Ohio's Model Curriculum | Mathematics

with Instructional Supports

Grade 4

Ohio | Department of Education

Mathematics Model Curriculum
with Instructional Supports
Grade 4

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

Standards for Mathematical Practice—Grade 4

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as they engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.

In Grade 4, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

Students might use an equation strategy to solve a word problem. For example, students could solve the problem "Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?" with the equation $3 \times \$12 + \$15 = a.$

Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

MP.2 Reason abstractly and quantitatively.

Fourth graders should recognize that a number represents a specific quantity. They connect the quality to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. Students might use base 10 blocks or drawings to demonstrate 154×6, as 154 added six times, and develop an understanding of the distributive property. For example: $154 \times 6 = (100 + 50 + 4) \times 6 = (100 \times 6) + (50 \times 6) + (4 \times 6) = 600 + 300 + 24 = 924$

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Standards for Mathematical Practice, continued

MP.3 Construct viable arguments and critique the reasoning of others.
In Grade 4, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. Students explain and defend their answers and solution strategies as they answer question that require an explanation. For example, "Vincent cuts 2 meters of string into 4 centimeter pieces for a craft. How many pieces of string does Vincent have? Explain your reasoning." Students ask appropriate questions and they decide if explanations make sense.

MP.4 Model with mathematics.
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense. For example, students may use money (i.e. dollars and coins) or base-10 blocks to solve the following problem: Elsie buys a drink for $1.39 and a granola bar for $0.89. How much change will she receive if she pays with a $5 bill?

MP.5 Use appropriate tools strategically.
Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper, a number line, or base 10 blocks to represent, compare, add, and subtract decimals to the hundredths. Students in fourth grade use protractors to measure angles. They use other measurement tools to understand the relative size of units within a given system and express measurements given in larger units in terms of smaller units.

MP.6 Attend to precision.
As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. For instance, they may use graph paper or a number line to represent, compare, add, and subtract decimals to the hundredths. Students in fourth grade use protractors to measure angles. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
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Standards for Mathematical Practice, continued

MP.7 Look for and make use of structure.
In Grade 4, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as arrays and area models to the multiplication principal of counting. They generate number or shape patterns that follow a given rule using two-column tables.

MP.8 Look for and express regularity in repeated reasoning.
Students in Grade 4 should notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Mathematics Model Curriculum
with Instructional Supports
Grade 4

STANDARDS	MODEL CURRICULUM
OPERATIONS AND ALGEBRAIC THINKING **Use the four operations with whole numbers to solve problems.** **4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Table 2, page 96. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) *Continued on next page*	**Expectations for Learning** In Grade 3, students built foundational understandings of multiplication and division (within 100) through work with equal groups, arrays, and area models. They solved two-step word problems using all four operations (with no remainders for division). Students used the commutative property to interpret products of whole numbers. In Grade 4, students develop their understanding of multiplication as a comparison, continuing to use symbols and letters for the unknown numbers. They continue to assess the reasonableness of solutions using mental computation and estimation strategies including rounding when solving multi-step word problems using all four operations with whole numbers. Students interpret remainders when dividing symbolically and in context. Using multiplicative comparison problems, students interpret the comparison based on one group being a particular multiple of the other (multiple copies). In Grade 5, students will apply and extend previous understandings of multiplication and division to multiply and divide fractions and decimals. **ESSENTIAL UNDERSTANDINGS** • Comparisons can be additive or multiplicative depending on the mathematical situation. • In multiplicative comparisons, the relationship between quantities is described in terms of how many times larger one is than the other. • Remainders can be interpreted numerically and in context. • Estimation strategies, including rounding, can be used to determine the reasonableness of answers. • Real-world mathematical situations can be represented using drawings and equations. • An unknown can be in any position of a multiplicative comparison problem. *Continued on next page*

4.OA.3 Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use grade-level appropriate mathematical language and notation to explain reasoning.
- Interpret word problems to determine the operation(s) to be used.
- Represent and solve real-world mathematical situations accurately.
- Justify mathematical models used.
- Reflect on whether the results are reasonable.

INSTRUCTIONAL FOCUS

- Explore and practice mental computation and estimation strategies including rounding.
- Use estimation as an entry point to make sense of situations.
- Create models, drawings, and equations to solve multiplication and division word problems involving multiplicative comparisons.
- In a multiplicative comparison, the underlying question is, *"What factor would multiply one quantity in order to result in another?"*
- Interpret and explain the use of remainders with respect to context.
- Assess the reasonableness of answers.
- Solve real-world problems accurately up to three steps and use numbers of easy (smaller values) and medium difficulty that are grade-level appropriate using the four operations.
- Represent verbal statements symbolically.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 4, Number 1, pages 23-24
- Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10

