This is the October 2013 version of the Kindergarten Model Curriculum for Mathematics. The current focus of this document is to provide instructional strategies and resources, and identify connections and misconceptions related to the clusters and standards. The Ohio Department of Education is working in collaboration with assessment consortia, national professional organizations and other multistate initiatives to develop common content elaborations and learning expectations.

Kindergarten	
Domain	**Cluster**
Counting and Cardinality	• Know number names and the count sequence. • Count to tell the number of objects. • Compare numbers.
Operations and Algebraic Thinking	• Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
Number and Operations in Base Ten	• Work with numbers 11–19 to gain foundations for place value.
Measurement and Data	• Describe and compare measurable attributes. • Classify objects and count the number of objects in each category.
Geometry	• Identify and describe shapes. • Analyze, compare, create, and compose shapes.

Kindergarten

Domain	Counting and Cardinality
Cluster	***Know number names and the count sequence.***
Standards	1. Count to 100 by ones and by tens. 2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). 3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Content Elaborations

Ohio has chosen to support shared interpretation of the standards by linking the work of multistate partnerships as the Mathematics Content Elaborations. Further clarification of the standards can be found through these reliable organizations and their links:

- Achieve the Core Modules, Resources
- Hunt Institute Video examples
- Institute for Mathematics and Education Learning Progressions Narratives
- Illustrative Mathematics Sample tasks
- National Council of Supervisors of Mathematics (NCSM) Resources, Lessons, Items
- National Council of Teacher of Mathematics (NCTM) Resources, Lessons, Items
- Partnership for Assessment of Readiness for College and Careers (PARCC) Resources, Items

Expectations for Learning

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Instructional Strategies and Resources

Instructional Strategies

The Counting and Cardinality domain in Kindergarten contains standard statements that are connected to one another. Examine the three samples in this domain at the same time to obtain a more holistic view of the content.

Provide settings that connect mathematical language and symbols to the everyday lives of kindergarteners. Support students' ability to make meaning and mathematize the real world. Help them see patterns, make connections and provide repeated experiences that give students time and opportunities to develop understandings and increase fluency. Encourage students to explain their reasoning by asking probing questions such as "How do you know?".

Students view counting as a mechanism used to land on a number. Young students mimic counting often with initial lack of purpose or meaning. Coordinating the number words, touching or moving objects in a one-to-one correspondence may be little more than a matching activity. However, saying number words as a chant or a rote procedure plays a part in students constructing meaning for the conceptual idea of counting. They will learn how to count before they understand cardinality, i.e. that the last count word is the amount of the set.

Counting on or counting from a given number conflicts with the learned strategy of counting from the beginning. In order to be successful in counting on, students must understand cardinality. Students often merge or separate two groups of objects and then re-count from the beginning to determine the final number of objects represented. For these students,

counting is still a rote skill or the benefits of counting on have not been realized. Games that require students to add on to a previous count to reach a goal number encourage developing this concept. Frequent and brief opportunities utilizing counting on and counting back are recommended. These concepts emerge over time and cannot be forced.

Like counting to 100 by either ones or tens, writing numbers from 0 to 20 is a rote process. Initially, students mimic the actual formation of the written numerals while also assigning it a name. Over time, children create the understanding that number symbols signify the meaning of counting. Numerals are used to communicate across cultures and through time a certain meaning. Numbers have meaning when children can see mental images of the number symbols and use those images with which to think. Practice count words and written numerals paired with pictures, representations of objects, and objects that represent quantities within the context of life experiences for kindergarteners. For example, dot cards, dominoes and number cubes all create different mental images for relating quantity to number words and numerals.

One way students can learn the left to right orientation of numbers is to use a finger to write numbers in air (sky writing). Children will see mathematics as something that is alive and that they are involved.

Students should study and write numbers 0 to 20 in this order: numbers 1 to 9, the number 0, then numbers 10 to 20. They need to know that 0 is the number items left after all items in a set are taken away. Do not accept “none” as the answer to “How many items are left?” for this situation.

