Ohio's Model Curriculum | Mathematics
with Instructional Supports

Grade 6

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Mathematics Model Curriculum
with Instructional Supports
Grade 6

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Introduction
PURPOSE OF THE MODEL CURRICULUM
Just as the standards are required by Ohio Revised Code, so is a model curriculum that supports the standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards. The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, possible connections between topics, and some common misconceptions.

To be noted, the emphasis of the model curriculum is on <u>instruction</u> not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Also, examples presented in this document may need to be rewritten to accommodate the needs of each individual classroom.

COMPONENTS OF THE MODEL CURRICULUM
The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.
- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.
Continued on next page

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Introduction, continued
COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

There are several icons that help identify various tips in the instructional strategies section:

    = a common misconception

    = a technology tip

    = a career connection

    TIP!   = a general tip which may include diverse leaner or English learner tips.

Standards for Mathematical Practice—Grade 6

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as they engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.

In Grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". Students can explain the relationships between equations, verbal descriptions, tables, and graphs. Mathematically proficient students check their answers to problems using different methods.

MP.2 Reason abstractly and quantitatively.

In Grade 6, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations or other meaningful moves. To reinforce students' reasoning and understanding, teachers might ask, "How do you know?" or "What is the relationship of the quantities?".

MP.3 Construct viable arguments and critique the reasoning of others.

In Grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

MP.4 Model with mathematics.

In Grade 6, students model problem situations symbolically, graphically, in tables, contextually, and with drawings of quantities as needed. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate and apply them to a problem context. Students should be encouraged to answer questions such as "What are some ways to represent the quantities?" or "What formula might apply in this situation?"

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Standards for Mathematical Practice, continued

MP.5 Use appropriate tools strategically.
Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in Grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to create dot plots, histograms, and box plots to visually compare the center and variability of the data. Visual fraction models can be used to represent situations involving division of fractions. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures. Students should be encouraged to answer questions such as "What approach did you try first?" or "Why was it helpful to use?"

MP.6 Attend to precision.
In Grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations, or inequalities. When using ratio reasoning in solving problems, students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. Students also learn to express numerical answers with an appropriate degree of precision when working with rational numbers in a situational problem. Teachers might ask, "What mathematical language, definitions, or properties can you use to explain ___?"

MP.7 Look for and make use of structure.
Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (e.g., $6 + 2n = 2(3 + n)$ by distributive property) and solve equations (e.g., $3c = 15$ so $c = 5$ by the Division Property of Equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume. Teachers might ask, "What do you notice when ___?" or "What parts of the problem might you eliminate, simplify, or ___?"

MP.8 Look for and express regularity in repeated reasoning.
In Grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. Given multiple opportunities to solve and model problems, they may notice that $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities. Students should be encouraged to answer questions such as, "How would we prove that ___?" or "How is this situation similar and/or different from other situations?"

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STANDARDS	MODEL CURRICULUM (6.RP.1-3)
RATIO AND PROPORTIONAL RELATIONSHIPS **Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."* *Continued on next page*	## Expectations for Learning The study of ratio and proportion extends student learning of multiplication, division, and measurement from previous grades. It is essential for students to make sense of quantities that involve proportional relationships within a context. This is the first interaction students have with ratios and ratio/rate reasoning including percents as ratios. The learning in this standard should be focused on developing an understanding of ratios and solving real-world problems through the use of visual models. In Grade 7, students recognize and represent proportional relationships and extend this reasoning to direct variation equations and more advanced percent problems. **ESSENTIAL UNDERSTANDINGS** **Ratios** • A ratio is a comparison used to describe relationships between two (or more) quantities. • The quantities in ratios may or may not have the same unit. • A ratio compares parts to parts or parts to a whole. • Fractions and percents are specific types of ratios which compare parts to a whole. • Ratios can be written in various forms, e.g., 3:1, 3 to 1, or $\frac{3}{1}$. • Rows (and columns) of ratio tables are multiples of each other. • Ratios are multiplicative relationships. • Ratios can be used to convert units of measure. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.RP.1-3)

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams[G], double number line diagrams[G], or equations.
 	
  a. Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity; solve problems involving finding the whole, given a part and the percent.
Continued on next page
	## Expectations for Learning, continued
	### ESSENTIAL UNDERSTANDINGS, CONTINUED
	Rates
	  • A unit rate is a comparison of two quantities where the second quantity (denominator) is one.
	  • Equivalent ratios have the same unit rate.
	  • The division line in a rate can mean "for every 1," "for each," and "per."
	
	Percents
	  • Percents are used to compare part-to-whole relationships.
	  • Percents can be found using different sized wholes.
	  • Percents are out of a 100.
	  • A percent is a specific type of ratio, which can be represented as a fraction, with a denominator of 100.
	  • All percent problems involve a part and a whole (100) measured in some unit and the same part and whole measured in hundredths.
	  • Benchmark percents can be used to estimate and calculate other percents.
	
	### MATHEMATICAL THINKING
	  • Use precise mathematical language to describe mathematical reasoning.
	  • Solve real-world problems accurately.
	  • Pay attention to and make sense of quantities.
	  • Make and modify a model to represent mathematical thinking.
	  • Recognize and use a pattern and structure to solve problems.
	  • Consider mathematical units in a problem.
	Continued on next page

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STANDARDS	MODEL CURRICULUM (6.RP.1-3)
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	## Expectations for Learning, continued **INSTRUCTIONAL FOCUS** **Ratios and Rates** - Use ratio or rate language to describe the relationship between two quantities. - Distinguish between part-to-part, part-to-whole, and whole-to-part comparisons. - Write a ratio symbolically to describe the relationship between two quantities. - Recognize ratios as multiplicative relationships. - Use models to solve problems involving ratios and unit rates such as ratio tables, tape diagrams, and double number lines. - Identify or create equivalent ratios. - Identify and write unit rates. - Use ratio reasoning to solve a variety of real-world problems. - Apply ratio reasoning to convert measurement units within the same system. - Solve real-life problems involving measurement units that need to be converted. - Make tables of equivalent ratios relating quantities with whole number measurements to do the following: - Find missing values; - Plot pairs of values in the first quadrant of the coordinate plane; - Compare ratios; and - Develop the concept of proportion without solving proportions explicitly. **Percents** - Represent percents using models, such as 100 grids, tape diagrams, and double number lines. - Use ratio reasoning to relate a percent of a quantity as a rate per 100. - Use benchmark percents (1%, 5%, 10%, 20%, 25%, 50%, and 100%) to compute other percents of a given whole number both mentally and with a model. - Identify the part, whole, and/or percent in a real-world or mathematical problem. - Use a model to find the percent when given the part and whole. - Use a model to find the whole when given the part and percent. - Solve real-world percent problems using a model.

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STANDARDS	MODEL CURRICULUM (6.RP.1-3)
	## Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 6, Number 1, page 36 • Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5 • Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7 • Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15 **CONNECTIONS ACROSS STANDARDS** Use variables to represent two quantities in a real-world problem that change in relation to one another (6.EE.9).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on <u>instruction</u> not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Quantities are things that can be measured such as length, distance, time, speed, temperature, etc. Some quantities can be measured directly such as distance or height whereas others are measured as a relation between two variables such as speed (distance/time) or taste (intensity of orange concentrate to water). Ratio reasoning is learning to pay attention to two quantities at the same time.

> **Standards for Mathematical Practice**
> *This cluster focuses on but is not limited to the following practices:*
> **MP.1** *Make sense of problems and perservere in solving them.*
> **MP.2** *Reason abstractly and quantitatively.*
> **MP.3** *Construct viable arguments and critique the reasoning of others.*
> **MP.4** *Model with mathematics.*
> **MP.5** *Use appropriate tools strategically.*
> **MP.7** *Look for and make use of structure.*

There are many differing opinions about the definitions of and differences between ratios and rates, so the emphasis should be on understanding ratios and rates instead of differentiating between the definitions. A ratio becomes a rate in a student's mind when he or she moves from thinking about two independent quantities to two quantities that change in relationship with one another (covarying). From there the student then needs to transition from viewing a rate as just a comparison between two quantities (distance and time) to one single quantity (speed).

Notice the cluster states "ratio reasoning" and not "proportional relationships" as it does in Grade 7. **Students are <u>not</u> required to set up proportions in Grade 6 nor should they use cross products.** Instead the focus is on reasoning and solving problems using different representations of ratios such as ratio tables, tape diagrams, double number lines, etc. Although students should be aware that ratios can be written as fractions, most ratio work in Grade 6 should avoid using fraction notation for ratios. Begin written representation of ratios with the words "out of" or "to" before using the symbolic notation of the colon. Then only after a lot of work with ratios, introduce the fraction bar, for example, 3 out of 5, 3 to 5, then 3:5 and finally $\frac{3}{5}$.

TIP! Sports sometimes use the words 6 to 4 to describe the score of a game. However, this is not a ratio because it is an additive, not multiplicative, relationship.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

RATIOS

Introduce ratios and rates with real-world experiences such as taste. For example, give students three different drinks with different ratios of orange juice concentrate to water and have them figure out which is the "orangiest" and explain why. They should have problems using a variety of contexts: measurement, prices, geometric contexts, miles per hour, constant speed, recipes, and portion per person.

Students need to develop the understanding that a ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percents are examples of part-to-whole ratios. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28. Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio 12 to 16. This form of a ratio is often used to compare an event to nonevents (something that can happen compared to the event that cannot happen). *Note: Fractions and ratios may represent different comparisons. Fractions always express a part-to-whole comparison, but ratios can express a part-to-whole comparison or a part-to-part comparison.*

TIP! Even though ratios and fractions express a part-to-whole comparison, the addition of ratios and the addition of fractions are distinctly different procedures. When adding ratios, the parts are added, the wholes are added, and then the total part is compared to the total whole. For example, (2 out of 3 parts) + (4 out of 5 parts) is equal to 6 parts out of 8 total parts (6 out of 8) if the parts are equal. When dealing with fractions, the procedure for addition is based on a common denominator: $\left(\frac{2}{3}\right) + \left(\frac{4}{5}\right) = \left(\frac{10}{15}\right) + \left(\frac{12}{15}\right)$ which is equal to $\left(\frac{22}{15}\right)$. Therefore, the addition process for ratios and for fractions is distinctly different. This is why it is best to avoid fraction notation until students have gained a thorough understanding of ratios.

Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used.

EXAMPLE

Have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

Have students create their own ratios. For example, a basketball player who makes $\frac{2}{3}$ shots in the first half and $\frac{4}{5}$ in the second half would have an overall success rate equal to $\frac{6}{8}$.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Ratio Language
It is also important that students use vocabulary associated with ratios such as "for every," "for each," "for each 1," and "per." To reinforce ratio reasoning ask students questions such as "How many times as many?" or "How many per?"

Comparison Problems
Not only should students use ratios to predict and to find missing elements, they should also use ratios to compare parts in real-world contexts. Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon, or students per bus. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

EXAMPLE
3 cans of pudding cost $2.48 at Store A, and 6 cans of the same pudding costs $4.50 at Store B, which store has the better buy?

Store A		Store B	
3 cans	6 cans	6 cans	3 cans
$2.48	$4.96	$4.50	$2.25

Discussion: Various strategies could be used to solve this problem.
- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking ½ of $4.50.

Additive vs Multiplicative Relationships
The first step in developing ratio reasoning is to distinguish between additive and multiplicative (times as much) relationships. Additive comparisons are not ratios but multiplicative comparisons are. Give students the opportunity to differentiate between the different types of comparisons.
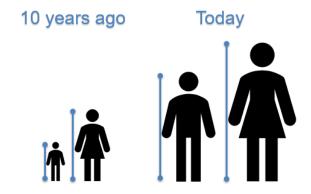
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

The pictures on the right show Jayce's and Jaqueesha's heights ten years ago and their heights today. Who grew the most?

Discussion: The class discussion should be designed to lead students from seeing an additive relationship towards seeing a multiplicative relationship. Differentiate between how much taller and how many times taller. Jaqueesha is 2 *times* as tall as she was 10 years ago, but Jayce is 3 *times* taller than he was 10 years ago. The students could cut out the pictures to see for themselves. Another way to represent this would be to align the two Jaqueeshas and the two Jayce's and use a ruler to show the steepness of the change in Jayce compared to Jaqueesha.

EXAMPLE

If Group A has 3 girls and 4 boys and Group B has 5 girls and 12 boys, which group has more girls?

EXAMPLE

Give students a 2-inch piece of straw and a full 10-inch straw. Ask students "How much longer is the full-length straw?" (8 inches) and "How many times longer is the full-length straw?" (5 times).
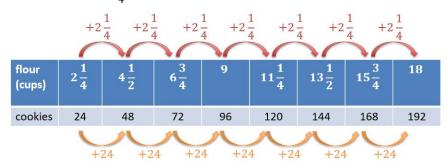
Building-Up and Breaking-Down Strategies

Building-Up and Breaking-Down are strategies where students take a ratio and either build it up using addition or take it down using subtraction to get a new equivalent ratio. Although, it is a good strategy to introduce ratios, it is not truly proportional reasoning because it primarily uses additive reasoning which does not take into account the constant ratio between the two quantities. However, it can be an important benchmark in understanding. To move students from additive reasoning to more multiplicative reasoning, ask students for more efficient ways to move across the ratio table stressing multiples. *Note: A ratio is still a multiplicative relationship even when students used additive reasoning. Students do not need to know the terms additive and multiplicative reasoning. These terms are used in this document to help teachers understand student thinking.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

A recipe calls for $2\frac{1}{4}$ cups of flour to make 24 cookies. If you need to make 192 cookies for a party, how many cups of flour will you need?

flour (cups)	$2\frac{1}{4}$	$4\frac{1}{2}$	$6\frac{3}{4}$	9	$11\frac{1}{4}$	$13\frac{1}{2}$	$15\frac{3}{4}$	18
cookies	24	48	72	96	120	144	168	192

(flour increases by $+2\frac{1}{4}$ each step; cookies increase by $+24$ each step)

Discussion: This problem will be shown several times in this document. The ratio table on the lefts shows a student applying additive reasoning to a ratio problem using the building up strategy. This type of reasoning, although not yet efficient, is an important developmental step in understanding ratios.

Ratio Tables

As students become fluent in Building-Up and Breaking-Down strategies move them towards ratio tables that emphasize multiplicative properties. Start with problems that use intuitive multiplicative strategies such as doubling, tripling, and halving, and then move towards less intuitive multiples. Ask them questions such as "Can you make your table more efficiently?" or "Do I have to write down all the numbers?" Have students discover that if one quantity is multiplied/divided by a factor, the other quantity must also be multiplied/divided by the same factor. This can be linked to the Identity Property of Multiplication.

Always link numbers in a ratio to what they each stand for, and draw explicit attention to the relationship between the two numbers and their units.

TIP!
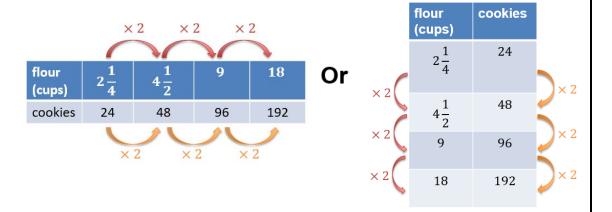
TIP! Using multiplication tables can help highlight multiples and common ratios in ratios tables. See the YouTube video Equivalent Ratios and Multiplication Tables by Mrmathblog for a demonstration.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

A recipe calls for $2\frac{1}{4}$ cups of flour to make 24 cookies. If you need to make 192 cookies for a party, how many cups of flour will you need?

Discussion: Notice that this example is the same as the one shown under Building-Up and Breaking Down. However, the level of sophistication in student reasoning has changed. Instead of using additive reasoning, the student is using early stages of multiplicative reasoning. Point out to students that although addition can sometimes be used in the process of completing a ratio table, ratios always have a multiplicative relationship. Have students focus on multiples and draw attention to the common ratio. *Note: Tables can be made both horizontally and vertically. Vertical tables more closely resemble the typical input-output tables whereas horizontal tables lend themselves to setting up proportions in Grade 7.*

flour (cups)	$2\frac{1}{4}$	$4\frac{1}{2}$	9	18
cookies	24	48	96	192

Or

flour (cups)	cookies
$2\frac{1}{4}$	24
$4\frac{1}{2}$	48
9	96
18	192
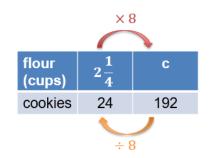
As students' reasoning evolves have them work with other multiples than halves and doubles. Also, they should have practice using multiples that involve fractions and decimals. Eventually students should build towards two-by-two ratios tables.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

A recipe calls for $2\frac{1}{4}$ cups of flour to make 24 cookies. If you need to make 192 cookies for a party, how many cups of flour will you need?

Discussion: Notice that this example is the same as the one shown twice previously. Student thinking has become even more sophisticated as he or she now only needs to make a two-by-two ratio table. The student is also able to discover that if you divide in one direction, you multiply in the other direction. It is important that students have many experiences using multiples and ratio tables and are not pushed too quickly to this more efficient form of thinking. A profitable class discussion after much work using ratio tables would be to put all three problems on the board and discuss why each works, and which one is more efficient.

		× 8	
flour (cups)	$2\frac{1}{4}$	c
cookies	24	192
		÷ 8	

Use problems involving large numbers where additive strategies such as Building-Up and Breaking-Down are inefficient. This should push hesitant students towards multiplicative reasoning.

EXAMPLE

Marcus's car has a 10-gallon tank. He can go 35.8 miles on a full tank of gas. If on a road trip he buys a total of 85 gallons of gas, how far can he go?

gallons	10 gallons			
miles	358 miles			
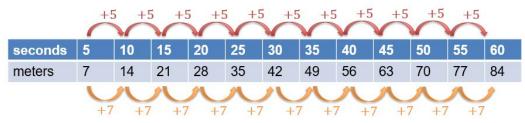
GRAPHS

A ratio can be written as an ordered pair. As students plot pairs of values from a contextual problem on the coordinate plane, emphasize the multiplicative and additive relationships. Emphasize the efficiency of multiplicative reasoning. Draw attention to the common ratio. Students should also come to the realization that all equivalent ratios make a line that goes through the origin. These proportional relationships can also be connected to 6.EE.9 where students write equations and identify the independent and dependent variable given a context.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE
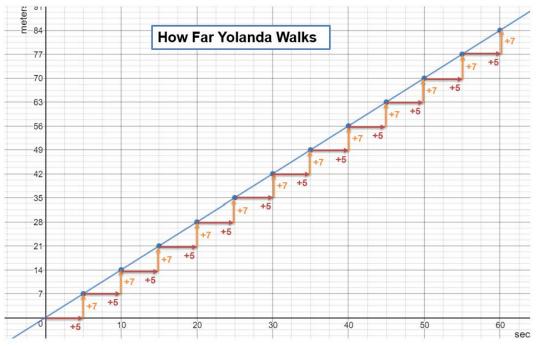
Yolanda walks 7 meters in 5 seconds. How far will she walk in a minute? Use a table and a graph to illustrate your answer. *Note: Students may use a horizontal or vertical table. Examples of both are shown below.*

Method 1: Additive Reasoning
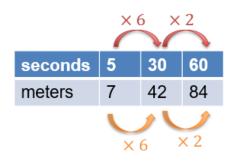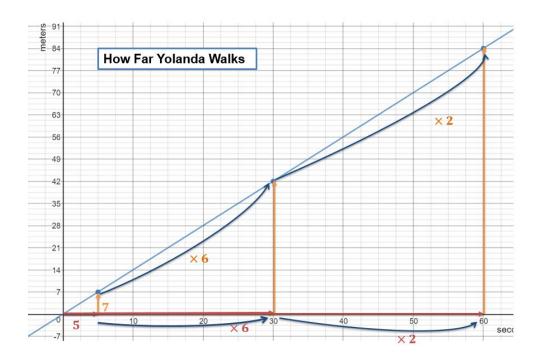
seconds	5	10	15	20	25	30	35	40	45	50	55	60
meters	7	14	21	28	35	42	49	56	63	70	77	84

seconds	meters
5	7
10	14
15	21
20	28
25	35
30	42
35	49
40	56
45	63
50	70
55	77
60	84

How Far Yolanda Walks

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Method 2: Multiplicative Reasoning

seconds	meters
5	7
30	42
60	84

seconds	5	30	60
meters	7	42	84

How Far Yolanda Walks

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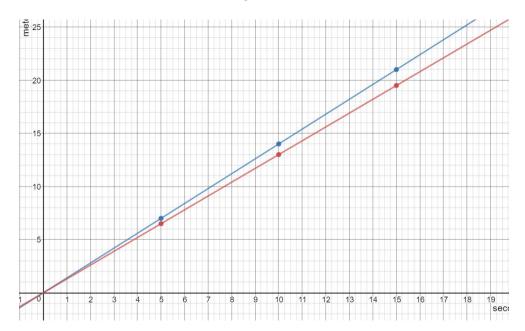
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Method 3: More Advanced Multiplicative Reasoning

seconds	meters
5	7
60	84

× 12   × 12

× 12

seconds	5	60
meters	7	84

× 12

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Students should also use graphs to compare rates by graphing two situations on the same coordinate grid.

