**Ohio** Department of Education

STUDENT READINESS ASSESSMENT ITEM RELEASE GUIDE TEACHER EDITION ALGEBRA 1

### Introduction

The state-mandated school closures through the end of the 2019-2020 school year not only changed the way schools delivered instruction but impacted how students were expected to learn grade-level and course content. As districts prepare for the 2020-2021 school year, the Ohio Department of Education is providing a series School Readiness Toolkits to help educators reflect on their instructional practices and support them in determining where their students are in their acquisition and retention of knowledge and skills.

### Background

The items contained in the School Readiness Toolkits have been selected from the pool of released items from previous spring administrations of Ohio's State Tests. The items are grouped together by Reporting Category and <u>Critical Area of Focus</u>. The collection of items as a whole is not representative of a single test form. The items presented are selected to offer a range of opportunity to work with each reporting category but do not comprise an actual test statistically. They are chosen to offer a range of experience with items of varying levels of difficulty or complexity. Items contained in School Readiness Toolkits are reflective of the 2017 Ohio's Learning Standards for Mathematics. All items satisfy the criteria set forth by the grade-level/course Test <u>Specifications</u> and Content Elaborations and Expectations for Learning established by the grade-level/course <u>Model Curriculum</u>.

### How These Items Can Be Used

The School Readiness Toolkits documents can be used to support instruction in a variety of ways. Districts can choose to administer the:

- Previous grade-level School Readiness Toolkits to acquire data and gather information on student understanding of previous grade-level content to begin the new school year;
- Current grade level School Readiness Toolkits items in sections as pre-assessments based on the grade level/course Critical Areas of Focus or local unit of study;
- Current grade-level School Readiness Toolkits in their entirety;
- Problems may be individually selected for use during local instruction; or
- Problems may be selected for use on local assessments.

### How to Identify Released Items

At the bottom of the page for each released item, there is a page number and year. For example: 3 (2018), identifies the item can be found on page 3 of the 2018 Released Item document for the grade level or course. Released Item Scoring Guides for Mathematics are available in the <u>Test Portal</u> in the <u>Student Practice</u> <u>Resources for Mathematics</u> folder under the <u>Student Practice Resources</u>.

The released item Release Scoring Guides sample responses and scoring rationales can help educators plan and deliver instruction by providing example responses for each question along with scoring rationales for each response.

- 2017 Item Release Scoring Guide Algebra 1
- 2018 Item Release Scoring Guide Algebra 1
- 2019 Item Release Scoring Guide Algebra 1

### **Using Released Test Items to Plan Instruction after COVID-19**

The following questions can be answered individually or as a teacher team in the review of the state-released items and subsequent reflection on the local curriculum, instructional practices and assessments (both formative and summative), along with the previous learning opportunities for students.

- What are the math concepts evident in the release item(s)?
  - o What is the math a student needs to know in this item?
  - o Specifically, what previous grade-level standards impact the ability to answer this item?
- What math strategies can a student use to answer the item?
  - o Identify examples of how these can be included in your instruction.
- Does the item focus on procedural fluency or conceptual understanding?
  - Procedural fluency follow-up:
    - What are the procedures and/or skills a student needs to know?
    - What experiences do students have to be prepared to demonstrate this learning?
    - What experiences can be used to move toward the desired outcome(s)?
- Conceptual understanding follow-up:
  - o What mathematical understanding is evident in the item?
  - o What tasks can be used to develop that mathematical understanding?
- Does the item require the student to make connections across standards? If so, what are they?
  - What previous grade-level expectations are evident in the item?
  - What experiences can improve a student's ability to demonstrate these learning expectations?
- Which Standards for Mathematical Practice are most evident in the item?
  - What types of experiences will improve student success?

### **Preparing for Instruction**

Identify a grade-level <u>Critical Area of Focus</u> or a mathematical topic of related standards. Think about what your typical instruction for this critical area of focus looks like. Determine whether changes are needed to grow all students mathematically.

- What was present in past instruction that helped students perform well?
  - Using the <u>Gap Analysis</u>, <u>Critical Area of Focus</u>, <u>Learning Progressions</u> and <u>Model Curriculum</u> documents, what previous learning is likely absent or weak?
  - What experiences would support bridging the gap(s)?
  - How could you strengthen the Standards for Mathematical Practice to help support or enhance learning?
  - o Specifically, what tasks would be used?
- What does typical instruction include?
  - Models/representations? What models or representations need introduction?
  - o One-step, two-step or multi-step problems? Is more experience needed? What?
  - Routine and non-routine problems? Is more experience needed? What?
  - Mathematical and real-world contexts? Is more experience needed? What rich tasks could incorporate multiple standards?
- Did the mathematical contexts use numbers and operations appropriate for the grade level?
  - How could those numbers be modified to highlight the mathematical understanding needed and increase access for all students?
- Were the real-world contexts familiar or unfamiliar to the students?
  - How do you know?
  - What is needed now?
- Did the instruction allow opportunities for student reasoning and communication?
  - Productive struggle?
  - o Student analysis of individual work, thinking and reasoning of others?
  - o Descriptions, explanations and justifications?
  - o Error analysis and reasonableness of answers?
  - What changes are needed to strengthen the Standards for Mathematical Practice?
- From this analysis, what overall changes are needed in instruction?
  - What instructional strategies should be maintained?
  - o What instructional strategies require modification?
  - o What needs to happen next to increase learning for all students?
    - Resources
    - Instructional strategies
    - Professional development
      - Do I need to seek out professional learning opportunities? What opportunities do I have for growing my own learning? What supports do I have to make these changes?

# Algebra 1 Reporting Category:

## Number, Quantities, Equations and Expressions

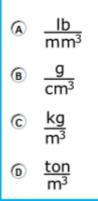
**CRITICAL AREA OF FOCUS #1** Relationships Between Quantities and Reasoning with Equations

CRITICAL AREA OF FOCUS #4 Expressions and Equations

Michelle holds a small rock in her hand.