Instructional Resources/Tools

Board games that require counting

Dot Card and Ten Frame Activities (pp. 1-6, 12-17) Numeracy Project, Winnipeg School Division, 2005-2006

Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity

Common Misconceptions

Some students might not see zero as a number. Ask students to write 0 and say *zero* to represent the number of items left when all items have been taken away. Avoid using the word *none* to represent this situation.

Diverse Learners

Strategies for meeting the needs of all learners including gifted students, English Language Learners (ELL) and students with disabilities can be found at this site. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.

Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. More information about this critical area of focus can be found by clicking here.

This cluster is connected to the other clusters in the Counting and Cardinality Domain and to *Classify objects and count the number of objects in each category* in Kindergarten, and to *Add and subtract within 20* and *Extend the counting sequence* in Grade 1.

Kindergarten

Domain	Counting and Cardinality
Cluster	***Count to tell the number of objects.***
Standards	4. Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. 5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Content Elaborations

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Expectations for Learning

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Instructional Strategies and Resources

Instructional Strategies

One of the first major concepts in a student's mathematical development is cardinality. Cardinality, knowing that the number word said tells the quantity you have and that the number you end on when counting represents the entire amount counted. The big idea is that number means amount and, no matter how you arrange and rearrange the items, the amount is the same. Until this concept is developed, counting is merely a routine procedure done when a number is needed. To determine if students have the cardinality rule, listen to their responses when you discuss counting tasks with them. For example, ask, "How many are here?" The student counts correctly and says that there are seven. Then ask, "Are there seven?" Students may count or hesitate if they have not developed cardinality. Students with cardinality may emphasize the last count or explain that there are seven because they counted them. These students can now use counting to find a matching set.

Students develop the understanding of counting and cardinality from experience. Almost any activity or game that engages children in counting and comparing quantities, such as board games, will encourage the development of cardinality. Frequent opportunities to use and discuss counting as a means of solving problems relevant to

kindergarteners is more beneficial than repeating the same routine day after day. For example, ask students questions that can be answered by counting up to 20 items before they change and as they change locations throughout the school building.

As students develop meaning for numerals, they also compare numerals to the quantities they represent. The models that can represent numbers, such as dot cards and dominoes, become tools for such comparisons. Students can concretely, pictorially or mentally look for similarities and differences in the representations of numbers. They begin to "see" the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities that are one larger. In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will build students' ability to break numbers into parts.

Provide a variety of experiences in which students connect count words or number words to the numerals that represent the quantities. Students will arrive at an understanding of a number when they acquire cardinality and can connect a number with the numerals and the number word for the quantity they all represent.

Instructional Resources/Tools

Dot Card and Ten Frame Activities (pp. 1-6, 12-17) Numeracy Project, Winnipeg School Division, 2005-2006

Common Misconceptions

Some students might think that the count word used to tag an item is permanently connected to that item. So when the item is used again for counting and should be tagged with a different count word, the student uses the original count word. For example, a student counts four geometric figures: triangle, square, circle and rectangle with the count words: one, two, three, four. If these items are rearranged as rectangle, triangle, circle and square and counted, the student says these count words: four, one, three, two.

Diverse Learners

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Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. More information about this critical area of focus can be found by clicking here.

This cluster is connected to the other clusters in the Counting and Cardinality Domain and to *Classify objects and count the number of objects in each category* in Kindergarten, and to *Add and subtract within 20* in Grade 1.

Kindergarten

Domain	Counting and Cardinality
Cluster	***Compare numbers.***
Standards	6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. 7. Compare two numbers between 1 and 10 presented as written numerals.

Content Elaborations

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Instructional Strategies and Resources

Instructional Strategies

As children develop meaning for numerals, they also compare these numerals to the quantities represented and their number words. The modeling numbers with manipulatives such as dot cards and five- and ten-frames become tools for such comparisons. Children can look for similarities and differences in these different representations of numbers. They begin to “see” the relationship of one more, one less, two more and two less, thus landing on the concept that successive numbers name quantities where one is larger. In order to encourage this idea, children need discussion and reflection of pairs of numbers from 1 to 10. Activities that utilize anchors of 5 and 10 are helpful in securing understanding of the relationships between numbers. This flexibility with numbers will greatly impact children’s ability to break numbers into parts.

