

Ohio's Model Curriculum |Mathematics with Instructional Supports

## Grade 3

## Mathematics Model Curriculum <br> with Instructional Supports <br> Grade 3

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## Introduction

## PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K-16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

## COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.
Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- Essential Understandings are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- Mathematical Thinking statements describe the mental processes and practices important to the cluster.
- Instructional Focus statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

## COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the Instructional Strategies and Instructional Tools/Resources sections which are designed to be fluid and improving over time, through additional research and input from the field. The Instructional Strategies are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The Instruction Tools/Resources are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision.

## Standards for Mathematical Practice - Grade 3

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

## MP. 1 Make sense of problems and persevere in solving them.

In Grade 3, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students may use concrete objects, pictures, or drawings to help them conceptualize and solve problems, such as "Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?" or "Describe another situation where there would be 5 groups of 3 or $5 \times 3$." Students may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.

## MP. 2 Reason abstractly and quantitatively.

Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. For example, students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given $4 \times \square=40$, students might think the following:

- 4 groups of some number is the same as 40 ;
- 4 times some number is the same as 40 ;
- I know that 4 groups of 10 is 40 , so the unknown number is 10 ; or
- The missing factor is 10 because 4 times 10 equals 40.

Teachers might ask, "How do you know?" or "What is the relationship between the quantities?" to reinforce students' reasoning and understanding.

## MP. 3 Construct viable arguments and critique the reasoning of others.

Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?" Students explain their thinking to others and respond to others' thinking. For example, after investigating patterns on the 100s chart, students might explain why the pattern makes sense. Continued on next page

## Standards for Mathematical Practice, continued

## MP. 4 Model with mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense. For example, students use various contexts and a variety of models (e.g., circles, squares, rectangles, fraction bars, and number lines) to represent and develop understanding of fractions. Students use models to represent both equations and story problems and can explain their thinking. They evaluate their results in the context of the situation and reflect on whether the results make sense. Students should be encouraged to answer questions, such as "What math drawing or diagram could you make and label to represent the problem?" or "What are some ways to represent the quantities?"

## MP. 5 Use appropriate tools strategically.

Third graders consider the available tools (including drawings and estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students should be encouraged to answer questions such as, "Why was it helpful to use $\qquad$ ?"

## MP. 6 Attend to precision.

As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.

## MP. 7 Look for and make use of structure.

Students look closely to discover a pattern or structure. For instance, students use properties of operations (e.g., commutative and distributive properties) as strategies to multiply and divide. Teachers might ask, "What do you notice when $\qquad$ ?" or "How do you know if something is a pattern?"

## MP. 8 Look for and express regularity in repeated reasoning.

Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they do not know. For example, if students are asked to find the product of $7 \times 8$, they might decompose 7 into 5 and 2 and then multiply $5 \times 8$ and $2 \times 8$ to arrive at $40+16$ or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense? Students should be encouraged to answer questions, such as, "What is happening in this situation?" or "What predictions or generalizations can this pattern support?"

# Mathematics Model Curriculum <br> with Instructional Supports Grade 3 

## STANDARDS

## OPERATIONS AND ALGEBRAIC

 THINKINGRepresent and solve problems involving multiplication and division.
3.OA. 1 Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that $a \times b$ means a groups of $b$ objects each; however, because of the commutative property, students may also interpret $5 \times 7$ as the total number of objects in 7 groups of 5 objects each).
3.OA. 2 Interpret whole number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.
Continued on next page

## MODEL CURRICULUM

## Expectations for Learning

In Grade 2, students worked with beginning multiplication concepts in the form of repeated addition, arrays, and skip counting. In Grade 3, students explore the meaning of multiplication, and develop the understanding of equal groups using multiplication and division within 100. They move from using arrays to the use of area models. Also, students are introduced to common multiplication and division situations involving equal groups, arrays, and areas. (See Table 2, page 96). Then, they explore the meaning of division types and how multiplication relates to division in missing factor situations. Likewise, students develop and interpret mathematical situations using the symbolic representations of multiplication and division. Equations include unknowns in all positions. This learning sets the foundation for multiplicative comparisons and fraction arithmetic in Grade 4.