Example
Yolanda walks 7 meters in 5 seconds. Miguel walks 13 meters in 10 seconds. Who walks faster? Explain your answer using a coordinate grid.

TAPE DIAGRAMS
Tape diagrams are a visual model that uses rectangles. They are a visual way to show proportions and are especially useful when comparing two or more quantities.
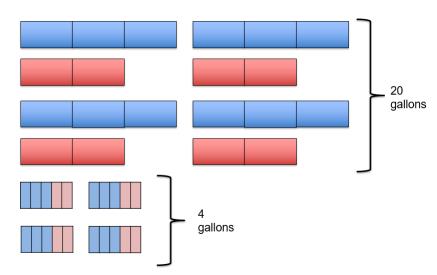
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

To get the perfect shade of purple Annie needs a ratio of blue to red of 3:2. If she needs 24 gallons to paint her house, how much of each color does she need? *Note: It may be helpful to start by giving students problems with easier numbers such as using 25 gallons of paint. However, as they advance in understanding, they should become more comfortable working with decimals and fractions*
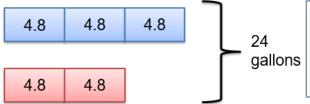
Method 1: Additive Reasoning

Discussion: This student represented each box as 1 gallon where the blue boxes represent blue paint and the red boxes represent red paint. He or she kept adding until there were no more whole groups that went into 24. (20 ÷ 5 = 4 complete groups with 4 gallons left over.) Then he or she divided the remaining 4 gallons into fifths and shaded $\frac{3}{5}$ of each gallon blue to represent blue paint and shaded $\frac{2}{5}$ of each gallon red to represent red paint. From there the student could add all the blue gallons to get

$(4 \times 3) + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = 14\frac{2}{5}$. Likewise the red gallons would be

$(4 \times 2) + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = 9\frac{3}{5}$..

Method 2: Multiplicative Reasoning

Discussion: Since 24 ÷ 5 = 4.8, each box is 4.8 gallons, so Annie needs 4.8 × 3 or 14.4 gallons of blue paint and 4.8 × 2 or 9.6 gallons of red paint. To move students from additive reasoning to multiplicative reasoning illustrate the connections between the two types of thinking using student work.
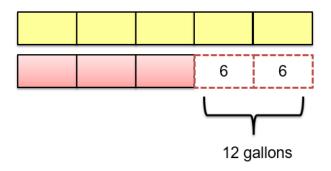
Since 24 ÷ 5 = 4.8, each box is 4.8 gallons. So they need 4.8 × 3 or 14.4 and 4.8 × 2 or gallons of blue paint 9.6 gallons of red paint.

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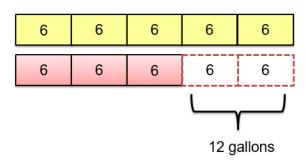
EXAMPLE

Yellow and red were mixed together in a ratio of 5:3 to get a shade of orange paint. Ted wanted his orange paint to have equal parts of yellow and red, so he added 12 gallons of red paint. How much red paint was there originally?

Step 1　　　　　　　　　　　　　　　　　　　**Step 2**

Discussion: Two parts of red need to be added to allow yellow and red to have equal parts. Since those two parts equals 12 gallons, each part equals 6 gallons. Since all parts are equivalent there are 6 x 3 or 18 gallons of red paint originally.
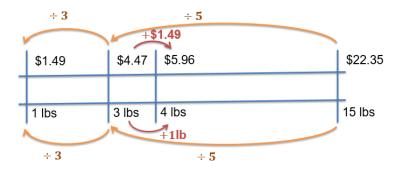
DOUBLE NUMBER LINES

A double number lines is a pair of parallel lines that are hinged at 0. The scaling of each number line is different, so a double number line visually illustrates a proportion. It is useful when two different units are being compared especially using proportions to illustrate measurement conversions. They can also be helpful representing ratios involving decimals and fractions.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

If 15 lbs of apples cost $22.35, how much will 4 lbs cost?
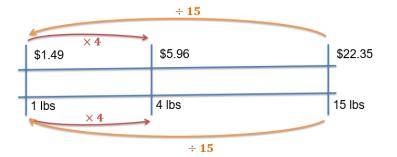
Method 1:

Discussion: This student used both multiplicative and additive reasoning flexibly. He or she recognized that 15 was divisible by 5, so he or she divided both 15 lbs and $22.35 by 5 to get 3 lbs for $4.47. Next, he or she divided that by 3 to get the unit rate, so the price of the apples is $1.49 per 1 pound. Then he or she added 1 lb to 3 lbs to get 4 lbs and added $1.49 to $4.47 to get $5.96.
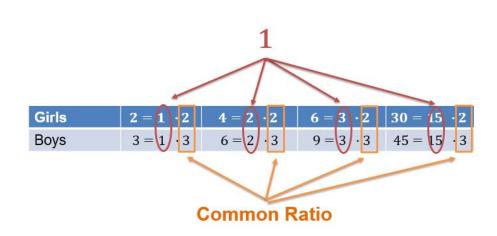
Method 2:

Discussion: This student used more advanced multiplicative reasoning thinking in terms of unit rate. He or she divided the money and pounds by 15, to get $1.49 for 1lb. Then he or she multiplied the unit rate by 4 to get the cost of 4 lbs which is $5.96.

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UNIT RATE/COMMON RATIO

As students move from additive reasoning to multiplicative reasoning, draw attention to the common ratio and connect it to the unit rate. This is the foundation for proportional reasoning that will be more solidified in Grade 7 and which extends to slope in Grade 8. Explain to students that the same or fixed ratio is associated with a quality such as steepness, flavor, or speed remains the same or fixed as the variable changes together. Ask students questions such as "How can qualities stay the same or fixed as quantities change?" They need practice in isolating the attribute associated with the fixed ratio from other measurable attributes. Proportional reasoning occurs when students understand that the ratio of the two quantities remains constant even though the corresponding values of the quantities may change.

Every rate has two unit rates. For example, the unit rate for buying 15 gallons of gas for $33.75 could be $2.25 per 1 gallon or 0.44 gallons per $1. Although students could use either rate for problem solving, one unit rate usually makes more sense than the other.

Equations

Another way to think of a ratio is a multiplicative comparison of two quantities. Revisiting the straw example: Give students a 2-inch piece of straw and a full 10-inch straw. Ask students "How many times longer is a full length straw?" Students could collaborate and come to the realization that it is 5 times longer. The "5 times" is the constant ratio or unit rate. Therefore they can write the equation $c = 5d$ to model the situation where c is the longer straw and d is the shorter straw.

Students should start writing equations using the unit rate. For example if students are using a table or graph, they should be able to identify the unit rate and write an equation. They should also be able to recognize a corresponding graph or table given an equation.

Unit Rate Method

One method for solving problems involving ratios is by finding the unit rate first. It is similar to using unit fractions to solve fraction problems. Students ask themselves "How many for one?". They recognize that a relationship exists, and then they calculate the rate so that one of the quantities is one. They can use this strategy to compare two rates, or they can multiply the unit rate by one of the quantities to find a missing value.

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EXAMPLE

Dietary guidelines recommend less than 25 grams of sugar a day. A 68 oz bottle of pop has 234 grams of sugar. Does an 8 oz cup of pop meet the recommended dietary guidelines?

	÷ 68	× 8
grams	234	3.44	27.52
ounces	68	1	8

÷ 68   × 8

CONVERTING UNITS OF MEASUREMENT

Students should solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with tools such as ratio tables, t-charts, double number line diagrams, or tape diagrams/bar models will help students internalize the size relationships between same system measurements. It also helps relate the process of converting measurements to the solution of a ratio. *Note: Students should be converting measurements by using ratio tables or double number lines. They should not be using unit analysis techniques.*

EXAMPLE

A 2-liter bottle of cola contains about 68 oz. If Julia needs to provide one cup of cola for 136 people, how many bottles does she need to buy? *Note: A cup is 8 oz.*

8 oz	68 oz	1088 oz
1 cup	8.5 cups	136 cups

68 oz	1088 oz
1 bottle	16 bottles

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE
   a. Measure two different objects using inches. Write a ratio comparing the two objects.
   b. Then measure the objects using centimeters. Write a ratio comparing the two objects.
   c. Does measuring with a different unit such as inches or centimeters change the ratio? Explain.

Discussion: This concept can be tied into scale models and could lend itself to a discussion on inverse ratios.

PERCENTS
This is the first time in the standards that students encounter percents. It would be helpful to have a discussion about where percents occur in real-life, why they are used, and how sometimes people use percents to disguise information. Percents—
   • compare sets of numbers of unequal size;
   • act as fractions comparing parts-to-wholes; or
   • act as ratios comparing two different objects in a set.

Students need to understand that a percent only has a meaning in relationship to its base. Also a percent can be a descriptor (30% are wearing blue) or a producer of data (30% discount). Percents are meaningless without a context, and their meaning can vary depending on the context:
   • 8 of 100;
   • 8 out of 100 (part-to-whole);
   • 8 of one thing for every 100 of the same thing (a ratio, interest rates, etc.);
   • a command to create a new number (sales tax);
   • slope (gradient);
   • probability; or
   • a comparison of two numbers.
Note: Many people define fraction, ratios and rates differently. Students do not need to differentiate between the different terminology.

Percents are often taught in relationship with learning fractions and decimals or a part-to-whole relationship; however, 6.RP.3c explicitly states that percents are also to be taught as a special type of rate, "per 100." That means the relational aspect of percent needs to be emphasized. Although percents have similarities with fractions and decimals, they also have nuanced differences which can cause confusion unless the instruction draws attention to the differences. Also, a percent is not necessarily a fraction or a decimal, Percent is always a comparison to 100. For example, although 25% is equivalent to $\frac{1}{4}$ numerically, it is not $\frac{1}{4}$ because it is a comparison of 25 units to 100, not 4, units. Converting between the forms of rational numbers is more of a computational convenience than a defining trait.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Common errors for students include viewing the percent sign as a label or ignoring the percent sign all together. Early U.S. and European texts referred to percents as $5 per cent (100). This related the interest of $5 to the amount borrowed, $100. The use of the quantity ($) and the percent helped people see the proportional relationship of the percent instead of misinterpreting the % sign as a label. Emphasize that the percent sign signals a comparison to 100.

Many students incorrectly believe that the percent sign to the right of the numeral can be replaced by a decimal point to the left of the numeral, e.g., 114% = 0.114 instead of 1.14 or 5% = 0.5 instead of 0.05. Having students write the percent over 100 before converting to a decimal may help them confront this misconception.

Percent as a Fraction
A percent viewed as a fraction emphasizes the part-whole relationship. Percents greater than 100% are not possible in a part/whole relationship. Examples of percents as a part-whole relationship include probability, circle graphs/pie charts, number of votes etc. Although still necessary, the standards deemphasize this viewpoint.

Percent as a Rate
A percent can be thought of as a rate. This viewpoint emphasizes its relational aspect as a comparison of two quantities. Many students who have a part/whole understanding of percent make mistakes when they need to apply percents to situations that are not part/whole such as percent change. For example, when solving the problem 80 = __% of 40, students will oftentimes give 50 for the answer. They just jump to dividing the larger number by the smaller number instead of reasoning through the process. Emphasizing percents as a rate or multiple will help alleviate this situation, since percents greater than 100 exist in the rate viewpoint. A percent as a rate can be used to show the relationship between the independent and dependent variable.

Representing Percents
The focus of percents in Grade 6 is on solving percent problems as equivalent ratios using models such as grids, pattern blocks, integer chips, tape diagrams, double number lines, and ratio tables. Since this is the first time students formally encounter percent, it may be helpful to spend some time representing percents. Students also need experience using different size wholes. Although the calculations in Grade 6 focus on percents within 100, when representing percents give examples of percents greater than 100 and less that one to prevent misconceptions from developing.

Pattern blocks are a good way to represent percents greater than 100%.

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EXAMPLE

If the figure is the whole, build
and/or draw the following:

    a. 25%
    b. 50%
    c. 75%
    d. 100%
    e. 125%
    f. 200%
    g. 225%
    h. What percent is one triangle?

Some manipulatives such as the hundreds board are useful as an introduction to percent, but they reinforce the part-whole relationship and make the rate relationship harder to see. When using the 100-grids it is important to have students identify the small square. Use the manipulatives to build proportional reasoning. NCTM Illuminations "Grid and Percent It" has several examples on how to model percents using 100-grids, but skip the percent change problems for Grade 6. *NCTM now requires a membership to view their lessons.*

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An example like this could be used after students have worked with representing percents using different sized grids.

EXAMPLE
Answer the following questions for each grid shown on the right:

Part 1
- Represent the fact that 40% of the class owns a dog on each of the grids.

Part 2
   a. If there are 100 kids in the class, what percent does each square represent?
   b. If there are 100 kids in the class, what number does each square represent?
   c. If there are 30 kids in the class, what percent does each square represent?
   d. If there are 30 kids in the class, what number does each square represent?
   e. If there are 120 kids in the class, what percent does each square represent?
   f. If there are 120 kids in the class, what number does each square represent?
   g. What do you notice about the percents? Explain why.
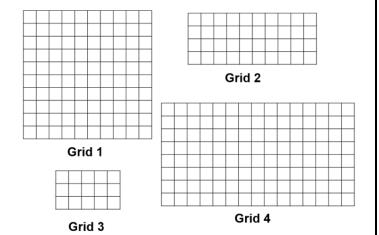
Part 3
   a. How many students own a dog if there are 100 students in the class?
   b. How many students own a dog if there are 30 students in the class?
   c. How many students own a dog if there are 120 students in the class?
   d. For situation a.-c., which grid did you prefer to use? Explain.
   e. Write ordered pairs representing (total boxes, shaded boxes), and graph on the same coordinate plane.
   f. Write ordered pairs representing (size of class, kids who own a dog), and graph on the same coordinate plane as part **e.**
   g. What do you notice about all the points on the two graphs? Explain why.
   h. Write an equation representing each situation. What does the constant ratio represent?
   i. Using your graph, find the number of students who own a dog if there are 60 students in the class.

TIP! It is important for students to see that grids do not have to be shaded in rows or columns, but that there can be a variety of equivalent shadings on the same grid. When modeling percents using grids, shade in different combinations of squares.

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Dual-figure models

By using two figures, one of which is a reference figure, it allows students to move beyond part-whole thinking to see one quantity as a percentage of another quantity. Double number lines, proportion bars, comparison scales, two shaded figures, and Cuisenaire rods with a reference 0-100% number line also help students develop proportional reasoning for percents.

Cuisenaire Rods

Image adapted from https://nzmaths.co.nz/content/cuisenaire-rod-fractions-level-3.

EXAMPLE

Figure A   Figure B

    a. Shade Figure B, so that it represents 25% of Figure A.
    b. Draw a Figure C that represents 125% of Figure A.

Discussion: Part **a.** is asking students to find 25% of 80% (or $\frac{4}{5}$), which is 20% or in this case 3 grid squares. Part **b.** is asking students to find 125% of 80% (or $\frac{4}{5}$) which is 100% or in this case 15 grid squares.

EXAMPLE

Figure A   Figure B

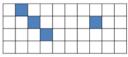   **a.** What percent is Figure B of Figure A?
    b. What percent is Figure A of Figure B?

Discussion: Part **a.** is asking students to find 40% (or $\frac{2}{5}$) is what percent of 60% (or $\frac{3}{5}$) which is $\frac{2}{3}$ or $66.\overline{6}\%$. Part **b.** is asking students to find 60% (or $\frac{3}{5}$) is what percent of 40% (or $\frac{2}{5}$) which is 150%.

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EXAMPLE

Figure A   Figure B
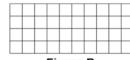
   a. Shade Figure B so that Figure A represents 20% of Figure B.
   b. Draw a Figure C so that Figure A is equal to 200% of Figure C.

Discussion: Figure A shows 10% (or $\frac{4}{40}$ or $\frac{1}{10}$), so Part **a.** is asking students to realize that if Figure A which is 4 blocks out of 40 is 10%, then for Figure A to be 20% of a new shaded region, the shaded region would have to be 20 blocks out of 40 shaded in. Part **b.** is asking students to realize that if Figures A is double (or 200% of Figure C), then Figure C must be half of Figure A or 2 squares.

Double number lines are useful to compare different percentages to a whole (100%). Tape diagrams show the connection between percents and fractions. Tape diagrams along with proportion bars are useful in solving problems involving finding what percent a part is of the whole.

TIP!

Instead of saying 75% = $\frac{3}{4}$, state that 75% of a quantity is $\frac{3}{4}$ of that quantity.

Benchmark Percents
Estimating percents is a skill that is needed in the real-world. Benchmark percents along with other strategies can help with this concept. Benchmark percents include 1%, 5%, 10%, 20%, 25%, 50%, and 100%. Another advantage of benchmark percents is that they highlight ratio reasoning to estimate. For example students can figure out 15% by finding 10%, halving 10% to get 5%, and then adding 10% and 5% to get 15%.

Using manipulatives such as integer chips, Cuisenaire rods, or grids, ask questions so students see the relationships between numbers. Connect benchmark percents to ratio tables to build ratio reasoning which allows students to view percent as a rate. Then to build number sense continually ask questions such as "Is 70% of 30 more than, less than, or equal to 30?" They should also practice using benchmark percents with decimals in the context of money or metric measurement.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

Using 10% as a benchmark, compute 5%, 10%, 15%, 20%, and 110% of 80, and solve the problems mentally.

Discussion:

- 10% of 80 is 8. (80 cut into 10 sections)
- Double 10% to get 20%. 20% would be 16. (8 doubled is 16.)
- Cut 10% in half to get 5%. 5% would be 4. (Half of 8 is 4.)
- 15% can be found by adding 10% and 5 %. 15% would be 12. (8 + 4 = 12)
- 100% of 80 is 80, and 10% of 80 is 8, so 110% of 80 is 88. (80 + 8 = 88).

EXAMPLE

Find an 18% tip of a $36 restaurant bill. Use benchmark percents and ratio tables to show your work.

	÷ 10	÷ 2	÷ 10		× 3
dollars	$36	$3.60	$1.80	$0.36	$1.08
percent	100%	10%	5%	1%	3%
	÷ 10	÷ 2	÷ 10		× 3

$$10\% + 5\% + 3\% = 18\%$$
$$\$3.60 + \$1.80 + \$1.08 = \$6.48$$

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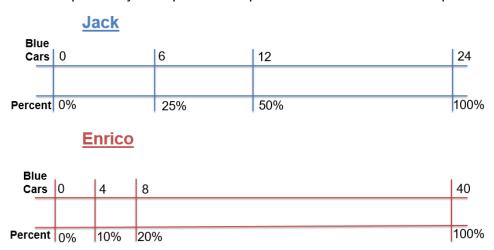
Percent Problems

Students should solve many problems involving percent. The focus is on solving percent problems using ratio reasoning with manipulatives or drawings. Using proportions and a percent equation is reserved for Grade 7. In Grade 6 they need experience using percents to compare and to find a missing part, whole, or percent. It is important in a percent problem to identify the part, the whole, and the percent. Students who rely exclusively on the word "of" to indicate the whole may struggle with percent problems as "of" has many meanings in the English language and does not always signal the whole. In addition there are many times where the word "of" is not even present.

