fee.”
Students write the equations letting y = the total charge and x = the number of movies.

Site A: $y = 1.25x + 6$ Site B: $y = 2x$

Students graph the solutions for each of the equations by finding ordered pairs that are solutions and representing them in a t-chart. Discussion should encompass the realization that the intersection is an ordered pair that satisfies both equations. And finally students should relate the solution to the context of the problem, commenting on the practicality of their solution.

Problems should be structured so that students also experience equations that represent parallel lines and equations that are equivalent. This will help them to begin to understand the relationships between different pairs of equations: When the slope of the two lines is the same, the equations are either different equations representing the same line (thus resulting in many solutions), or the equations are different equations representing two not intersecting, parallel, lines that do not have common solutions.

System-solving in Grade 8 should include estimating solutions graphically, solving using substitution, and solving using elimination. Students again should gain experience by developing conceptual skills using models that develop into abstract skills of formal solving of equations. Provide opportunities for students to change forms of equations (from a given form to slope-intercept form) in order to compare equations.

Instructional Resources/Tools

SMART Board’s new tools for solving equations
Graphing calculators
Index cards with equations/graphs for matching and sorting
Supply and Demand This activity focuses on having students create and solve a system of linear equations in a real-world setting. By solving the system, students will find the equilibrium point for supply and demand. Students should be familiar with finding linear equations from two points or slope and *y*-intercept. This lesson was adapted from the October 1991 edition of *Mathematics Teacher.*

Common Misconceptions

Students think that only the letters x and y can be used for variables.
Students think that you always need a variable = a constant as a solution.
The variable is always on the left side of the equation.
Equations are not always in the slope intercept form, $y=mx+b$
Students confuse one-variable and two-variable equations.

Diverse Learners

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Content Standards and Model Curriculum Development webpage. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Grade 8 Critical Area of Focus #1, **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equaitons.** More information about this critical area of focus can be found by clicking here.

This cluster also builds upon the understandings in Grades 6 and 7 of Expressions and Equations, Ratios and Proportional Relationships, and utilizes the skills developed in the previous grade in The Number System.

Grade 8

Domain	Functions
Cluster	***Define, evaluate, and compare functions.***
Standards	1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. 2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* 3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Content Elaborations (in development)

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Instructional Strategies and Resources

Instructional Strategies

In grade 6, students plotted points in all four quadrants of the coordinate plane. They also represented and analyzed quantitative relationships between dependent and independent variables. In Grade 7, students decided whether two quantities are in a proportional relationship. In Grade 8, students begin to call relationships functions when each input is assigned to exactly one output. Also, in Grade 8, students learn that proportional relationships are part of a broader group of linear functions, and they are able to identify whether a relationship is linear. Nonlinear functions are included for comparison. Later, in high school, students use function notation and are able to identify types of nonlinear functions.

To determine whether a relationship is a function, students should be expected to reason from a context, a graph, or a table, after first being clear which quantity is considered the input and which is the output. When a relationship is not a function, students should produce a counterexample: an “input value” with at least two “output values.” If the relationship is a function, the students should explain how they verified that for each input there was exactly one output. The “vertical line test” should be avoided because (1) it is too easy to apply without thinking, (2) students do not need an efficient strategy at this point, and (3) it creates misconceptions for later mathematics, when it is useful to think of functions more broadly, such as whether x might be a function of y.

“Function machine” pictures are useful for helping students imagine input and output values, with a rule inside the machine by which the output value is determined from the input.

Notice that the standards explicitly call for exploring functions numerically, graphically, verbally, and algebraically (symbolically, with letters). This is sometimes called the “rule of four.” For fluency and flexibility in thinking, students need experiences translating among these. In Grade 8, the focus, of course, is on linear functions, and students begin to recognize a linear function from its form y = mx + b. Students also need experiences with nonlinear functions, including functions given by graphs, tables, or verbal descriptions but for which there is no formula for the rule, such as a girl’s height as a function of her age.