CONNECTIONS ACROSS STANDARDS

- Know relative sizes of measurement units within one system of units (4.MD.1).
- Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money (4.MD.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
OPERATIONS AND ALGEBRAIC THINKING **Gain familiarity with factors and multiples.** **4.OA.4** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	**Expectations for Learning** In Grade 3, students worked with factors in multiplication problems within 100. In Grade 4, students find all factor pairs for any given whole number between 1 and 100. They identify whole numbers with only one factor pair as *prime* and whole numbers with more than one factor pair as *composite*. Students extend their understanding of decomposition to multiplication and learn to use the term *multiple.* Any whole number is a *multiple* of each of its factors. Also, they determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. In later grades, students will find greatest common factors and least common multiples when computing with multi-digit numbers. **ESSENTIAL UNDERSTANDINGS** • A number can be multiplicatively decomposed into factor pairs and expressed as a product of these factor pairs. • A prime number has only two factors: one and itself (only one factor pair). • A composite number has more than two factors (more than one factor pair). • Any whole number is a *multiple* of each of its factors. **MATHEMATICAL THINKING** • Recognize and make connections about the relationship between factors and multiples. • Make conjectures about patterns and structures. • Use grade-level appropriate mathematical language and notation to explain reasoning. • Justify mathematical models used. • Reflect on whether the results are reasonable. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Explore and discuss factor pairs for whole numbers in the range 1-100.
- Use models to develop the understanding that a prime number only has the number one and itself as factors (only one factor pair).
- Use models to develop the understanding that a composite number has more than two factors (more than one factor pair).
- Use models to explore if a given whole number is a multiple of another given a one-digit number.
- Use models to explain and justify if a given whole number in the range 1-100 is prime or composite.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 4, Number 1, pages 23-24
- Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10

CONNECTIONS ACROSS STANDARDS

- Use multiplication and division with whole numbers to solve problems and make multiplicative comparisons (4.OA.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
OPERATIONS AND ALGEBRAIC THINKING **Generate and analyze patterns.** **4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	**Expectations for Learning** In Grade 3, students learned to identify arithmetic patterns and explained them using the properties of operations. In Grade 4, students reason about number or shape patterns and generate terms that follow a given rule. They connect numerical sequences to addition and multiplication concepts by exploring rules of repeatedly adding the same whole number or of repeatedly multiplying by the same whole number. In later grades, understanding patterns will be fundamental to algebraic thinking. **ESSENTIAL UNDERSTANDINGS** • Patterns can be classified as growing or as repeating sequences. • Features can be identified from patterns generated from a given rule. **MATHEMATICAL THINKING** • Make and analyze mathematical conjectures related to patterns. • Justify mathematical models used. • Reflect on whether the results are reasonable. • Use grade-level appropriate mathematical language and notation to explain reasoning. • Explore and generalize concepts based on patterns and structures. **INSTRUCTIONAL FOCUS** • Generate a geometric or numeric pattern that follows a given rule. • Explore patterns that consist of repeated sequences of shapes. • Explore patterns that consist of growing sequences of designs. • Explore patterns that consist of repeatedly adding the same whole number or repeatedly multiplying by the same whole number. • Identify features of given or generated patterns. • Make and describe generalizations about patterns. • Connect rules and terms of patterns to numerical concepts. *Continued on next page*

	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 4, Number 2, pages 25-26 • Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10 **CONNECTIONS ACROSS STANDARDS** • Extend the understanding of fraction equivalence (4.NF.1- 2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
NUMBER AND OPERATIONS IN BASE TEN **Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.** **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division. **4.NBT.2** Read and write multi-digit whole numbers using standard form, word form, and expanded form[G]. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. **4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.	**Expectations for Learning** In previous grades, students used place value understanding to compare numbers using >, =, and < symbols. In Grade 3, students rounded numbers within 1,000 to the nearest 10 or 100. Also, they explored patterns and structures when multiplying a one-digit whole number by a multiple of 10. Now in Grade 4, students apply place value understandings to numbers within 1,000,000. They recognize that a digit in one place represents ten times what it represents in the place to its right. Also, students will round numbers to any given place value within 1,000,000. In future grades, students will further these understandings to include rational numbers. **ESSENTIAL UNDERSTANDINGS** • In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. • Each period of three digits separated by commas is read as hundreds, tens, and ones, followed (when appropriate) by the name of the period, e.g., 123,456 is read as one hundred twenty-three thousand, four hundred fifty-six. • Numbers can be expressed in standard form, word form, and expanded form. • Rounding helps solve problems mentally and assess the reasonableness of an answer. **MATHEMATICAL THINKING** • Pay attention to and make sense of quantities. • Use grade-level appropriate mathematical language, models, and notation to explain and justify reasoning. • Determine reasonableness of results. • Explore and generalize concepts based on patterns and structures. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Generalizing Place Value

- Relate multiplication and division to place value understanding.
- Explore multiplication or division with numbers to 1,000,000 using place value:
  - A digit in the tens place represents a number that is ten times more than the number resulting from the same digit in the ones place.
  - A digit in the hundreds place represents a number that is ten times more than the number resulting from the same digit in the tens place.
  - A digit in the thousands place represents a number that is ten times more than the number resulting from the same the digit in the hundreds place, etc.
- Compare the value of a numeral in a number to the same numeral in a different place in a different number, e.g., Given 342 and 432 compare the value of 3.
- Use patterns in the place value system to read and write numbers.
- Represent, read, and write whole numbers in various forms (standard, word, expanded) within 1,000,000.
- Generalize that each period of three digits separated by commas is read as hundreds, tens, and ones, followed by the name of the appropriate unit.

