



Sample Mathematics Item: Algebra I/Mathematics I

“Popcorn Inventory”

November 2013

The Main Street Cinema gets a food delivery every Friday morning. On Thursday, Hannah checks the computer to determine what to order the next morning. The computer shows the amount of popcorn seed and boxes remaining at the end of each day.

Date	Popcorn Seed (in cups)	Large Boxes (20 cups)	Medium Boxes (16 cups)	Small Boxes (11 cups)
Sun. 3/1	393.3	818	539	581
Mon. 3/2	331.3	784	525	557
Tues. 3/3	285.4	758	516	542
Wed. 3/4	179.5	699	495	504
Thurs. 3/5	69.7	638	472	464

Sales Sunday through Thursday are relatively consistent. Friday and Saturday are busier days, and on each of those days they sell between 200 and 300 large boxes of popcorn. On Friday and Saturday, they also sell about twice as many small and medium boxes of popcorn as they do on the other days.

She also knows that $\frac{1}{3}$ -cup of popcorn seed makes 8 cups of popcorn, and she must buy enough popcorn seed to last until the next delivery on the following Friday.

Estimate the amount of popcorn seed that Hannah should order this Friday so that there are between 100 and 200 cups of popcorn seed remaining next Friday morning. Show or explain the reasoning you used to determine your estimate.

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HS	Popcorn Inventory
Type	Type III 6 Points
Evidence Statement	HS.D.1-1: Solve multi-step contextual problems with degree of difficulty appropriate to the course, requiring application of knowledge and skills articulated in 7.RP.A, 7.NS.3, 7.EE, and/or 8.EE.
Most Relevant Standards for Mathematical Content	<p>7.RP.2: Recognize and represent proportional relationships between quantities.</p> <p>a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p> <p>b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>c. Represent proportional relationships by equations. <i>For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.</i></p> <p>d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.</p> <p>7.RP.3: Use proportional relationships to solve multistep ratio and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p> <p>These standards are major content in the seventh grade based on the PARCC Model Content Frameworks.</p>
Most Relevant Standards for Mathematical Practice	Students creating reasoned estimates must be able to reason abstractly and quantitatively in order to build a model of the situation that is accurate enough for the given situation (MP.2 and MP.4). Working with ambiguity is an important part of the modeling skills expected at high school, and this requires students to productively engage with the quantities given in the context. Modeling is a critical component of the high school standards, and this item requires students to create a model from a complex situation to make a real-world estimate given an unscaffolded situation where a model is a useful tool. To make this model, students will have to reason with the given quantities, their units and the proportional relationships between them. This will require students to understand how to use the numbers mathematically, and then be able to periodically check their own understanding of what those numbers mean.
Item Description and Assessment Qualities	<p>This application task requires students to create a reasoned estimate in response to solve a real-world problem. Students must first wrestle with the data displayed on the Popcorn Inventory page. They should recognize that the amounts in the table are decreasing because each day boxes and popcorn seeds are used. This means that students should recognize that they are using about 15 medium boxes each day and about 25-30 small boxes each day.</p> <p>In order to address the amount of popcorn sold over the weekend, students must first create a viable estimate of the number of cups of popcorn sold, then use the ratio $\frac{1}{3}$</p>

cups of popcorn seed:8 cups of popcorn. Students may choose to use this method to estimate the amount of popcorn seed used Sunday through Thursday; however, other methods could be used to determine the amount of popcorn seed used each day. Students using this method must be sure to account for the amount of popcorn seed used on Sunday because the original information starts from end of day on Sunday.

The final estimate requires students to use the current amount of popcorn seed, the amount used Friday and Saturday, and the amount used Sunday through Thursday in order to estimate the amount of popcorn seeds she should purchase so there are 100-200 pounds left over next Friday morning.

Note that ratio and proportional relationships are key skills required for college and career readiness, and this item provides a strong application of that content. Unlike traditional multiple choice, it is difficult to guess the correct answer or use a choice elimination strategy.

Scoring Rubric for HS.D.1-1

Task is worth 6 points. Task can be scored as 0, 1, 2, 3, 4, 5, or 6.

Scoring consists of 2 points for calculation and 4 points for modeling.

Structure (6 points total):

- 2 points for correctly addressing the cups of popcorn seed needed for Sunday-Thursday
 - o 1 calculation point for adequate estimate
 - o 1 modeling point for adequate estimation strategy
- 3 points for correctly addressing the cups of popcorn seed needed for Friday and Saturday.
 - o 1 modeling point for adequate estimation strategy for addressing two sizes of boxes for both days.
 - o 1 modeling point for accurate use of the proportion of popcorn seed to popcorn
 - o 1 calculation point for adequate estimate
- 1 point for correctly estimating the amount of popcorn seed that should be ordered
 - o 1 modeling point for adequate estimation strategy (the calculation is not as important as the strategy)

Example student response:

On Friday and Saturday, they will sell about 500 large boxes ($250 + 250 = 500$). I found

Scoring
Information

that they sold about 17 medium boxes ($\frac{539 - 472}{4} = 16.75$) and about 30 small boxes ($\frac{581 - 464}{4} = 29.75$) each day in the table, so they would sell about 68 (2 x 17 for both days) medium boxes and 120 (2 x 30 for both days) small boxes on Friday and Saturday combined.

That means they need to pop:

- Large: $500 \cdot 20$ cups = 10,000 cups of popcorn
- Medium (approx.): $68 \cdot 16$ cups = 1088 cups of popcorn
- Small (approx.): $120 \cdot 11$ cups = 1,320 cups of popcorn

I added the three amounts of popcorn to find that they will need about 12,400 cups of popcorn over the weekend.

Since $\frac{1}{3}$ cups of popcorn seed makes 8 cups of popcorn, I know that 1 cup of popcorn seed will make 24 cups of popcorn. That means that they need about $\frac{12,400}{24} \approx 517$ cups of popcorn seed for Friday and Saturday. So, they will need about 525 cups of popcorn seed for the weekend.

According to the table, they used about 80 cups of popcorn seed each of the remaining days of the week ($\frac{393 - 70}{4} = 80.75$). They will need 80 x 5 or 400 cups of popcorn seed for Sunday-Thursday.

I made this list to make sure she buys enough:

(69.7 cups currently)
525 cups of popcorn seed for Friday and Saturday
400 cups of popcorn seed for Sunday-Thursday
+ 100 extra cups to make sure she is between 100 and 200 cups on Friday morning
1,025 cups of popcorn seed to order in the morning

NOTE: There are a wide variety of estimation strategies that can receive full credit.