Density (D) can be found using the formula  $D = \frac{\text{mass}}{\text{volume}}$ 

Which unit would be the **most** appropriate for calculating the density of the rock in Michelle's hand?



Points Possible: 1

Content Cluster: Reason quantitatively and use units to solve problems.

Content Standard: Define appropriate quantities for the purpose of descriptive modeling. (N.Q.2)

Depth of Knowledge: Level 2

e. Compare and/or contrast figures or statements

Select the most appropriate unit for each situation.

	<u>feet</u> minute	square feet minute	cubic feet minute
Rate of walking to school			
Rate of painting a bedroom wall			
Rate of filling a bucket with water			
Rate of mopping the kitchen floor			

Points Possible: 1

**Content Cluster:** Reason quantitatively and use units to solve problems.

Content Standard: Define appropriate quantities for the purpose of descriptive modeling. (N.Q.2)

Trent plants a sunflower that is 6 inches tall. The sunflower is expected to grow at an average rate of 1.5 inches per day during the next month.

- A. Create an equation that Trent can use to find the number of days, x, it will take the sunflower to grow to a height of 45 inches.
- B. How many days will it take the sunflower to grow to a height of 45 inches?

4. B.								d	ays										
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			sin	cos	tan	arcsin	arco	os a	rctan		1.4								

#### Points Possible: 2

Content Cluster: Create equations that describe numbers or relationships

**Content Standard:** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. *(A.CED.1)* 

A factory has two assembly lines, M and N, that make the same toy. On Monday, only assembly line M was functioning, and it made 900 toys.

On Tuesday, both assembly lines were functioning for the same amount of time. Line M made 300 toys per hour and line N made 480 toys per hour. Line N made as many toys on Tuesday as line M did over both days.

Write an equation that can be used to find the number of hours, t, that the assembly lines were functioning on Tuesday.

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			sin	cos	tan	arcsin	arccos	arctan			

#### Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

**Content Standard:** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (A.CED.1)* 

Stephanie adds pennies, nickels and quarters to a scale until the mass of the combined coins is 75 grams. Each penny has a mass of 2.5 grams, each nickel has a mass of 5 grams and each quarter has a mass of 5.7 grams.

Create an equation to model this situation, where x is the number of pennies, y is the number of nickels and z is the number of quarters that Stephanie can put on the scale so that the mass of the combined coins is exactly 75 grams.

$\bullet \bullet \bullet \bullet \\ \blacksquare$
123 x y z
4 5 6 + - • ÷
789 < ≤ = ≥ >
$0  -  \exists  \Box^{0}  \Box_{0}  ()      \sqrt{\Box}  \forall \overline{\Box}  \pi  i$
sin cos tan arcsin arccos arctan

#### Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships

**Content Standard:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (A.CED.2)

Tim is sorting his book collection into groups. He places each group onto bookshelves that can each hold a maximum of 25 pounds. His collection includes hardcover books that weigh 3 pounds each and softcover books that weigh 2 pounds each.

Select all of the possible numbers of hardcover books that could be on one bookshelf.

3	
4	
8	
9	
12	

Points Possible: 1

**Content Cluster:** Create equations that describe numbers or relationships.

**Content Standard:** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (A1, M1) (A.CED.3)

#### Depth of Knowledge: Level 2

d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts

I. Select a procedure according to criteria and perform it

The equation shown is used to find the force of gravity, F, between two objects, where

- · G is the gravitational constant,
- m1 and m2 are the masses of the two objects, and
- r is the distance between the two objects.

$$F = \frac{Gm_1m_2}{r^2}$$

Which equation correctly shows the distance between the two objects?

(A) 
$$r = \frac{\sqrt{F}}{Gm_1m_2}$$
  
(B)  $r = \frac{\sqrt{Gm_1m_2}}{F}$   
(C)  $r = \sqrt{\frac{F}{Gm_1m_2}}$   
(D)  $r = \sqrt{\frac{Gm_1m_2}{F}}$ 

#### Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

**Content Standard:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (*A.CED.4*)

An equation is given.  $A = 4 \pi r^{2}$ Solve the equation for *r*. (A)  $r = \sqrt{\frac{4 \pi}{A}}$ (B)  $r = \sqrt{\frac{A}{4 \pi}}$ (C)  $r = \frac{4 \pi A}{2}$ (D)  $r = \frac{A}{2 \pi}$ 

#### Points Possible: 1

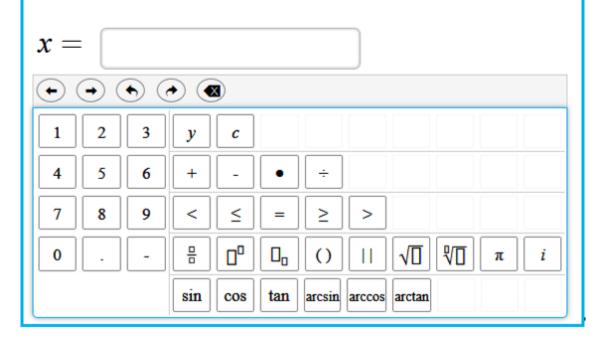
Content Cluster: Create equations that describe numbers or relationships.

**Content Standard:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. (A.CED.4)

An equation is given.

y = 3x + c

Create an equivalent equation by solving for x in terms of y and c.



Points Possible: 1

Content Cluster: Create equations that describe numbers or relationships.

**Content Standard:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law V=IR to highlight resistance R, or rearrange the formula for the area of a circle  $A=(pi)r^2$  to highlight radius r. (A1) (A.CED.4)

#### Depth of Knowledge: Level 2

I. Select a procedure according to criteria and perform it

d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts

### **Scoring Guidelines**

Exemplar Response

•  $X = \frac{y-c}{3}$ 

Other Correct Responses Any equivalent equation For this item, a full-credit response includes

• a correct equation (1 point).