## ESSENTIAL UNDERSTANDINGS

- Multiplication is repeated addition.
- The product is the result of multiplication.
- Factors are the numbers being multiplied together.
- For whole numbers $A$ and $B$, the product $A \times B$ is represented by $A$ groups with $B$ objects in each group.
- Although visually 5 groups of 7 objects looks different than 7 groups of 5 objects, the products are equivalent because multiplication is commutative.
- There are two major division situations: fair sharing (group size unknown) and repeated subtraction (number of groups unknown). See Table 2, page 96.
- The dividend divided by the divisor is the quotient.
- There is a relationship between multiplication and division.
- Real-world mathematical situations can be represented using drawings and equations.
- An unknown can be in any position of a mathematical situation.
3.0A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Table 2, page 96. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
3.OA. 4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times \square=48 ; 5=\square \div 3 ; 6 \times 6=\square$.


## Expectations for Learning, continued

## MATHEMATICAL THINKING

- Interpret word problems to determine the operation(s) to be used.
- Represent and solve real-world mathematical situations accurately.
- Justify mathematical models used.
- Reflect on whether the results are reasonable.
- Use grade-level appropriate mathematical language to explain reasoning.


## INSTRUCTIONAL FOCUS

- Build on repeated addition models and rectangular arrays used in Grade 2 to illustrate multiplication.
- Explore the commutative aspect of multiplication by building arrays and then area models.
- Interpret the "x" symbol as meaning equal groups of objects (multiplication).
- Interpret the " $\div$ " symbol as meaning partitioning the total into equal groups or an equal number in each group (division).
- Explain the relationship between multiplication and division.
- Model and solve multiplication and division in real-world situations.
- Interpret a word problem using models to illustrate.
- Describe a context when given an expression or equation.
- Compute using strategies and models.
- Create a word problem when given an expression or equation.
- Determine an unknown number in any position that makes an equation true.


## Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 1, pages 17-18
- Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10


## CONNECTIONS ACROSS STANDARDS

- Multiply side lengths to find areas of rectangles (3.MD.7).
- Represent the understanding of equal groups to create scaled graphs (3.MD.3).
- Understand properties of multiplication and the relationship between multiplication and division (3.OA.5-6).

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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
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Instructional Tools/Resources
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## STANDARDS <br> OPERATIONS AND ALGEBRAIC THINKING

Understand properties of multiplication and the relationship between multiplication and division.
3.OA. 5 Apply properties of operations as strategies to multiply and divide. For example, if $6 \times 4=24$ is known, then $4 \times 6=24$ is also known (Commutative Property of Multiplication); $3 \times 5 \times 2$ can be found by $3 \times 5=15$, then $15 \times 2=30$, or by $5 \times 2=10$, then $3 \times 10=30$ (Associative Property of Multiplication); knowing that $8 \times 5=$ 40 and $8 \times 2=16$, one can find $8 \times 7$ as $8 \times(5+2)=(8 \times 5)+(8 \times 2)=40$ $+16=56$ (Distributive Property). Students need not use formal terms for these properties.
3.OA. 6 Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

## MODEL CURRICULUM

## Expectations for Learning

In previous grades students explored properties of addition and rectangular arrays to build a foundation for multiplication. Students used composing and decomposing of numbers for addition and subtraction. In Grade 3, students connect their understanding of the relationship between addition and multiplication to develop a conceptual understanding of additive and multiplicative properties, such as ordering and rearranging numbers to find the product. They explore decomposing factors to develop efficient strategies for solving multiplication and division situations. To be noted students do not need to know the names of the properties as they use formal terminology in later grades. Also, they use the relationship between multiplication and division to solve unknown-factor division problems. In Grade 4, students will apply these understandings as they work with multi-digit numbers.

## ESSENTIAL UNDERSTANDINGS

- The order of numbers in multiplication does not change the product.
- Numbers can be regrouped in a multiplication problem without changing the product.
- In multiplication, one factor can be decomposed into parts; each part is multiplied separately by the other factor, then the results are added.
- There is a relationship between multiplication and division.


## MATHEMATICAL THINKING

- Use grade-level appropriate mathematical language to explain and justify reasoning
- Determine reasonableness of results.
- Use informal reasoning.
- Recognize and make use of structure.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
This section is under revision.

Instructional Tools/Resources
This section is under revision.

## STANDARDS <br> OPERATIONS AND ALGEBRAIC THINKING

Multiply and divide within 100.
3.OA. 7 Fluently ${ }^{G}$ multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that $8 \times 5=40$, one knows $40 \div 5=8$, or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two onedigit numbers.