Comparing Numbers

- Compare numbers based on place-value understanding—
  - with the same number of digits;
  - with the same leading digits; and
  - with different leading digits and different number of digits.
- Connect the mathematical language to the use of symbols >, =, and < when describing the relationship between the numbers.
- Write two true inequality statements using symbols and words for a pair of numbers, e.g., 3,012 < 4,542 and 4,542 > 3,012.
- Create numbers given specific criteria, e.g., Create a number that has 3 in the thousands place, 5 in the millions place, etc.

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Expectations for Learning, continued

INSTRUCTIONAL FOCUS, CONTINUED

Rounding

- Explore rounding by using the location of a given number on a model, e.g., number line, number chart, etc.
- Round numbers based on place-value understanding.
- Explain and reason when rounding.
- Develop and generalize rounding rules for larger numbers.
- Identify or create numbers that will round to a chosen number, e.g., Create a number that will round to 10,500.
- Explore the purposes of rounding.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 4, Number 1, pages 23-24
- Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5

CONNECTIONS ACROSS STANDARDS

- Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3).
- Solve problems involving metric measurement and conversions from a larger unit to a smaller unit (4.MD.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
NUMBER AND OPERATIONS IN BASE TEN **Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.** **4.NBT.4** Fluently[G] add and subtract multi-digit whole numbers using a standard algorithm[G]. **4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.6** Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**Expectations for Learning** This cluster extends work in previous grades with the use of place value understanding and properties of operations to perform multi-digit arithmetic. In Grade 3, students fluently added and subtracted within 1,000; fluently multiplied and divided within 100; and developed mental strategies for all products of two one-digit numbers. Also, they multiplied one-digit whole numbers by multiples of 10. In Grade 4, students fluently add and subtract within 1,000,000 using a standard algorithm. They expand on multiplication of up to two four-digit numbers as they apply equations, area models, and arrays to illustrate and explain their strategies based on place value and properties of operations. The multiplication equations, area models, and arrays used in Grade 3 are now expanded to include the multiplication of up to four-digit numbers by one-digit numbers and the multiplication of two two-digit numbers. Students apply equations, area models, and arrays to illustrate and explain division of up to four-digit dividends and one-digit divisors using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. In future grades, students will use standard algorithms to fluently calculate with whole numbers and decimal numbers in real-world situations. *(Fluency is the ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests)*. **ESSENTIAL UNDERSTANDINGS** **Addition/Subtraction** • There are different algorithms that can be used to add or subtract. • Fluency is being efficient, accurate, and flexible with strategies. *Continued on next page*

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Multiplication/Division

- When multiplying and dividing, patterns and structures can be generalized.
- The *product* is the result of multiplication.
- *Factors* are the numbers being multiplied together.
- There is a relationship between multiplication and division.
- Equations, rectangular arrays, and/or area models can be used to illustrate and explain multiplication and division.
- There are two major division situations: fair sharing (group size unknown) and repeated subtraction (number of groups unknown). See Table 2, page 96.
- The *dividend* divided by the *divisor* is the *quotient.*
- A remainder can be stated, can be discarded, or can force the quotient to increase to the next whole number depending on the context.

MATHEMATICAL THINKING

- Justify mathematical models used.
- Reflect on whether the results are reasonable.
- Use grade-level appropriate mathematical language to illustrate and explain reasoning.
- Compute accurately, efficiently, and flexibly with grade-level numbers.
- Explore and generalize concepts based on patterns and structures.

Continued on next page

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Addition/Subtraction

- Estimate the solution of an addition or subtraction situation.
- Connect a standard algorithm to an efficient strategy.
- Explain and justify the reasoning used in a standard algorithm.
- Analyze other students' use of a standard algorithm and explain any errors.
- Determine reasonableness of a solution and compare to initial estimation.
- Use an efficient standard algorithm accurately and flexibly.

Multiplication/Division

- Explore number relationships and look for patterns.
- Multiply a whole number of up to four digits by a one-digit whole number.
- Multiply two two-digit numbers.
- Illustrate and explain the relationship between multiplication and division.
- Solve using strategies that may include the following*:* decomposing factors; using the relationship between multiplication and division; creating equivalent but easier or known products, doubles, and properties of operations, etc.
- Apply the conceptual understanding of properties to multiplication and division.
- Estimate the solution of a multiplication or division problem.
- Use visual representations such as area models and arrays to draw connections to equations.
- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
- Explore and explain how zeroes affect division: in the dividends; within the process of dividing; and in the quotient.
- Determine reasonableness of a solution, and compare it to initial estimation.
- Explore division problems that result in remainders.
- Determine whether the remainder is left alone, is discarded, or forces the quotient to increase.