In the elementary grades, students explore number and shape patterns (sequences), and they use rules for finding the next term in the sequence. At this point, students describe sequences both by rules relating one term to the next and also by rules for finding the nth term directly. (In high school, students will call these recursive and explicit formulas.) Students express rules in both words and in symbols. Instruction should focus on additive and multiplicative sequences as well as sequences of square and cubic numbers, considered as areas and volumes of cubes, respectively.

When plotting points and drawing graphs, students should develop the habit of determining, based upon the context, whether it is reasonable to “connect the dots” on the graph. In some contexts, the inputs are discrete, and connecting the dots can be misleading. For example, if a function is used to model the height of a stack of *n* paper cups, it does not make sense to have 2.3 cups, and thus there will be no ordered pairs between n = 2 and n = 3.

Provide multiple opportunities to examine the graphs of linear functions and use graphing calculators or computer software to analyze or compare at least two functions at the same time. Illustrate with a slope triangle where the run is "1" that slope is the "unit rate of change." Compare this in order to compare two different situations and identify which is increasing/decreasing as a faster rate.

Students can compute the area and perimeter of different-size squares and identify that one relationship is linear while the other is not by either looking at a table of value or a graph in which the side length is the independent variable (input) and the area or perimeter is the dependent variable (output).

Instructional Resources/Tools

Graphing calculators
Graphing software (including dynamic geometry software)

Common Misconceptions

Some students will mistakenly think of a straight line as horizontal or vertical only.

Some students will mix up *x*- and *y*-axes on the coordinate plane, or mix up the ordered pairs. When emphasizing that the first value is plotted on the horizontal axes (usually x, with positive to the right) and the second is the vertical axis (usually called y, with positive up), point out that this is merely a convention: It could have been otherwise, but it is very useful for people to agree on a standard customary practice.

Diverse Learners

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Connections:

This Cluster is connected to the Grade 8 Critical Area of Focus #2, **Grasping the concept of a function and using functions to describe quantitative relationships.** More information about this critical area of focus can be found by clicking here.

Expressions and Equations: Linear equations in two variables can be used to define linear functions, and students can use graphs of functions to reason toward solutions to linear equations.
Geometry: Similar triangles are used to show that the slope of a line is constant.
Statistics and Probability: Bivariate data can often be modeled by a linear function.

Domain	Functions
Cluster	***Use functions to model relationships between quantities.***
Standards	4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

In Grade 8, students focus on linear equations and functions. Nonlinear functions are used for comparison.

Students will need many opportunities and examples to figure out the meaning of $y = mx + b$. What does *m* mean? What does *b* mean? They should be able to "see" m and b in graphs, tables, and formulas or equations, and they need to be able to interpret those values in contexts. For example, if a function is used to model the height of a stack of *n* paper cups, then the rate of change, *m*, which is the slope of the graph, is the height of the "lip" of the cup: the amount each cup sticks above the lower cup in the stack. The "initial value" in this case is not valid in the context because 0 cups would not have a height, and yet a height of 0 would not fit the equation. Nonetheless, the value of b can be interpreted in the context as the height of the "base" of the cup: the height of the whole cup minus its lip.

Use graphing calculators and web resources to explore linear and non-linear functions. Provide context as much as possible to build understanding of slope and y-intercept in a graph, especially for those patterns that do not start with an initial value of 0.

Give students opportunities to gather their own data or graphs in contexts they understand. Students need to measure, collect data, graph data, and look for patterns, then generalize and symbolically represent the patterns. They also need opportunities to draw graphs (qualitatively, based upon experience) representing real-life situations with which they are familiar. Probe student thinking by asking them to determine which input values make sense in the problem situations.

Provide students with a function in symbolic form and ask them to create a problem situation in words to match the function. Given a graph, have students create a scenario that would fit the graph. Ask students to sort a set of "cards" to match a graphs, tables, equations, and problem situations. Have students explain their reasoning to each other.

From a variety of representations of functions, students should be able to classify and describe the function as linear or non-linear, increasing or decreasing. Provide opportunities for students to share their ideas with other students and create their own examples for their classmates.