EXAMPLE

Enrico and Jack collect Bakugan toys. They get into an argument about who has more blue Bakugan toys. Jack has 24 toys and 6 are blue. Enrico has 40 toys and 8 are blue. Enrico says that he has more because he has 2 more than Jack. Jack says that he has more, because he has more comparatively. Use percents to prove the Jack has more comparatively. Whose argument do you think is a better comparison?
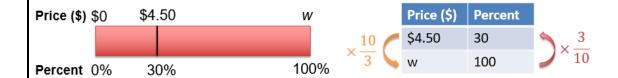
Students in Grade 6 do not set up proportions nor use methods such as cross product, since such methods are not appropriate at this level. Instead one method that may be helpful to solve percent problems is using percent bars coupled with ratio tables. This will also set students up for representing and solving percent proportions in Grade 7. Students should be able to fluidly move around the table using inverse relationships such as multiplying by 3 in one direction or dividing by 3 in the other (or perhaps multiplying by 3 in one direction and multiplying by $\frac{1}{3}$ in the other). They can also use any other ratio method to solve problems.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

A 30% discount of a shirt is $4.50.
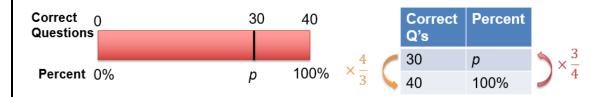
    a. How much does the shirt originally cost?
    b. What is the discounted price?

Price ($) $0 $4.50 w

Percent 0%   30%   100%

$\times \dfrac{10}{3}$

Price ($)	Percent
$4.50	30
w	100

$\times \dfrac{3}{10}$

Discussion: If students are having difficulty with the inverse relationships encourage them to make ratio tables using benchmark percents. For example, they may want to find 10% of $4.50 which is $1.50 and then multiply that by 10 to get the whole. It is more important at this level for students to be able to reason through percents than it is for them to use a specific method.

EXAMPLE

On a 40-question test, Jason got 30 questions correct. What percent did he get correct?
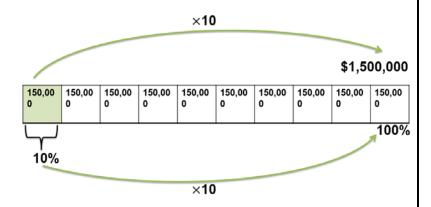
Correct Questions 0   30   40

Percent 0% p 100%

$\times \dfrac{4}{3}$

Correct Q's	Percent
30	p
40	100%

$\times \dfrac{3}{4}$

Discussion: If students are having difficulty with the inverse relationships encourage them to make ratio tables. For example, they may recognize that 20 correct questions is 50% and that 10 correct questions is 25%, so 30 correct questions would be 75%. It is more important at this level for students to be able to reason through percents than it is for them to use a specific method.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

EXAMPLE

Mr. Jones wanted to invest $150,000 in a company that sells dog food. The dog food company said that they would give him 10% ownership in the company. How much is the company worth?

EXAMPLE

About 60% of students wear glasses.
  a. Using the graph, name some equivalent ratios that compare students who wear glasses to the class size.
  b. Make a prediction:
      • If the class size is 25 students, how many wear glasses?
      • If 12 students wear glasses, how big is the class?

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Part-whole Methods
Part-whole methods can be used to solve percent problems. Although, not wrong, it can limit students understanding of percents. That is why the standards emphasize defining a percent in terms of a rate.

EXAMPLE
Jonathan bought a used car for $14,130 which was 75% of the original price. How much did the car originally cost?

$14{,}130 \div 75 = 188.40$

Discussion: This is an example of a part-whole model of a student. Students may choose to use a 100's grid. They can shade in 75 out of the 100 cells. Then they can divide 14,130 by 75 and see that each cell is 188.40. Since all cells are the same amount, they can multiply 188.40 by 100 to get $18,840.

188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40
188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40	188.40

Circle Graph
The standards never require students to use or make a circle graph/pie chart. If a district wishes to incorporate this concept into their curriculum, this may be an appropriate place. Be aware though that although a pie chart is helpful if the percent is smaller than 100 (since it represents a part-whole relationship), it may hinder students from conceptualizing that a percent can be greater than 100 such as in percent of increase or decrease problems in Grade 7. Therefore, make sure to also use other models with students where they can visualize percents greater than 100%.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- 100 grids (10 x 10) for modeling percents
- Ratio tables – to use for proportional reasoning
- Bar Models – for example, 4 red bars to 6 blue bars as a visual representation of a ratio and then expand the number of bars to show other equivalent ratios
- Connecting cubes in multiple colors
- Cuisenaire rods
- Play money - act out a problem with play money
- Advertisements in newspapers
- Unlimited manipulatives or tools (Do not restrict the tools to one or two models, instead give students many options.)
- *If You Hopped Like a Frog* by David M. Schwartz is a picture book that introduces the concepts of ratio and proportion by comparing what humans would be able to do if they had the capabilities of different animals. (ISBN-13: 978-0590098571)

Ratios

- Games at Recess by Illustrative Mathematics is where students explore ratios and are required to write their answers in complete sentences.
- Bag of Marbles by Illustrative Mathematics explores the relationship between fractions and ratios.
- Many Ways to Say It by Illustrative Mathematics helps students understand and use ratio language.
- Apples to Apples by Illustrative Mathematics connects multiplicative reasoning to ratios.
- Converting Square Units by Illustrative Mathematics has students critique an argument about converting length to the square units.
- New and Improved Thinking Blocks by Math Playground has videos, a game, and an applet to use thinking blocks to solve a variety of ratio story problems. Thinking blocks are similar to tape diagrams.
- Episode 9: the Queen of Tarts is an episode from the British TV series Math Mansion about ratios.
- Represent Rates on Coordinate Grids—Feet Per Second by Mobius Math is a worksheet that integrates tables, graphs, and double number lines with word problems.
- Learning to Think Mathematically with the Ratio Table: A Resource for Teachers, A Tool for Young Children by Jeff Frykholm is a book in pdf form that prepares teachers with the theoretical basis, practical knowledge, and expertise to use the ratio table as a mode of learning. It also includes student activity sheets.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Equivalent Ratios
- Equivalent Ratios and Multiplication Tables by MrMathBlog is a video that shows how to use multiplication tables to find equivalent ratios.

Applying Ratio Reasoning to Real-World Contexts
- Something Fishy by PBS Learning Media has students estimating the size of a large population by applying the concepts of ratio and proportion through the capture-recapture statistical procedure.
- How Many Noses Are in Your Arm? by PBS Learning Media has students applying the concept of ratio and proportion to determine the length of the Statue of Liberty's torch-bearing arm.
- Nana's Paint Mixup by Dan Meyer is a 3-Act Tast where students use equivalent ratios to determine how to fix a paint mixup.
- Ratio Activities with real-world contexts from Yummy Math
  - Is This Promo a Good Deal?
  - April's Calf Was Born!
  - Boston Marathon
  - Greatest March Madness Program
  - Super Bowl Cheesy Pretzel Poppers
  - Longest NHL Overtime Games In History
  - Not Enough Mashed Potatoes
  - Hiking the Appalachian Trail
  - Big Burger
  - McDonald's Moves Towards Antibiotic-free Chicken
  - Losing Team in the Playoffs, 2015
  - Olympic Reflection
  - Which is Most Expensive?
  - Which Sweethearts are the Best Deal?
  - Delicious Pumpkin Pie
  - An Ounce of Cola
  - Shipping Crunch
  - Halloween Candy Sales
  - Which is the Best Candy Deal?
  - Water Saving Toilets

Unit Rates
- Price per Pound and Pounds per Dollar by Illustrative Mathematics helps develop the concepts of unit rates.
- Mangos for Sale by Illustrative Mathematics generates classroom discussion about rates and unit rates.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

Percents
- Security Camera by Illustrative Mathematics uses the context of a security camera to have students explore percent and area.
- Dana's House by Illustrative Mathematics has students connect area and percent.
- Episode 8: The Whole Class Should Be Expelled from the British TV series Math Mansion about percentages.
- Use Double Number Lines to Determine Percent of a Number by Learn Zillion is a video for teachers to gain background on using a double number line.
- Benchmark Percents on Bars by Mobius Math has students use percent bars and ratio reasoning when calculating percents. See page 2.
- Benchmark Percent Practice by Geogebra is an applet that has students explore benchmark percents.
- Grid and Percent It by NCTM Illuminations is an exploratory activity about percents. *NCTM now requires a membership to view their lessons.*

Curriculum and Lessons from Other Sources
- EngageNY, Grade 6, Module 1, Topic A, Lesson 1: Ratios, Lesson 2: Ratios, Lesson 3: Equivalent Ratios, Lesson 4: Equivalent Ratios, Lesson 5: Solving Problems by Finding Equivalent Ratios, Lesson 6: Solving Problems by Finding Equivalent Ratios, Lesson 7: Associated Ratios and the Value of Ratio and Lesson 8: Equivalent Ratios Defined Through the Value of a Ratio are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 1, Topic B, Lesson 9: Tables of Equivalent Ratios, Lesson 10:The Structure of Ratio Tables—Additive and Multiplicative, Lesson 11: Comparing Ratios Using Ratio Tables, Lesson 12: From Ratio Tables to Double Number Line Diagrams, Lesson 13: From Ratio Tables to Equations Using the Value of the Ratio, Lesson 14: From Ratio Tables, Equations, and Double Number Line Diagrams to Plots on the Coordinate Plane, Lesson 15: A Synthesis of Representations of Equivalent Ratio Collections are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 1, Topic C, Lesson 16: From Ratios to Rates, Lesson 17: From Rates to Ratios, Lesson 18: Finding a Rate by Dividing Two Quantities, Lesson 19: Comparison Shopping-Unit Price and Related Measurement Units, Lesson 20: Comparison Shopping-Unit Price and Related Measurement Units, are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 1, Topic D, Lesson 24: Percents and Rates per 100, Lesson 25: A Fraction as a Percent, Lesson 26: Percent of a Quantity, Lesson 27: Solving Percent Problems, Lesson 28: Solving Percent Problems, Lesson 29: Solving Percent Problems are lessons that pertain to this cluster.
- Georgia Standards of Excellence Curriculum Frameworks, Grade 6, Unit 2: Rate, Ratio, and Proportional Reasoning Using Equivalent Fractions has many tasks that address this cluster.
- Illustrative Mathematics has two units pertaining to this cluster, Unit 6.2 Introducing Ratios and Unit 6.3: Unit Rates and Percentages.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

General Resources

- Arizona 6-7 Progressions on Ratios and Proportional Relationships is an informational document for teachers. This cluster is addressed on pages 5-7.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.RP.1-3)

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STANDARDS	MODEL CURRICULUM (6.NS.1)
THE NUMBER SYSTEM **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models[G] and equations to represent the problem. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ pound of chocolate equally? How many $3/4$ cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?*	## Expectations for Learning In previous grades, students divided unit fractions by whole numbers and divided whole numbers by unit fractions. This is the first time students divide fractions by fractions. The learning in this standard should be focused on interpreting and computing quotients using visual models (not an algorithm) and to develop an understanding of the division of fractions and solving real-world problems. In future grades, students will extend this understanding to computation with rational numbers. ### ESSENTIAL UNDERSTANDINGS • There are two meanings of division: partitive and measurement. • Partitive problems are sharing problems and rate problems. (Rate problems are not always partitive problems.) • Measurement problems are repeated subtraction or equal groups. • There is a relationship between multiplication and division that can be seen using visual models. ### MATHEMATICAL THINKING • Draw a picture or create a model to make sense of mathematical and real-world problems. • Compute using strategies or models. • Interpret and explain a model to solve mathematical and real-world problems. • Use a pattern or structure. • Compute accurately and efficiently with grade-level numbers. ### INSTRUCTIONAL FOCUS **Visual Models** • Recognize and interpret a visual model for division of a fraction by a fraction. • Create and use a visual model for division of a fraction by a fraction. • Divide a fraction by a fraction using visual models. • Use visual models to show the relationship between the multiplication and division of fractions. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.NS.1)
	## Expectations for Learning, continued **INSTRUCTIONAL FOCUS, CONTINUED** **Equations** - Create a story context to demonstrate understanding of dividing a fraction by a fraction. - Use numerical equations to show the relationship between the multiplication and division of fractions. - Use a numerical equation to represent a real-world problem involving the division of a fraction by a fraction. - Explore patterns and visual models of dividing a fraction by a fraction to discover the relationship between multiplication and division to explain that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$. ## Content Elaborations - Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38 - Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5 - Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7 - Ohio's K-8 Learning Progressions, The Number System, pages 16-17 **CONNECTIONS ACROSS STANDARDS** - Write expressions and equations to solve real-world problems involving non-negative rational numbers (6.EE.7).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on <u>instruction</u> not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

> **Standards for Mathematical Practice**
> *This cluster focuses on but is not limited to the following practices:*
> **MP.1** *Make sense of problems and perservere in solving them.*
> **MP.2** *Reason abstractly and quantitatively.*
> **MP.4** *Model with mathematics.*
> **MP.5** *Use appropriate tools strategically.*

Computation with fractions is best understood when it builds upon the familiar understandings of whole numbers and is paired with visual representations. Students can solve a simpler problem with whole numbers, and then use the same steps to solve a fraction divided by a fraction. Looking at the problem through the lens of "How many groups?" or "How many in each group?" helps visualize what is being sought. For example, $12 \div 3$ means: "How many groups of three would make 12?" or "How many in each of the 3 groups would make 12?" Thus $\frac{7}{2} \div \frac{1}{4}$ can be solved the same way. Dividing by a quarter means to find how many $\frac{1}{4}$s there are in $\frac{7}{2}$?

It may be beneficial to begin instruction on the division of fractions by reviewing the Grade 5 concept of dividing a whole number by a fraction. Moving forward to fractions divided by a fraction with the same denominators, and then to fractions divided by fractions with different denominators. It is vital that students have a chance to use models and see fractional problems within a real-world context, so they can see the relationships between the numbers.

This cluster should be connected to 6.EE.7 where students solve equations involving fraction division.

TIP! Avoid teaching "invert and multiply" or "keep, change, flip" as it leads to confusion as to when to apply the shortcut. Student oftentimes invert the dividend instead of the divisor or even invert both numbers. Instead build understanding of the "why" behind the division of fractions.

Students may incorrectly believe that dividing by $\frac{1}{2}$ is the same as dividing in half. Dividing by half means to find how many $\frac{1}{2}$s there are in a quantity, whereas, dividing in half means to take a quantity and split it into two equal parts. Thus 7 divided by $\frac{1}{2}$ = 14 and 7 divided in half equals $3\frac{1}{2}$. Have students model these two types of problems, so they see the difference.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Students often incorrectly think that multiplication makes something bigger and division makes something smaller; however, this thinking does not hold true for fraction operations. See 4.NF and 5.NF Model Curriculum for more information.

FRACTIONAL LANGUAGE

It has been found that language surrounding fractions can create misconceptions. For example, when teachers describe a fraction as reduced, students may think that the fraction is smaller and not necessarily equivalent. This also holds for simplified form, for what is simple in some situations is not always the simplest form for another situation. Formerly, fractions used to be called "proper" and "improper," but there is nothing improper about fractions where the numerator is greater than the denominator. In fact, fractions where the numerator is greater than the denominator is one of the "proper" ways of writing slope. Instead of using language that creates misconceptions, emphasize equivalent fractions. See https://mathematicalmusings.org/forums/topic/the-term-improper-fractions/ for more information.

ATTENDING TO THE REFERENT UNIT (UNIT WHOLE)

Unlike the addition and subtraction of fractions, multiplication and division of fractions require students to conceptualize multiple units and view a quantity in two ways. A barrier to understanding fractions can be that students have difficulty recognizing that a given amount can be simultaneously referring to two different referent units. Therefore, it is important to help students attend to the referent unit. One way to do this is to explicitly draw attention to the whole when discussing fractions.

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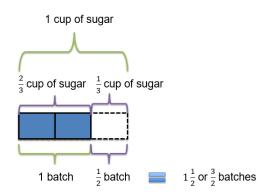
EXAMPLE
Part 1
Chandra wants to make cookies for the school's bake sale and only has 1 cup of sugar in the pantry. She found eight different recipes for a batch of cookies where each requires a different amount of sugar.

- Recipe 1: $\frac{1}{2}$ cup
- Recipe 2: $\frac{1}{4}$ cup
- Recipe 3: $\frac{2}{3}$ cup
- Recipe 4: $\frac{3}{5}$ cup

- Recipe 5: 2 cups
- Recipe 6: 3 cups
- Recipe 7: $1\frac{1}{3}$ cups
- Recipe 8: $2\frac{3}{4}$ cups

- **a.** For each recipe, draw a model to illustrate the situation.
- **b.** Identify the units and the whole(s) in each problem.
- **c.** Find out exactly how many batches she can make for each recipe using 1 cup of sugar.
- **d.** Explain how to figure out the exact batches without drawing a model? Explain.

Discussion: Students who struggle with this problem often have difficulty viewing one segment as part of a cup of sugar and also part of a batch simultaneously. The same quantity is represented in two ways. Once students have discovered the patterns of the reciprocals, ask students to generalize, "How many $\frac{a}{b}$s are in 1?" or "$1 \div \frac{a}{b} = ?$" Students should come to the conclusion that the reciprocal tells how many of each original number are in one whole.
Example continued on next page

1 cup of sugar

$\frac{2}{3}$ cup of sugar $\frac{1}{3}$ cup of sugar

1 batch $\frac{1}{2}$ batch $1\frac{1}{2}$ or $\frac{3}{2}$ batches

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Part 2
Use the pictures to explain the following equations based on the context of Part 1. Make sure you identify the whole.

Discussion: From here, have students explore other dividends besides 1. Eventually move them towards fractional dividends divided by fractional divisors.

Recipe 1: 1-whole cup of sugar divided into $\frac{1}{2}$-cup needed for the batch makes 2 batches.

Recipe 2: 1-whole cup of sugar divided into $\frac{1}{4}$-cup needed for the batch makes 4 batches.

Recipe 3: 1-whole cup of sugar divided into $\frac{2}{3}$ -cup needed for the batch makes $1\frac{1}{2}$ (or $\frac{3}{2}$) batches.

Recipe 4: 1-whole cup of sugar divided into $\frac{3}{5}$ -cup needed for the batch makes $\frac{2}{3}$ (or $\frac{5}{3}$) batches.

Recipe 5: 1 whole cup of sugar divided into 2 cups needed for the batch makes $1\frac{2}{3}$ (or $\frac{5}{3}$) batches.

Recipe 6: 1 whole cup of sugar divided into 3 cups needed for the batch makes $1\frac{2}{3}$ (or $\frac{5}{3}$) batches.

Recipe 7: 1 whole cup of sugar divided into $1\frac{1}{3}$ cups needed for the batch makes $1\frac{2}{3}$ (or $\frac{5}{3}$) batches.

Recipe 8: 1 whole cup of sugar divided into $2\frac{3}{4}$ cups needed for the batch makes $1\frac{2}{3}$ (or $\frac{5}{3}$) batches.

When students are ready move them from the context of sugar and batches to more abstract descriptions such as 1-whole divided into $\frac{2}{3}$ makes $1\frac{1}{2}$ (or $\frac{3}{2}$) $\frac{2}{3}$-sized pieces.

Recipe 1: $1 \div \frac{1}{2} = 2$

Recipe 2: $1 \div \frac{1}{4} = 4$

Recipe 3: $1 \div \frac{2}{3} = \frac{3}{2}$

Recipe 4: $1 \div \frac{3}{5} = \frac{5}{3}$
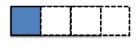
Recipe 5: $1 \div 2 = \frac{1}{2}$

Recipe 6: $1 \div 3 = \frac{1}{3}$

Recipe 7: $1 \div 1\frac{1}{3} = \frac{3}{4}$

Recipe 8: $1 \div 2\frac{3}{4} = \frac{4}{11}$

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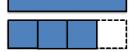
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

Jack has $4\frac{1}{4}$ hours before he has to go to practice. If it takes him $\frac{3}{4}$ of an hour to play a video game. How many video games can he play before he has to leave?

Example: $4\frac{1}{4} \div \frac{3}{4}$

Step 1: Represent the problem.