Children demonstrate their understanding of the meaning of numbers when they can justify why their answer represents a quantity just counted. This justification could merely be the expression that the number said is the total because it was just counted, or a “proof” by demonstrating a one to-one match, by counting again or other similar means (concretely or pictorially) that makes sense. An ultimate level of understanding is reached when children can compare two numbers from 1 to10 represented as written numerals without counting.

Students need to explain their reasoning when they determine whether a number is greater than, less than, or equal to another number. Teachers need to ask probing questions such as “How do you know?” to elicit their thinking. For students, these comparisons increase in difficulty, from greater than to less than to equal. It is easier for students to identify differences than to find similarities.

Instructional Resources/Tools

Board games
Dot Card and Ten Frame Activities Numeracy Project, Winnipeg School Division, 2005-2006

Diverse Learners

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Connections:

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This cluster is connected to the other clusters in the Counting and Cardinality Domain and *to Classify objects and count the number of objects in each category* in Kindergarten, and to *Add and subtract within 20* and *Extend the counting sequence* in Grade 1.

Grade Kindergarten

Domain	Operations and Algebraic Thinking
Cluster	***Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.***
Standards	1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. 2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. 3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). 4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. 5. Fluently add and subtract within 5.

Content Elaborations

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Instructional Strategies and Resources

Instructional Strategies

Provide contextual situations for addition and subtraction that relate to the everyday lives of kindergarteners. A variety of situations can be found in children's literature books. Students then model the addition and subtraction using a variety of representations such as drawings, sounds, acting out situations, verbal explanations and numerical expressions. Manipulatives, like two-color counters, clothespins on hangers, connecting cubes and stickers can also be used for modeling these operations. Kindergarten students should see addition and subtraction equations written by the teacher. Although students might struggle at first, teachers should encourage them to try writing the equations. Students' writing of equations in Kindergarten is encouraged, but it is not required.

Create written addition or subtraction problems with sums and differences less than or equal to 10 using the numbers 0 to 10 and Table 1 on page 88 of the *Common Core State Standards (CCSS) for Mathematics* for guidance. It is important to use a problem context that is relevant to kindergarteners. After the teacher reads the problem, students choose their own method to model the problem and find a solution. Students discuss their solution strategies while the teacher represents the situation with an equation written under the problem. The equation should be written by listing the numbers and symbols for the unknown quantities in the order that follows the meaning of the situation. The teacher

and students should use the words *equal* and *is the same as* interchangeably.

Have students decompose numbers less than or equal to 5 during a variety of experiences to promote their fluency with sums and differences less than or equal to 5 that result from using the numbers 0 to 5. For example, ask students to use different models to decompose 5 and record their work with drawings or equations. Next, have students decompose 6, 7, 8, 9, and 10 in a similar fashion. As they come to understand the role and meaning of arithmetic operations in number systems, students gain computational fluency, using efficient and accurate methods for computing.

The teacher can use backmapping and scaffolding to teach students who show a need for more help with counting. For instance, ask students to build a tower of 5 using 2 green and 3 blue linking cubes while you discuss composing and decomposing 5. Have them identify and compare other ways to make a tower of 5. Repeat the activity for towers of 7 and 9. Help students use counting as they explore ways to compose 7 and 9.

Instructional Resources/Tools

Colored cubes
Linking cubes

Students can use a Part-Part-Whole Mat and objects to model problem situations and find solutions. This mat is divided into three sections and the labels for the sections in order are Part, Part, and Whole.

Dot Card and Ten Frame Activities (pp. 7-11, 18-37) Numeracy Project, Winnipeg School Division, 2005-2006

Common Core State Standards for Mathematics: Common addition and subtraction situations
Table 1 on page 88 in the *Common Core State Standards for School Mathematics* illustrates 12 addition and subtraction problem situations.

ORC # 1129 From the National Council of Teachers of Mathematics: Exploring adding with sets
This lesson builds on the previous two lessons in the unit *Do It with Dominoes* and encourages students to explore another model for addition, the set model.