Use the slope of the graph and similar triangle arguments to call attention to not just the change in x or y, but also to the rate of change, which is a ratio of the two.

Emphasize key vocabulary. Students should be able to explain what key words mean: e.g., model, interpret, initial value, functional relationship, qualitative, linear, non-linear. Use a "word wall" to help reinforce vocabulary.

Instructional Resources/Tools

Graphing calculators
Graphing software for computers, including dynamic geometry software
Data-collecting technology, such as motion sensors, thermometers, CBL's, etc.
Graphing applets online

Common Misconceptions

Students often confuse a recursive rule with an explicit formula for a function. For example, after identifying that a linear function shows an increase of 2 in the values of the output for every change of 1 in the input, some students will represent the equation as $y = x + 2$ instead of realizing that this means $y = 2x + b$. When tables are constructed with increasing consecutive integers for input values, then the distinction between the recursive and explicit formulas is about whether you are reasoning vertically or horizontally in the table. Both types of reasoning—and both types of formulas—are important for developing proficiency with functions.

When input values are not increasing consecutive integers (e.g., when the input values are decreasing, when some integers are skipped, or when some input values are not integers), some students have more difficulty identifying the pattern and calculating the slope. It is important that all students have experience with such tables, so as to be sure that they do not overgeneralize from the easier examples.

Some students may not pay attention to the scale on a graph, assuming that the scale units are always "one." When making axes for a graph, some students may not using equal intervals to create the scale.

Some students may infer a cause and effect between independent and dependent variables, but this is often not the case.

Some students graph incorrectly because they don't understand that x usually represents the independent variable and y represents the dependent variable. Emphasize that this is a convention that makes it easier to communicate.

Diverse Learners

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Connections:

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Expressions and Equations: Linear equations in two variables can be used to define linear functions, and students can use graphs of functions to reason toward solutions to linear equations.
Geometry: Similar triangles are used to show that the slope of a line is constant.
Statistics and Probability: Bivariate data can often be modeled by a linear function.

Grade 8

Domain	Geometry
Cluster	***Understand congruence and similarity using physical models, transparencies, or geometry software.***
Standards	*1.* Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. 3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. 5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Content Elaborations (in development)

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Expectations for Learning (in development)

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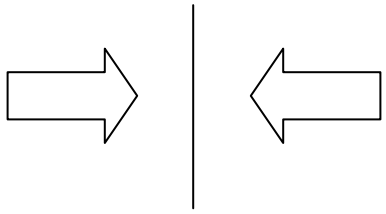
Instructional Strategies and Resources

Instructional Strategies

A major focus in Grade 8 is to use knowledge of angles and distance to analyze two- and three-dimensional figures and space in order to solve problems. This cluster interweaves the relationships of symmetry, transformations, and angle relationships to form understandings of similarity and congruence. Inductive and deductive reasoning are utilized as students forge into the world of proofs. Informal arguments are justifications based on known facts and logical reasoning. Students should be able to appropriately label figures, angles, lines, line segments, congruent parts, and images (primes or double primes). Students are expected to use logical thinking, expressed in words using correct terminology. They are NOT expected to use theorems, axioms, postulates or a formal format of proof as in two-column proofs.

Transformational geometry is about the effects of rigid motions, rotations, reflections and translations on figures. Initial work should be presented in such a way that students understand the concept of each type of transformation and the effects that each transformation has on an object before working within the coordinate system. For example, when reflecting over a line, each vertex is the same distance from the line as its corresponding vertex. This is easier to visualize when not using regular figures. Time should be allowed for students to cut out and trace the figures for each step in a series of transformations. Discussion should include the description of the relationship between the original figure and its image(s) in regards to their corresponding parts (length of sides and measure of angles) and the descriptionof the movement, including the atributes of transformations (line of symmetry, distance to be moved, center of rotation, angle of rotationand the amount of dilation).The case of distance – preserving transformation leads to the idea of congruence.

