

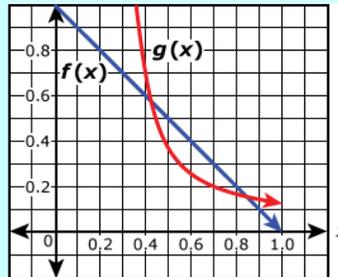


**Sample Mathematics Item: Algebra II/Mathematics III**

**“Graphs of Functions”**

**November 2013**

The functions  $f(x) = 1 - x$  and  $g(x) = \frac{0.11}{x^2}$  are defined for all values of  $x > 0$ . The graphs are shown in the coordinate plane.



Part A

Explain how you can use the graph to find the solution(s) of the equation  $f(x) = g(x)$ . In your answer, provide the approximate value(s) of the solution(s).

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Part B

Write the value(s) of  $f(x)$  when  $x$  equals the solution(s) from Part A.

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Part C

Let the function  $h(x)$  be defined as  $h(x) = f(x) - g(x)$ .

What are the coordinates of the point(s) on the graph of  $h(x)$  when  $x$  equals the solution(s) from Part A? Explain your reasoning.

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HS	Graphs of functions
Type	Type II 4 Points
Evidence Statement	<p><b>HS.C.6.2:</b> Base explanations/reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.</p> <p>Content scope: A-REI.D</p>
Most Relevant Standards for Mathematical Content	<p><b>A-REI.D. Represent and solve equations and inequalities graphically</b></p> <p>10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> <p>11. Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p> <p>These standards are major content in the course based on the PARCC Model Content Frameworks.</p>
Most Relevant Standards for Mathematical Practice	<p>PARCC highlights MP.3 through Subclaim C. These items require students to construct viable chains of reasoning through precise mathematical statements. This item requires students to reason with the structure of the graph to determine the number of solutions and the correct value of those solutions (MP.7).</p>
Item Description and Assessment Qualities	<p>This item assesses students' ability to use a graph to find solutions to the equation <math>f(x) = g(x)</math>. Students are initially presented with the graphs of two functions, and then asked to find the solution to the equation when the functions are set equal to each other. The question does not clue students that there are two solutions to the equation, and expects students to explain how the graph is used. Students must recognize the nature of the solutions using the graph.</p> <p>Given the content mastered by students at this level, the graph is the best way for students to find the solutions to <math>f(x) = g(x)</math>. Students use the graph to estimate these values, recognizing that the solution of two functions set equal to each other is the <math>x</math>-coordinate of the two points of intersection.</p> <p>Part B requires students to determine values of <math>f(x)</math>. Students could use the graph from Part A or substitute the values of <math>x</math> into the function</p> $f(x) = 1 - x.$

	<p>In Part C, students need to consider a new function <math>h(x)</math> given as the difference of <math>f(x)</math> and <math>g(x)</math>. They must then recognize and explain the solution values of <math>f(x) = g(x)</math> as the zeros of <math>h(x)</math>.</p> <p>Unlike traditional multiple choice, it is difficult to guess the correct answer or use a choice elimination strategy.</p>
<p>Scoring Information</p>	<p>Scoring Rubric for Sample HS.C.6.2</p> <p>Task is worth 4 points. Task can be scored as 0, 1, 2, 3 or 4.</p> <p>The scoring consists of two components: Computation for 2 points and Reasoning for 2 points.</p> <p>Part A: 1 point for reasoning, 1 point for computation</p> <ul style="list-style-type: none"> <li>• Reasoning: Correct explanation that the <math>x</math>-coordinates of the points where the graphs of the functions <math>f(x)</math> and <math>g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>.</li> <li>• Computation: Based on the graph, the solutions to the equation are within the range <math>0.4 \leq x \leq 0.5</math> and <math>0.8 \leq x \leq 0.9</math>.</li> </ul> <p>Sample student response: The solutions to the equation <math>f(x) = g(x)</math> can be found by looking at the <math>x</math>-coordinates of the points where the graphs of <math>f(x)</math> and <math>g(x)</math> intersected. I determined that the <math>x</math>-coordinates of those two points were approximately 0.45 and 0.85.</p> <p>Part B: 1 point for computation</p> <ul style="list-style-type: none"> <li>• Computation: 1 point for both values of <math>f(x)</math>.</li> </ul> <p>Sample Student Response Based on the graph, the values of the function <math>f(x)</math> for the solutions are within the range <math>0.5 \leq f(x) \leq 0.6</math> and <math>0.1 \leq f(x) \leq 0.2</math>.</p> <p>Part C: 1 point for reasoning</p> <p>Sample Student Response In part A, I found that <math>f(x) = g(x)</math> when <math>x \approx 0.45</math> and <math>x \approx 0.85</math>, so I know that when <math>x \approx 0.45</math> or when <math>x \approx 0.85</math> that <math>h(x)</math> will have values near zero. So, the graph will go through these two approximate points: <math>(0.45, 0)</math> and <math>(0.85, 0)</math>.</p> <p><b>Task score:</b> The task score is the sum of the points awarded in each component.</p>