ORC # 4319 From the National Council of Teachers of Mathematics: Links Away
In the unit *Links Away* (lessons 2, 4, 5, and 7) students explore models of subtraction (counting, sets, balanced equations, and inverse of addition) and the relation between addition and subtraction using links. Students also write story problems in which subtraction is required.

ORC # 4269 From the National Council of Teachers of Mathematics: More and More Buttons
In this lesson, students use buttons to create, model, and record addition sentences.

Common Misconceptions

Students may over-generalize the vocabulary in word problems and think that certain words indicate solution strategies that must be used to find an answer. They might think that the word *more* always means to add and the words *take away* or *left* always means to subtract. When students use the words *take away* to refer to subtraction and its symbol, teachers need to repeat students' ideas using the words *minus* or *subtract*. For example, students use addition to solve this Take from/Start Unknown problem: Seth took the 8 stickers he no longer wanted and gave them to Anna. Now Seth has 11 stickers *left*. How many stickers did Seth have to begin with?

If students' progress from working with manipulatives to writing numerical expressions and equations, they skip using pictorial thinking. Students will then be more likely to use finger counting and rote memorization for work with addition and subtraction. Counting forward builds to the concept of addition while counting back leads to the concept of subtraction. However, counting is an inefficient strategy. Teachers need to provide instructional experiences so that students progress from the concrete level, to the pictorial level, then to the abstract level when learning mathematical concepts.

Diverse Learners Strategies for meeting the needs of all learners including gifted students, English Language Learners (ELL) and students with disabilities can be found at this site. Additional strategies and resources based on the Universal Design for Learning principles can be found at www.cast.org.
Connections: This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. More information about this critical area of focus can be found by clicking here. This cluster is connected to *Work with numbers 11-19 to gain foundations for place value* in Kindergarten, and to all clusters in the Operations and Algebraic Thinking Domain in Grade 1.

Kindergarten

Domain	Number and Operations in Base Ten
Cluster	***Work with numbers 11–19 to gain foundations for place value.***
Standards	1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Content Elaborations

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Instructional Strategies and Resources

Instructional Strategies

Kindergarteners need to understand the idea of *a ten* so they can develop the strategy of adding onto 10 to add within 20 in Grade 1. Students need to construct their own base-ten ideas about quantities and their symbols by connecting to counting by ones. They should use a variety of manipulatives to model and connect equivalent representations for the numbers 11 to19. For instance, to represent 13, students can count by ones and show 13 beans. They can anchor to five and show one group of 5 beans and 8 beans or anchor to ten and show one group of 10 beans and 3 beans. Students need to eventually see *a ten* as different from 10 ones.

After the students are familiar with counting up to 19 objects by ones, have them explore different ways to group the objects that will make counting easier. Have them estimate before they count and group. Discuss their groupings and lead students to conclude that grouping by ten is desirable. *10 ones make 1 ten* makes students wonder how something that means a lot of things can be one thing. They do not see that there are 10 single objects represented on the item for ten in pre-grouped materials, such as the rod in base-ten blocks. Students then attach words to materials and groups without knowing what they represent. Eventually they need to see the rod as *a ten* that they did not group themselves. Students need to first use groupable materials to represent numbers 11 to 19 because a group of ten such as a bundle of 10 straws or a cup of 10 beans makes more sense than *a ten* in pre-grouped materials.

Kindergarteners should use proportional base-ten models, where a group of ten is physically 10 times larger than the model for a one. Non-proportional models such as an abacus and money should not be used at this grade level. Students should impose their base-ten concepts on a model made from groupable and pre-groupable materials (see Resources/Tools). Students can transition from groupable to pre-groupable materials by leaving a group of ten intact to be reused as a pre-grouped item. When using pre-grouped materials, students should reflect on the ten-to-one

relationships in the materials, such as the "tenness" of the rod in base-ten blocks. After many experiences with pre-grouped materials, students can use dots and a stick (one tally mark) to record singles and a ten.