## MODEL CURRICULUM

## Expectations for Learning

In Grade 2, students developed fluency within 100 using addition and subtraction. In Grade 3, students develop fluency with products of two one-digit numbers and multiplication and division within 100 (without remainders) using strategies and properties of operations. In 3.NBT.2, students fluently add and subtract within 1,000. In Grade 4, students will develop fluency with multi-digit multiplication and division using multi-digit dividends. (Fluency is the ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests).

## ESSENTIAL UNDERSTANDINGS

- Fluency means being efficient, accurate, and flexible with strategies.
- There is an inverse relationship between multiplication and division.


## MATHEMATICAL THINKING

- Compute using strategies within 100.
- Compute using mental strategies with products of two one-digit numbers.
- Recognize some strategies may be more efficient than others.
- Explore and generalize concepts based on patterns and structures.


## INSTRUCTIONAL FOCUS

- Explore number relationships and look for patterns.
- Apply the conceptual understanding of properties to multiplication and division.
- Use strategies to multiply and divide within 100 (without remainders).
- Use efficient mental strategies to compute accurately and flexibly with products of two one-digit numbers.
- Explain why some strategies may be more efficient than others.
- Encourage the use of mental strategies which may include the following: decomposing factors; using the relationship between multiplication and division; creating equivalent but easier or known products, doubles, and properties of operations, etc.
Continued on next page



## Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 1, pages 17-18
- Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10


## CONNECTIONS ACROSS STANDARDS

- Use all operations and algebraic thinking standards for Grade 3 (3.OA.1-6, 8-9).
- Relate area to the operations of multiplication and addition (3.MD.7)

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Instructional Strategies
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Instructional Tools/Resources
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## STANDARDS <br> OPERATIONS AND ALGEBRAIC THINKING

Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.OA. 8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.
3.OA. 9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

## MODEL CURRICULUM

## Expectations for Learning

In Grade 2, students used addition and subtraction, within 100, to solve multi-step problems. In addition, they assessed the reasonableness of solutions and used estimation when measuring lengths. In Grade 3, students continue to assess the reasonableness of solutions through the use of estimation and rounding strategies when solving two-step word problems using all four operations with whole numbers. Also, they are introduced to the use of a letter to represent an unknown and may use parentheses for clarification within expressions (algebraic order of operations is not expected). In addition, students identify and explain patterns using properties of operations. In Grade 4, students will solve multi-step word problems and represent unknowns as letters, and then they will generate a number or shape pattern from a given rule.

## ESSENTIAL UNDERSTANDINGS

- A letter or symbol can represent an unknown quantity.
- Estimation strategies, including rounding, can be used to determine the reasonableness of answers.
- In solving a two-step word problem, one or more of the four operations may be needed.
- Arithmetic patterns can be identified and explained using properties of operations.


## MATHEMATICAL THINKING

- Interpret word problems to determine the operation(s) to be used.
- Represent and solve real-world mathematical situations accurately.
- Justify mathematical models used.
- Reflect on whether the results are reasonable.
- Use grade-level appropriate mathematical language and notation to explain reasoning.
- Explore and generalize concepts based on patterns and structures.

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|  | Expectations for Learning, continued <br> INSTRUCTIONAL FOCUS <br> - Solve two-step, real-world problems accurately using the four operations. <br> - Assess the reasonableness of answers. <br> - Explore and practice mental computation and estimation strategies including rounding. <br> - Scaffold representation of unknowns from symbols to letters. <br> - Use and explain different strategies or properties of operations. <br> - Find the numerical relationships within a pattern. <br> - Connect properties of operations to arithmetic patterns. <br> Content Elaborations <br> - Ohio's K-8 Critical Areas of Focus, Grade 3, Number 1, pages 17-18 <br> - Ohio's K-8 Critical Areas of Focus, Grade 3, Number 5, page 22 <br> - Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10 <br> CONNECTIONS ACROSS STANDARDS <br> - Use multiplication and division within 100 in word problems involving equal groups, arrays, and measurement quantities (3.OA.3). |
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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
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Instructional Tools/Resources
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## STANDARDS <br> NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.
3.NBT. 1 Use place value understanding to round whole numbers to the nearest 10 or 100 .
3.NBT. 2 Fluently add and subtract within 1,000 using strategies and algorithms ${ }^{G}$ based on place value, properties of operations, and/or the relationship between addition and subtraction.
3.NBT. 3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., $9 \times 80,5 \times 60$ using strategies based on place value and properties of operations.