This item has three parts.

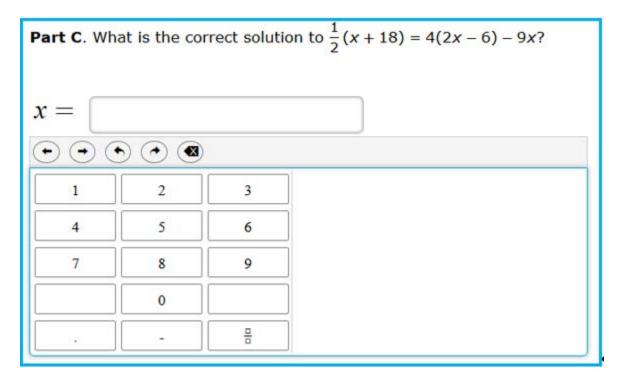
Eleanor incorrectly solves the equation  $\frac{1}{2}(x+18) = 4(2x-6) - 9x$ .

Part A. Select the first equation in which Eleanor makes an error.

Step	Equation
Given	$\frac{1}{2}(x+18)=4(2x-6)-9x$
1.	x + 18 = 8(2x - 6) - 9x
2.	x + 18 = 16x - 48 - 9x
3.	x + 18 = 7x - 48
4.	66 = 6x
5.	<i>x</i> = 11

Part B. Create an equation to correct Eleanor's error identified in Part A.

)(	$   \mathbf{\bullet} \mathbf{\bullet} $	$\odot$									
1	2	3	x								
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	2	>				
0		-	010		0,	0	11	$\sqrt{\Box}$	₩	π	i



#### Points Possible: 2

**Content Cluster:** Understand solving equations as a process of reasoning and explain the reasoning.

**Content Standard:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (A.REI.1)

What is t	ne solutio	on to the	equation $12(x + 5) = 4x$ ?
<i>x</i> = [			
$\bullet \bullet$	$\textcircled{\bullet} \textcircled{\bullet}$		
1	2	3	
4	5	6	
7	8	9	
	0		
	-		

Points Possible: 1

**Content Cluster:** Solve equations and inequalities in one variable.

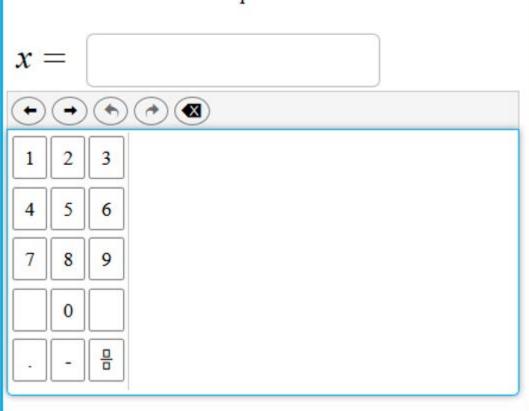
**Content Standard:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A.REI.3)

**Depth of Knowledge:** Level 1 m. Solve linear equations

An equation is shown.

$$3x + \frac{4}{5} = 7 - 2x$$

What is the solution to the equation?



Points Possible: 1

Content Cluster: Solve equations and inequalities in one variable

**Content Standard:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (A.REI.3)

n equation is shown.	
$6x^2 + 10x - 27 = -6x + 5$	
What are the solutions to this equation?	
x =	
x =	
$\bullet \bullet \bullet \bullet \bullet \bullet$	]
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4 5 6 < ≤ = ≥ >	
7 8 9 Ξ □□ □□ ()    √□ √□ π i	
0 sin cos tan arcsin arccos arctan	

#### Points Possible: 1

**Content Cluster:** Solve equations and inequalities in one variable.

**Content Standard:** Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers *a* and *b*. (*A.REI.4b*)

A system of equations is given.  $y = x^2 - 9$  y = -2x - 1What is one solution to the system of equations?

Points Possible: 1

Content Cluster: Solve systems of equations.

**Content Standard:** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle  $x^2 + y^2 = 3$ . (A.REI.7)

Lena sells custom-printed T-shirts at her shop. She can sell 120 T-shirts in a month if she charges \$14 for each T-shirt. She plans to change the price per T-shirt by a fixed amount *n* times. The revenue, in dollars, after *n* changes in price can be modeled by the expression (120 + 5n)(14 - 3n).

Based on the information, complete the sentence about Lena's T-shirt sales.

For each	•	in the price per T-shirt by \$	•	, Lena
will sell		Γ-shirts in a month.		

#### Drop down choices:

For each	•	in the price per T-shirt by \$	🔹 , Lena
	increase		3
	decrease		5
		_	9
			14
			17
will sell	-	T-shirts in a month.	
	3 more		
	5 more		
	14 more		
	3 fewer		
	5 fewer		
	9 fewer		

#### Points Possible: 1

Content Cluster: Interpret the structure of expressions.

**Content Standard:** Interpret expressions that represent a quantity in terms of its context. *(A.SSE.1) b.* Interpret complicated expressions by viewing one or more of their parts as a single entity.

#### Depth of Knowledge: Level 2

j. Translate between tables, graphs, words and symbolic notation

k. Make direct translations between problem situations and symbolic notation

### **Scoring Guidelines**

Exemplar Response

• For each **decrease** in the price per T-shirt by **\$3**, Lena will sell **5 more** T-shirts in a month.

Other Correct Responses

• N/A

For this item, a full-credit response includes

• the correctly completed statement (1 point).

Samantha sells two types of wristbands, rope or beaded. She charges more for beaded wristbands than for rope wristbands. The amount of money, in dollars, that she collects from selling x wristbands of one type and y wristbands of the other type can be modeled by the expression 5x + 8y.

What does the variable y represent in this situation?