Encourage students to use base-ten language to describe quantities between 11 and 19. At the beginning, students do not need to use *ones* for the singles. Some of the base-ten language that is acceptable for describing quantities such as18 includes *one ten and eight, a bundle and eight*, *a rod and 8 singles* and *ten and eight more.* Write the horizontal equation 18 = 10 + 8 and connect it to base-ten language. Encourage, but do not require, students to write equations to represent quantities.

Instructional Resources/Tools

Groupable models

- Dried beans and small cups for holding groups of 10 dried beans
- Linking cubes
- Plastic chain links

Pregrouped materials

- Strips (ten connected squares) and squares (singles)
- Base-ten blocks
- Dried beans and bean sticks (10 dried beans glued on a craft stick)
- Five-frame and Ten-frame
- Place-value mat with ten-frames

Common Misconceptions

Students have difficulty with *ten* as a singular word that means 10 things. For many students, the understanding that a group of 10 things can be replaced by a single object and they both represent 10 is confusing. Help students develop the sense of 10 by first using groupable materials then replacing the group with an object or representing 10. Watch for and address the issue of attaching words to materials and groups without knowing what they represent. If this misconception is not addressed early on it can cause additional issues when working with numbers 11-19 and beyond.

Diverse Learners

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Connections:

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This cluster is connected to *Understand addition as putting together and adding to*, *and understand subtraction as taking apart and taking from* in Kindergarten, and to *Add and subtract within 20* and *Understand place value* in Grade 1.

Kindergarten

Domain	Measurement and Data
Cluster	***Describe and compare measurable attributes.***
Standards	1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. 2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Content Elaborations

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Instructional Strategies and Resources

Instructional Strategies

It is critical for students to be able to identify and describe measureable attributes of objects. An object has different attributes that can be measured, like the height and weight of a can of food. When students compare shapes directly, the attribute becomes the focus. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: Which box will hold the most? Which box will hold least? Will they hold the same amount? Students can decide to fill one box with dried beans then pour the beans into the other box to determine the answers to these questions.

Have students work in pairs to compare their arm spans. As they stand back-to-back with outstretched arms, compare the lengths of their spans, then determine who has the smallest arm span. Ask students to explain their reasoning. Then ask students to suggest other measureable attributes of their bodies that they could directly compare, such as their height or the length of their feet.

Connect to other subject areas. For example, suppose that the students have been collecting rocks for classroom observation and they wanted to know if they have collected typical or unusual rocks. Ask students to discuss the measurable attributes of rocks. Lead them to first comparing the weights of the rocks. Have the class chose a rock that seems to be a "typical" rock. Provide the categories: *Lighter Than Our Typical Rock* and *Heavier Than Our Typical Rock*. Students can take turns holding a different rock from the collection and directly comparing its weight to the weight of the typical rock and placing it in the appropriate category. Some rocks will be left over because they have about the same weight as the typical rock. As a class, they count the number of rocks in each category and use these counts to

order the categories and discuss whether they collected “typical” rocks.

Instructional Resources/Tools
Two-and three-dimensional real-world objects
Dried beans
Rice

ORC # 4330 From the National Council of Teachers of Mathematics: The Weight of Things
This lesson introduces and provides practice with the measurable attribute of weight.

Diverse Learners
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Connections:

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This cluster is connected to *Measure lengths indirectly and by iterating length units* in Grade 1.

Kindergarten

Domain	Measurement and Data
Cluster	***Classify objects and count the number of objects in each category.***
Standards	3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

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- Partnership for Assessment of Readiness for College and Careers (PARCC) Resources, Items

Expectations for Learning

Ohio has selected PARCC as the contractor for the development of the Next Generation Assessments for Mathematics. PARCC is responsible for the development of the framework, blueprints, items, rubrics, and scoring for the assessments. Further information can be found at Partnership for Assessment of Readiness for College and Careers (PARCC). Specific information is located at these links:

- Model Content Framework
- Item Specifications/Evidence Tables
- Sample Items
- Calculator Usage
- Accommodations
- Reference Sheets

Instructional Strategies and Resources

Instructional Strategies

Provide categories for students to use to sort a collection of objects. Each category can relate to only one attribute, like *Red* and *Not Red* or *Hexagon* and *Not Hexagon,* and contain up to 10 objects. Students count how many objects are in each category and then order the categories by the number of objects they contain.