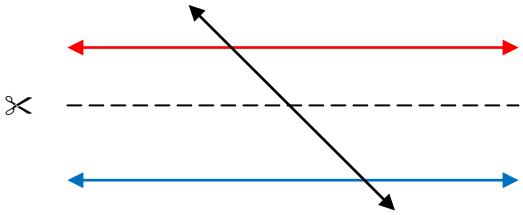
It is these distance-preserving transformations that lead to the idea of congruence.

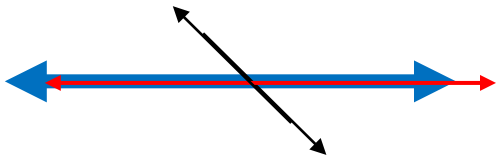


Work in the coordinate plane should involve the movement of various polygons by addition, subtraction and multiplied changes of the coordinates. For example, add 3 to *x*, subtract 4 from *y*, combinations of changes to *x* and *y*, multiply coordinates by 2 then by $\frac{1}{2}$. Students should observe and discuss such questions as 'What happens to the polygon?' and 'What does making the change to all vertices do?'. Understandings should include generalizations about the changes that maintain size or maintain shape, as well as the changes that create distortions of the polygon (dilations). Example dilations should be analyzed by students to discover the movement from the origin and the subsequent change of edge lengths of the figures. Students should be asked to describe the transformations required to go from an original figure to a transformed figure (image). Provide opportunities for students to discuss the procedure used, whether different procedures can obtain the same results, and if there is a more efficient procedure to obtain the same results. Students need to learn to describe transformations with both words and numbers.

Through understanding symmetry and congruence, conclusions can be made about the relationships of line segments and angles with figures. Students should relate rigid motions to the concept of symmetry and to use them to prove congruence or similarity of two figures. Problem situations should require students to use this knowledge to solve for missing measures or to prove relationships. It is an expectation to be able to describe rigid motions with coordinates.

Provide opportunities for students to physically manipulate figures to discover properties of similar and congruent figures, for example, the corresponding angles of similar figures are equal. Additionally use drawings of parallel lines cut by a transversal to investigate the relationship among the angles. For example, what information can be obtained by cutting between the two intersections and sliding one onto the other?



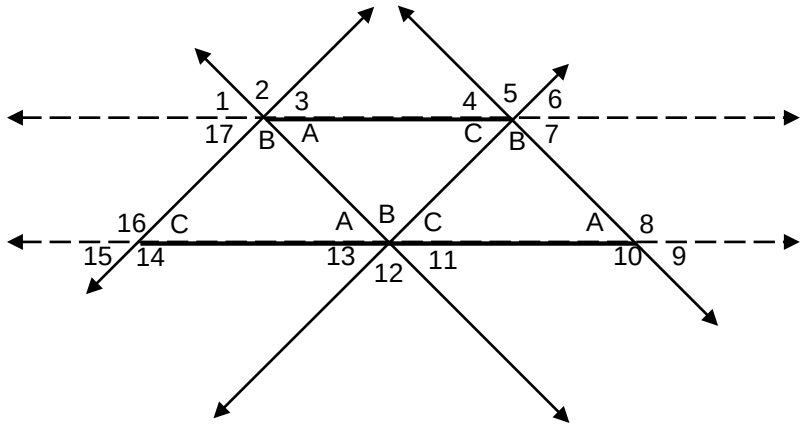


In Grade 7, students develop an understanding of the special relationships of angles and their measures (complementary, supplementary, adjacent, vertical). Now, the focus is on learning the about the sum of the angles of a triangle and using it to, find the measures of angles formed by transversals (especially with parallel lines), or to find the measures of exterior angles of triangles and to informally prove congruence.

By using three copies of the same triangle labeled and placed so that the three different angles form a straight line, students can:

- explore the relationships of the angles,
- learn the types of angles (interior, exterior, alternate interior, alternate exterior, corresponding, same side interior, same side exterior), and
- explore the parallel lines, triangles and parallelograms formed.

Further examples can be explored to verify these relationships and demonstrate their relevance in reallife.



Investigations should also lead to the Angle-Angle criterion for similar triangles. For instance, pairs of students create two different triangles with one given angle measurement, then repeat with two given angle measurements and finally with three given angle measurements. Students observe and describe the relationship of the resulting triangles. As a class, conjectures lead to the generalization of the Angle-Angle criterion.