- A the number of rope wristbands sold
- (8) the number of beaded wristbands sold
- © the selling price of one rope wristband
- It the selling price of one beaded wristband

#### Points Possible: 1

Content Cluster: Interpret the structure of expressions.

**Content Standard:** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. *(A.SSE.1a)* 

Henry places x marbles into an empty bucket. Each marble has the same weight.

The weight, in ounces, of the bucket and marbles can be calculated using the expression shown.

3x + 8

What does the term 8 represent in this expression?

- A the weight of each marble
- (B) the weight of the empty bucket
- © the number of marbles in the bucket
- (D) the total weight of the bucket and marbles

#### Points Possible: 1

Content Cluster: Interpret the structure of expressions

Content Standard: Interpret expressions that represent a quantity in terms of its context.\*

(A.SSE.1)

a. Interpret parts of an expression, such as terms, factors, and coefficients.

\*An asterisk appears next to any standard or group of standards linked with Modeling as a conceptual category.

An expression is given.

(2x + 8)(5x - 7)

Which expression is equivalent to the given expression?

- A 36x 56
- B 10x<sup>2</sup> 56
- $\bigcirc 10x^2 + 26x 56$
- (b)  $10x^2 + 54x + 56$

#### Points Possible: 1

Content Cluster: Perform arithmetic operations on polynomials.

**Content Standard:** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) (*A.APR.1*)

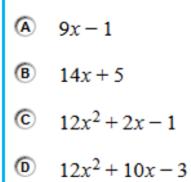
#### Depth of Knowledge: Level 1

b. Apply/compute a well-known algorithm (e.g., sum, quotient)

An expression is shown.

(2x-3) + [4x(3x+2)]

Which expression is equivalent to the given expression?



#### Points Possible: 1

Content Cluster: Perform arithmetic operations on polynomials.

**Content Standard:** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. *(A.APR.1)* 

# Algebra 1 Reporting Category:

## **Functions**

**CRITICAL AREA OF FOCUS #2** Linear and Exponential Relationships

CRITICAL AREA OF FOCUS #5 Quadratics Functions and Modeling

A total of 330 children and adults attended a school play. There were 21 times as many children in attendance as there were adults.

This situation is modeled by the given system of equations, where a represents the number of adults and c represents the number of children.

c = 21aa + c = 330

How many children attended the play?

$\bullet \bullet \bullet$	•		
1	2	3	
4	5	6	
7	8	9	
	0		
	-	0	

Points Possible: 1

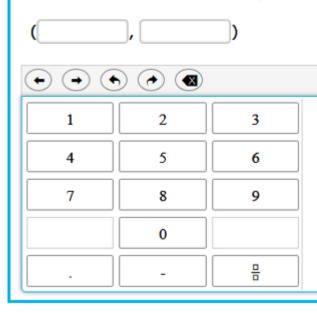
Content Cluster: Solve systems of equations.

**Content Standard:** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A.REI.6)

A system of equations is given.

$$\begin{cases} y+2 = 3 (x-1) \\ y = -2x + 10 \end{cases}$$

What is the solution to the system?



#### Points Possible: 1

Content Cluster: Solve systems of equations.

**Content Standard:** Solve systems of linear equations algebraically and graphically. a. Limit to pairs of linear equations in two variables. (A1, M1) (A.REI.6)

#### Depth of Knowledge: Level 2

d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts

I. Select a procedure according to criteria and perform it

What two numbers have a sum of 217 and a difference of 85?			
$\bullet \bullet \bullet \bullet \bullet \bullet$			
1	2	3	
4	5	6	
7	8	9	
	0		
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<u> </u>			·

Points Possible: 1

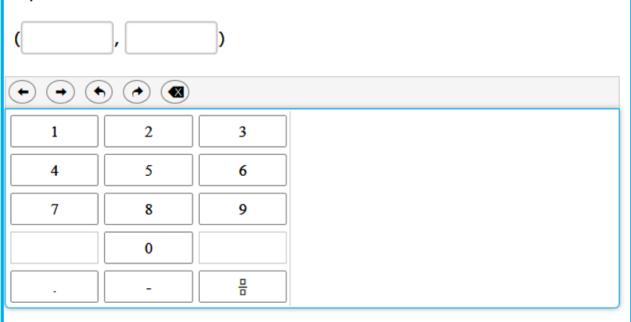
Content Cluster: Solve systems of equations

**Content Standard:** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (A.REI.6)

An equation is given.

5y - 2x = 5

Create an ordered pair that represents one point on the graph of the equation.



#### Points Possible: 1

**Content Cluster:** Represent and solve equations and inequalities graphically.

Content Standard: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)

#### Depth of Knowledge: Level 1

b. Apply/compute a well-known algorithm (e.g., sum, quotient)

The points (0, 1) and (1, 4) are contained in the graph of an equation with only two variables, x and y.
Select all of the true statements.
There is exactly one equation in the form y = mx + b that contains these points.
There are two equations in the form y = mx + b that contain these points.
There are no equations in the form y = a • b<sup>x</sup> that contain these points.
There is exactly one equation in the form y = a • b<sup>x</sup> that contains these points.
There is exactly one equation in the form y = a • b<sup>x</sup> that contains these points.
There is exactly one equation in the form y = a • b<sup>x</sup> that contains these points.

#### Points Possible: 1

**Content Cluster:** Represent and solve equations and inequalities graphically

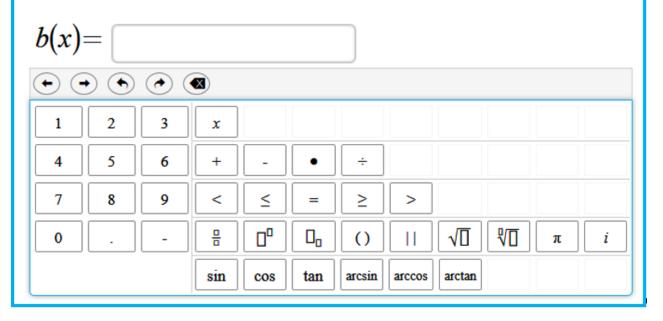
**Content Standard:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (A.REI.10)

A linear function is given.

a(x) = 26 - 12.4x

The function *b* is also linear. The equation a(x) = b(x) has exactly one solution at x = 5.