Step 2:

Discussion: Students should choose which model to use keeping track of the units. One type of model is shown. Notice how $\frac{3}{4}$ of an hour equals one game is clearly marked. There are 5 full $\frac{3}{4}$-hours (or full games) and $\frac{2}{3}$ of $\frac{3}{4}$-hour (or game). Therefore, Jack can play $5\frac{2}{3}$ games.

Difficulties in fraction division problems appear when students have to relate the leftover amount to the amount per group. Students may confuse the unit of measure for a group with other units of measure present in the problem. Help students learn to reason about the relationship between the leftover amount and the amount per group. Sequence problems so that they increase in complexity.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

Create a model to represent the situation.

a. It takes $\frac{5}{8}$ can of paint to paint a wall. How many walls can I paint with $5\frac{5}{8}$ cans of paint?

b. It takes $\frac{5}{8}$ can of paint to paint a wall. How many walls can I paint with $4\frac{1}{2}$ cans of paint?

c. It takes $\frac{5}{8}$ can of paint to paint a wall. How many walls can I paint with $10\frac{3}{8}$ cans of paint?

d. It takes $\frac{5}{8}$ can of paint to paint a wall. How many walls can I paint with $5\frac{1}{2}$ cans of paint?

e. It takes $\frac{5}{8}$ can of paint to paint a wall. How many walls can I paint with $10\frac{11}{16}$ cans of paint?

(Empson & Levi, 2011, Extending Children's Mathematics: Fractions and Decimals)

Discussion:

- For part **a.** 9 walls can be painted with no leftover paint.
- For part **b.** $7\frac{1}{5}$ walls can be painted. After 7 walls have been painted, $\frac{1}{8}$ can is leftover ($\frac{1}{8}$ is $\frac{1}{5}$ of $\frac{5}{8}$).
- For part **c.** $16\frac{3}{5}$ walls can be painted. After 16 walls have been painted $\frac{3}{8}$ can is left over ($\frac{3}{8}$ is $\frac{3}{5}$ of $\frac{5}{8}$).
- For part **d.** $8\frac{4}{5}$ walls can be painted. After 8 walls have been painted, $\frac{1}{2}$ of a can is left over ($\frac{1}{2}$ is $\frac{4}{5}$ of $\frac{5}{8}$).
- For part **e.** $17\frac{1}{10}$ walls can be painted. After 17 walls have been painted, there is $\frac{1}{16}$ can of paint left over ($\frac{1}{16}$ is $\frac{1}{10}$ of $\frac{5}{8}$).

MODELS
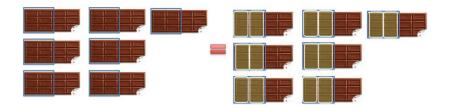
A variety of models are available to students. It is important for students to be exposed to many different types of models, and they should be allowed to use the tool that best models the problem and makes sense to them. It may be helpful for students to use manipulatives such as snap cubes or fraction bars to begin exploring fraction division. Using grid paper for modeling or folding strips could also be helpful for students to make the mathematics visible to them.

Area models can be used for division of fractions. (See 5.NF.3-7 for more information about area models for fraction multiplication and division.) Pattern blocks are another example of an area model. Length models can also be used. Fraction bars are an example of a length model. Fractions bars transition nicely into a tape diagram. Division of fractions can also be modeled using a number line.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

Maya's little sister ate half of 7 Hershey's bars. Maya has $\frac{7}{2}$ of Hershey bars left. If she wants to use $\frac{1}{4}$ of a bar for each s'more she is making. How many s'mores can she make?

Discussion: This picture shows $\frac{7}{2} \div \frac{1}{4}$. The picture on the left side of the equal sign represents $\frac{7}{2}$. The image on the right side of the equal sign represents those pieces and were taken and broken into $\frac{1}{4}$-sized pieces. Therefore, 14 smore's can be made from $\frac{7}{2}$ Hershey bars.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

$2\frac{2}{3} \div \frac{1}{2}$ can be interpreted as "How many $\frac{1}{2}$'s are there in $2\frac{2}{3}$?"

Step 1: Model $2\frac{2}{3}$ using pattern blocks.

Step 2: Figure out how many $\frac{1}{2}$s there are in $2\frac{2}{3}$.

Step 3: Interpret the remainder.

Therefore, $2\frac{2}{3} \div \frac{1}{2} = 5\frac{1}{3}$.
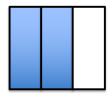
Discussion: Emphasize that the hexagon is the whole. Clarify in words that $2\frac{2}{3} \div \frac{1}{2}$ is five halves and $\frac{1}{3}$ of one half.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE
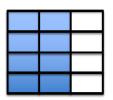
$\frac{2}{3} \div \frac{3}{4}$ can be interpreted as "How many $\frac{3}{4}$'s are there in $\frac{2}{3}$?"

Method 1

Step 1: Represent the whole.
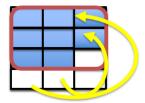
Step 2: Divide the whole into fourths to get a common denominator.

Step 3: Ask "How many parts is $\frac{3}{4}$ of 1 whole?"

Step 4: Ask "How many $\frac{3}{4}$ are in $\frac{2}{3}$?" or "How many $\frac{9}{12}$ are in $\frac{8}{12}$?" or "How many 9s are in 8?
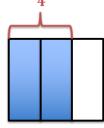
Therefore, $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

Discussion: Encourage students to use grid paper. There is not a whole $\frac{3}{4}$ in $\frac{2}{3}$. There is only part of a $\frac{3}{4}$ in $\frac{2}{3}$, $\frac{8}{9}$ parts to be exact. Relate back to the equivalent multiplication problem with an accompanying area model.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)
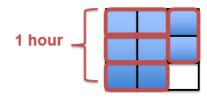
Method 2

It takes $\frac{3}{4}$ of an hour to do $\frac{2}{3}$ of your homework, which can be represented by $\frac{2}{3} \div \frac{3}{4}$. How much homework can you do in an hour?
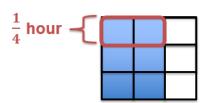
Step 1: Represent the whole. $\frac{3}{4}$ **hour**

Step 3: Multiply by 4 to find the whole hour.

1 hour

Therefore, $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

Step 2: Divide the whole by 3 to find $\frac{1}{4}$ hour.

$\frac{1}{4}$ **hour**

Discussion: An area model makes more sense when presented in a context. Notice this is the same division problem as the previous one, but it is represented differently. This shows that the same division situation may even be modeled differently (even using the same type of model) depending on the context or the way a student thinks about a problem. This is why it is important to use contexts when teaching the division of fractions and allow students to use their own strategies. Notice that both models illustrate something different about the division of fractions. The model in Method 1 illustrated the common denominator method. The model in Method 2 illustrates leads to the traditional algorithm strategy.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

$2\frac{1}{3} \div \frac{1}{2}$ can be interpreted as "How many $\frac{1}{2}$'s are there in $2\frac{1}{3}$?"

Step 1: Model $2\frac{1}{3}$ using fraction bars.

1	1	$\frac{1}{3}$
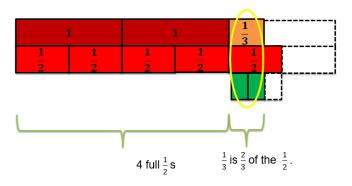
Step 2: Figure out how many $\frac{1}{2} - sized$ pieces there are in $2\frac{2}{3}$.

4 full $\frac{1}{2}$ s   Part of a $\frac{1}{2}$

> There are 4 full $\frac{1}{2} - sized$ pieces and part of a $\frac{1}{2} - sized$ piece

Step 3: Figure out how much of the part of a $\frac{1}{2} - sized$ piece is there.

4 full $\frac{1}{2}$ s $\frac{1}{3}$ is $\frac{2}{3}$ of the $\frac{1}{2}$.

> Therefore, there are $4\frac{2}{3}$ $\frac{1}{2} - sized$ pieces in $2\frac{1}{3}$

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE
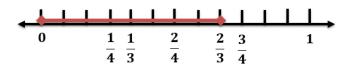
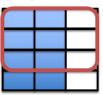$\frac{2}{3} \div \frac{3}{4}$ can be interpreted as "How many $\frac{3}{4}$'s are there in $\frac{2}{3}$?"

Step 1: Show $\frac{2}{3}$ on a number line.

Step 2: Mark off $\frac{3}{4}$ on the same number line. (To put $\frac{2}{3}$ and $\frac{3}{4}$ on the same line divide the line into $\frac{1}{12}$ s which is the common denominator.)

Step 3: Highlight $\frac{3}{4}$.

Step 4: Ask "How much of $\frac{3}{4}$ is $\frac{2}{3}$?"

Therefore, $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

ESTIMATING WITH DIVISION OF FRACTIONS

It might be helpful to have students estimate before dividing. This will allow them to develop number sense. Ask them questions such as "Will the solution (or quotient) to the division problem be greater than or less than the dividend? Explain how you know." Or "Will the solution (or quotient) to the division problem be greater than or less than 1? Explain how you know."

RELATIONAL UNDERSTANDING OF FRACTIONS

Fractional quantities are relational. Understanding relational thinking with respect to fractions is essential. Teachers need to recognize and foster this type of thinking. Relational thinking involves expressing a number in terms of other numbers and/or operations (applying the properties of operation and equality). In previous grades students focused on the relational understanding of unit fractions and began exploring the relational understanding of fractions as composites especially with respect to multiplication. Now students are expanding their relational thinking towards the division of a fraction by a fraction.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Emphasizing the part-to-whole relationship of fractions can lead to confusion when students encounter fractions greater than one including the form where the numerator is greater than the denominator.

It is important to explicitly draw attention to relational thinking. The key to fostering relational thinking is to—
- Choose tasks intentionally and thoughtfully including contexts students can relate to.
- Focus on particular strategies used in combination with the mathematics.
- Facilitate communication among students.
- Continually assess student thinking (informal and ongoing), and use it as a guide for further instruction.

As relational thinking develops students should move towards multiplicative thinking.
(Empsen & Levi, 2011, Extending Children's Mathematics)
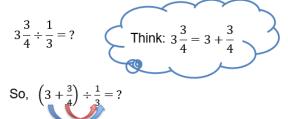
EXAMPLE
- Akino had an 8-inch by 10-inch photograph. She reduced it to $\frac{3}{4}$ of its original size. What are its new dimensions?
- Her mom forgot that she reduced it and bought a frame for the original size. What fraction of its present size should she program the computer to restore it to its original size.
- How do the two fractions relate to each other? Explain.

Discussion: When Akino reduces the photograph the new dimensions become 6-inch by $7\frac{1}{2}$-inch photograph. Many students will think that they can just multiply the reduced dimensions by $\frac{3}{4}$, but instead they have to divide by $\frac{3}{4}$ which is the same as multiplying by $\frac{4}{3}$.

PROPERTIES OF OPERATION AND FRACTIONAL DIVISION
Remind students that they should still apply the properties of operation when dividing fractions. They can compose and recompose numbers using the Distributive Property to help them solve problems mentally.

$$3\frac{3}{4} \div \frac{1}{3} = ?$$

Think: $3\frac{3}{4} = 3 + \frac{3}{4}$

So, $\left(3 + \frac{3}{4}\right) \div \frac{1}{3} = ?$

$3 \div \frac{1}{3} = 9$ **and** $\frac{3}{4} \div \frac{1}{3} = \frac{9}{4}$ *or* $2\frac{1}{4}$

Therefore, $9 + 2\frac{1}{4} = 11\frac{1}{4}$

TIP! Students often incorrectly apply the Commutative Property to division.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

USING REAL-WORLD CONTEXTS

Problems involving the division of fractions should be given in real-world contexts, so that students can create meaning about dividing a fraction by a fraction. Students should also be challenged to write stories given a division expression. Incorporate vocabulary terms such as numerator, denominator, product, quotient, factor, dividend, and divisor into instruction.

EXAMPLE

Devante has $\frac{4}{5}$ of a cell phone battery left, and it drains another $\frac{1}{4}$ every hour. How many hours will his battery last?

EXAMPLE

A rectangle is $3\frac{1}{3}$ square centimeters. If the length is $1\frac{1}{4}$ centimeters, find the width.

EXAMPLE

Amelia runs $6\frac{1}{2}$ miles per hour. How long time will it take her to run a half-Marathon, which is $13\frac{1}{10}$ mile?

Discussion: The solution is $2\frac{1}{65}$ hours. Discuss the meaning of $\frac{1}{65}$. Ask students what unit it is and discuss whether $\frac{1}{65}$ of an hour is more or less than one minute?

It may be beneficial to give students many opportunities to write fractional division word problems. Some students may not truly understand what a fractional division problem looks like and may write problems that are actually multiplication, subtraction, or even addition problems. Students should share their stories during discussion. This is a good opportunity for both the teacher and the student to evaluate students' understanding of division.

EXAMPLE

Write a story illustrating $\frac{3}{4} \div \frac{1}{3}$. Then draw a model to solve your problem. Be sure to interpret your remainder in the context of your story.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

TYPES OF DIVISION SITUATIONS

Division can be approached from two perspectives. See Ohio's Learning Standards Table 2. The first is Group Size Unknown (partitive). Group Size Unknown problems can be thought of as sharing or rate problems. They ask questions such as "How many will each person get?" "How much is one?" "How much will one friend get?" "How many kilometers did she go in one hour?" The second is Number of Groups Unknown (measurement or repeated subtraction). They ask questions such as "How many groups are there? "How many times as much does the popcorn cost compared to the Snowcaps?" Students should see examples of both. Number of Groups Unknown problems can be thought of as repeated subtraction or equal groups. Different models lend themselves to either partitive or measurement interpretations of division. *Note: Students do not need to know the names of the two different types nor be able to distinguish between the two types. This information is to help teachers ensure that they give students a variety of division problems.*

Group Size Unknown Interpretation of Division (Partitive).

Group Size Unknown (partitive) problems are known as equal sharing or rate problems because the amount per group is unknown. Students solve by distributing or dealing out. *Note: Rate problems are not always partitive problems.*

TIP!   Students who are taught to write the equations before solving conceptually tend to struggle with partitive division problems.

EXAMPLE

Kenzie has $8\frac{2}{3}$ yards of fabric. If she needs to make 6 purses, how much fabric can she use for each purse?

Discussion: Despite the fact that this example has a measure of fabric, this is an example of the Group Size Unknown (partitive) interpretation of division since the question answers "How much for one purse?"

EXAMPLE

A pitcher containing $4\frac{2}{3}$ cups of Kool-Aid will fill $2\frac{1}{2}$ glasses. How much will each glass hold?

EXAMPLE

A rubber band is stretched to $13\frac{3}{4}$ cm long and that is $2\frac{1}{2}$ times as long as it was initially. How long was the rubber band initially?

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

TIP! Using only the Group Size Unknown (partitive) model of division in earlier grades leads to student misunderstandings. They may think that the divisor must be a whole number; the divisor must be less than the dividend; or the quotient must be less than the dividend. This creates misunderstandings for students when trying to apply division to situations involving fractions.

After having students do several examples of Group Size Unknown problems. Have them discuss how the problems look alike.

Number of Groups Unknown Interpretation of Division (Measurement or Repeated Subtraction)
Number of Groups Unknown (Measurement, Repeated Subtraction or Quotative) problems are known as repeated subtraction or equal groups. In Number of Groups Unknown interpretation of division, the amount per group is known and the number of groups is unknown. *Note: A measurement situation is different than the measurement interpretation of division, which is why it may be better to refer to this situation as the Number of Groups Unknown.*

EXAMPLE

A retirement home serves pudding for dessert. Each guest gets $\frac{2}{3}$ cup of pudding. If containers hold 6 cups, how many guests can be served from one container?

Discussion: This is an example of a division problem that uses the measurement interpretation because the number of groups is unknown. It answers questions such as "How many?"

EXAMPLE

Julia is making bows. If she has $3\frac{1}{4}$ feet of ribbon and each bow takes $\frac{5}{8}$ feet of ribbon, how many bows can she make?

Discussion: This is a good opportunity to compare the mathematical response to the real-world situation. Mathematically, the solution is 5 and $\frac{1}{5}$; However, in the real-world the remainder would be discarded. Many students will also misinterpret the remainder and think that she can make 5 bows and have $\frac{1}{5}$ of foot left over, where she can really make 5 and $\frac{1}{5}$ bows.

EXAMPLE

A rubber band is initially $5\frac{1}{2}$ cm long. Now it is stretched to $13\frac{3}{4}$ cm. How many times as long is the rubber band now as it was initially?

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION

It is important to illustrate the relationship between multiplication and division. Reviewing concepts from previous grades involving whole numbers and unit fractions might be helpful.

EXAMPLE
   a. Draw a picture using grid paper representing each pair of equations.
   b. How do your pictures compare?

- $24 \div 3 =?$ And $24 \times \frac{1}{3} =?$
- $24 \div 4 =?$ And $24 \times \frac{1}{4} =?$
- $24 \div 6 =?$ And $24 \times \frac{1}{6} =?$
- $24 \div \frac{2}{3} =?$ And $24 \times \frac{3}{2} =?$
- $24 \div \frac{3}{4} =?$ And $24 \times \frac{4}{3} =?$
- $24 \div \frac{6}{5} =?$ And $24 \times \frac{5}{6} =?$

Discussion: It may be beneficial to have students make their own rectangles on graph paper, so they can keep track of the unit referent when they multiply by a number greater than 1. It may also be helpful to modify this activity to include real-world contexts such as two dozen donuts. Then students can compare how both the multiplication and division equation represent the situation. Another model that could be used to illustrate the same point would be fraction bars or tape diagrams. Use a variety of models in the classroom to enable students to make connections.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

   a. Draw a picture to solve each situation, and write a corresponding equation to model the situation.

       o If everyone eats $\frac{1}{5}$ of a pizza, how much will 8 people eat all together?

       o If $1\frac{3}{5}$ pizzas are shared equally among 8 people, how much will everyone get?

       o If $1\frac{3}{5}$ pizzas are shared so each person gets $\frac{1}{5}$ of a pizza, how many people will get a share.

   b. How are the three problems related? Explain.
   c. How is whole-number division and fractional division related? Explain.
   d. How is multiplication related to division? Explain.
   e. Could you use multiplication to solve a division problem? Explain.

See Model Curriculum 5.NF.4-7 for more information about the multiplication and division of fractions.

Give students many opportunities to discover connections between multiplication and division. For example, $\frac{5}{8} \div \frac{3}{4} = \frac{5}{6}$ because $\frac{3}{4}$ of $\frac{5}{6}$ is $\frac{15}{24}$ or $\frac{5}{8}$. There are several ways to relate multiplication to division.

To reinforce the relationship between multiplication and division, have students write the division equation and its corresponding multiplication equation that represents each situation. It may be helpful to have them write a 3rd statement that shows the equivalence of the two expressions.

Through exploration students should make the connection when dividing a number by the fraction $\frac{b}{c}$, the denominator indicates how many halves, thirds, fourth, etc. there are, and the numerator indicates the size. Therefore, the process involves dividing the dividend by the numerator b and then multiplying the dividend by the denominator c. Group Size Unknown (partitive) examples illustrate this nicely.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

Asher has $1\frac{1}{2}$ oranges which makes $\frac{3}{5}$ of an adult serving. How many oranges (and parts of an orange) make up 1 adult serving?

(Taken from Kribs-Zaleta, 2008)

Example: $\frac{3}{2} \div \frac{3}{5}$

Step 1: Represent the dividend.   **Step 2:** Divide by the numerator.   **Step 3:** Multiply by the denominator.

$\frac{3}{5}$ **of a serving**

$\frac{1}{5}$ **of a serving** $\div 3$

How much is $\frac{1}{5}$ of a serving? I can divide by 3.
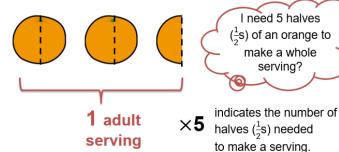
indicates the size of $\frac{1}{5}$ which is $\frac{1}{2}$ (or $\frac{1}{3}$ of the oranges).

