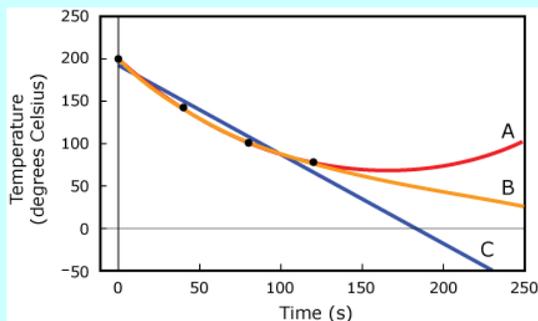


A scientist is studying the cooling patterns of a particular material over time. Her research requires heating a sample of the material to 200°C. She records the temperature of the sample as it is cooled to 0°C. The table shows the data collected during the first 2 minutes of the cooling process.

Time material is cooling (seconds)	0	40	80	120
Temperature (°C)	200	141	101	74

The figure shows the scientist's data (data points are plotted as large dots). Three possible models for the data are also shown: a linear model, a quadratic model, and an exponential model.



Part A

- Which model is linear? Which model is quadratic? Which model is exponential?
- Which model is best for the range of times $0 \leq t \leq 250$?
- Explain why the other models do not fit the data very well for the range of times $0 \leq t \leq 250$.

Cut
Paste
Undo
Redo

Part B

Construct a function using the type of model you decided is best (linear, quadratic, or exponential). Show your work and use function notation when entering your answer.

Cut
Paste
Undo
Redo

Algebra II	Temperature Changes
Item Type	Type III – 3 points
Evidence Statement	<p>HS.D.2-10 with content scope of F.BF.A</p> <p>HS.D.2-10: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, A-CED.1, A-SSE.3, F-IF.B, F-IF.7.</p> <p>Clarifications for HS.D.2-10: i) F-BF.A is the primary content; other listed content elements may be involved in tasks as well.</p>
Most Relevant Standards for Mathematical Content	<p>F-BF.A Build a function that models a relationship between two quantities</p> <ol style="list-style-type: none"> Write a function that describes a relationship between two quantities. <ol style="list-style-type: none"> Determine an explicit expression, a recursive process, or steps for calculation from a context. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. <p>F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</p> <p>These standards are major content in the course based on the PARCC Model Content Frameworks.</p>
Most Relevant Standards for Mathematical Practice	<p>Students apply their knowledge of functions to construct an accurate model of the situation (MP.4). Students first reason with the context and the potential models (MP.1 and MP.2), then they explain why the other models are inadequate and explain why their model is the best choice (MP.3). Tools and structure are important elements that students can access through the graphing tool (MP.5 – Use appropriate tools strategically and MP.7 – Look for and make use of structure).</p>

<p>Item Description and Assessment Qualities</p>	<p>This three-point application task requires students to identify and explain which type of function is appropriate to model the situation. The item provides three graphical models and students must identify whether each model is linear, quadratic or exponential. Then, students must reason abstractly with the graph and the context to evaluate each model. Students should explain that the linear model does not fit the data as well and allows for negative values of $f(t)$ within the given range. The quadratic model is also flawed because the temperature begins to increase when t is greater than 165 seconds. The exponential model best matches this data because it has good visual fit and the values of $f(t)$ will never be less than 0.</p> <p>Students may reach these conclusions and base their explanations on the graph and/or the data as provided in the table to evaluate the function types and create a function that models the data. The rubric makes it clear that there are a variety of possible models that could accurately fit this data, and all should receive credit. Note that other versions of this task could require students to use their model to solve for $f(t) = 1$.</p>
<p>New Scoring Information</p>	<p>Task is worth 3 points. Task can be scored as 0, 1, 2, or 3.</p> <ol style="list-style-type: none"> Part A: 1 modeling point: Student correctly classifies models as Model A (red): Quadratic Model, Model B (yellow): Exponential Model, and Model C (blue): Linear Model; identifies model B as best; and rejects models A and C for valid reasons. For example: Model A fits the data well, but the temperature of the material should fall to zero, and this model shows that the temperature starts to rise before the temperature reaches zero. Model C doesn't fit the data as well as model B, and Model C also says that the temperature reaches negative values, which isn't what the experiment says. Part B: 2 modeling points: Student creates a function that adequately models the data. Not every step has to be justified, but the student's method should be perceptible with its key steps shown. For example, an idealized solution that does justify each step is shown. (Other approaches besides this approach are possible.) My model is $f(t) = A b^t$. To make my model, I started by finding the ratios of the data points: $141/200 = 0.71$, $101/141 = 0.72$, $74/101 = 0.73$. They are pretty close. So I made the ratio in my model be 0.72 when the difference in time is 40 seconds:

	$B^{t+40}/B^t = 0.72$ <p>I know the properties so</p> $B^t B^{40}/B^t = 0.72$ $B^{40}/B^t = 1$ $B^{40} = 0.72$ <p>Solve for B:</p> $B = 0.72^{1/40} = 0.992.$ <p>So my model is $f(t) = A(0.992)^t$. To find the A, I made it fit the data at the beginning:</p> $A(0.992)^0 = 200$ $(0.992)^0 = 1$ $A = 200.$ <p>So my model is $f(t) = 200(0.992)^t$.</p> <p>NOTE: Accept other valid methods. If the student uses exponential regression on the calculator, they will get a function of $f(t) = 198(0.992)^t$.</p> <p>Additional notes: A student can earn a maximum of 2 points if they choose an incorrect model and use it correctly. A maximum of 1 point will be deducted if a computation mistake is made.</p> <p>Task score: The task score is the sum of the points awarded in each component.</p>
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