Create a possible equation for function b.



#### Points Possible: 1

Content Cluster: Represent and solve equations and inequalities graphically.

**Content Standard:** Explain why the *x*-coordinates of the points where the graphs of the equation y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations. (*A.REI.11*)

#### Depth of Knowledge: Level 3

- c. Make and/or justify conjectures
- e. Use concepts to solve non-routine problems
- i. Formulate a mathematical model for a complex situation
- m. Translate between a problem situation and symbolic notation that is not a direct translation

### **Scoring Guidelines**

Exemplar Response

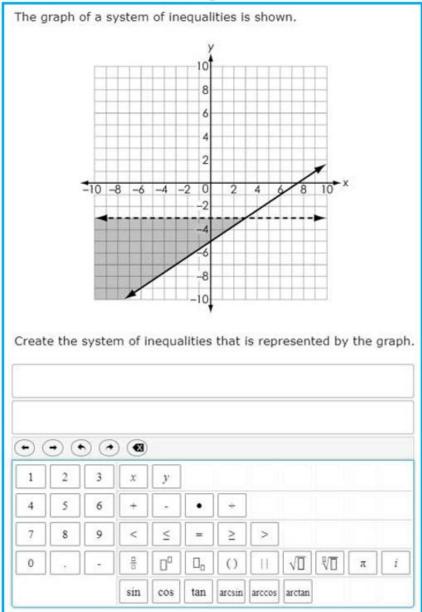
•  $b(x) = (-36/_5)x$ 

#### Other Correct Responses

- any linear equation that:
  - is not equivalent to a(x)
  - has an x-coefficient other than -12.4
  - $\circ$  has (5, -36) as a solution

For this item, a full-credit response includes

• a correct function (1 point).



## Points Possible: 2

Content Cluster: Represent and solve equations and inequalities graphically.

**Content Standard:** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (*A.REI.12*)

## **Scoring Guidelines**

Exemplar Response

- y < -3
- $y \ge \frac{2}{3}x 5$

Other Correct Responses

- Any equivalent system of inequalities For this item, a full-credit response includes:
  - A correct inequality (1 point) AND
  - Another correct inequality (1 point).

For this item, a partial-credit response includes:

• One correct inequality (1 point)

The first four terms of an arithmetic sequence are given.

27, 32, 37, 42, ...

## What is the 60th term of the sequence?



#### Points Possible: 1

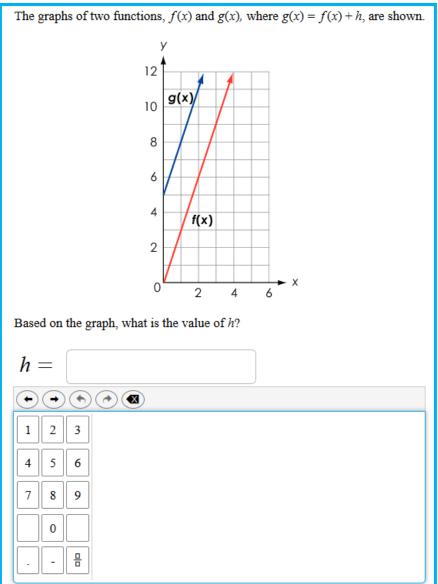
**Content Cluster:** Build a function that models a relationship between two quantities.

**Content Standard:** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (*F.BF.2*)

#### Depth of Knowledge: Level 2

c. Use models to represent mathematical concepts

d. Solve a routine problem requiring multiple steps/decision points, or the application of multiple concepts



## Points Possible: 1

Content Cluster: Build new functions from existing functions

**Content Standard:** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.

Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(F.BF.3)* 

Ryan works for a delivery service. The function f(n) is used to calculate his daily pay, in dollars, on a day when he makes n deliveries.

Number of Deliveries	Daily Pay (dollars)
0	
5	
	145

f(n) = 7n + 96

Use the function to complete the table shown.

## Points Possible: 1

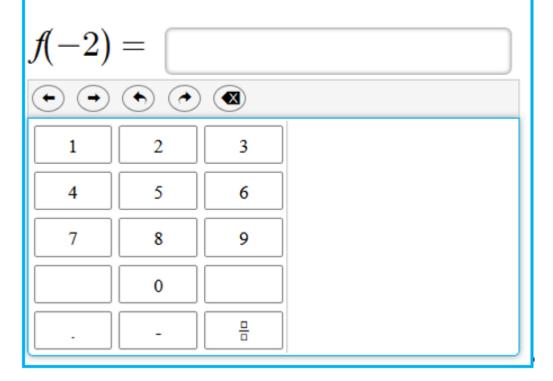
Content Cluster: Understand the concept of a function and use function notation

**Content Standard:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (*F.IF.2*)

A function is given.

$$f(x) = 2^x + 3$$

What is the value of f(-2)?



Points Possible: 1

**Content Cluster:** Understand the concept of a function and use function notation.

**Content Standard:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (*F.IF.2*)

Question 12							
The values of several terms in a sequence are shown in the table.							
			1				
	Term	Value					
	Second	5					
	Fourth	12					
	Seventh	22.5					
Find the first term, $f(1)$ . $f(1) =$ $(\bullet \circ \bullet) \circ \otimes$							
1 2	2 3						
4 5	6						
7 8	3 9						
0.	-						

Points Possible: 1

**Content Cluster:** Understand the concept of a function and use function notation.

**Content Standard:** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1) for  $n \ge 1$ . (*F.IF.3*)

## Depth of Knowledge: Level 3

d. Use evidence to develop logical arguments for a concept

g. Generalize a pattern

A sequence is shown.

