



Sample Mathematics Item: Algebra I/Mathematics II

“Mini-Golf Prices”

November 2013

A local mini-golf course charges \$5 per person to play a round of golf, and the course sells 120 rounds of golf per week. The manager of the course studied the effect of raising the price to increase revenue and found the following data.

The table shows the price, number of rounds of golf, and weekly revenue for different numbers of \$0.25 increases in price.

Number of \$0.25 price increases, n	0	1	2	3	4
Price of a round of golf, $p(n)$	\$5.00	\$5.25	\$5.50	\$5.75	\$6.00
Number of rounds of golf sold, $s(n)$	120	117	114	111	108
Weekly revenue, $r(n)$	\$600	\$614.25	\$627	\$638.25	\$648

Part A

Based on the data, write a linear function to model the price of one round of golf, $p(n)$, in terms of n , the number of \$0.25 increases.

Based on the data, write a linear function to model the number of rounds of golf sold in a week, $s(n)$, in terms of n , the number of \$0.25 increases.

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Part B

Based on the data, write a quadratic function for the weekly revenue in a week, $r(n)$, in terms of n , the number of \$0.25 increases.

Use your quadratic function to determine the weekly revenue in a week when tickets cost \$6.25.

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Part C

The maximum possible weekly revenue is what percent greater than the weekly revenue with no price increases? Justify your answer graphically or algebraically.

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HS	Mini-golf prices
Type	Type III 6 points
Evidence Statement	<p>HS.D.2-9: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course -level knowledge and skills articulated in F-BF.1a, F-BF.3, A-CED.1, A-SSE.3, F-IF.B, F-IF.7, limited to linear and quadratic functions</p> <p>Clarification: i) F-BF.1a is the primary content; other listed content elements may be involved in tasks as well.</p>
Most Relevant Standards for Mathematical Content	<p>F-BF. 1. Write a function that describes a relationship between two quantities.</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>This standard is major content in the course based on the PARCC Model Content Frameworks.</p>
Most Relevant Standards for Mathematical Practice	<p>Students must model with mathematics (MP.4) in order to create the functions and determine the percent increase of the maximum weekly revenue. The question in Part C is not as scaffolded as the previous parts and will confront students with a novel concept that will take time to ascertain how to begin a correct solution method (MP.1). In order to do this, students must decontextualize the given information and use it to construct a viable solution (MP.2).</p>
Item Description and Assessment Qualities	<p>This application task requires students to create functions to interpret a situation. Then, students must use those functions to address application questions based on this context. This content is complex, the model used in Parts B and C is quadratic, but the item provides some scaffolding in Part A so that students can gain familiarity with the underlying structure of the components. Students first write functions that model various related quantities in the task. Then, students show that they can use these functions correctly by calculating a value using the quadratic function. Then, students use their model to find the maximum weekly revenue, and the percent increase that change represents over the revenue with no price increases. Students may choose to solve this problem algebraically or with the graphing tool. However, students must explain the steps they took to determine the percent increase.</p> <p>One of the reasons for modeling with quadratics is that, unlike linear or exponential functions, quadratic functions can model situations with local optimums. Optimization is a common and important use of mathematics.</p>

Graphing technology is available to support student's work on this item.

Scoring
Information

Scoring Rubric for Sample HS.D.2-9

Task is worth 6 points. Task can be scored as 0, 1, 2, 3, 4, 5, or 6.

Task has three parts: Part A is worth 2 points, Part B is worth 2 points, and Part C is worth 2 points.

Part A: 2 point

- 1 point is earned for a correct function of $p(n)$: $p(n) = 5 + 0.25n$ or an equivalent expression (e.g., $\frac{n}{4} + 5$).
- 1 point is earned for a correct function of $s(n)$: $s(n) = 120 - 3n$ or an equivalent expression (e.g., $120 + \left(\frac{120-117}{0-1}\right)n$).

Part B: 2 points

- 1 point is earned for a correct function of $r(n)$:
 $r(n) = (5 + 0.25n)(120 - 3n)$ OR an equivalent expression (e.g., $600 + 15n - 0.75n^2$). The student may use quadratic regression to determine their function.
- 1 point is earned for the correct weekly revenue for a price of \$6.25: \$656.25.

Possible student work (not scored):

$$p(n) = \$6.25, \text{ when } n = 5 \text{ because } 6.25 = 5 + 0.25(5).$$
$$r(5) = (5 + 0.25(5))(120 - 3(5))$$
$$r(5) = 6.25(105)$$
$$r(5) = 656.25$$

NOTE: Students can receive 1 or 2 points on Part B if they use incorrect functions from Part A to correctly address Part B.

Part C: 2 points

- 1 point is earned for stating that the maximum weekly revenue is 12.5% greater than the weekly revenue with no increases.
- 1 point is earned for adequate supporting work that has a valid solution method.

Sample Student Response 1

I graphed my function and saw that the vertex is the maximum value. It's at $n = 10$, so I calculated $p(10) = \$7.50$ and $s(10) = 90$, so I know that the maximum weekly revenue will be \$675. That would be a \$75 increase from \$600, $\frac{\$75}{\$600} = 0.125$. So, the average weekly revenue would increase 12.5%.

OR

I graphed my function and saw that the vertex is the maximum value. It's at

$n = 10$, so I calculated $p(10) = \$7.50$ and $n(10) = 90$, so I know that the maximum weekly revenue will be \$675. $\frac{\$675}{\$600} = 1.125$. This shows a 12.5 % increase.

Sample Student Response 2

The vertex form of $r(n)$ can be found by completing the square from the standard form:

$$r(n) = -0.75n^2 + 15n + 600$$

$$r(n) = -0.75(n^2 - 20n) + 600$$

$$r(n) = -0.75(n^2 - 20n + 100) + 75 + 600$$

$$r(n) = -0.75(n - 10)^2 + 675$$

I know that $n = 10$ will maximize the value of the equation, and can see that $r(10) = 675$ because when $n = 10$ then $n - 10 = 0$, so the value of the whole expression is 675. That is a

$$\frac{675 - 600}{600} = 12.5\% \text{ increase.}$$

NOTE: There are other methods for getting $r(n)$ into vertex form. For example, one could use the formula for finding the axis of symmetry, $\frac{-b}{2a}$, to obtain the value of n when at the parabola's vertex:

$$n = \frac{-15}{2(-0.75)} = \frac{-15}{-1.5} = 10.$$

Task score: The task score is the sum of the points awarded in each component.