Ask questions to initiate discussion about the attributes of shapes. Then have students sort a collection of two-dimensional and three-dimensional shapes by their attributes. Provide categories like *Circles* and *Not Circles* or *Flat* and *Not Flat*. Have students count the objects in each category and order the categories by the number of objects they contain.

Have students infer the classification of objects by guessing the rule for a sort. First, the teacher uses one attribute to sort objects into two loops or regions without labels. Then the students determine how the objects were sorted, suggest labels for the two categories and explain their reasoning.

Instructional Resources/Tools

Attribute blocks
Yarn for loops
Large paper to draw loops
A variety of objects to sort

Diverse Learners

Strategies for meeting the needs of all learners including gifted students, English Language Learners (ELL) and students with disabilities can be found at this site. Additional strategies and resources based on the Universal Design for

Learning principles can be found at www.cast.org.
Connections: This cluster is connected to the Kindergarten Critical Area of Focus #1, **Representing and comparing whole numbers, initially with sets of objects**. More information about this critical area of focus can be found by clicking here. This cluster is connected to *Know number names and the count sequence* and *Count to tell the number of objects* in Kindergarten, and to *Represent and interpret data* in Grade 1.

Kindergarten

Domain	Geometry
Cluster	***Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).***
Standards	1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. 2. Correctly name shapes regardless of their orientations or overall size. 3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Content Elaborations

Ohio has chosen to support shared interpretation of the standards by linking the work of multistate partnerships as the Mathematics Content Elaborations. Further clarification of the standards can be found through these reliable organizations and their links:

- Achieve the Core Modules, Resources
- Hunt Institute Video examples
- Institute for Mathematics and Education Learning Progressions Narratives
- Illustrative Mathematics Sample tasks
- National Council of Supervisors of Mathematics (NCSM) Resources, Lessons, Items
- National Council of Teacher of Mathematics (NCTM) Resources, Lessons, Items
- Partnership for Assessment of Readiness for College and Careers (PARCC) Resources, Items

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Instructional Strategies and Resources

Instructional Strategies

Develop spatial sense by connecting geometric shapes to students' everyday lives. Initiate natural conversations about shapes in the environment. Have students identify and name two- and three-dimensional shapes in and outside of the classroom and describe their relative position.

Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see, e.g. *This rectangle (a poster) is over the sphere (globe)*. Teachers can use a digital camera to record these relationships.
Hide shapes around the room. Have students say where they found the shape using positional words, e.g. *I found a triangle UNDER the chair*.

Have students create drawings involving shapes and positional words: *Draw a window ON the door* or *Draw an apple UNDER a tree*. Some students may be able to follow two- or three-step instructions to create their drawings.

Use a shape in different orientations and sizes along with non-examples of the shape so students can learn to focus on defining attributes of the shape.

Manipulatives used for shape identification actually have three dimensions. However, Kindergartners need to think of these shapes as two-dimensional or "flat" and typical three-dimensional shapes as "solid." Students will identify two-dimensional shapes that form surfaces on three-dimensional objects. Students need to focus on noticing two and three dimensions, not on the words *two-dimensional* and *three-dimensional.*

Career Connection

Provide students with real-work examples that demonstrate the use of shapes in various work places (e.g., circle wheels on a car, rectangle door on a house, triangle suspensions on a bridge). Students will identify the shapes within the context. Then, lead a discussion of careers related to the examples (e.g., architects, designers, engineers).

Instructional Resources/Tools

Common two- and three-dimensional items
Digital camera
Pattern blocks
Die cut shapes
Three-dimensional models
Assorted shapes
Tangrams

ORC # 4459 From the International Reading Association and the National Council of Teachers of English: Going on a Shape Hunt: Integrating Math and Literacy
In this unit, students are introduced to the idea of shapes through a read-aloud session with an appropriate book. They then use models to learn the names of shapes, work together and individually to locate shapes in their real-world environment.

