

Michelle wanted to investigate the effect on the vertex of the function $f(x) = x^2 + 6x$ when $f(x)$ is replaced by $f(x + k)$.

Michelle graphed functions of the form $f(x + k)$ for $k = 1, 2, 3$ and 4 . For each of the functions she graphed, the x -coordinate of the vertex was negative and different for each value of k , but the y -coordinate of the vertex was the same for each value of k . Michelle made three conjectures based on her results.

1. The x -coordinate of the vertex depends on the value of k .
2. The x -coordinate of the vertex is negative for all values of k .
3. The y -coordinate of the vertex is independent of the value of k .

Determine if each of Michelle's three conjectures are true. Justify each answer.

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Algebra I	Michelle's Conjectures
Item Type	Type II – 3 points
Evidence Statement	HS.C.9.1 with content scope F-BF.3 HS.C.9.1: Express reasoning about transformations of functions. Content scope: F-BF.3, limited to linear and quadratic functions. Tasks will not involve ideas of even or odd functions.
Most Relevant Standards for Mathematical Content	F-BF.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. This standard is additional content in the course based on the PARCC Model Content Frameworks.
Most Relevant Standards for Mathematical Practice	PARCC highlights MP.3 and MP.6 through Subclaim C. These items require students to construct viable arguments through precisely written mathematical statements. This item requires students to look for and make use of structure of the function to create clear proofs (MP.7).

	<p>Students will need to use tools to address Conjecture #3 like completing the square; in addition, there is a graphing calculator available in this item to allow for students to work with the tools needed for this exploration. (MP.5).</p>
<p>Item Description and Assessment Qualities</p>	<p>This conceptual understanding task requires students to evaluate three conjectures using their conceptual understanding of functions as well as a graphing calculator that can help students find the underlying structure in the changing parameters of the item. The Standards require High School students to express precise mathematical reasoning and this task assesses students' ability to reason well with function concepts.</p> <p>The first proof can be verified by understanding how the graph of the function is changing using the graphing calculator or other methods to justify that the x-coordinate of the vertex changes as k changes. The second conjecture is incorrect so students may disprove the conjecture algebraically or provide a counterexample. The final conjecture requires students to justify that the y-coordinate of the vertex does not change for any value of k. Each of these responses requires significant mathematical reasoning and the use of precise mathematical language.</p> <p>The graphing calculator available in this item allows for students to work with the tools needed for this exploration.</p>
<p>Scoring Information</p>	<p>Task is worth 3 points. Task can be scored as 0, 1, 2, or 3.</p> <ul style="list-style-type: none"> • Students will earn 1 reasoning point for an adequate proof that conjecture 1 is correct. <ul style="list-style-type: none"> ○ For example: As k increases, the vertex moves so I know that the x-coordinate of the vertex will change as I change k. • Students will earn 1 reasoning point for an adequate proof that conjecture 2 is incorrect. <ul style="list-style-type: none"> ○ For example: I figured that when k is much less than 1, then the vertex would be positive. I solved used $k = -5$ and found that $f(x - 5) = x^2 - 4x - 5 = (x - 2)^2 - 9$, and the vertex is at $(2, -9)$. OR ○ For example: $f(x + k) = (x - (-k - 3))^2 - 9$. Therefore, when $-k - 3 > 0$, i.e., when $k < -3$, the x-coordinate of the vertex is $-k - 3$, which is positive. • Students will earn 1 reasoning point for an adequate algebraic proof based on completing the square that conjecture 3 is correct. <ul style="list-style-type: none"> ○ For example: Because the completed equation is $f(x + k) = (x + (k + 3))^2 - 9$. The vertex of the graph is $(-k - 3, -9)$ and the y-coordinate of the vertex will always be -9. It is

independent of k .

Task Score: The task score is the sum of the points awarded in each component.