3, 6, 12, 24, 48, ...

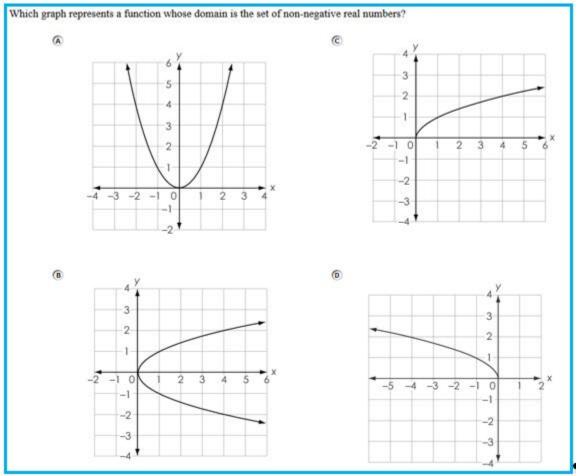
Which function, f(n), represents the *n*th term of the sequence, where f(1) = 3?

(a) 
$$f(n) = 2 \cdot 3^{n-1}$$
  
(b)  $f(n) = 3 \cdot 2^{n-1}$   
(c)  $f(n) = 3 \cdot 2^{n}$   
(d)  $f(n) = 6^{n}$ 

## Points Possible: 1

**Content Cluster:** Understand the concept of a function and use function notation.

**Content Standard:** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for  $n \ge 1$ . (F.IF.3)



Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context

**Content Standard:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of personhours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.\* (*F.IF.5*)

\*An asterisk appears next to any standard or group of standards linked with Modeling as a conceptual category.

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, x.

f(x) = (x - 22)(53 - x)

A. What is the minimum price, in dollars, to avoid a loss?

<sup>B.</sup> What is the maximum price, in dollars, to avoid a loss?

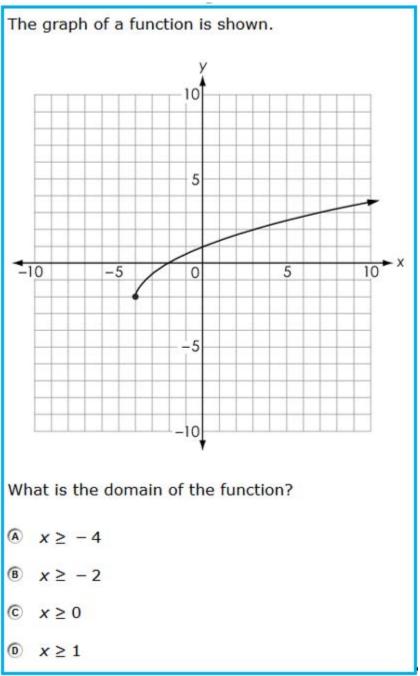
C. What is the price, in dollars, that results in the greatest profit?

A. \$			
<i>B</i> . \$			j
<i>C</i> . \$			
$\bullet \bullet \bullet$	ی ک		
1	2	3	
4	5	6	
7	8	9	
	0		
	-	<u><u></u></u>	

## Points Possible: 2

Content Cluster: Interpret functions that arise in applications in terms of the context.

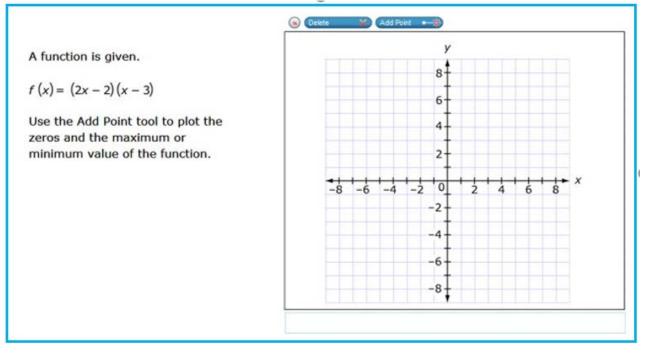
**Content Standard:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)* 



#### Points Possible: 1

Content Cluster: Interpret functions that arise in applications in terms of the context.

**Content Standard:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of personhours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (F.IF.5)



#### Points Possible: 1

**Content Cluster:** Analyze functions using different representations.

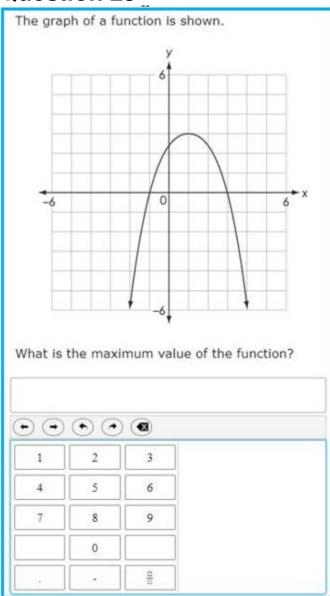
**Content Standard:** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. (*F.IF.7*)

b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2)

#### Depth of Knowledge: Level 2

j. Translate between tables, graphs, words and symbolic notation

I. Select a procedure according to criteria and perform it



## Points Possible: 1

Content Cluster: Analyze functions using different representations.

**Content Standard:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. *(F.IF.7a)* 

Which situation describes a quantity that increases by a constant percent rate?

- A The size of one photo is 15% larger than the size of another photo.
- The number of plants in a pond is 85% of the number from the previous year.
- © The population of one city is 85% greater than the population of another city.
- The number of magazine subscribers each year is 15% greater than the previous year.