Students should solve mathematical and real-life problems involving understandings from this cluster. Investigation, discussion, justification of their thinking, and application of their learning will assist in the more formal learning of geometry in high school.

Instructional Resources/Tools

pattern blocks or shape sets
mirrors - Miras
Geometry software like Geometer's Sketchpad, Cabri Jr or GeoGebra
graphing calculators
grid paper
patty paper

From the National Library of Virtual Manipulatives
Congruent Triangles – Build similar triangles by combining sides and angles.
Geoboard - Coordinate – Rectangular geoboard with x and y coordinates.
Transformations - Composition – Explore the effect of applying a composition of translation, rotation, and reflection transformations to objects.
Transformations - Dilation – Dynamically interact with and see the result of a dilation transformation.
Transformations - Reflection – Dynamically interact with and see the result of a reflection transformation.
Transformations - Rotation – Dynamically interact with and see the result of a rotation transformation.
Transformations - Translation – Dynamically interact with and see the result of a translation transformation.

Common Misconceptions

Students often confuse situations that require adding with multiplicative situations in regard to scale factor. Providing experiences with geometric figures and coordinate grids may help students visualize the difference.

Diverse Learners

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Connections:

This cluster is connected to the Grade 8 Critical Area of Focus #3, **Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.** More information about this critical area of focus can be found by clicking here.

This cluster builds from Grade 7 Geometry, Ratios and Proportional Relationships, and prepares students for more formal work in high school geometry.

Grade 8

Domain	Geometry
Cluster	***Understand and apply the Pythagorean Theorem.***
Standards	6. Explain a proof of the Pythagorean Theorem and its converse. 7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

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Instructional Strategies and Resources

Instructional Strategies

Previous understanding of triangles, such as the sum of two side measures is greater than the third side measure, angles sum, and area of squares, is furthered by the introduction of unique qualities of right triangles. Students should be given the opportunity to explore right triangles to determine the relationships between the measures of the legs and the measure of the hypotenuse. Experiences should involve using grid paper to draw right triangles from given measures and representing and computing the areas of the squares on each side. Data should be recorded in a chart such as the one below, allowing for students to conjecture about the relationship among the areas within each triangle.

Triangle	Measure of Leg 1	Measure of Leg 2	Area of Square on Leg 1	Area of Square on Leg 2	Area of Square on Hypotenuse
1					

Students should then test out their conjectures, then explain and discuss their findings. Finally, the Pythagorean Theorem should be introduced and explained as the pattern they have explored. Time should be spent analyzing several proofs of the Pythagorean Theorem to develop a beginning sense of the process of deductive reasoning, the significance of a theorem, and the purpose of a proof. Students should be able to justify a simple proof of the Pythagorean Theorem or its converse.

Previously, students have discovered that not every combination of side lengths will create a triangle. Now they need situations that explore using the Pythagorean Theorem to test whether or not side lengths represent right triangles. (Recording could include Side length *a*, Side length *b*, Sum of $a^2 + b^2$, c^2, $a^2 + b^2 = c^2$, Right triangle? Through these opportunities, students should realize that there are Pythagorean (triangular) triples such as (3, 4, 5), (5, 12, 13), (7, 24, 25), (9, 40, 41) that always create right triangles, and that their multiples also form right triangles. Students should see how similar triangles can be used to find additional triples. Students should be able to explain why a triangle is or is not a right triangle using the Pythagorean Theorem.

The Pythagorean Thereom should be applied to finding the lengths of segments on a coordinate grid, especially those segments that do not follow the vertical or horizontal lines, as a means of discussing the determination of distances between points. Contextual situations, created by both the students and the teacher, that apply the Pythagorean theorem and its converse should be provided. For example, apply the concept of similarity to determine the height of a tree using the ratio between the student's height and the length of the student's shadow. From that, determine the distance from the tip of the tree to the end of its shadow and verify by comparing to the computed distance from the top of the student's head to the end of the student's shadow, using the ratio calculated previously. Challenge students to identify additional ways that the Pythagorean Theorem is or can be used in real world situations or mathematical problems, such as finding the height of something that is difficult to physically measure, or the diagonal of a prism.