ORC # 3336 From the National Council of Teachers of Mathematics: Investigating Shapes (Triangles)
Students will identify and construct triangles using multiple representations in this unit.

ORC # 423 From the National Council of Teachers of Mathematics: I've Seen That Shape Before
Students will learn the names of solid geometric shapes and explore their properties at various centers or during multiple lessons.

Common Misconceptions

Students many times use incorrect terminology when describing shapes. For example students may say a *cube* is a *square* or that a *sphere* is a *circle*. The use of the two-dimensional shape that appears to be part of a three-dimensional shape to name the three-dimensional shape is a common misconception. Work with students to help them understand that the two-dimensional shape is a part of the object but it has a different name.

Diverse Learners

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Connections:

This cluster is connected to the Kindergarten Critical Area of Focus #2, **Describing shapes and space**. More information about this critical area of focus can be found by clicking here.

This cluster is connected to *Analyze, compare, create and compose shape*s in Kindergarten, and to *Reason with shapes and their attributes* in Grade 1.

Kindergarten

Domain	Geometry
Cluster	***Analyze, compare, create and compose shapes***
Standards	4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). 5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. 6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Content Elaborations

Ohio has chosen to support shared interpretation of the standards by linking the work of multistate partnerships as the Mathematics Content Elaborations. Further clarification of the standards can be found through these reliable organizations and their links:

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Instructional Strategies and Resources

Instructional Strategies

Use shapes collected from students to begin the investigation into basic properties and characteristics of two- and three-dimensional shapes. Have students analyze and compare each shape with other objects in the classroom and describe the similarities and differences between the shapes. Ask students to describe the shapes while the teacher records key descriptive words in common student language. Students need to use the word *flat* to describe two-dimensional shapes and the word *solid* to describe three-dimensional shapes.

Use the sides, faces and vertices of shapes to practice counting and reinforce the concept of one-to-one correspondence.

The teacher and students orally describe and name the shapes found on a Shape Hunt. Students draw a shape and build it using materials regularly kept in the classroom such as construction paper, clay, wooden sticks or straws.

Students can use a variety of manipulatives and real-world objects to build larger shapes with these and other smaller shapes: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. Kindergarteners can manipulate cardboard shapes, paper plates, pattern blocks, tiles, canned food, and other common items.

Have students compose (build) a larger shape using only smaller shapes that have the same size and shape. The sides of the smaller shapes should touch and there should be no gaps or overlaps within the larger shape. For example, use

one-inch squares to build a larger square with no gaps or overlaps. Have students also use different shapes to form a larger shape where the sides of the smaller shapes are touching and there are no gaps or overlaps. Ask students to describe the larger shape and the shapes that formed it.

Instructional Resources/Tools

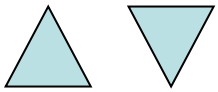
Pattern blocks
Tangrams
Colored tiles
Cubes
Three-dimensional models
Cans of food
Carpet squares or rectangles
Paper plates
Balls
Boxes that are cubes
Floor tiles
Straws
Wooden sticks
Clay
Construction paper

ORC # 4258 From NCTM: Building with triangles: what can you build with two triangles?

The first lesson in this unit includes the *Just Two Triangles* activity worksheet where students are asked to form different larger shapes with two triangles.

Common Misconceptions

One of the most common misconceptions in geometry is the belief that orientation is tied to shape. A student may see the first of the figures below as a triangle, but claim to not know the name of the second.



Students need to have many experiences with shapes in different orientations. For example, in the *Just Two Triangles* activity referenced above, ask students to form larger triangles with the two triangles in different orientations.

Another misconception is confusing the name of a two-dimensional shape with a related three-dimensional shape or the shape of its face. For example, students might call a *cube* a *square* because the student sees the face of the cube.

Diverse Learners

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Connections

This cluster is connected to the Kindergarten Critical Area of Focus #2**, Describing shapes and space**. More information about this critical area of focus can be found by clicking here.

This cluster is connected *to Identify and descibe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)* in Kindergarten, and to *Reason with shapes and their attributes* in Grade 1.