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	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 4, Number 1, pages 23-24 • Ohio K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Multiply or divide to solve word problems involving multiplicative comparisons (4.OA.2). • Solve multi-step word problems with whole numbers and assess the reasonableness of answers using mental computation and estimation strategies (4.OA.3). • Develop strategies to determine the area and perimeter of rectangles in real-world situations (4.MD.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

NUMBER AND OPERATIONS—FRACTIONS

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.1 Explain why a fraction $^{a}/_{b}$ is equivalent to a fraction $^{(n \times a)}/_{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $^{1}/_{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Expectations for Learning

Previously students partitioned shapes into equal parts; described those parts with words and fraction symbols; and solved simple equal sharing problems involving the divisors (denominators) 2, 3, 4, 6, and 8. In Grade 3, students used length models (e.g., number lines and fraction strips) and visual models (e.g., area models, set models, and pattern block models) to explore how different fraction names can describe the same size part, (e.g., $\frac{1}{2}$ and $\frac{2}{4}$), beginning the understanding of equivalent fractions. They compared fractions with the same numerator or with the same denominator and recorded those comparisons using >, =, and < symbols. In Grade 4, students formalize the understanding of equivalence of fractions by using length models and visual models to develop the conceptual understanding that $\frac{a}{b}$ is equivalent to $\frac{n \times a}{n \times b}$. They use common denominators, numerators, or a comparison to a benchmark fraction to compare two fractions (with unlike numerators and denominators). This learning sets the foundation for adding and subtracting fractions (with like denominators in Grade 4 and unlike denominators in Grade 5).

ESSENTIAL UNDERSTANDINGS

- The denominator describes the number of equal parts the whole is divided into; the more equal fractional parts used to make a whole, the smaller the size of the parts.
- Equivalent fractions use different sized fractional parts to describe the same amount, e.g., $\frac{1}{2} = \frac{2}{4}$.
- Visual models, such as rectangular area models, arrays (e.g., egg cartons) and length models (including fraction strips and number lines), can be used to represent and compare fractions.
- To compare fractions using models, each fraction should be represented with the same visual model and the same sized whole.
- There is a multiplicative relationship between the number of equal parts in a whole and the size of the parts.
- Multiplying the numerator and the denominator by the same number will result in an equivalent fraction.

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Expectations for Learning, continued

MATHEMATICAL THINKING

- Use mathematical models to solve problems.
- Explore and generalize concepts based on patterns and structures.
- Use grade-level appropriate mathematical language and notation to illustrate and explain reasoning.
- Compute accurately, efficiently, and flexibly with grade-level numbers.
- Reflect on whether results are reasonable.

INSTRUCTIONAL FOCUS

- Recognize and generate equivalent fractions on number lines or using other length models (include wholes and values greater than a whole; use denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100).
- Given pairs of length models divided into different numbers of equal parts, explore how both of the models can illustrate equivalent quantities.
- Given pairs of length models divided into different numbers of equal parts with a fraction for each model, explore and explain which fraction is larger, smaller, or equivalent. Represent the comparison of the fractions using >, =, or <.
- Represent fractions with different numerators and denominators given pairs of visual models such as shaded area models (with the same model unit):
    - Determine if the shaded portions are equal.
    - Represent the comparison of the fractions using >, =, or <.
    - Justify answers.
- Explore the use of benchmark fractions of 0, $\frac{1}{2}$, and 1 to compare two fractions with different numerators and different denominators.
- Use models to explore what is required to maintain equivalent fractions when a denominator is multiplied by a number, e.g., doubling or tripling the number of parts.
- Use models to explore what is required to maintain equivalent fractions when a numerator is multiplied by a number.
- Generate equivalent fractions and explain why the fractions are equivalent.

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	Content Elaborations • Ohio's K-8 Critical Area of Focus, Grade 4, Number 2, pages 25-26 • Ohio's K-8 Learning Progressions, Number and Operations – Fractions, pages 6-7 **CONNECTIONS ACROSS STANDARD** • Gain familiarity with factors and multiples (4.OA.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
NUMBER AND OPERATIONS—FRACTIONS **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified).** **4.NF.3** Understand a fraction $^{a}/_{b}$ with $a > 1$ as a sum of fractions $^{1}/_{b}$. **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model[G]. *Examples:* $^{3}/_{8} = {}^{1}/_{8} + {}^{1}/_{8} + {}^{1}/_{8}$; $^{3}/_{8} = {}^{1}/_{8} + {}^{2}/_{8}$; $2\,{}^{1}/_{8} = 1 + 1 + {}^{1}/_{8} = {}^{8}/_{8} + {}^{8}/_{8} + {}^{1}/_{8}$. **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	**Expectations for Learning** In Grade 3, the instructional focus was on exploring the meaning and relationships in fractions; the significance of the whole; the unit fraction; and the initial understanding of equivalence of fractions using models. These explorations used denominators of 2, 3, 4, 6, and 8. In Grade 4, students explore fractions with like or unlike numerators and denominators, using the denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. They apply whole number multiplication understandings to fractions. They utilize the renaming of fractions to solve real-world word problems involving the addition or subtraction of fractions (with like denominators) or the multiplication of a fraction by a whole number. (Solutions are permitted to be expressed un-simplified.) In future grades, students will formalize rules and procedures for fraction operations. **ESSENTIAL UNDERSTANDINGS** • A unit fraction is the building block of fractions. • A unit fraction is any fraction where the numerator is 1, and the denominator is a whole number not equal to zero. • A whole can be divided into any number of equal sized parts and re-combined to make a whole again. • In a problem, the unit is the amount counted as the whole or one. • It is necessary to determine the unit in a problem to add or subtract correctly. • Unit fractions can be combined from multiple wholes if all the wholes are the same size. • Fractions can be added and subtracted when the wholes are the same size. • Fractions with the same denominators can be added and subtracted using visual models, properties of operations, and relationships of addition and subtraction of whole numbers. • Mixed numbers can be written as fractions, e.g., $\frac{14}{3} = 4\frac{2}{3}$, and can be added or subtracted in this form. • Equivalent fractions can be used to add and subtract fractions. • Multiplication is repeated addition, i.e., just as $4 \times 3 = 3 + 3 + 3 + 3$, $5 \times \frac{1}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ which equals $\frac{5}{8}$. *Continued on next page*