1 adult serving $\times 5$

I need 5 halves ($\frac{1}{2}$s) of an orange to make a whole serving?

indicates the number of halves ($\frac{1}{2}$s) needed to make a serving.
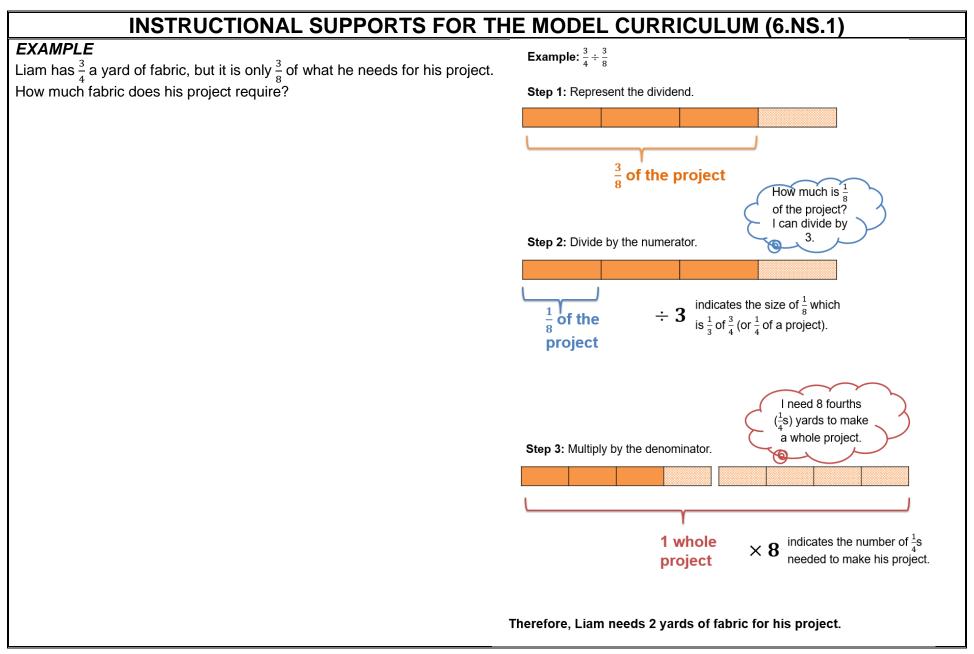
Therefore, $\frac{5}{2}$, or $2\frac{1}{2}$ oranges make 1 adult serving.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

Tawanna can mow $\frac{1}{4}$ acre in $\frac{2}{3}$ of an hour. How much can she mow in a full hour?

Example: $\frac{1}{4} \div \frac{2}{3}$

Step 1: Represent the dividend.

$\frac{2}{3}$ **of an hour**

How much is $\frac{1}{3}$ of an hour? I can divide by 2.

Step 2: Divide by the numerator.

$\frac{1}{3}$ **of an hour**　　$\div\, 2$　indicates the size of $\frac{1}{3}$ which is $\frac{1}{2}$ of $\frac{1}{4}$ (or $\frac{1}{8}$ of an acre).

I need 3 eighths ($\frac{1}{8}$s) of an acre to make a whole hour.

Step 3: Multiply by the denominator.

1 whole hour　　$\times\, 3$　indicates the number of halves ($\frac{1}{2}$s) need to make a full hour.

Therefore, Tawanna can mow $\frac{3}{8}$ of an acre in 1 hour.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE

Liam has $\frac{3}{4}$ a yard of fabric, but it is only $\frac{3}{8}$ of what he needs for his project.
How much fabric does his project require?

Example: $\frac{3}{4} \div \frac{3}{8}$

Step 1: Represent the dividend.

$\frac{3}{8}$ **of the project**

How much is $\frac{1}{8}$ of the project? I can divide by 3.

Step 2: Divide by the numerator.

$\frac{1}{8}$ **of the project**

$\div\ 3$ indicates the size of $\frac{1}{8}$ which is $\frac{1}{3}$ of $\frac{3}{4}$ (or $\frac{1}{4}$ of a project).

I need 8 fourths ($\frac{1}{4}$s) yards to make a whole project.

Step 3: Multiply by the denominator.

1 whole project

$\times\ 8$ indicates the number of $\frac{1}{4}$s needed to make his project.

Therefore, Liam needs 2 yards of fabric for his project.

Grade 6

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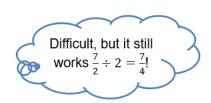# INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

TIP! Substituting whole numbers in place of fractions in a problem temporarily may help students clarify to themselves which operation to use: multiplication or division.

METHODS FOR DIVIDING FRACTIONS
Dividing Across Fractions
Like multiplying fractions, it is legitimate to divide across any two fractions (numerator divided by numerator and denominator divided by denominator). This is an intuitive approach, since students are used to multiplying across the numerators and denominators. Start by giving students problems in real-world contexts that have common denominators or that are easily divisible. Encourage students to draw a model and connect it to division. Once students become proficient at dividing across, give them a more difficult problem that is not as easily divisible. Students will realize the need for another strategy and some may even come up with the common denominator strategy, which will allow them to continue to divide across numerators and denominators. *Note: Students at this level can leave fractions in the form of $\frac{3.5}{2}$.*

$$\frac{9}{20} \div \frac{3}{4} = \frac{3}{5}$$

$$\frac{6}{7} \div \frac{2}{7} = \frac{3}{1}$$

$$\frac{8}{9} \div \frac{4}{3} = \frac{2}{3}$$

$$\frac{7}{10} \div \frac{2}{5} = \frac{7}{2}$$

Difficult, but it still works $\frac{7}{2} \div 2 = \frac{7}{4}$!

$$\frac{7}{9} \div \frac{2}{3} = ?$$

$$\frac{7}{9} \div \frac{4}{9} = ?$$

$$7 \div 4 = \frac{7}{4}$$

The Common Denominator Method
One viable method for dividing fractions is converting the dividend and divisor to a common denominator before dividing. This is an intuitive method that makes sense to many students.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE
   a. Divide. Write each of your solutions as a number and in words.
   b. Rewrite each division problem as fractions, and then divide.
   c. What patterns do you notice?

- $12 \div 3 =$ _____   ___ $\div$ ___ = ___
- $12\ ones \div 3\ ones =$ _____   ___ $\div$ ___ = ___
- $12\ tens \div 3\ tens =$ _____   ___ $\div$ ___ = ___
- $12\ hundreds \div 3\ hundreds =$ _____   ___ $\div$ ___ = ___
- $12\ millions \div 3\ millions =$ _____   ___ $\div$ ___ = ___
- $12\ tenths \div 3\ tenths =$ _____   ___ $\div$ ___ = ___
- $12\ hundredths \div 3\ hundreths =$ _____   ___ $\div$ ___ = ___
- $12\ fourths \div 3\ fourths =$ _____   ___ $\div$ ___ = ___
- $12\ fifteenths \div 3\ fifteenths =$ _____   ___ $\div$ ___ = ___

Discussion: For example, for the third bullet students could write "12 tens divided into groups of 3 tens" or "12 tens divided into 3 groups."

Dividing Fractions by Multiplying by the Inverse
It is *unhelpful* to teach students to flip and multiply. When students learn the algorithm without understanding why it works, they oftentimes misapply it. It is more beneficial to have students discover the algorithm through exploration using situations that drive them to divide by the numerator and then multiply by the denominator. Partitioning activities can make this concept evident. After doing enough of these types of activities, students should naturally discover the connection to multiplying by the inverse. Once students come to that realization, highlight the connection to the Inverse Property of Multiplication.

Provide a series of tasks and have students look for patterns. Begin with situations where the divisor is a unit fraction. Then move towards problems where the divisor is not a unit fraction but the denominators are the same. Then progress towards more difficult situations.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE
Part 1

- How many $\frac{1}{2}$ -cup servings are in a 3-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{3}$ -cup servings are in a 3-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{4}$ -cup servings are in a 3-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{5}$-cup servings are in a 3-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{8}$ -cup servings are in a 3-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{10}$ -cup servings are in a 3-cup container?   ___ ÷ ___ = ___

    a. For the following situations write a corresponding division equation. Then solve the equation using a model. (Write your solutions as a fraction that is not a mixed number.)
    b. Without drawing a picture, predict how many $\frac{1}{35}$-servings would be in a 3-cup container.
    c. Without drawing a picture, what do you predict the answer would be for the equation $3 \div \frac{1}{9} =$ ___? Explain how you made your prediction.
    d. Without drawing a picture, what do you predict the answer would be for the equation $5 \div \frac{1}{6} =$ ___? Explain how you made your prediction.
    e. Write a rule for dividing a whole number by a unit fraction. Explain why it works.

Example continued on next page

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Part 2

- How many $\frac{1}{4}$-cup servings are in a $\frac{3}{4}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{5}$-cup servings are in a $\frac{3}{5}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{5}$-cup servings are in a $\frac{4}{5}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{8}$-cup servings are in a $\frac{3}{8}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{8}$-cup servings are in a $\frac{5}{8}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{8}$-cup servings are in a $\frac{7}{8}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{1}{10}$-cup servings are in a $\frac{9}{10}$-cup container?   ___ ÷ ___ = ___

   a. For the following situations write a corresponding division equation. Then solve using a model. (Write your solutions as fractions that are not mixed numbers.)
   b. Without drawing a picture, predict how many $\frac{1}{12}$-cup servings would be in a $\frac{7}{12}$-cup container.
   c. Without drawing a picture, what do you predict the answer would be for the equation $\frac{6}{11} \div \frac{1}{11} =$ ___? Explain how you made your prediction.
   d. Without drawing a picture, what do you predict the answer would be for the equation $\frac{4}{7} \div \frac{1}{7} =$ ___? Explain how you made your prediction.
   e. Would your methods for finding the solution for **c.** and **d.** work for a problem such as $\frac{4}{7} \div \frac{4}{5} =$ ___? Explain.
   f. Write a rule for dividing a fraction by a unit fractions. Explain.
   g. How does your rule compare to your rule in Part 1 and Part 2?

Example continued on next page

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Part 3

- How many $\frac{3}{4}$ -cup servings are in a 3 -cup container?   ____ ÷ ____ = ____
- How many $\frac{3}{5}$ -cup servings are in a 3 -cup container?   ____ ÷ ____ = ____
- How many $\frac{3}{10}$ -cup servings are in a 3-cup container?   ____ ÷ ____ = ____
- How many $\frac{2}{3}$ -cup servings are in a 2 -cup container?   ____ ÷ ____ = ____
- How many $\frac{2}{5}$ -cup servings are in a 2-cup container?   ____ ÷ ____ = ____
- How many $\frac{4}{5}$ -cup servings are in a 3-cup container?   ____ ÷ ____ = ____

   a. For the following situations write a corresponding division equation. Then solve using a model. (Write your solutions as fractions that are not mixed numbers.)
   b. Without drawing a picture, predict how many $\frac{5}{8}$ -cup servings would be in a 5-cup container.
   c. Without drawing a picture, what do you predict the answer would be for the equation $6 \div \frac{6}{11} =$ ____? Explain how you made your prediction.
   d. Without drawing a picture, what do you predict the answer would be for the equation $4 \div \frac{4}{7} =$ ____? Explain how you made your prediction.
   e. Would your methods for finding the solution for **c.** and **d.** work for a problem such as $3 \div \frac{4}{5} =$ ____?
   f. Write a rule for dividing a whole number by a fraction where the numerator in the dividend matches the whole number. Explain why it works.
   g. How does your rule compare to your rule in Part 1 and Part 2?

Example continued on next page

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Part 4

- How many $\frac{3}{4}$-cup servings are in a $3\frac{3}{4}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{3}{5}$-cup servings are in a $1\frac{4}{5}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{3}{10}$-cup servings are in a $2\frac{1}{10}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{2}{3}$-cup servings are in a $3\frac{1}{3}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{2}{5}$-cup servings are in a $1\frac{4}{5}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{4}{5}$-cup servings are in a $4\frac{2}{5}$-cup container?   ___ ÷ ___ = ___

   a. For the following situations write a corresponding division equation. Then solve using a model. (Write your solutions as fractions that are not mixed numbers.)

   b. Without drawing a picture, predict how many $\frac{5}{8}$-cup servings would be in a $3\frac{1}{8}$-cup container.

   c. Without drawing a picture, what do you predict the answer would be for the equation $1\frac{7}{11} \div \frac{6}{11} =$ ___? Explain how you made your prediction.

   d. Without drawing a picture, what do you predict the answer would be for the equation $2\frac{6}{7} \div \frac{4}{7} =$ ___? Explain how you made your prediction.

   e. Would your methods for finding the solution for **c.** and **d.** work for a problem such as $3\frac{1}{8} \div \frac{4}{5} =$ ___?

   f. Write a rule for dividing a whole number by a fraction where the denominator in the dividend matches the denominator in the divisor. Explain why it works.

   g. How does your rule compare to your rule in Part 1, Part 2, and Part 3?

Example continued on next page

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Part 5

- How many $\frac{2}{3}$-cup servings are in a $\frac{3}{4}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{3}{4}$-cup servings are in a $\frac{2}{3}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{2}{3}$-cup servings are in a $\frac{4}{5}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{4}{5}$-cup servings are in a $\frac{2}{3}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{2}{3}$-cup servings are in a $\frac{4}{5}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{3}{10}$-cup servings are in $\frac{2}{5}$-cup container?   ___ ÷ ___ = ___

- How many $\frac{4}{5}$-cup servings are in $\frac{3}{10}$-cup container?   ___ ÷ ___ = ___

   a. For the above situations write a corresponding division equation. Then solve using a model. (Write your solutions as fractions that are not mixed numbers in the division equation.)
   b. Without drawing a picture, what do you predict the answer would be for the equation $\frac{2}{3} \div \frac{4}{9} =$ ___? Explain how you made your prediction.
   c. Without drawing a picture, what do you predict the answer would be for the equation $\frac{4}{9} \div \frac{2}{3} =$ ___? Explain how you made your prediction.
   d. What do you notice about your solutions to part **b.** and part **c.** Explain why this happens.
   e. Write a rule for dividing a fraction by a fraction. Explain why it works.
   f. How does your rule compare to your rules in Part 1, Part 2, Part 3, and Part 4?
   g. Will your rule work for dividing improper fractions? Explain.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

EXAMPLE
Part 1

- I have $\frac{1}{3}$ of a whole cake. I want to divide it equally into 3 containers. How much cake will be in each container?

- I have $\frac{1}{3}$ of a whole cake. I want to divide it equally into 4 containers. How much cake will be in each container?

- I have $\frac{1}{3}$ of a whole cake. I want to divide it equally into 8 containers. How much cake will be in each container?

- I have $\frac{2}{3}$ of a whole cake. I want to divide it equally into 2 containers. How much cake will be in each container?

- I have $\frac{2}{3}$ of a whole cake. I want to divide it equally into 3 containers. How much cake will be in each container?

- I have $\frac{3}{4}$ of a whole cake. I want to divide it equally into 2 containers. How much cake will be in each container?

Part 2

- I have $\frac{1}{3}$ of a whole cake. It fills up exactly $\frac{1}{2}$ of my container. How much cake will fit in 1 whole container?

- I have $\frac{1}{3}$ of a whole cake. It fills up exactly $\frac{1}{4}$ of my container. How much cake will fit in 1 whole container?

- I have $\frac{3}{4}$ of a whole cake. It fills up exactly $\frac{1}{2}$ of my container. How much cake will fit in 1 whole container?

Example continued on next page

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Part 3

- I have 3 whole cakes. They fill up exactly $\frac{2}{3}$ of my container.

    o How much cake will fit in $\frac{1}{3}$ of my container?

    o How much cake will fit in 1 whole container?

- I have $\frac{1}{2}$ of a cake. It fills up exactly $\frac{3}{4}$ of my container.

    o How much cake will fit in $\frac{1}{4}$ of the container?

    o How much cake will fit in 1 whole container?

- I have $\frac{3}{4}$ of a cake. It fills up exactly $\frac{2}{3}$ of my container.

    o How much cake will fit in $\frac{1}{3}$ of the container?

    o How much cake will fit in 1 whole container?

(Taken from Gregg & Gregg, 2007, Measurement and Fair-Sharing Models for Dividing Fractions)

In both the Number of Groups Unknown or Group Size Unknown interpretation of division, the denominator indicates how many thirds, fourths, fifths, or tenths there are, and the numerator indicates the size of the serving or how many servings. This should lead students to the idea that although one can divide the numerator and multiply the denominator, it may be easier just to flip the divisor.

FRACTIONS AS RATES

A fraction symbol $\frac{c}{d}$ has several meanings:

- Division ($c \div d$)

- Rational Number ($\frac{c}{d}$ of a mile)

- Ratio (c parts of mile compared to d number of hours)

- Rate (c miles per d hours)

Note: There are many differing opinions about the definitions of and differences between ratios and rates, so the emphasis should be on understanding fractions as ratios and rates instead of differentiating between the definitions.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Connect the division of fractions to rates to build proportional understanding in support of 6.RP.1-3.
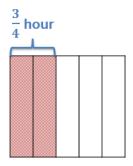
TIP! Students who understand equivalence typically use more sophisticated strategies related to fractions. Emphasize concepts of equivalence in the classroom to help students become more advanced in their thinking.
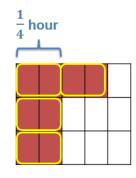
EXAMPLE

It takes one robot vacuum $\frac{3}{4}$ of an hour to clean $\frac{2}{5}$ of the house. What fraction of the house can be cleaned per hour?
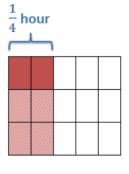
Step 1: Represent the problem.

$\frac{3}{4}$ hour

hours	house
$\frac{3}{4}$	$\frac{2}{5}$

Step 2: Find $\frac{1}{4}$ of an hour.

$\frac{1}{4}$ hour

hours	house
$\frac{3}{4}$	$\frac{2}{5}$
$\frac{1}{4}$	$\frac{2}{15}$

$\div 3$ $\div 3$

Step 3: Find an hour.

$\frac{1}{4}$ hour

hours	house
$\frac{3}{4}$	$\frac{2}{5}$
$\frac{1}{4}$	$\frac{2}{15}$
1	$\frac{8}{15}$

$\div 3$ $\div 3$

$\times 4$ $\times 4$

Therefore, $\frac{2}{5} \div \frac{3}{4} = \frac{8}{15}$.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

OPERATOR INTERPRETATION OF RATIONAL NUMBERS

In the operator interpretation, rational numbers such as fractions can be interpreted as functions. This is different than the part-whole relationships that many students and teachers are used to. The important relationship is the comparison between the two quantities. Rational numbers as operators multiply and divide, shrink (reduce) and enlarge, or contract and expand. (This interpretation sets the stage for mappings in Grade 8.) Operators as transformers—

- Lengthen or shorten line segments;
- Increase or decrease the numbers of items in a set of discrete objects; and/or
- Take a figure in a geometric plane and map it onto a similar figure in the plane.
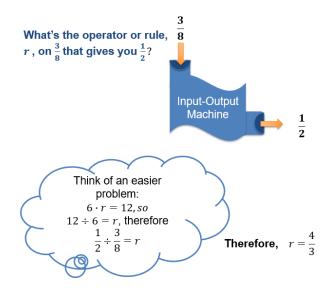
It is different than a part-whole interpretation of fractions. The significance is that it is action oriented.

An input-output table or function machine may also help understand equations that relate multiplication of fractions to division.

The operator explains the relationship between sets.

input	8	12	16	20
output	6	9	12	15

Output = $\frac{3}{4}$ input

What's the operator or rule, $\frac{3}{8}$
r, on $\frac{3}{8}$ that gives you $\frac{1}{2}$?