## MODEL CURRICULUM

## Expectations for Learning

In Grade 2, students composed and decomposed tens and hundreds using concrete models or drawings. In Grade 3, students use efficient strategies to fluently add and subtract within 1,000. In 3.OA.7, students fluently multiply and divide within 100 with all products of two one-digit numbers. (Fluency is the ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests). Students are introduced to rounding whole numbers to the nearest 10 or 100. In addition, they multiply one-digit whole numbers by multiples of 10 in the range of 10-90. In Grade 4, students will use a standard algorithm to add and subtract. Also, they will multiply up to four-digit by one-digit numbers and two-digit by two-digit numbers. In future grades, rounding strategies and place value concepts will be expanded to include larger numbers and decimal notation.

## ESSENTIAL UNDERSTANDINGS

- Rounding helps solve problems mentally and assess the reasonableness of an answer.
- Fluency is being efficient, accurate, and flexible with addition and subtraction strategies.
- Generalizing patterns and structures can be used when multiplying by a multiple of ten.


## MATHEMATICAL THINKING

- Pay attention to and make sense of quantities.
- Use grade-level appropriate mathematical language, models, and notation to explain and justify reasoning.
- Determine reasonableness of results.
- Explore and generalize concepts based on patterns and structures.

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## Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 1, pages 17-18
- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 5, page 22
- Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5


## CONNECTIONS ACROSS STANDARDS

- Solve two-step word problems and assess the reasonableness of answers using mental computation and estimation strategies (3.OA.8).
- Identify arithmetic patterns, and explain them using properties of operations (3.OA.9).
- Solve word problems with adding and subtracting money and measurement within 1,000 (3.MD.1-2).
- Represent and interpret data (3.MD.3).

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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
This section is under revision.
Instructional Tools/Resources
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## STANDARDS <br> NUMBER AND OPERATIONS- <br> FRACTIONS

Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators $2,3,4,6$, and 8 .
3.NF. 1 Understand a fraction ${ }^{1 / b}$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a_{b}$ as the quantity formed by a parts of size $1 / b$.
3.NF. 2 Understand a fraction as a number on the number line; represent fractions on a number line diagram ${ }^{G}$.
a. Represent a fraction $1_{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1 / b$ and that the endpoint of the part based at 0 locates the number ${ }^{1 / b}$ on the number line.
b. Represent a fraction ${ }^{2} / b$ (which may be greater than 1) on a number line diagram by marking off a lengths $1 / b$ from 0 . Recognize that the resulting interval has size ${ }^{a} / b$ and that its endpoint locates the number ${ }^{a} / b$ on the number line.
Continued on next page

## MODEL CURRICULUM

## Expectations for Learning

In grades 1 and 2, students partitioned shapes into two, three, and four equal shares. They described those parts using fraction language of halves, thirds, and fourths. In Grade 3, the learning shifts to developing a more formal understanding of equal parts including the usage of mathematical symbols and number line diagrams. The instructional focus is on exploring the meaning and relationships in fractions; the significance of the whole; the unit fraction; and the initial understanding of equivalence of fractions using models and comparison of fractions (with like numerators or like denominators). In future grades, students will expand their understanding of fractions to formally solving problems with unlike denominators.

## ESSENTIAL UNDERSTANDINGS

## Understanding Fractions

- A fraction is a number showing a relationship between the parts and the whole.
- Fractional parts have names that tell how many parts of a size are needed to make the whole ( 3 parts - thirds; 4 parts - fourths, etc.).
- Fractional parts can be described with words and symbols.
- Fractions can be represented with visual models such as rectangular area models, arrays, and length models including number lines.
- On a number line, the size of the part is measured by the distance from zero to the numbered point.
- A unit fraction represents one piece of the equal-sized pieces that make a whole $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}\right)$.
- A unit fraction is the building block for fractions just as 1 is the building block for whole numbers.
- A whole can be divided into any number of parts and recombined to make a whole.
- When a fraction is written symbolically, there is a top number and a bottom number.
o The bottom number (the denominator) describes how many equal parts the whole is divided into.
o The top number (the numerator) describes how many of that size part there are.
3.NF. 3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., $1 / 2=2 / 4,4 / 6=2 / 3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model ${ }^{G}$.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = $3 / 1$; recognize that $6 / 1=6$; locate $4 / 4$ and 1 at the same point of a number line diagram.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, $=$, or <, and justify the conclusions, e.g., by using a visual fraction model.