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction $^{a}/_{b}$ as a multiple of $^{1}/_{b}$. *For example, use a visual fraction model to represent $^{5}/_{4}$ as the product $5 \times (^{1}/_{4})$, recording the conclusion by the equation $^{5}/_{4} = 5 \times (^{1}/_{4})$ or $^{5}/_{4} = (^{1}/_{4}) + (^{1}/_{4}) + (^{1}/_{4}) + (^{1}/_{4}) + (^{1}/_{4})$.*

b. Understand a multiple of $^{a}/_{b}$ as a multiple of $^{1}/_{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (^{2}/_{5})$ as $6 \times (^{1}/_{5})$, recognizing this product as $^{6}/_{5}$. (In general, $n \times (^{a}/_{b}) = ^{(n \times a)}/_{b}$.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $^{3}/_{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use mathematical models to solve problems.
- Explore and generalize concepts based on patterns and structures.
- Use grade-level appropriate mathematical language and notation to illustrate and explain reasoning.
- Compute accurately, efficiently, and flexibly with grade-level numbers.
- Reflect on whether results are reasonable.

INSTRUCTIONAL FOCUS

- Express a fraction as a unit fraction and a factor.
- Use models to illustrate renaming a fraction as a mixed number and vice versa.
- Explore and explain decomposing a fraction into a sum of fractions with the same denominator in more than one way.
- Write an equation to show the decomposition of a fraction.
- Add and subtract fractions with like denominators using models.
- Add and subtract mixed numbers with like denominators using models.
- Explore equal sharing problems involving 2, 3, 4, 5, 6, 8, 10, and 12 sharers and having answers that are whole numbers, mixed numbers, or fractions less than 1, e.g., Ten children are sharing 14 liters of punch. If they each are to have the same amount, how much punch should each child get? Between what two whole numbers will your answer lie?
- Solve multiple groups involving fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{10}$ in each group including problems using mixed numbers, e.g., Five children are having lunch. Each child is to get $1^{3}/_{4}$ of a sandwich. How many sandwiches are needed? Between what two whole numbers does your answer lie?
- Represent real-world problems with visual models and with equations (involving addition or subtraction of fractions with like denominators or multiplication of a fraction by a whole number), and justify the solutions.
- Solve real-world problems that will result in various equivalent answers, having students explain why the solutions are equivalent.
- Solve problems where the unit or the amount counted as the whole represents various amounts, e.g., using pattern blocks, make the hexagon a whole, 1 trapezoid a whole, 2 trapezoids a whole, 2 pizzas the whole, etc.

Continued on next page

	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 4, Number 2, pages 25-26 • Ohio's K-8 Learning Progressions, Number and Operations – Fractions, pages 6-7 **CONNECTIONS ACROSS STANDARDS** • Interpret and represent multiplicative comparisons (4.OA.1). • Determine whether a whole number is a multiple on another whole number (4.OA.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
NUMBER AND OPERATIONS—FRACTIONS **Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.** **4.NF.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $^{3}/_{10}$ as $^{30}/_{100}$, and add $^{3}/_{10} + {}^{4}/_{100} = {}^{34}/_{100}$.* In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade. **4.NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as $^{62}/_{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **4.NF.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	**Expectations for Learning** This is the first time that students are working with decimal notation. In Grade 4, students relate their understanding of place value, the meaning of fractions, and equivalent fractions to rename tenths as hundredths. Using the fraction notation for tenths and hundredths, they solve addition and subtraction problems and compare fractions In addition, students change fractions with denominators of 10 and 100 to decimals and vice versa. The previous cluster 4.NF.3-4 information applies to denominators of 10 and 100, so it is also useful in this cluster. Students compare two decimals to hundredths, record the results using >, =, or <, and justify their conclusions. Also addition and subtraction of decimals to hundredths is informally addressed in 4.MD.2 when money is modeled and applied using a dollar sign and decimal point when dealing with dollars and cents. Formal place value understanding of decimals will occur in grade 5. **ESSENTIAL UNDERSTANDINGS** • Using equivalent fractions, any fraction with a denominator of ten can be renamed as a fraction with a denominator of 100. • The place value system of whole numbers can be expanded to represent numbers less than 1. • A fraction with a denominator of 10 or 100 can be written using decimal notation. • A number can be written as a fraction, e.g., $\frac{17}{100}$, or as a decimal, e.g., 0.17. • A decimal point or horizontal bar can be used to show where the unit is located, e.g., $\frac{35}{100} = 0.35$. • Fractions with denominators of 10 or 100 are compared with the same strategies as with other fractions. • Decimals can only be compared when the decimals being compared refer to the same whole. • Decimals written as tenths or hundredths can be compared using equivalent fractions. • Fractions with denominators of 10 or 100 can be compared using equivalent decimals. *Continued on next page*

Expectations of Learning, continued

MATHEMATICAL THINKING

- Use mathematical models to solve problems.
- Explore and generalize concepts based on patterns and structures.
- Use grade-level appropriate mathematical language and notation to illustrate and explain reasoning.
- Compute accurately, efficiently, and flexibly with grade-level numbers.
- Reflect on whether results are reasonable.