## Points Possible: 1

**Content Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems.

**Content Standard:** Distinguish between situations that can be modeled with linear functions and with exponential functions. (*F.LE.1*)

#### Depth of Knowledge: Level 2

- c. Use models to represent mathematical concepts
- e. Compare and/or contrast figures or statements
- j. Translate between tables, graphs, words and symbolic notation

Some values for a function are shown in the table.

x	<i>f</i> ( <i>x</i> )
0	0
2	25
3	50

Which statement best describes the function?

(A) It is linear because f(x) increases by a constant amount compared to x.

(B) It is linear because f(x) increases by a constant percentage compared to x.

 $\bigcirc$  It is not linear because f(x) does not increase by a constant amount compared to x.

It is not linear because f(x) does not increase by a constant percentage compared to x.

#### Points Possible: 1

**Content Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems

**Content Standard:** Distinguish between situations that can be modeled with linear functions and with exponential functions. *(F.LE.1)* 

*b.* Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Emerson has \$120. Each week, he saves an additional \$15.

Write a function f(x) that models the total amount of money Emerson has after x weeks.

1	2	3	x								
4	5	6	+	-	•	÷					
7	8	9	<	≤	=	2	>				
0		121	8			0	11	$\sqrt{\Box}$	٧Ū	π	i
			sin	cos	tan	arcsin	arccos	arctan			

#### Points Possible: 1

**Content Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems.

**Content Standard:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). (*F.LE.2*)

An incomplete table of values for an exponential function is shown. The exponential function is of the form  $y = a \cdot b^x$ , where a is a real number such that  $a \neq 0$  and b is a positive real number not equal to 1.

Complete the table with possible values for the exponential function.

Y         96	x	0	1	2	3
	У	96			

## Points Possible: 1

**Content Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems.

**Content Standard:** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (*F.LE.2*)

## Depth of Knowledge: Level 3

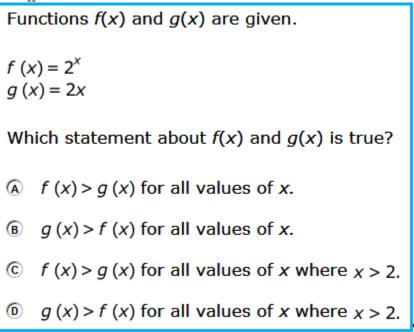
- e. Use concepts to solve non-routine problems
- f. Perform procedure with multiple steps and multiple decision points
- m. Translate between a problem situation and symbolic notation that is not a direct translation

The manager of a company uses the function shown to model its daily profit based on the price of a product in dollars, <i>x</i> .							
f(x) = (x - 22)(53 - x)							
A. What is the	A. What is the minimum price, in dollars, to avoid a loss?						
<sup>B.</sup> What is the	maximum pric	e, in dollars, t	o avoid a loss?				
C. What is the	price, in dollar	s, that results	in the greatest profit?				
A. \$	4. \$						
<i>B</i> . \$			)				
<i>C</i> . \$			]				
$\bigcirc \bigcirc \bigcirc \bigcirc$	• •						
1	2	3					
4	5	6					
7	8	9					
	0						
	-	<u>n</u>					

#### Points Possible: 2

Content Cluster: Interpret functions that arise in applications in terms of the context.

**Content Standard:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F.IF.4)* 



## Points Possible: 1

**Content Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems.

**Content Standard:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (A1, M2) (*F.LE.3*)

## Depth of Knowledge: Level 2

- c. Use models to represent mathematical concepts
- e. Compare and/or contrast figures or statements
- j. Translate between tables, graphs, words and symbolic notation

A fitness club charges members an initial fee and a separate monthly membership fee. The equation of the function given models the total fee, f(x), in dollars, that a person pays for x months of membership.

f(x) = 30x + 25

What does the number 30 represent in this situation?

- A the initial membership fee
- B the monthly membership fee
- © the number of months that a person is a member
- the total amount that a member pays in monthly fees

Points Possible: 1

Content Cluster: Interpret expressions for functions in terms of the situation they model.

**Content Standard:** Interpret the parameters in a linear or exponential function in terms of a context. *(F.LE.5)* 

## Depth of Knowledge: Level 2

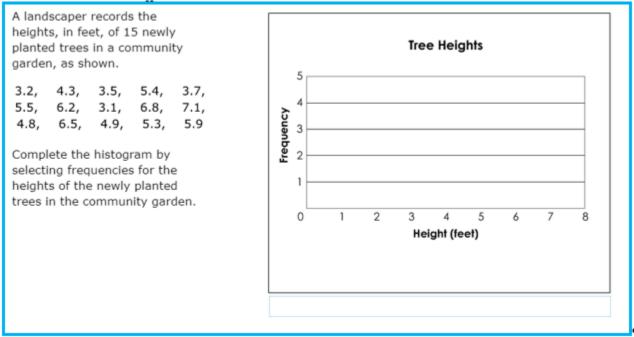
c. Use models to represent mathematical concepts

j. Translate between tables, graphs, words and symbolic notation

# Algebra 1 Reporting Category:

## **Statistics**

CRITICAL AREA OF FOCUS #3 Descriptive Statistics



#### Points Possible: 1

**Content Cluster:** Summarize, represent, and interpret data on a single count or measurement variable.

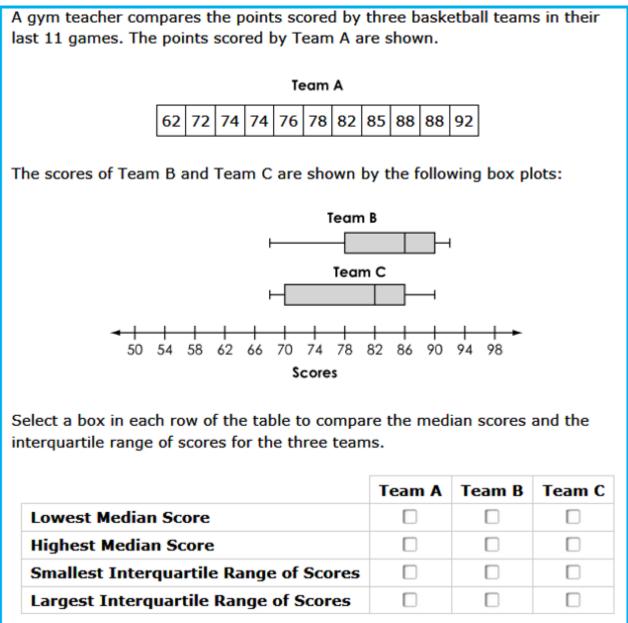
**Content Standard:** Represent data with plots on the real number line (dot plots, histograms, and box plots). (*S.ID.1*)