Input-Output Machine

$\frac{1}{2}$

Think of an easier problem:
$6 \cdot r = 12, so$
$12 \div 6 = r$, therefore
$\frac{1}{2} \div \frac{3}{8} = r$

Therefore, $r = \frac{4}{3}$

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Instructional Tools/Resources
These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology
- Fraction bars
- Pattern blocks
- Grid paper
- Snap cubes
- Divide Fractions is an applet by Visual Fractions that has number line and circle models

Division of Fractions
- Father's Day Blueberry Surprise by Yummy Math is an activity that has students multiply and divide using unit fractions.
- The Kool-Aid Kid by Graham Fletcher is a 3-act task where students can apply the division of fractions.
- Dan's Division Strategy by Illustrative Mathematics is a task that explores a strategy for dividing fractions with the same denominator.
- How Many Containers in One Cup/Cups in One Container? by Illustrative Mathematics is a task that requires students to divide fractions in an opposite order to probe them to think carefully about the meaning of fraction division.
- Making Hot Cocoa Variation 1 and Making Hot Cocoa Variation 2 by Illustrative Mathematics are two tasks that try to make the connection between diagrams and fraction division.
- Cup of Rice by Illustrative Mathematics is a task that requires students to interpret the remainder.
- How Many _____ are in _____? by Illustrative Mathematics is a task where students explore the division of fractions through a logical progression leading them toward the invert-and-multiply algorithm.
- Dividing Fractions by Open Middle is an activity that has students fill in the numerators and denominators of a division expression to find the smallest (or largest) quotient.
- Dividing Mixed Numbers by Open Middle is an activity that has students fill in the blanks of a mixed number a division expression to find the smallest (or largest) quotient.
- Undefined Quotient with Fraction Division by Open Middle is an activity that has students fill in the numerators and denominators of a division expression to find an undefine quotient.
- Nana's Lemonade by Dan Meyer is a 3-Act Task that may encourage students to apply fraction division.
- Traffic James by Illustrative Mathematics is a division of fraction problem that has continuous quantities.
- Standing in Line by Illustrative Mathematics is a task with multiple entry points including dividing a whole number by a unit fraction.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

Division of Fractions, continued

- Interpretations of Fractions published by Shasta County Office of Education that quotes Math Matter from Suzanne H. Chapin and Art Johnson is a teacher resource document that explains the different interpretations of fractions.
- Water in Containers by Open Up Resources is a video that models a situation that can be solved by division of fractions. Questions could include "How can the water be both $\frac{3}{4}$ and $\frac{2}{5}$?" or "How many liters of water fit in the dispenser?"
- Models for the Multiplication and Division of Fractions by Annenberg Learner is a lesson that uses an area model to multiply and divide fractions.
- Dividing Fractions and Dividing Fractions by NZMaths is a lesson on dividing fractions.
- Division of Fractions: Connections Between Fractions and Division by PBS Learning Media is a video that uses fraction bars to divide fractions. It emphasizes the dividing across and the common denominator method. This is for teacher use.
- Use Models for Division of Fractions by Fraction by Learnzillion is a teacher resource to show how to divide fractions using tape diagrams.

Curriculum and Lessons from Other Sources

- EngageNY, Grade 6, Module 2, Topic A, Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models, Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models, Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models, Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models, Lesson 5: Creating Division Stories, Lesson 6: More Division Stories, Lesson 7: The Relationship Between Visual Fraction Models and Equations, and Lesson 8: Dividing Fractions and Mixed Numbers are lessons that pertain to this cluster.
- Georgia Standards of Excellence Curriculum Frameworks, Grade 5, Unit 1: Number System Fluency has many tasks that pertain to this cluster. This cluster is addressed on pages 65-116.
- Illustrative Mathematics, Grade 6, Unit 4: Dividing Fractions has many lessons that pertain to this cluster. In order to view these lessons, a free teacher account must be created.

General Resources

- Arizona Progression 6-High School Progression on the Number System is an informational document of teachers. This cluster is addressed on pages 5-6.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.1)

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STANDARDS	MODEL CURRICULUM (6.NS.2-4)
THE NUMBER SYSTEM **Compute fluently with multi-digit numbers and find common factors and multiples.** **6.NS.2** Fluently[G] divide multi-digit numbers using a standard algorithm[G]. **6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. **6.NS.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*	## Expectations for Learning Place value has been a major emphasis in the elementary standards. In grade 6, students become fluent in the use of a standard division algorithm, continuing to use their understanding of place value to describe what they are doing. This standard is the end of this progression to address students' understanding of place value. Previously students have had exposure to addition, subtraction, multiplication, and division of whole numbers. Students also have had prior exposure to addition and subtraction of decimals and multiplication and division of whole numbers by decimals and decimals by whole numbers. The learning in these standards should be focused on multiplying and dividing decimals by decimals and developing fluency with grade-level decimals. Students have had experience with common multiples and common factors in earlier grades in relation to fractions. The learning in this standard should focus on finding and using least common multiple and greatest common factor to solve mathematical and real-world problems. Composing and decomposing numbers efficiently is a precursor to operating with algebraic expressions. ### ESSENTIAL UNDERSTANDINGS **Whole Number and Decimal Operations** • There are several acceptable standard algorithms for operations involving addition, subtraction, multiplication, and division. • When adding and subtracting, tenths are added/subtracted to tenths, and hundredths are added/subtracted to hundredths. • When adding and subtracting decimals, line up the decimal point to align the place values of the numbers. • A remainder can be expressed as a fraction or a decimal. **Greatest Common Factors** • Factors are the numbers being multiplied together. • Greatest Common Factor (GCF) is useful in expressing the numbers using the distributive property. • Composing and decomposing numbers can help solve routine mathematical and real-world problems. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.NS.2-4)
	## Expectations for Learning, continued

MATHEMATICAL THINKING
- Compute accurately and efficiently.
- Use different properties of operations flexibly.
- Determine the reasonableness of results.
- Attend to precision when performing mathematical operations.
- Solve multi-step problems accurately.

INSTRUCTIONAL FOCUS

Whole Number and Decimal Operations
- Estimate sum, differences, products, or quotients before computing.
- Use estimation to determine the reasonableness of solutions.
- Add, subtract, multiply, and divide multi-digit whole numbers and decimals using a standard algorithm.
- Explain and justify the steps in a standard algorithm.
- Develop an understanding of and determine where to place the decimal point in products and quotients.
- Evaluate whether the location of the decimal point makes sense in a given context.
- Adjust the precision of the decimal to match the context.
- Apply and adapt a variety of appropriate strategies to solve routine and non-routine mathematical and real-world problems involving whole numbers and decimals.

Greatest Common Factor
- Find the least common multiple of two numbers less than or equal to 12.
- Find the greatest common factor of two whole numbers less than or equal to 100.
- Use the greatest common factor and distributive property to rewrite the sum of two whole numbers 1-100.

Continued on next page.

STANDARDS	MODEL CURRICULUM (6.NS.2-4)
	## Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38 • Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5 • Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7 • Ohio's K-8 Learning Progressions, The Number System, pages 16-17 **CONNECTIONS ACROSS STANDARDS** • Apply the distributive property to algebraic expressions (6.EE.3).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.

As students study whole numbers in the elementary grades, a foundation is laid in the conceptual understanding of each operation. Discovering and applying multiple strategies for computing creates connections which evolve into the proficient use of a standard algorithm.

FLUENCY

Fluency is the ability to use efficient, accurate, and flexible methods for computing something that develops over time; it develops when students build from their deep conceptual understanding of each operation combined with number sense. With fluency, students are able to confidently compose and decompose numbers using this understanding. Opportunities to determine when to use paper pencil algorithms, mental math, or a computing tool is also a necessary skill and should be provided in problem solving situations.

Efficiency is in the eye of the beholder. What may be efficient for one student may not be efficient for all students. It is more important that students use a method that they understand and provides them accurate results consistently than it is for each student to use the same method. Students will oftentimes invent their own strategies that make sense to them. These can be built upon to help students become fluent in a way that makes sense to them.

ESTIMATION

When performing computations using algorithms it is important to have students use estimation before implementing procedures to promote number sense and judge the reasonableness of the computation. It is especially crucial with division as students need to estimate the divisor in order to divide efficiently.

PROPERTIES OF OPERATIONS

When working with whole number and decimal computations, reinforce the properties of operations composing and decomposing numbers. Draw special attention to the usefulness of the Distributive Property.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

A STANDARD ALGORITHM

There are many standard algorithms. An algorithm is a step-by-step procedure for solving a problem or accomplishing an end goal. There are different variations in the way the steps are orchestrated to reach the desired end goal. Algorithms depend on representations. For example a division algorithm for fractions looks different than it does for decimals. The preferred standard algorithm is the one that is the most understandable and efficient for each child. Therefore, several students in a classroom may approach a computation problem with a different standard algorithm. Students should be able to explain why their preferred standard algorithm works, reinforcing concepts of place value.

It may be beneficial to expose students to multiple standard algorithms and have them try each. After exposure to several algorithms, students can choose which algorithm makes sense to them. Use class discussion to help students create understanding explaining why their preferred algorithm works.

TIP!   Grid paper helps students organize their computational work. It is highly encouraged that all students have access to grid paper.

Comparing Algorithms

It may be helpful to compare different student-created algorithms in order to generate discussion about why each algorithms works the way that it does.

DIVIDING WHOLE NUMBERS

In Grade 5 students worked with a standard algorithm for multiplying whole numbers but still used models and strategies to divide whole numbers. This is the first time students are required to be working with a division algorithm.

There are two interpretations of division problems: partitive and measurement. See Model Curriculum 7.NS.1 for more information regarding the two interpretations. Although the focus of this cluster is on performing a standard algorithm, it is still important to give students contexts in which a standard algorithm can be applied.

Although students should be able to use any number of digits for divisors based on reasoning through patterns, in order for computation to avoid being cumbersome focus on one- or two-digit divisors for situations without a context and one-, two-, and three-digit divisors for division problems represented within a context.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Partial Quotients

Partial Quotient is similar to the traditional algorithm, but it allows for friendly factors. This strategy may help students who struggle with their multiplication facts. Over time students should move toward the most efficient partial quotients algorithm.
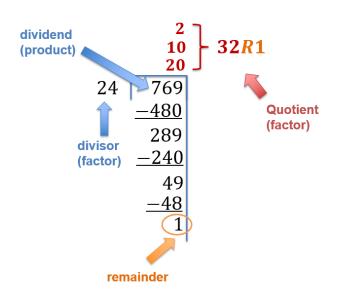
EXAMPLE

- Approximate the solution to $769 \div 24$.
- Solve $769 \div 24$.
- Check with multiplication.

Method 1   **Method 2**

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

The Area Method
An area model is a useful way for students to understand division. It relates to prior learning involving multiplication and the Distributive Property, and it can also be applied to division problems in Algebra that include factoring.

An area model whole length is truly representative of place-value dimensions (where each dimension would be 10 times bigger than the one on the right) would look something like this:

Explain to students that although this rectangle's length is representative of place-value, it can be difficult to use since the rectangles on the right are so small. Therefore, we do not always draw our rectangles to scale.
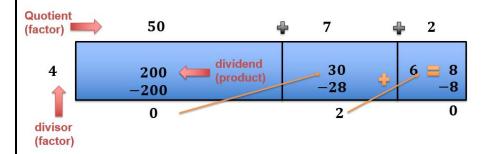
Another benefit of the area model is that it reinforces the concept of area and easily connects multiplication and division. The dimensions of the rectangles are the factors (quotient and divisor) and the area is the dividend or the product.

There are several variations of using an area model to divide.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

EXAMPLE
- Approximate the solution to $236 \div 4$.
- Solve $236 \div 4$.
- Check with multiplication.

Discussion: An area model is useful for it models real-life applications of multiplication and division, and it allows students decompose numbers to make smaller numbers that are easier to work with. One large area can be broken into several smaller rectangles. This variation of the area model mimics those that students are used to when applying the Distributive Property as 236 is decomposed into 200 + 30 + 6. This also models how students can use the area model when multiplying and dividing polynomials in high school. The dividend is the area and is place inside the box. The divisor is the length of the box. In this example,
- The dividend is decomposed into 200 + 30 + 6, and each number is placed into a subdivided box.
- The student needs to figure out how man 4s are in 200, which is 50 with nothing left over. So, 50 is the length of the subdivided box.
- If anything was left over, it would be added to the next subdivided box on the right.
- Then the student needs to figure out how many times 4s are in 30, which is 7, so 7 is the length of the middle subdivide box, since 7 times 4 is 28, 28 is subtracted from the 30 which results in 2.
- The 2 is then added to the 6 in the next subdivided box to get 8, and the student needs to figure out how many 4s are in 8, which is 2, so 2 is the length of the last subdivided box.
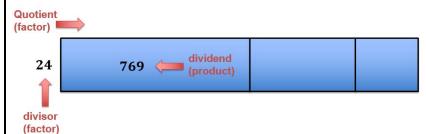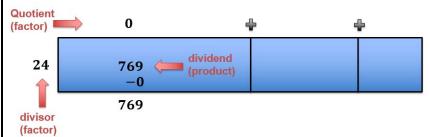- To find the quotient the student needs to add all the lengths which is 50 + 7 + 2 to get 59.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

EXAMPLE

- Approximate the solution to $769 \div 24$.
- Solve $769 \div 24$.
- Check with multiplication.

Method 1:

Step 1:

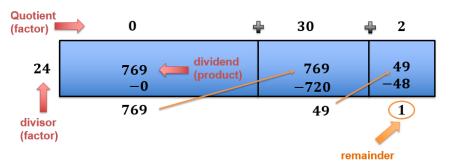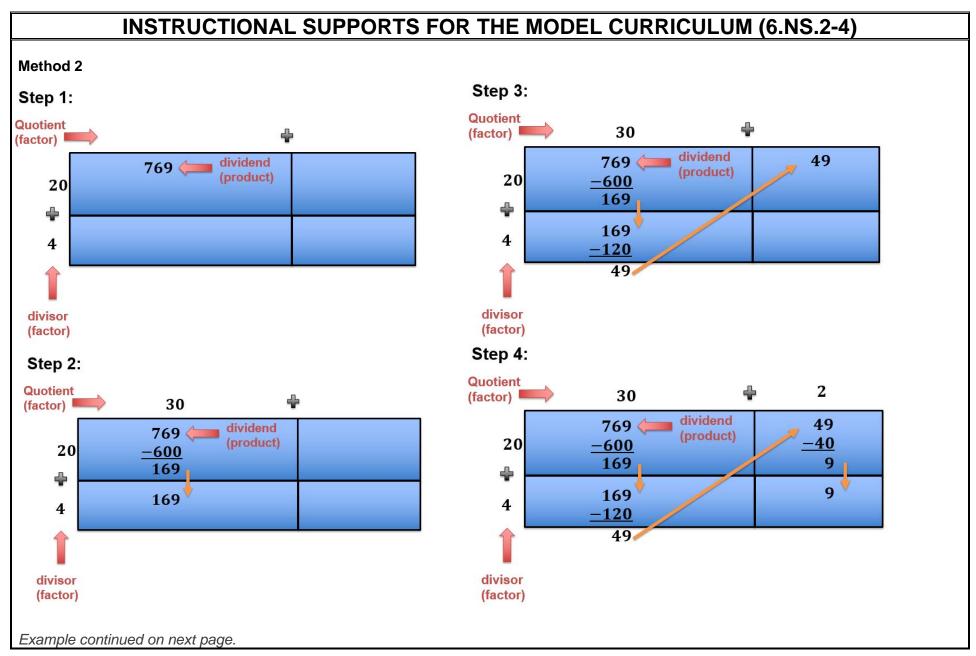**Step 2:**

Step 3:

Step 4:

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Method 2

Step 1:

Step 2:

Step 3:

Step 4:

Example continued on next page.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Step 5:

Quotient (factor) → 30   +   2

| 20 | 769 ← dividend (product) −600 169 | 49 −40 9 |
| 4 | 169 −120 49 | 9 −8 (1) |

divisor (factor)

remainder
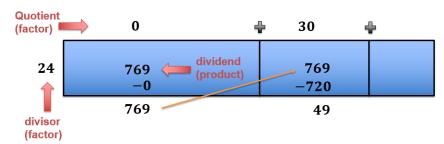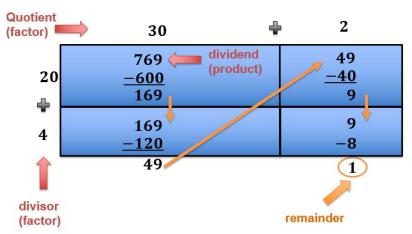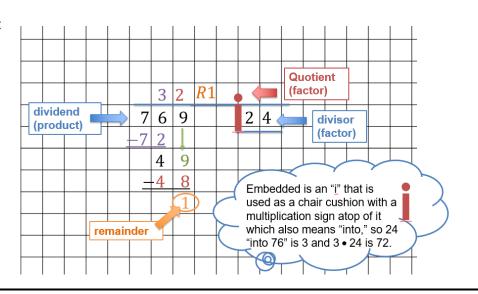
Discussion: Here is a video describing this process:
https://www.youtube.com/watch?v=N7boECP9BAE

The Chair That Is Laying Down

This is a method used in Sweden. The advantage to this method is that it prevents the mistake that students often make between switching the divisor and the dividend when performing division problems.

Example

- Approximate the solution to $769 \div 24$.
- Solve $769 \div 24$.
- Check with multiplication.

dividend (product) → | 3 2 R1 | Quotient (factor)
7 6 9 | 2 4 divisor (factor)
−7 2
4 9
−4 8
(1)

remainder

Embedded is an "i" that is used as a chair cushion with a multiplication sign atop of it which also means "into," so 24 "into 76" is 3 and 3 • 24 is 72.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)
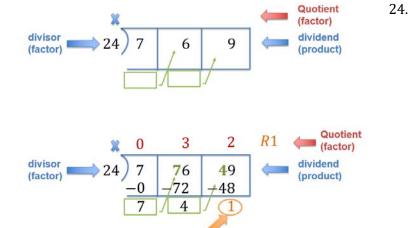
Explicit-Trade Method
This method is a cross between the traditional standard algorithm and the area model. Its benefit is that it helps students keep track of their work. Place value may not be as evident to some students; make sure to highlight place value concepts.

EXAMPLE
- Approximate the solution to 769 ÷ 24.
- Solve 769 ÷ 24.
- Check with multiplication.

The Traditional Standard Algorithm
The traditional standard algorithm is familiar to teachers and parents and is efficient for some. However, students often apply it incorrectly because they do not understand why it works. Many students have a difficult time connecting the algorithm to their place value understanding. The one big benefit to the traditional standard algorithm is that it can be transferred to dividing polynomials in high school.

EXAMPLE
- Approximate the solution to 769 ÷ 24.
- Solve 769 ÷ 24.
- Check with multiplication.
-

Discussion: Have students estimate by thinking about the multiplication focusing on tens and simple facts.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

OPERATIONS WITH DECIMALS
Students learned to add, subtract, multiply, and divide decimals in Grade 5 using models and strategies. Multiplication and division of decimals was limited to whole numbers by decimals and decimals by whole numbers, but not decimals by decimals. Therefore this is the first time students are using any standard algorithms for decimal computation.

Specific rules for decimal computation are not necessary if a student has a firm understanding of place value. This is especially true if there is an understanding of how decimals are related to fractions. Before computing with decimals, students should be fluent at estimating decimal computations. This allows students to develop number sense, so that when they eventually transfer to a calculator for decimal computations, they can realize if their answer is in a reasonable range.

Connecting Decimal Operations with Fractional Operations
Make sure to intentionally connect decimal notation and computation with fraction notation and computation. Students can relate their thinking in multiplying and dividing decimals to multiplying and dividing fractions over 10, 100 or 1000. For example, 0.15 x 0.6 = 15/100 x 6/10 = 90/1000 = 0.090 This will help students to understand where the digits go in relation to the decimal point.

TIP!   Use decimal place-value language such as "zero and three-tenths" instead of saying "zero point 3."

A common misconception is applying whole-number strategies when doing computations with decimals based on rules for the traditional algorithm. For example, a student without place value understanding may get the answer 0.44 for the problem 0.38 + 0.6 because they see the problem as 38 + 6 and are used to adding all digits from right to left instead of seeing it as $\frac{38}{100}$ and $\frac{6}{10}$.