## Expectations for Learning, continued

## ESSENTIAL UNDERSTANDINGS, CONTINUED

## Understanding Fractions, continued

- A whole number can be expressed as a fraction, e.g., $1=\frac{1}{1}, 2=\frac{2}{1}, 3=\frac{3}{1}$, or $\frac{4}{4}=1, \frac{2}{2}=1$, etc.
- A fraction can be greater than one whole, e.g., $\frac{3}{2}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=1+\frac{1}{2}$ or $\frac{8}{2}=4 \mathrm{etc}$.
- A quantity greater than one (mixed number) can be represented, e.g., $2 \frac{1}{3} ; \frac{14}{3}=4+\frac{2}{3}$ or $4 \frac{2}{3}$.
- Two parts are equal if they represent the same size part of the same whole.
- Equivalent fractions use different sized fractional parts to describe the same amount.
- Equivalent fractions are represented by the same point on the number line.
- The more fractional parts used to make a whole, the smaller the size of the part.


## Comparing Fractions

- Two fractions can be compared when the two fractions refer to the same whole.
- When comparing fractions with the same denominator, the fraction with the greater numerator is greater because more unit fractions are needed to make up the part.
- When comparing unit fractions with different denominators, the fraction with the larger denominator is smaller because it takes more equal sized pieces to make the whole.
- When comparing fractions, if the numerators are the same, then compare the denominators.
- When comparing fractions, if the denominators are the same, then compare the numerators.
- The symbols >, =, and < are used with fractions just as they are used with whole numbers.
Continued on next page





## Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 2, page 19
- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 3, page 20
- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 4, page 21
- Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7
- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14


## CONNECTIONS ACROSS STANDARDS

- Identify and use multiplicative relationships between numbers (3.OA.2, 3, and 9).
- Measure with a ruler marked in halves and fourths of an inch (3.MD.4).
- Connect the use of area models for multiplication to fractions (3.MD.7).

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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
This section is under revision.
Instructional Tools/Resources
This section is under revision.
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## STANDARDS

## MEASUREMENT AND DATA

Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.
3.MD. 1 Work with time and money.
a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock.
b. Solve word problems by adding and subtracting within 1,000 , dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $\$$ and $\mathbb{C}$ symbol appropriately (not including decimal notation).
3.MD. 2 Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve onestep word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Table 2, page 96.

## MODEL CURRICULUM

## Expectations for Learning

This cluster builds on the previous conceptual understanding and skills developed with respect to money, time, and measurement. In Grade 2, students found the value of a collection of quarters, nickels, dimes, and pennies and added and subtracted dollars with dollars and cents with cents (not using dollars and cents simultaneously) within 100. Additionally, students learned to tell time to the nearest 5 minutes, using a.m. and p.m. as appropriate. Also, they measured and estimated lengths using inches, feet, centimeters, and meters. In Grade 3, students add and subtract money (dollars with dollars and cents with cents, not using dollars and cents simultaneously) to solve word problems within 1,000 . Furthermore, they extend telling time to the nearest minute and measure elapsed time within 90 minutes. They are introduced to measuring masses and liquid volumes with grams, kilograms, and liters. Then in Grade 4, students will add and subtract intervals of time in hours and minutes. They will add and subtract money using decimal notation. Students will solve length, mass, and liquid volume problems using metric units.

## ESSENTIAL UNDERSTANDINGS

## Time

- Time is measured in hours and minutes.
- Time can be measured to the nearest minute.
- Elapsed time measures the duration of an event.


## Money

- Money is added and subtracted using whole number strategies.
- A dollar symbol, \$, is used to represent dollars.
- A cent symbol, $\mathbb{C}$, is used to represent cents.
- The dollar symbol and cent symbol are not used simultaneously, i.e., do not use decimal notation. Note: Decimal notation, e.g., $\$ 1.33$, will be used in 4th grade to represent values beyond 100 cents.


## Mass and Liquid Volume

- Mass is measured in kilograms or grams.
- Liquid volume is measured in liters.
- Mass and liquid volume word problems are solved using whole number strategies.



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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
This section is under revision.
Instructional Tools/Resources
This section is under revision.
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## STANDARDS

## MEASUREMENT AND DATA

Represent and interpret data.
3.MD. 3 Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.
3.MD. 4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot $^{G}$, where the horizontal scale is marked off in appropriate unitswhole numbers, halves, or quarters.