Which statistical measure changes when every number in a data set is increased by 10?
(A) range
(B) mean
(C) standard deviation
(D) interquartile range

## Points Possible: 1

**Content Cluster:** Summarize, represent, and interpret data on a single count or measurement variable

**Content Standard:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *(S.ID.2)* 



## Points Possible: 1

**Content Cluster:** Summarize, represent, and interpret data on a single count or measurement variable.

**Content Standard:** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. (*S.ID.2*)

## Depth of Knowledge: Level 2

- I. Select a procedure according to criteria and perform it
- n. Compare, classify, organize, estimate or order data

## **Scoring Guidelines**

Exemplar Response

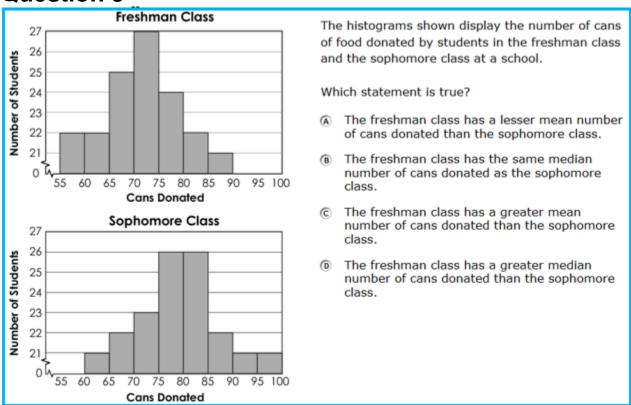
	Team A	Team B	Team C
Lowest Median Score			
Highest Median Score		~	
Smallest Interquartile Range of Scores		~	
Largest Interquartile Range of Scores			~

## Other Correct Responses

• N/A

For this item, a full-credit response includes

• the correctly matched table (1 point).



#### Points Possible: 1

**Content Cluster:** Content Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

**Content Standard:** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *(S.ID.3)* 

Alton studies the growth patterns of sassafras trees and yellow birch trees in the 88 counties in Ohio. He finds that

- approximately 27% of the counties have both species of trees,
- yellow birch only grows in 24 counties, and
- 1 out of 11 counties grows neither species.

Complete the table to show the relationship between the number of counties where sassafras trees and yellow birch trees grow.

	Sassafras Grow	Sassafras Do Not Grow	Total
Yellow Birch Grow			
Yellow Birch Do Not Grow			
Total			88

#### Points Possible: 2

**Content Cluster**: Summarize, represent, and interpret data on two categorical and quantitative variables.

**Content Standard:** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. *(S.ID.5)* 

#### Depth of Knowledge: Level 3

m. Translate between a problem situation and symbolic notation that is not a direct translation

- d. Use evidence to develop logical arguments for a concept
- e. Use concepts to solve non-routine problems

The population of a town has grown by an average of 2,000 people per year over the last 10 years.

Which equation could represent an appropriate linear model of the population?

- (a) y = 25,000x + 2,000
- (B) y = 2,000x + 25,000
- $\bigcirc y = -25,000x + 2,000$
- **(b)** y = -2,000x + 25,000

#### Points Possible: 1

Content Cluster: Interpret linear models.

**Content Standard:** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (*S.ID.7*)

#### Depth of Knowledge: Level 2

- c. Use models to represent mathematical concepts
- j. Translate between tables, graphs, words and symbolic notation

Select all of the correlation coefficients that represent a linear model with a weak correlation.

-0.982
-0.618
-0.103
0.204
0.907
1

## Points Possible: 1

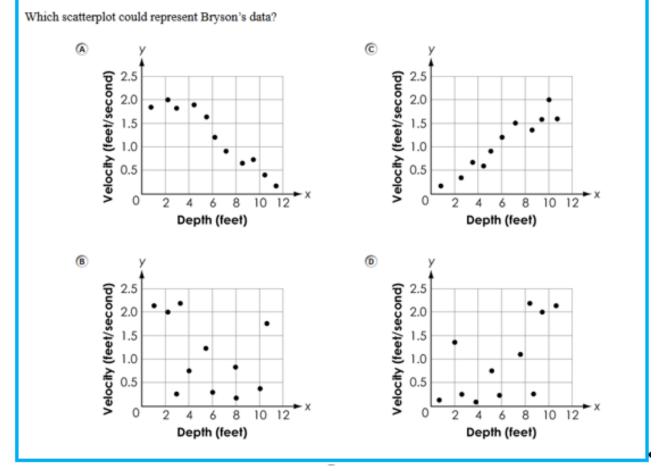
Content Cluster: Interpret linear models.

**Content Standard:** Compute (using technology) and interpret the correlation coefficient of a linear fit. (*S.ID.8*)

## Depth of Knowledge: Level 1

a. Recall, observe, or recognize a fact, definition, term, or property

Bryson collects data on the depth of a river at various points and the velocity of the river at those points. His data have a correlation coefficient of -0.9382.



Points Possible: 1

Content Cluster: Interpret linear models

**Content Standard:** Compute (using technology) and interpret the correlation coefficient of a linear fit. (S.ID.8)