Adding and Subtracting Decimals
When adding and subtracting with decimals continue to emphasize place value. Using decimal squares or base ten blocks to connect the algorithm with the models may aid in student understanding. Using money as a context helps students make connections to place value.
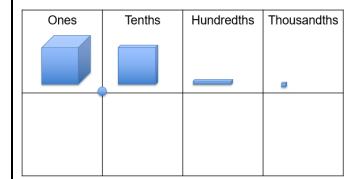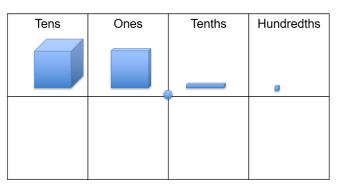
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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)
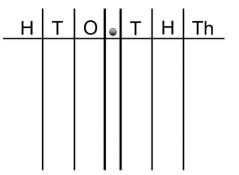
EXAMPLE

Solve $54.2 - 8.9 + 15.75$

- Estimate and explain how you made your estimate.
- Compute and explain your process.
- Solve an equivalent expression using fractional notation.
- Compare your procedures for part **b.** and part **c.**

For students struggling with addition and subtracting with decimals, connect the algorithm with models such as base-ten blocks, decimal squares, or number lines. Models help reinforce why it is important to line up the place value positions before adding and subtracting. Connecting decimals to fractions that need to have common denominators before adding or subtracting also reinforces the need to line up place value positions. When using models, it may be helpful to periodically change the whole, so students see the relationships between the place value digits instead of just memorizing a model.

Ones	Tenths	Hundredths	Thousandths

Tens	Ones	Tenths	Hundredths

H | T | O | T | H | Th

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

EXAMPLE
Solve $2.75 + 1.8$

- Estimate your solution.
- Model your solution two different ways.
- Explain how you could solve without using a model.
- Explain why your method works.
- How does $0.275 + 0.18$ differ from your solution? Explain.
- How does $27.5 + 0.18$ differ from your solution? Explain.

Typically adding begins on the right and moves left from right to left using the traditional algorithm; however, it may make more sense to some students adding from left to right. Many adults do the same thing mentally. Both ways are acceptable as long as place value understanding is being applied.

$$\begin{array}{r} 3.026 \\ +4.79 \\ \hline 7.7 \\ 0.11 \\ +0.006 \\ \hline 8.816 \end{array}$$

Clearing Decimals (Connection to 6.EE. 7)
One strategy for solving equations with decimals is to clear the equation of decimals by creating an equivalent equation without decimals. For example, $5 - 0.17 = x$ is equivalent to $500 - 17 = 100x$ by multiplying each side of the equation by 100. This strategy may work well for some students who struggle with decimal computation. The resulting answer would be $483 = 100x$ or $x = \frac{483}{100}$ or 4.83.

Multiplying Decimals
To make sense of decimal multiplication it may be useful to think of multiplication in terms of measuring instead of equal groups.

Placing the Decimal Point When Multiplying
To make sense of placing the decimal point in multiplication algorithms, where one multiplies the two factors as whole numbers and then counts the number of zeros or 10s in the factors, and then replaces it back in the product, try connecting it to equivalent equations and tying it into 6.EE.7. Also tie it back to what students learned in 6th grade about zeros and multiplication. Grid paper can help students organize their work.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

EXAMPLE

- Approximate your solution to $8.37 \cdot 1.2 = p$
- Solve $8.37 \cdot 1.2 = p$.

$$\begin{array}{l} 8.37 \times 100 \\ \underline{\times 1.2} \times 10 \end{array} = \begin{array}{l} p \times 100 \\ \times 10 \end{array}$$

Discussion: Students should estimate before computing to develop numbers sense. This also will help them place the decimal point, which will be especially useful when using technology so they can determine if their answer is reasonable. They may consider that 8.37 is slightly above 8 and 1.2 is slightly above 1, so the product should be a little greater than 8 × 1 or 8. Therefore, when they get 10044, they should determine that the decimal needs to go after the 10, since 10 is slightly above 8, therefore $8.37 \cdot 1.2 = 10.044$.

$$\begin{array}{l} 837 \\ \underline{\times 12} \\ 10044 \end{array} = 1{,}000p$$

$$\frac{10044}{1000} = p$$

$$10.044 = p$$

Have students explore where the decimal point should be placed using technology in patterns. Then have the predict where the decimal point should be based on their observations.

EXAMPLE

If $837 \cdot 12 = 10044$, predict what the solutions will be for the following equations without calculating.

- $8.37 \cdot 12 = p$
- $8.37 \cdot 1.2 = p$
- $8.37 \cdot 0.12 = p$
- $8.37 \cdot 1200 = p$
- $83.7 \cdot 1.2 = p$
- $83.7 \cdot .12 = p$
- $83.7 \cdot 120 = p$

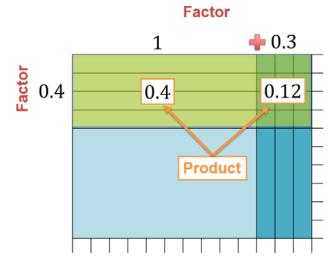## INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

TIP! Connect the idea of renaming decimals by connecting the concept to measurement units. Solve multiplication problems using the metric measurement system. The context can help explain the placement of the decimal point. For example a length of 3.56 meters is the same is 356 centimeters.

Area Model Algorithm

Connect the area models that students use in multiplying fractions to decimal multiplication. Start by using concrete models that represent the dimensions of the decimals, and then once students understand the more accurate model, they can move to boxes where they can convert the decimals to whole numbers and then place the decimal at the appropriate spot at the end. Students can check their work by estimating that 1 times $\frac{1}{2}$ is about $\frac{1}{2}$.
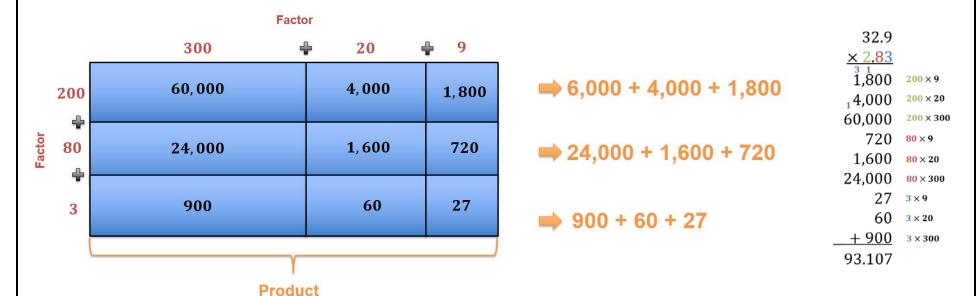
EXAMPLE

- Approximate your solution to 1.3(0.4).
- Solve 1.3(0.4).

Therefore, the product is 0.52

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

EXAMPLE

- Approximate your solution to 32.9(2.83).
- Solve 32.9(2.83).

Factor

	300	+	20	+	9
200	60,000		4,000		1,800
80	24,000		1,600		720
3	900		60		27

Factor

➡ **6,000 + 4,000 + 1,800**

➡ **24,000 + 1,600 + 720**

➡ **900 + 60 + 27**

Product

$$\begin{array}{r} 32.9 \\ \times\ 2.83 \\ \hline \end{array}$$

$$
\begin{array}{rl}
1,800 & \text{200 × 9} \\
4,000 & \text{200 × 20} \\
60,000 & \text{200 × 300} \\
720 & \text{80 × 9} \\
1,600 & \text{80 × 20} \\
24,000 & \text{80 × 300} \\
27 & \text{3 × 9} \\
60 & \text{3 × 20} \\
+\ 900 & \text{3 × 300} \\
\hline
93.107 &
\end{array}
$$

Discussion: Notice the similarity to the partial products algorithm. The area model allows students to keep track of the different products, but the procedure is basically the same.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Partial Products Algorithm

Partial products is beneficial for students because it illustrates the Distributive Property and reinforces place value. It also closely imitates the traditional standard algorithm, so it could also be used a scaffold to the traditional standard algorithm. To be noted partial products can be done left to right or right to left. See the example in the discussion about the Lattice Algorithm. Grid paper can help students organize their work.

EXAMPLE

- Approximate your solution to 32.9(2.83).
- Solve 32.9(2.83).

32.9	
× 2.83	
1 3 1	
60,000	200×300
4,000	200×20
1,800	200×9
24,000	80×300
1,600	80×20
720	80×9
900	3×300
60	3×20
+27	3×9
93.107	

32.9	
× 2.83	
27	3×9
₁60	3×20
900	3×300
₃720	80×9
₁1,600	80×20
24,000	80×300
1,800	200×9
4,000	200×20
+60,000	200×300
93.107	

Grade 6

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)
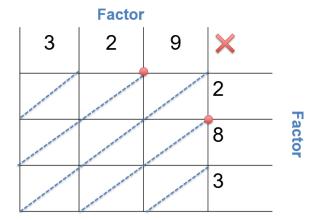
Lattice Algorithm

Although the lattice method is easy to use and many students are successful using this method, place value is not as evident compared to other algorithms. If students desire to use the lattice method, connect it to partial products (from right to left) to help students understand why it works. Encourage students who favor this method to explain why it always works.
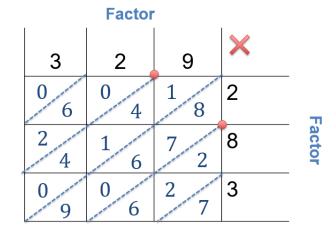
EXAMPLE

- Approximate your solution to 32.9(2.83).
- Solve 32.9(2.83).

Step 1: Draw a lattice.
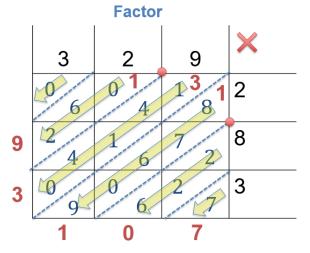
Step 2: Multiply each row by each column. Put the ten's place in the top-left of each square and the one's place in the bottom-right of each square.

Example continued on next page

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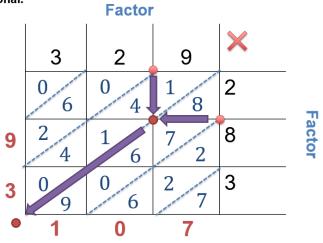
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Step 3: Add diagonally, carrying to the next diagonal when necessary.
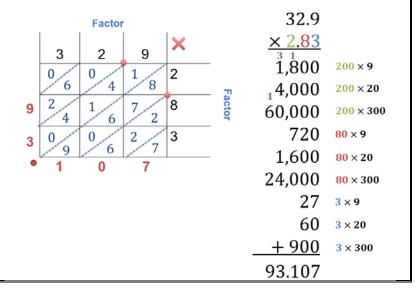
Step 4: Connect the decimals and then slide along the diagonal.

Therefore, the product is 93.107

Discussion: Draw students back to their original approximation. Since 32.9 is about 33 and 2.83 is about 3, 32.9 x 2.83 should be slightly less than 33 x 3, which is 99. Therefore in 93107, the decimal should be placed after 93. Encourage students to justify why the lattice method works. For students who like this method, compare it to partial products from right to left to emphasize place value. Grid paper is very helpful with the lattice method.

$$\begin{array}{r} 32.9 \\ \times\ 2.83 \\ \hline 1,800 \\ 4,000 \\ 60,000 \\ 720 \\ 1,600 \\ 24,000 \\ 27 \\ 60 \\ +\ 900 \\ \hline 93.107 \end{array}$$

1,800	200 × 9
4,000	200 × 20
60,000	200 × 300
720	80 × 9
1,600	80 × 20
24,000	80 × 300
27	3 × 9
60	3 × 20
900	3 × 300

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Traditional Multiplication Algorithm

Many children tend to make mistakes using the traditional algorithm because they may have a difficult time making sense of it and connecting it to place value. One of the most common mistakes is that the student forgets to write in the place-holder zeros when they start multiplying with tens and hundreds. Grid paper can help students keep track of their work.

EXAMPLE

- Approximate your solution to
- Solve 32.9(2.83).

$$
\begin{array}{r}
\overset{1}{}\overset{7}{} \\
\overset{2}{}\overset{2}{} \\
32.9 \\
\times\ 2.83 \\
\hline
987 \\
26{,}320 \\
+65{,}800 \\
\hline
93.107
\end{array}
$$

32.9(2.83).

Dividing Decimals

Although students should be able to use any-number of digits for divisors based on reasoning through patterns, in order for computation to avoid being cumbersome focus on one- or two-digit divisors for situations without a context and one-, two-, and three-digit divisors for division problems represented within a context.

The previous algorithms for division with whole numbers can also be extended to decimals.

Connect equivalent fractions to the placing of the decimal point in division algorithms. *Note: Emphasize to students that the digits move, not the decimal point.*

EXAMPLE

Solve $8.37 \div 1.2 = p$.

$$\frac{8.37}{1.2} \frac{\times 10}{\times 10} = \frac{83.7}{12}$$

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Partial Quotients Algorithm
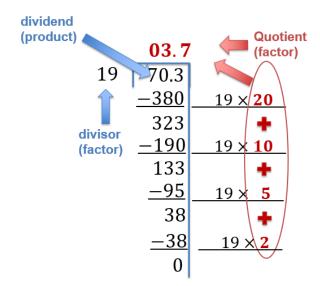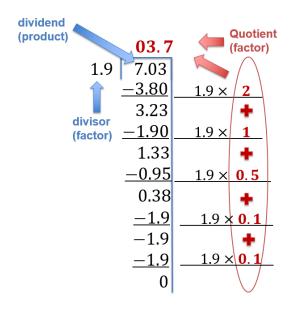
EXAMPLE
- Approximate your solution to $7.03 \div 1.9$.
- Solve $7.03 \div 1$.

Method 1: Converting the divisor to a whole number and then bringing up the decimal point:
- $1.9 \cdot 10 = 19$
- $7.03 \cdot 10 = 70.3$
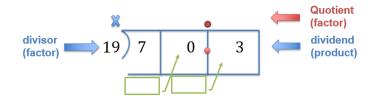
Method 2: Using the decimal.

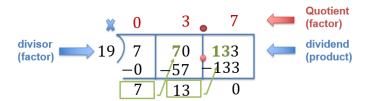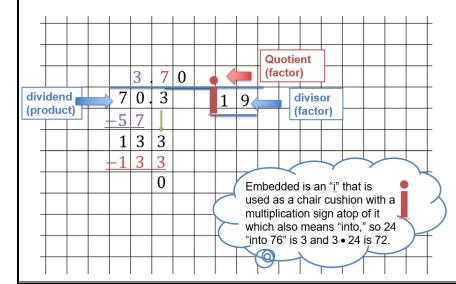# INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Explicit Trade Algorithm

EXAMPLE
- Approximate your solution to $7.03 \div 1.9$.
- Solve $7.03 \div 1.9$

Convert the divisor to a whole number.
- $1.9 \cdot 10 = 19$
- $7.03 \cdot 10 = 70.3$

The Chair That Is Laying Down

EXAMPLE
- Approximate your solution to $7.03 \div 1.9$.
- Solve $7.03 \div 1.9$

Convert the divisor to a whole number.
- $1.9 \cdot 10 = 19$
- $7.03 \cdot 10 = 70.3$

Embedded is an "i" that is used as a chair cushion with a multiplication sign atop of it which also means "into," so 24 "into 76" is 3 and $3 \cdot 24$ is 72.

Discussion: The precision of accuracy for the decimal must be determined before you begin. The Swedish people usually set it up to allow for two blank spaces.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Traditional Algorithm
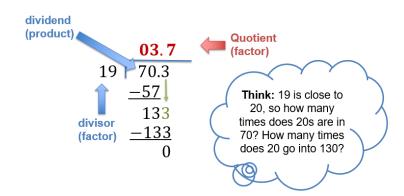
Partial products is similar to the traditional algorithm. The benefit to the partial products approach is that it reinforces the Distributive Property (that will be useful for extending to multiplying and dividing polynomials in Algebra) and reinforces place value.

EXAMPLE

- Approximate your solution to $7.03 \div 1.9$.
- Solve $7.03 \div 1.9$

Convert the divisor to a whole number.

- $1.9 \cdot 10 = 19$
- $7.03 \cdot 10 = 70.3$

dividend (product)

Quotient (factor)

$$03.7$$

$$19 \overline{\smash{)}70.3}$$
$$-57$$
$$133$$
$$-133$$
$$0$$

divisor (factor)

Think: 19 is close to 20, so how many times does 20s are in 70? How many times does 20 go into 130?

FACTORS AND MULTIPLES

Greatest common factor and *least common multiple* are usually taught as a means of combining fractions with unlike denominators. This cluster builds upon the previous learning of the multiplicative structure of whole numbers, as well as prime and composite numbers in Grade 4. Although the process is the same, the point is to become aware of the relationships between numbers and their multiples.

EXAMPLE

If two numbers are multiples of four, will the sum of the two numbers also be a multiple of four?

Discussion: Being able to see and write the relationships between numbers will be beneficial as further algebraic understandings are developed.

Students should be able compose and decompose numbers into factors and multiples which could be done with prime factorization.
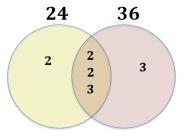
EXAMPLE

- Use a factor tree to write the prime factorization of 136.
- Using the prime factorization, list as many factors that you can of 136.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Another focus is to be able to see how the Greatest Common Factor (GCF) is useful in expressing the numbers using the Distributive Property, (36 + 24) = 12(3 + 2), where 12 is the GCF of 36 and 24. This concept will be extended in Expressions and Equations as work progresses from understanding the number system and solving equations to simplifying and solving algebraic equations in Grade 7. Students should also be able to find the Least Common Multiple (LCM) of two whole numbers less than or equal to 12. This can be done with a variety of methods. A Venn diagram may be useful in using prime factors to compare factors and multiples.

$$36 = 2 \cdot 3 \cdot 2 \cdot 3$$
$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

It may be useful to analyze even and odd numbers and divisibility rules to deepen understanding of factors and multiples.

EXAMPLE
- What happens when you add two even numbers? Explain.
- What happens when you add two odd numbers? Explain.
- What happens when you add an even and an odd number? Explain.

MATH IS ACTIVELY CHANGING

Facebook invented a new unit of time called a "flick" which is derived from "frame-tick." It is $\frac{1}{705,600,000}$ second. The reason that new unit of time was invented is because it's a number that is easily divisible by a number of factors: $\frac{1}{8}, \frac{1}{16}, \frac{1}{22.05}, \frac{1}{24}, \frac{1}{25}, \frac{1}{30}, \frac{1}{32}, \frac{1}{44.1}, \frac{1}{48}, \frac{1}{50}, \frac{1}{60}, \frac{1}{90}, \frac{1}{100}, \frac{1}{120}$. All of those factors are framerates or frequencies that are used in media such as films or music such as 24 frames per second, 120 hertz Tvs, 44.1 KHz sample rate audio. Before the "flick" there were a lot of tricky decimals to work with, so things had to be rounded. Using a "flick" allows people who work with these numbers to be more precise since the measurements can be converted to whole numbers. People think it will be especially useful in developing virtual reality products and making sure videos stay in sync. See the article by Devin Coldewey and BBC for more information.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

Instructional Tools/Resources
This section is under revision and will be published in 2018.

Manipulatives/Technology
- Grid paper

Dividing Whole Numbers
- Use an Area Model for Division of 4-Digit Dividends by 2-Digit Divisors by Julie McGough from LearnZillion is an instructional video that shows how to divide whole numbers using an area model.
- Legos by YummyMath is a 3-Act task that requires students to compute using whole numbers.
- Interpreting a Division Computation by Illustrative Mathematics is a task where students interpret the standard division algorithm.
- How Many Staples? by Illustrative Mathematics is a task where students use long division with a remainder in a context.
- Batting Averages by Illustrative Mathematics is a task where students use division with a decimal answer to analyze the division of whole numbers in a sports context.
- Algorithms by Everyday Mathematics has video discussing different algorithms including Partial Quotients and Column Division.
- Galley Division is a YouTube video explaining the method of Galley Division.

Computation with Decimals
- Use an Area Model to Multiply Decimals by Decimals by Julie McGough from LearnZillion is an instructional video that shows how to multiply decimals by decimals using an area model.
- Getting the Decimal (Point) with Blocks: Multiplying Two Decimals Using Base Ten Blocks by NCTM Illuminations is a lesson where students model the multiplication of decimals using base-ten blocks. *NCTM now requires a membership to view their lessons.*
- How Should I Cook My Turkey? by YummyMath is an activity that requires students to compute using decimals.
- Cheap-otle? by YummyMath is an activity that requires students to compute using decimals.
- How High Can a Gymnastic Score Be? by YummyMath is an activity that requires students to compute using decimals.
- The Boston Marathon by YummyMath is an activity that requires students to compute using decimals.
- How Many Pennies Did Otha Save? by YummyMath is a 3-Act task that requires students to compute using decimals.
- Free Pizza for a Whole Stadium? by YummyMath is a 3-Act task that requires students to compute using decimals.
- Cost of Light Bulbs-Updated! by YummyMath is an activity that requires students to compute using decimals.