## MODEL CURRICULUM

## Expectations for Learning

In Grade 2, students created picture graphs and bar graphs (with up to four categories) when single unit scales were provided and solved one-step problems using graphs. Also, they generated measurement data and created line plots using a whole unit scale. In Grade 3, students create scaled picture graphs or bar graphs (with several categories), and solve two-step compare problems using information presented in scaled graphs. Further, students generate measurement data and create line plots using scales that include whole numbers, halves, or fourths/quarters. In Grade 4, students will display and interpret data in graphs and solve grade appropriate problems.

## ESSENTIAL UNDERSTANDINGS

- Data can be organized and represented in a picture graph, a bar graph, or a line plot.
- The key of a picture graph tells how many items each picture or symbol represents.
- A scaled graph (bar graph or line plot) is labeled using equal-sized intervals along the axes.
- The scale of a bar graph varies depending on the data set.
- Length measurement data can be generated and used to create a line plot.
- The scale of a line plot can be whole numbers, halves, or quarters.
- Symbols used in picture graphs and line plots should be consistently spaced and sized for visual accuracy.
- Information presented in a graph can be used to solve problems involving the data in the graph.
Continued on next page




## Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 2, page 19
- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 5, page 22
- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14

CONNECTIONS ACROSS STANDARDS

- Solve problems involving multiplication (3.0A.3).
- Solve two-step problems using the four operations (3.OA.8).
- Understand a fraction as a number on a number line (3.NF.2).

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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
This section is under revision.
Instructional Tools/Resources
This section is under revision.
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## STANDARDS

## MEASUREMENT AND DATA

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD. 5 Recognize area as an attribute of plane figures and understand concepts of area measurement.
a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units.
3.MD. 6 Measure areas by counting unit squares (square cm , square m , square in, square ft, and improvised units).
3.MD. 7 Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving realworld and mathematical problems, and represent whole number products as rectangular areas in mathematical reasoning.
Continued on next page

## MODEL CURRICULUM

## Expectations for Learning

This cluster introduces the formal study of area. In previous grades, students used linear measurement with no gaps or overlaps. In Grade 2, students used addition to find the total number of objects arranged in rectangular arrays (up to $5 \times 5$ ). Now in Grade 3, students measure area in square units. They first count unit squares in tilings (concrete) and later use multiplication and addition of whole numbers to find areas of figures composed of rectangles in real-world situations. Students move from using arrays to the use of area models, and they use visual models to represent the distributive property when finding area. Then in Grade 4, students will use rectangular arrays and/or area models to develop efficiency in multiplication and extend this knowledge to develop more efficient strategies and formulas to find the area and perimeter of rectangles.

## ESSENTIAL UNDERSTANDINGS

- Area is an attribute of plane figures that is measured using square units.
- Area is found by covering the inside of a two-dimensional plane figure with square units without gaps or overlaps, and then counting the number of square units used.
- The process of finding area shifts from tiling and counting, to the multiplication of side lengths.
- The area of a rectangle can be found by multiplying the lengths of two adjacent sides of the rectangle.
- The area of a rectangle can be found by being decomposed into two rectangular parts; finding the areas of the two smaller rectangles; and then adding the two smaller areas to find the total area.
- A figure composed of rectangles may be decomposed into rectangles whose areas may be added to find the area of the figure.
Continued on next page
c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths $a$ and $b+c$ is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model).
d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into nonoverlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.


## Expectations for Learning, continued

 MATHEMATICAL THINKING- Pay attention to and make sense of quantities.
- Compute accurately and efficiently with grade-level numbers using whole number strategies.
- Draw a picture or create a model to make sense of a problem.
- Solve real-world problems accurately and consider the reasonableness of the solution(s).
- Measure using appropriate tools and units; justify mathematical models used.
- Use grade-level appropriate mathematical language and notation to explain reasoning.