INSTRUCTIONAL FOCUS

- Model and explore fractions with denominators of 10 or 100.
- Explore and generate an equivalent fraction in tenths as a fraction in hundredths.
- Describe and express a fraction with a denominator of 10 or 100 as a decimal.
- Explore and express a decimal up to hundredths as a fraction.
- Compare, discuss, and justify two fractions with denominators of 10 or 100 using visual models.
- Recognize that decimals can only be compared when the decimals being compared refer to the same whole.
- Compare two decimals using visual models to hundredths; record the results using >, =, or <; and justify the conclusions.
- Add and subtract fractions with denominators of 10 and 100.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 4, Number 2, pages 25-26
- Ohio's K-8 Learning Progressions, Number and Operations – Fractions, pages 6-7

CONNECTIONS ACROSS STANDARDS

- Use units to solve measurement problems (4.MD.1-2).
- Generalize place value understanding (4.NBT.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
MEASUREMENT AND DATA **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** **4.MD.1** Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. *For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...* *Continued on next page*	**Expectations for Learning** Building on a Grade 2 understanding of meters and centimeters and a Grade 3 understanding of grams, kilograms, and liters, Grade 4 students use the metric units kilometer, millimeter, and milliliter. They express larger measurement units in terms of smaller units. Students use a two-column table to perform metric conversions using units within the same metric type, e.g., meters with centimeters, kilograms with grams, etc. In Grade 5, students will apply this understanding of the metric system through place value and decimals using both multiplication and division. They will also explore U.S. customary units. Grade 4 builds on the knowledge and skills of Grade 3 concepts of money, time, mass, and liquid volume using metric units. • In Grade 3, students added and subtracted money amounts within 1,000 without using decimal notation. Grade 4 students use decimal notation to add and subtract in order to solve real-world problems using this notation. Grade 5 students will apply this understanding by solving real-world problems using all four operations. • In Grade 3, students learned to tell time to the nearest minute and measured elapsed time within 90 minutes. Grade 4 students add and subtract intervals of time in hours and minutes using models (number line diagrams, clocks, etc.) in real-world situations. In Grade 5, students will apply this knowledge when solving real-world problems involving hours, minutes, and seconds. • In Grade 3, students solved one-step, real-world problems involving liquid volumes (liters), and masses of objects (kilograms and grams). Grade 4 students solve real-world addition, subtraction, and multiplication problems involving distances (kilometers, meters, centimeters, and millimeters), liquid volumes (liters and milliliters), and masses (kilograms and grams). In Grade 5, students will apply this understanding of the metric system using place value, division operation, and decimals. *Continued on next page*

4.MD.2 Solve real-world problems involving money, time, and metric measurement.
- **a.** Using models, add and subtract money and express the answer in decimal notation.
- **b.** Using number line diagrams[G], clocks, or other models, add and subtract intervals of time in hours and minutes.
- **c.** Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.

4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. *For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.*

Expectations for Learning, continued

Grade 3 introduced the formal study of area and perimeter, and Grade 4 extends these concepts.

- In Grade 3, students counted unit squares in tilings and related area to the operations of multiplication and addition in order to find areas of figures composed of rectangles. In Grade 4, students use rectangular arrays and area models to develop efficiency in multiplication thereby extending this knowledge to the development of more efficient strategies for area of rectangles. In Grade 5, students will be introduced to volume.
- Perimeter was first introduced in Grade 3. Those students used the measurement of side lengths to solve for the perimeter of polygons and explored the relationships between area and perimeter of rectangles, i.e., the same perimeters but different areas or the same areas but different perimeters. In Grade 4, students extend this knowledge to develop more efficient strategies for perimeter of rectangles. In Grade 5, students will be introduced to volume.

ESSENTIAL UNDERSTANDINGS

Metric Units

- Larger units can be expressed in terms of smaller units.
- The number of units used to measure an object will depend on the size of the unit of measure.
- The larger the unit, the smaller the measurement reads; the smaller the unit, the larger the measurement reads.
- Metric units are related by powers of ten.
  - 1 kilometer = 1,000 meters, 1 meter = 100 centimeters, 1 centimeter = 10 millimeters;
  - 1 kilogram = 1,000 grams; and
  - 1 liter = 1,000 milliliters.

Money

- Answers to money problems can include the dollar symbol, $, and decimal point placed appropriately in decimal notation.
- The dollar symbol and the cent symbol are not used simultaneously.

Continued on next page

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Time

- Answers to time problems should include a.m. and p.m. as appropriate.
- Noon is 12:00 p.m.
- Midnight is 12:00 a.m.

Area and Perimeter

- The area of a rectangle can be found by multiplying the lengths of adjacent sides (length and width) of the rectangle.
- Given an area or a perimeter of a rectangle and one side length, the adjacent side length can be determined.