Instructional Resources/Tools

From the National Library of Virtual Manipulatives

Pythagorean Theorem – Solve two puzzles that illustrate the proof of the Pythagorean Theorem.

Right Triangle Solver – Practice using the Pythagorean theorem and the definitions of the trigonometric functions to solve for unknown sides and angles of a right triangle.

Common Misconceptions

Diverse Learners

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Connections:

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Domain	Geometry
Cluster	***Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.***
Standards	9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve-real world and mathematical problems.

Content Elaborations (in development)

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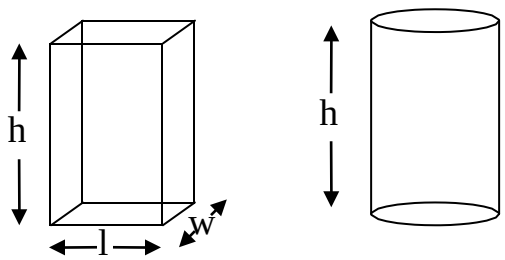
Instructional Strategies and Resources

Instructional Strategies

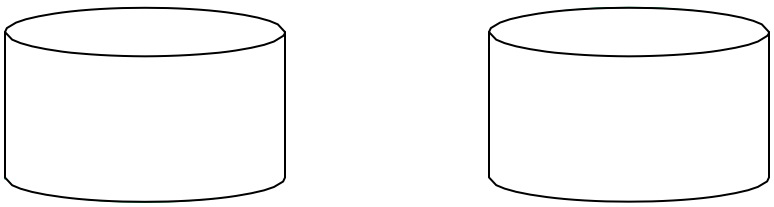
Begin by recalling the formula, and its meaning, for the volume of a right rectangular prism: V = l ×w ×h. Then ask students to consider how this might be used to make a conjecture about the volume formula for a cylinder:



Most students can be readily led to the understanding that the volume of a right rectangular prism can be thought of as the area of a "base" times the height, and so because the area of the base of a cylinder is πr^2 the volume of a cylinder is $V_c = \pi r^2 h$.

To motivate the formula for the volume of a cone, use cylinders and cones with the same base and height. Fill the cone with rice or water and pour into the cylinder. Students will discover/experience that 3 cones full are needed to fill the cylinder. This non-mathematical derivation of the formula for the volume of a cone, $V = 1/3\ \pi r^2 h$, will help most students remember the formula.

In a drawing of a cone inside a cylinder, students might see that that the triangular cross-section of a cone is 1/2 the rectangular cross-section of the cylinder. Ask them to reason why the volume (three dimensions) turns out to be *less* than 1/2 the volume of the cylinder. It turns out to be 1/3.



For the volume of a sphere, it may help to have students visualize a hemisphere "inside" a cylinder with the same height and "base." The radius of the circular base of the cylinder is also the radius of the sphere and the hemisphere. The area of the "base" of the cylinder and the area of the section created by the division of the sphere into a hemisphere is πr^2. The height of the cylinder is also r so the volume of the cylinder is πr^3. Students can see that the volume of the hemisphere is less than the volume of the cylinder and more than half the volume of the cylinder. Illustrating this with concrete materials and rice or water will help students see the relative difference in the volumes. At this point, students can reasonably accept that the volume of the hemisphere of radius r is $2/3\ \pi r^3$ and therefore volume of a sphere with

radius r is twice that or 4/3 π r^3. There are several websites with explanations for students who wish to pursue the reasons in more detail. (Note that in the pictures above, the hemisphere and the cone together fill the cylinder.)

Students should experience many types of real-world applications using these formulas. They should be expected to explain and justify their solutions.

Instructional Resources/Tools

Ohio Resource Center
#8421 "The Cylinder Problem - Students build a family of cylinders, all from the same-sized paper, and discover the relationship between the dimensions of the paper and the resulting cylinders. They order the cylinders by their volumes and draw a conclusion about the relationship between a cylinder's dimensions and its volume.