## INSTRUCTIONAL FOCUS

- Explore and develop the conceptual understanding of "a unit square" with area "one square unit."
- Use tiling (without gaps or overlaps) to find the area of a rectangle by counting unit squares.
- Use appropriate units (square cm , square m , square in, square ft, and improvised units).
- Discover by tiling that the area is the same as would be by multiplying the side lengths; use whole number side lengths.
- Represent the distributive property using visual models.
- Explore finding the area of a rectangle by decomposing into two rectangular parts; finding the areas of the two smaller rectangles; and then adding to find the total area ( $a$ and $b+c$ is the sum of $a \times b$ and $a \times c$ (distributive property)).
- Explore and explain decomposing a figure composed of rectangles into nonoverlapping rectangles in order to find the area of the figure by adding areas of rectangles.
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## Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 1, pages 17-19
- Ohio's K-8 Critical Areas of Focus, Grade 3, Number 3, page 20
- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14


## CONNECTIONS ACROSS STANDARDS

- Solve problems involving multiplication (3.0A.3).
- Solve two-step problems using the four operations (3.OA.8).
- Apply properties of operations (3.OA.5).
- Partition shapes into parts with equal areas (3.G.2).
- Determine the unknown whole number in a multiplication of division equation (3.OA.4).

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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
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Instructional Tools/Resources
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## STANDARDS

## MEASUREMENT AND DATA

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
3.MD. 8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

## MODEL CURRICULUM

## Expectations for Learning

This cluster introduces perimeter. In Grade 2, students measured and counted linear lengths. In Grade 3, students use the measurement of side lengths to solve for the perimeter of polygons, including problems with an unknown side length. They solve real-world problems involving perimeters. Students explore the relationships between area and perimeter of rectangles, i.e., the same perimeters but different areas or the same areas but different perimeters. In Grade 4, students will extend this knowledge to develop more efficient strategies and formulas for area and perimeter of rectangles.

## ESSENTIAL UNDERSTANDINGS

- Perimeter is an attribute of plane figures that is measured using linear units (inches, feet, centimeter, meter, etc.).
- Perimeter is found by adding all the outside (exterior) side lengths of a polygon.
- An unknown side length of a polygon can be computed when given the perimeter and other side lengths or properties of the polygon.
- Different rectangles may have the same perimeter but different areas.
- Different rectangles may have the same area but different perimeters.


## MATHEMATICAL THINKING

- Pay attention to and make sense of quantities.
- Compute accurately and efficiently with grade-level numbers using whole number strategies.
- Draw a picture or create a model to make sense of a problem.
- Solve real-world problems accurately and consider the reasonableness of the solution(s).
- Use grade-level appropriate mathematical language and notation to explain reasoning.
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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
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Instructional Tools/Resources
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## STANDARDS

## GEOMETRY

Reason with shapes and their attributes.
3.G.1 Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).
3.G. 2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $1 / 4$ of the area of the shape.

## MODEL CURRICULUM

## Expectations for Learning

This cluster builds upon K-2 experiences with shapes. In Grade 2, students identified two-dimensional shapes (triangles, quadrilaterals, pentagons, and hexagons) based on the number of sides or vertices. In Grade 3, students draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of right angles. Students partition shapes into equal areas and express the individual parts as unit fractions of the whole (with denominators of $2,3,4,6$, and 8 ). In Grade 4, students will recognize and classify two-dimensional figures by their properties (lines or angles).

Student understanding begins to progress from Visualization (Level 0) to Analysis (Level 1) of van Hiele's Model of Geometric Thinking.

## ESSENTIAL UNDERSTANDINGS

- Polygons are closed two-dimensional shapes with straight sides.
- Polygons can be described by the number of sides.
- Polygons can be described by the presence or absence of square corners/right angles.
- Some shapes can be partitioned into parts with equal areas.
- When shapes are partioned into equal areas, the area of each part is the unit fraction of the whole.


## MATHEMATICAL THINKING

- Use spatial reasoning.
- Create models and drawings to represent shapes.
- Recognize and use a pattern or structure.
- Make and test conjectures about polygons; then justify reasoning.
- Use grade-level appropriate mathematical language to explain reasoning.

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|  | Expectations for Learning, continued <br> INSTRUCTIONAL FOCUS <br> - Explore classifying triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles). <br> - Draw polygons with up to 8 sides. <br> - Partition shapes into parts with equal areas. <br> - Express each equal-area part of a shape as a unit fraction of the whole. <br> Content Elaborations <br> - Ohio's K-8 Critical Areas of Focus, Grade 3, Number 3, page 20 <br> - Ohio's K-8 Critical Areas of Focus, Grade 3, Number 4, page 21 <br> - Ohio's K-8 Learning Progressions, K-5 Geometry, page 11 <br> CONNECTIONS ACROSS STANDARDS <br> - Understand that a unit fraction is the quantity formed when a whole is partitioned into equal parts and one part is selected (3.NF.1). |
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    INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies
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