MATHEMATICAL THINKING

- Use spatial reasoning.
- Solve real-world problems accurately, and consider the reasonableness of the solution(s).
- Draw a picture or create a model to make sense of a problem.
- Recognize a pattern or structure when using different types of measurement.
- Measure using appropriate tools and units; justify mathematical models used.
- Pay attention to and make sense of quantities.
- Make and test conjectures about conversions; then justify reasoning.
- Explore and generalize concepts based on patterns and structures.
- Create and use models and drawings to represent measurements.
- Reflect on whether the results are reasonable.
- Use grade-level appropriate mathematical language and notation to explain reasoning.

Continued on next page

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Metric Units

- Explore the metric system using appropriate tools (metric rulers, meter sticks, balance scales, etc.)
- Explore the units *millimeter* and *milliliter.*
- Know relative sizes of the metric measurement units within one system of units:
  - length – kilometer, meter, centimeter, and millimeter;
  - mass – kilogram and gram; and
  - liquid volume – liter and milliliter.
- Explore and explain larger measurement units in terms of smaller units, e.g., 5 kilograms = 5,000 grams.
- Explore, record, and look for patterns when doing conversions in a two-column table.
- Add, subtract, and multiply whole numbers to solve metric measurement situations involving distances, liquid volumes, and masses of objects.

Money

- Use models to illustrate addition and subtraction of money.
- Explore and explain the use of a decimal to separate dollars and cents.
- Express the answer in decimal notation (Conventionally, the $ is placed before the numerical amount.) when solving money word problems.
- The dollar symbol and the cent symbol are not used simultaneously.

Time

- Explore using models (number lines, clocks, etc.) to add and subtract intervals of time in hours and minutes.

Area and Perimeter

- Explore and explain efficient strategies used to determine the perimeter and area of rectangles in real-world and mathematical situations.
- Explore finding an unknown side length in a mathematical situation involving area or perimeter.

	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 4, Number 1, pages 23-24 • Ohio's K-8 Critical Areas of Focus, Grade 4, Number 2, pages 25-26 • Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14 **CONNECTIONS ACROSS STANDARDS** • Generalize place value understanding for multi-digit whole numbers (4.NBT.1 - 2). • Use place value operations and properties of operations to perform multi-digit arithmetic (4.NBT.5). • Use the four operations with whole numbers to solve problems (4.OA.2 – 3). • Build fractions from unit fractions (4.NF.3-4). • Understand decimal notation for fractions, and compare decimal fractions (4.NF.5 – 7).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
MEASUREMENT AND DATA **Represent and interpret data.** **4.MD.4** Display and interpret data in graphs (picture graphs, bar graphs, and line plots[G]) to solve problems using numbers and operations for this grade.	**Expectations for Learning** In Grade 3, students created scaled picture and bar graphs with several categories; created line plots (using wholes, halves, and fourths); and solved two-step problems using graphs. Grade 4 students display and interpret data in graphs and solve grade-level appropriate problems. In Grade 5, students will display and interpret data in graphs (including metric and U.S. customary units) and solve grade-level appropriate problems. **ESSENTIAL UNDERSTANDINGS** • Data can be organized and represented in a picture graph, a bar graph, or a line plot. • The key of a picture graph tells how many items each picture or symbol represents. • The scale of a line plot can be whole numbers, halves, quarters, tenths, or hundredths. • The scale of a bar graph varies depending on the data set. • Symbols used in picture graphs and line plots should be consistently spaced and sized for visual accuracy. • Information presented in a graph can be used to solve problems involving the data in the graph. **MATHEMATICAL THINKING** • Interpret word problems to determine the operation(s) to be used. • Use a graph to organize, represent, and solve real-world mathematical situations accurately. • Measure using appropriate tools and units; justify mathematical models used. • Pay attention to and make sense of quantities. • Reflect on whether the results are reasonable. • Use grade-level appropriate mathematical language and notation to explain reasoning. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Picture Graphs

- Display and interpret data using real-world problems.
  - Use units of halves and quarters for data sets in situations where these fractions are appropriate.

Bar Graphs

- Display and interpret data using real-world problems (including money and metric measures).
  - Use whole number units for a large variety of data sets.
  - Use units of halves and quarters for data sets in situations where these fractions are appropriate.

Line Plots

- Display and interpret data using real-world problems (including money and metric measures).
  - Use whole number units for a large variety of data sets.
  - Use units of halves, quarters, tenths, or hundredths for data sets in situations where these fractions are appropriate.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 4, Number 2, pages 25-26
- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14