National Library of Virtual Manipulatives
"How High" is an applet that can be used to take an inquiry approach to the formula for volume of a cylinder or cone.

NCTM
Finding Surface Area and Volume
Blue Cube, 27 Little Cubes (Stella Stunner)
Volume of a Spheres and Cones (Rich Problem)

Common Misconceptions

A common misconception among middle grade students is that "volume" is a "number" that results from "substituting" other numbers into a formula. For these students there is no recognition that "volume" is a measure – related to the amount of space occupied. If a teacher discovers that students do not have an understanding of volume as a measure of space, it is important to provide opportunities for hands on experiences where students "fill" three dimensional objects. Begin with righ- rectangular prisms and fill them with cubes will help students understand why the units for volume are cubed. See Cubes http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

Diverse Learners

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Domain	Statistics and Probability
Cluster	***Investigate patterns of association in bivariate data.***
Standards	1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. 3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*. 4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Content Elaborations (in development)

This section will provide additional clarification and examples to aid in the understanding of the standards. To support shared interpretations across states, content elaborations are being developed through multistate partnerships organized by CCSSO and other national organizations. This information will be included as it is developed.

Expectations for Learning (in development)

As the framework for the assessments, this section will be developed by the CCSS assessment consortia (SBAC and PARCC). Ohio is currently participating in both consortia and has input into the development of the frameworks. This information will be included as it is developed.

Instructional Strategies and Resources

Instructional Strategies

Building on the study of statistics using univariate data in Grades 6 and 7, students are now ready to study bivariate data. Students will extend their descriptions and understanding of variation to the graphical displays of bivariate data.

Scatter plots are the most common form of displaying bivariate data in Grade 8. Provide scatter plots and have students practice informally finding the *line of best fit*. Students should create and interpret scatter plots, focusing on outliers, positive or negative association, linearity or curvature. By changing the data slightly, students can have a rich discussion about the effects of the change on the graph. Have students use a graphing calculator to determine a linear regression and discuss how this relates to the graph. Students should informally draw a line of best fit for a scatter plot and informally measure the strength of fit. Discussion should include "What does it mean to be above the line, below the line?"

The study of the line of best fit ties directly to the algebraic study of slope and intercept. Students should interpret the slope and intercept of the line of best fit in the context of the data. Then students can make predictions based on the line of best fit.

Instructional Resources/Tools

This lesson - Glued to the Tube or Hooked to the Books? - provides step-by-step instructions for using the graphing calculator to construct a scatter plot of class data and a line of best fit.

From the National Council of Teachers of Mathematics, Illuminations: Impact of a Superstar - This lesson uses technology tools to plot data, identify lines of best fit, and detect outliers. Then, students compare the lines of best fit when one element is removed from a data set, and interpret the results.

From the National Council of Teachers of Mathematics, Illuminations: Exploring Linear Data - In this lesson, students construct scatter plots of bivariate data, interpret individual data points, make conclusions about trends in data,

especially linear relationships, and estimate and write equation of lines of best fit.

The Ohio Resource Center –the tutorial video Lines of Fit shows how to determine a line of best fit for a set of data.

Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report, American Statistical Association

Common Misconceptions

Students may believe:

Bivariate data is only displayed in scatter plots. 8.SP.4 in this cluster provides the opportunity to display bivariate, categorical data in a table.

In general, students think there is only one correct answer in mathematics. Students may mistakenly think their lines of best fit for the same set of data will be exactly the same. Because students are informally drawing lines of best fit, the lines will vary slightly. To obtain the exact line of best fit, students would use technology to find the line of regression.

Diverse Learners

Information and instructional strategies for gifted students, English Language Learners (ELL), and students with disabilities is available in the Introduction to Universal Design for Learning document located on the Revised Academic Content Standards and Model Curriculum Development webpage. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This Cluster is connected to the grade 8 Critical Area of Focus #1, **Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.** More information about this critical area of focus can be found by clicking here.