CONNECTIONS ACROSS STANDARDS

- Solve grade-level appropriate problems using the four operations (4.OA.1-3).
- Extend the understanding of fraction equivalence and ordering (4.NF.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
MEASUREMENT AND DATA **Geometric measurement: understand concepts of angle and measure angles.** **4.MD.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. **a.** Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $^1/_{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. **b.** Understand an angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.6** Measure angles in whole number degrees using a protractor. Sketch angles of specified measure. *Continued on next page*	**Expectations for Learning** In Grade 3, students classified objects based on the presence or absence of right angles (square corners). In Grade 4, students are introduced to angle measurement (4.G.1) and the use of a protractor. Also students build conceptual understanding of 1 degree as $\frac{1}{360}$ of a circle and *n* degrees as *n* one-degree angles, e.g., 45 one-degree angles = 45 °. Students measure angles and sketch angles to specified measures. They decompose an angle into non-overlapping parts, recognizing that the measure of the angle is the sum of the measures of its parts. Then students solve addition and subtraction angle measurement problems in real-world and mathematical situations using an equation with a symbol for the unknown angle measure. In Grade 5, students will use their knowledge of angles to identify and describe commonalities and differences between types of triangles and quadrilaterals. **ESSENTIAL UNDERSTANDINGS** • Angles are formed when two rays share a common endpoint; the common endpoint of the rays is called a vertex. • Angles are measured in degrees. • A protractor is a tool used to measure angles. • There are 360 degrees in a circle. • One degree is $\frac{1}{360}$ of a circle. • Angles can be decomposed into unit angles. (*n* degrees is *n* one-degree angles.) • A straight angle has a measurement of 180 degrees. • A right angle has a measurement of 90 degrees. **MATHEMATICAL THINKING** • Use spatial reasoning. • Measure using appropriate tools and units; justify mathematical models used. • Create models and sketches to represent angles. • Recognize and use structure. • Make and test conjectures about angles; then justify reasoning. • Use grade-level appropriate mathematical language and notation to explain reasoning.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Draw two rays with a common endpoint to form an angle.
- Explore circles explaining one-degree as $\frac{1}{360}$ of a circle.
- Explore and explain what happens as multiple one-degree angles are accumulated with a common vertex to form a larger angle.
- Identify angles as right, acute, obtuse, or straight.
- Explore and explain which angle measures to use when using a protractor.
- Measure angles in whole number degrees using a protractor.
- Sketch angles of a specified measure using a protractor.
- Explore and explain decomposing an angle into parts, e.g., 45 degrees = 15 degrees + 30 degrees.
- Use addition and subtraction to solve for unknown angles in real-world and mathematical problems.
- Use an equation with a symbol for an unknown angle measure.

Content Elaborations

- Ohio's K-8 Critical Area of Focus, Grade 4, Number 3, page 27
- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14

CONNECTIONS ACROSS STANDARDS

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles (4.G.1-2).
- Understand fraction equivalence and ordering (4.NF.1-2).
- Use the four operations to solve problems (4.OA.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
GEOMETRY **Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** **4.G.1** Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.	**Expectations for Learning** In Grade 1, students composed rectangles, squares, triangles, and trapezoids. In Grade 2, students recognized and identified triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. In Grade 3, students drew and described triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles). In Grade 4, students transition to classification of figures (shapes) based on explicit use of angle size: acute, right, or obtuse. Students classify figures based on the presence or absence of parallel and perpendicular sides. Drawing and identifying figures lends itself to the development of geometric vocabulary with the use of terms such as points, lines, line segments, rays, angles (acute, right, and obtuse), and perpendicular lines and parallel lines. In Grade 5, students will distinguish between different types of triangles and quadrilaterals. They will also use perpendicular lines to develop the coordinate grid system. The student understanding of this cluster aligns with a van Hiele Level 1 (Analysis). **ESSENTIAL UNDERSTANDINGS** • A *point* is a location in space; it has no length, width, or height. • A *line* is a continuous straight path that extends indefinitely in two opposite directions. • A *line segment* is a continuous straight path between two points. • A *ray* is a continuous straight path that extends indefinitely in one direction from one point. • Angles are made of two rays with the same endpoint; the endpoint is called the *vertex.* • A *right angle* has a measure of 90˚. • An *acute angle* has a measure of less than 90˚. • An *obtuse angle* has a measure between 90˚ and 180˚ • A *plane* is a flat surface that extends infinitely in all directions. • Two lines (or two line segments) in a plane are *perpendicular* if the angle between them is a right angle. • Two lines (or two line segments) in a plane are *parallel* if they do not intersect. • Two-dimensional figures can be classified (based on the presence or absence of parallel or perpendicular lines or presence or absence of angles of a specified size).

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use spatial reasoning.
- Create models and drawings to represent figures.
- Recognize and use a pattern or structure.
- Make and test conjectures about the classification of polygons; then justify reasoning.
- Use grade-level appropriate mathematical language to explain reasoning.

INSTRUCTIONAL FOCUS

- Draw points, lines, line segments, rays, and perpendicular and parallel lines.
- Identify points, lines, line segments, rays, and perpendicular and parallel lines.
- Identify points, line segments, and perpendicular and parallel line segments in two-dimensional figures.
- Draw angles (right, acute, and obtuse).
- Identify angles (right, acute, and obtuse).
- Identify angles (right, acute, and obtuse) in two-dimensional figures.
- Draw a shape based on properties of angles, number of sides, or parallel and/or perpendicular sides.
- Use correct language when discussing points, lines, line segments, rays, and angles.
- Classify two dimensional figures based on the following:
    - presence or absence of acute, right, and/or obtuse angles;
    - presence or absence of parallel and/or perpendicular sides; and/or
    - presence or absence of symmetry.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 4, Number 3, page 27
- Ohio's K-8 Learning Progressions, K-5 Geometry, page 11

CONNECTIONS ACROSS STANDARDS

- Students can use recognition of angle measures to classify two-dimensional figures (4.MD.5 and 4.MD.6).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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*(WG) refers to a member of the Working Group in the Standards Revision Process.