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Computation with Decimals, continued

- Cruising by YummyMath is an activity that requires students to compute using decimals.
- Reasoning with Multiplication and Division and Place Value, Part 1 and Reasoning with Multiplication and Division and Place Value, Part 2 by Illustrative Mathematics is a task where students use reasoning and estimating strategies to perform computations.
- Gifts from Grandma, Variation 3 by Illustrative Mathematics is a task that has three different types of problems using a similar context.
- 2 Units Wide and 3 Units Long by Illustrative Mathematics is a task where students use an area model to notice where the decimal point is placed. 12 Rectangular Units follows up this task.
- Tenths of Tenths and Hundredths of Hundredths by Illustrative Mathematics is a task where students connect multiplication, area, and fractions to multiplying decimals.
- Adding Base Ten Numbers, Part 1, Adding Base Ten Numbers, Part 2, and Adding Base Ten Numbers, Part 3 by Illustrative Mathematics are a series of tasks generalizing the traditional addition algorithm for decimals.
- What Is the Best Way to Divide? by Illustrative Mathematics is a task where students need to choose the best strategy to divide instead of automatically resorting to the algorithm.
- Changing Currency by Illustrative Mathematics is a task where students use patterns of 10 to divide which helps them understand the rules for moving the decimal point in long division.
- Open Middle: The Number System has many open-ended problems where students have to apply their understandings of decimal operations.

Factors and Multiples

- Cicada Swarmaggedon by Yummy Math is an activity that looks at factors and prime numbers in the cyclic nature of cicadas.
- Lunar New Year, 2018 by Yummy Math is an activity that has students explore the least common multiple of two Chinese cycles.
- Adding Multiples by Illustrative Mathematics is a task where students analyze patterns of multiples.
- Bake Sale by Illustrative Mathematics is a task where students find common factors in a context.
- The Florist Shop by Illustrative Mathematics is a task where students find common multiples in a context.
- Models for Multiples and Factors by Annenberg Learner uses an area model for finding GCF and LCM.

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Curriculum and Lessons from Other Sources

- EngageNY, Grade 6, Module 2, Topic B, Lesson 9: Sums and Differences of Decimals, Lesson 10: The Distributive Property and the Products of Decimals, Lesson 11: Fraction Multiplication and the Products of Decimals are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 2, Topic C, Lesson 12: Estimating Digits in a Quotient, Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm, Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions, Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 2, Topic D, Lesson 16: Even and Odd Numbers, Lesson 17: Divisibility Tests for 3 and 9, Lesson 18: Least Common Multiple and Greatest Common Factor, Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm are lessons that pertain to this cluster.
- Georgia Standards of Excellence Curriculum Frameworks, Grade 6, Unit 1: Number System Fluency has many tasks that pertain to this cluster. This cluster is addressed on pages 15-64 and 117.
- Illustrative Mathematics, Grade 6, Unit 5: Arithmetic in Base Ten is a unit on using efficient algorithms.

General Resources

- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- Arizona 6-HS Progression on the Number System is an informational document for teachers. This cluster is addressed on pages 6-7

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.2-4)

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STANDARDS	MODEL CURRICULUM (6.NS.5-8)
THE NUMBER SYSTEM **Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., − (−3) = 3, and that 0 is its own opposite. *Continued on next page*	## Expectations for Learning Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. In previous grades, students worked with positive fractions, decimals, and whole numbers on the number line and in quadrant one of the coordinate plane. In sixth grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (e.g., thermometer) which facilitates the movement from number lines to coordinate grids. Students begin graphing in all four quadrants of the coordinate plane. They will explore absolute value, opposite numbers, comparing numbers, and ordering rational numbers using the number line or coordinate plane. This will lead to operations with integers in future grades. **ESSENTIAL UNDERSTANDINGS** **Rational Numbers** • A number line can show magnitude (quantity) and direction. • Negative numbers are to the left of zero on a horizontal number line and below zero on a vertical number line. • A number and its opposite are the same distance from zero on a number line. • Zero is its own opposite. • The negative sign means the "opposite of," so $-p$ is the opposite of p. • The set of integers consists of positive whole numbers, their opposites, and 0. • The opposite of the opposite of the number is the number itself. • Although fractions and decimals can be negative, they are not necessarily integers, e.g., $-\frac{3}{5}$ and -3.2 are not integers, but $-\frac{4}{1}$ and -3.0 are integers. • In a fraction the negative sign can be written in the numerator, the denominator, or out front, e.g., $\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$. • As the magnitude of a negative number increases (moves to the left or downward on a number line), the value of the number decreases. • The absolute value of a number is the distance from zero. • The absolute value of zero is zero. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.NS.5-8)
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.7** Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write −3°C > −7°C to express the fact that −3°C is warmer than −7 °C.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of −30 dollars, write $\lvert -30 \rvert = 30$ to describe the size of the debt in dollars.* *Continued on next page*	# Expectations for Learning, continued ## ESSENTIAL UNDERSTANDING, CONTINUED **Coordinate Plane** - The coordinate plane is a plane formed by the intersection of a horizontal number line with a vertical number line. - The *x*-axis is the horizontal number line in a coordinate plane. - The *y*-axis is the vertical number line in a coordinate plane. - In an ordered pair (*x*, *y*), *x* represents the horizontal position and *y* represents the vertical position in the coordinate plane. - The ordered pair gives a precise location in the coordinate plane. - Quadrants are numbered counter-clockwise with the top-right quadrant being Quadrant I. - Coordinate points can have fractional or decimal units. ## MATHEMATICAL THINKING - Draw or use a picture, model, or graph to make sense of a problem. - Apply mathematical vocabulary to describe real-world contexts. - Represent a concept symbolically. - Pay attention to and make sense of quantities. - Communicate mathematical ideas ## INSTRUCTIONAL FOCUS **Rational Numbers** - Find and position positive and negative rational numbers on a horizontal or vertical number line. - Use positive and negative numbers to represent quantities on a number line and/or in real-world contexts. - Explain the meaning of zero and opposite numbers in a real-world situation. - Use number lines to compare and order positive and negative rational numbers in the same form and in different forms. - Interpret and use the absolute value of quantities in real-world situations. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.NS.5-8)
d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars.* **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	## Expectations for Learning, continued **INSTRUCTIONAL FOCUS, CONTINUED** **Coordinate Plane** • Graph ordered pairs in all four quadrants. • Name an ordered pair given a point on a graph. • Represent real-world and mathematical problems by graphing points in all four quadrants. • Determine the distance between two points with the same x or y coordinates by counting, using absolute value, or other strategies. • Reflect an ordered pair across the x-axis, to discover that in the resulting image, the x-coordinate remains the same and the y-coordinate is its opposite, i.e., (x, y) and $(x, -y)$. • Reflect and ordered pair across the y-axis, to discover that in the resulting image, the y-coordinate remains the same the x-coordinate is its opposite, i.e., (x, y) and $(-x, y)$. ## Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38 • Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5 • Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7 • Ohio's K-8 Learning Progressions, The Number System, pages 16-17 **CONNECTIONS ACROSS STANDARDS** • Write and graph on a number line inequalities of the form $x > c$ or $x < c$ (6.EE.8). • Draw a polygon in the coordinate plane and determine lengths of (horizontal and vertical) segments in the coordinate plane (6.G.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

The purpose of this cluster is to begin the study of the existence of negative numbers, their relationship to positive numbers, and the meaning and uses of absolute value. This cluster should connect with 6.EE.5-8.

> ***Standards for Mathematical Practice***
> *This cluster focuses on but is not limited to the following practices:*
> **MP.1** *Make sense of problems and persevere in solving them.*
> **MP.2** *Reason abstractly and quantitatively.*
> **MP.4** *Model with mathematics.*
> **MP.6** *Attend to precision.*
> **MP.7** *Look for and make use of structure.*

THE NUMBER LINE

Each number can be depicted as a point on a number line. It is the first time that students see the number line extending in both directions. Since a number line shows both quantity and direction, there are now two different numbers that have the same quantity such as 4 and -4. Therefore, students shift from using line segments to show value on the number line to using arrows which show both quantity (magnitude) and direction. Make sure students are exposed to both horizontal and vertical number lines. Scales are not limited to 1.

Demonstration of understanding of positives and negatives involves translating among words, numbers, and models. For example, given the words "7 degrees below zero," students could show the number on a thermometer and write -7. Given the number -4 on a number line, students could write a real-life example and state that it is -4.

When making number lines, reinforce the idea of scale.

EXAMPLE

- Draw a number line with points 0 and 1 marked on it.
- Plot and label the following points: $-1, \frac{1}{2}, 3, -4, 6, 3.5$.
- Use a ruler or compass to measure the distance between 0 and 3; -1 and -4; 3 and 6; $\frac{1}{2}$ and 3.5. Describe the distance between the sets of numbers. Explain why.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

The Negative Sign
Many students struggle with the negative sign. Historically, the concept of the negative sign that we use today did not even emerge until 1867 because of its conceptual difficulties.

The negative sign can mean several things:
- A sign attached to a number to form negative numbers;
- A subtraction sign; or
- An indication to take the opposite of.

EXAMPLE
   a. Which is larger $--5$ or 5?
   b. Which is larger $-x$ or x?
   c. Which is larger -6 or x?
   d. Which is larger $x + x$ or x?
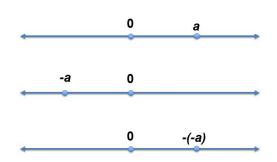
Discussion: Students have difficulties with these types of problems because they have difficulty viewing negative signs as "the opposite of." In parts **b-d.,** students have a hard time recognizing that they do not have enough information. For example, in part **b.** if x is a positive number, x would be larger. If x is a negative number, $-x$ is larger, and if x were 0, the two statements would be equal. To help counter these difficulties, it may be helpful in instruction to read $-x$ as "the opposite of x" instead of "negative x." That way students will not incorrectly infer from the language that "negative x" could indeed represent a positive number. For part **c.**, you could challenge students to place the two numbers on a number line and show how x could appear on either side of the number line whether or not the variable had a negative sign attached to it. See Model Curriculum 7.NS.1-3 for more information on difficulties with the negative sign.

Many students incorrectly think that any symbol with a negative sign is a negative number, whereas $-x$ could in essence equal a positive number such as $-x = 3$. To prevent this misconception, let students see example where $-x$ is a positive number. For example if $x = -4$, then $-x = -(-4) = 4$.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

Opposite Numbers
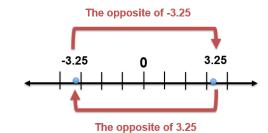
It is helpful for students to understand that putting a negative sign in front of a number place the number on the opposite side of zero on the number line. This leads to the idea of opposites.

Starting with examples of gaining/owing and above/below zero sets the stage for understanding that there is a mathematical way to describe opposites. Students should already be familiar with the counting numbers (positive whole numbers and zero), as well as with fractions and decimals (also positive). They are now ready to understand that every number has an opposite. These special numbers can be shown on vertical or horizontal number lines, which then can be used to solve simple problems. This cluster connects with standard 6.EE.6 using variables as an application.

EXAMPLE

a. Jessica has $3.25. Plot that on a number line.
b. Mike has the opposite situation as Jessica. Describe his situation in words, and plot it on the number line.
c. Julie has the opposite situation as Mike's situation. Describe her situation in words, and plot the situation on the number line.
d. How does Julie's situation compare with Jessica's situation? Explain.

EXAMPLE

Plot -6.1 and its opposite. How far is each number away from 0? Explain.

Plotting variables on a Number Line (connects to 6.EE.6)

To help students make the connection that variables are used to represent numbers (6.EE.6), students should plot variables on a number line. The use of variables will also help students make algebraic generalizations about numbers with respect to opposites, quantity, direction, and ordering. Whereas the number 1 represents one of anything, the variable x, represents an unknown quantity of anything.

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EXAMPLE

a. Plot and label the points $2m, \frac{m}{2}, \frac{m}{3}, -m$, and $-2m$ on each number line.

b. Use the number line to plot and label the points $-m, -(-m), -n,$ and $-(-n)$.
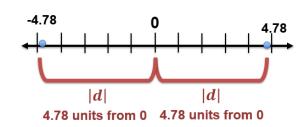
Absolute Value

Number lines give the opportunity to model absolute value as the distance from zero. In Grade 6, students should learn that the absolute value of a number does not take into account sign or direction; it is only a measure of distance (magnitude) from 0. Discourage students from saying that the "answer is always positive or 0" since that will lead to misconceptions when students encounter problems such as $|4x - 2| = 18$ in high school. Instead emphasize that it is the "distance from 0." This is why the value of something like $|x| = -5$ has no solution, since distance cannot be negative. Instead students should come to the understanding that a number and its opposite have the same distance from zero. Students should also distinguish comparisons of absolute value from statements of order. Finally, absolute values should be used to relate contextual problems to their meanings and solutions.

EXAMPLE
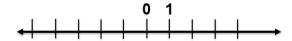
If $|d| = 4.78$, plot the possible solutions to the equation on a number line. Explain your reasoning.

Discussion: A student may come up with the following explanation: *d* can equal 4.78 or -4.78, since the distance can equal 4.78 units in either direction.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

EXAMPLE

A flea is jumping around on the number line.
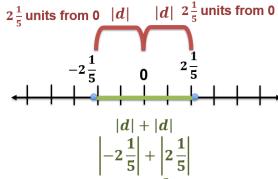
a. If he starts at 1 and jumps 3 units to the right, then where is he on the number line? How far away from zero is he?
b. If he starts at 1 and jumps 3 units to the left, then where is he on the number line? How far away from zero is he?
c. If the flea starts at 0 and jumps 5 units away, where might he have landed?
d. If the flea jumps 2 units and lands at zero, where might he have started?
e. The absolute value of a number is the distance it is from zero. The absolute value of the flea's location is 4, and he is to the left of zero. Where is he on the number line?

EXAMPLE
a. Choose a rational number.
b. Plot the number and its opposite on a number line.
c. How can you find the distance between both points?
d. How does the distance of the 2 numbers from 0 relate to the absolute value of the numbers?
e. How can you use absolute value to find the distance between two points?
f. Could you use your method to find the distance between any two points? Explain.
g. Does it matter whether the two points have the same signs or different signs? Explain.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)
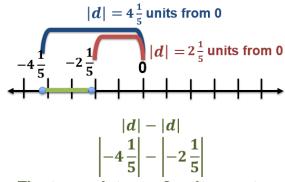
Finding the distance between points with *opposite* signs.

$2\frac{1}{5}$ units from 0 $|d|$ $|d|$ $2\frac{1}{5}$ units from 0

$-2\frac{1}{5}$ 0 $2\frac{1}{5}$

$|d| + |d|$

$\left|-2\frac{1}{5}\right| + \left|2\frac{1}{5}\right|$

The two points are $4\frac{2}{5}$ units apart, so the distance is $4\frac{2}{5}$ units.

Finding the distance between points with the *same* sign.

$|d| = 4\frac{1}{5}$ units from 0

$-4\frac{1}{5}$ $-2\frac{1}{5}$ 0 $|d| = 2\frac{1}{5}$ units from 0

$|d| - |d|$

$\left|-4\frac{1}{5}\right| - \left|-2\frac{1}{5}\right|$

The two points are 2 units apart, so the distance is 2 units.

ORDERING AND COMPARING RATIONAL NUMBERS

Using the number line, students order and compare rational numbers. They use number lines, the equal sign, and inequality symbols: >, <, and =. The number line allows students to see that as a number moves to the left (or down) its value decreases and as it moves to the right (or up) its value increases. Students need to shift from seeing value only in terms of magnitude, but also direction, e.g., $-100 < 1$ even though $|-100| > 1$. They should also explain statements of order in real-world contexts.

EXAMPLE

In a football game the Bengals lost 5 yards on their first offensive play. The Browns lost 8 yards on their first offensive play. Write an inequality comparing the total yardage of the two teams, and then explain the situation in terms of its context. Then write another equivalent inequality statement.

TIP!   Have students regularly write two inequalities for a situation to help students develop a conceptual understanding of the inequality sign. For example, $-5.15 < -5.13$ is the same as $-5.13 > -5.15$.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

EXAMPLE
John and Matt are friends. John owes his brother $15.48, and Matt owes his brother $22.25.
   a. If that is all the money each has, does John or Matt have more money? Explain.
   b. Write an inequality showing who has more money.
   c. Does John or Matt have greater debt? Explain.
   d. Write an inequality showing who has greater debt.
   e. How do the two inequalities differ? Compare.

Discussion: Students should come to the realization that John has more money: $-22.25 < -15.48$, but that Matt has greater debt: $|-15.48| < |-22.25|$. This comparison can lead to a good discussion about the difference in magnitude and direction.

EXAMPLE
A diver is less than 30 feet below sea level.
   a. Graph the diver's position on a number line.
   b. Name and plot 3 different points on the number line where the diver could be located.
   c. Describe the diver's position in words.
   d. Write an inequality describing the diver's location, *y*.

EXAMPLE
   a. Order the numbers from least to greatest using a number line: $\frac{1}{5}, \frac{1}{8}, \frac{5}{6}$.

   Write an inequality describing the numbers.
   b. Order the numbers from least to greatest using a number line: $-\frac{1}{5}, -\frac{1}{8}, -\frac{5}{6}$.

   Write an inequality describing the numbers.
   c. Compare the inequalities in part **a.** and part **b.** What do you notice?
   d. Will your observation in part **c.** always be true for any sequence of numbers and their opposites? Explain.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

EXAMPLE

a. Order the numbers from least to greatest using a number line: $|0|, \left|3\frac{1}{4}\right|, \left|-3\frac{3}{8}\right|, |3.5|$.

- What do you notice about all the numbers?

b. Order the numbers from least to greatest using a number line: $0, 3\frac{1}{4}, 3\frac{3}{8}, 3.5$.

- How do the numbers in part **b.** compare to the numbers in part **a.**?

c. Order the numbers from least to greatest using a number line: $0, -3\frac{1}{4}, -3\frac{3}{8}, -3.5$.

- How do the numbers in part **c.** compare to the numbers in part **a.**?
- How do the numbers in part **c.** compare to the numbers in part **b.**?

EXAMPLE

a. If $-7 < -6$ and $-6 < -4$, use an inequality symbol to compare -7 and -4.

b. If $3.2 < 3.21$ and $3.21 < 3.211$, use an inequality symbol to compare 3.2 and 3.211.

c. If $-3 < 6$ and $6 < 9$, use an inequality symbol to compare -3 and 9.

d. If $-0.75 < -0.745$ and $-0.745 < -0.74$, use an inequality symbol to compare -0.75 and -0.74.

e. If $a < b$ and $b < c$, use an inequality symbol to compare a and c. Explain how you know.

f. If $a > b$ and $b > c$, use an inequality symbol to compare a and c. Explain how you know.

g. If $a < b$ and $b > c$, can you use an inequality symbol comparing a and c? Explain.

Situation 1: *a* and *b* are positive numbers.

Discussion: This example can be used to develop the understanding of inequalities and number by making generalizations.

EXAMPLE

a. Represent any number a and b so that $a < b$ on a number line.

b. Now place $-a$ and $-b$ on the same number line.

c. Use an inequality symbol to compare $-a$ and $-b$.

d. Compare $a < b$ with the inequality statement you wrote for part **c.** Will what you found for the two inequalities statements hold true for any inequality statement written with two numbers? Explain.

Situation 2: *a* and *b* are negative numbers.

Discussion: Through discussion draw attention to the three situations that can be represented by a and b and $-a$ and $-b$ as shown in the diagram on the right. This example like the last one can be used to develop the understanding of inequalities and number by making generalizations.
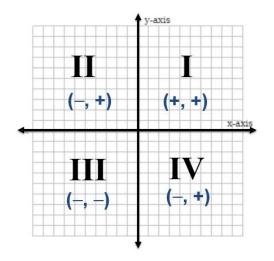
Situation 3: *a* is negative and *b* is positive.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

COORDINATE PLANE

In Grade 5 students only plotted points in Quadrant I of a coordinate grid. Now students learn to plot points in all four quadrants. Scales should not be limited to 1. When applying the concepts of opposites to the coordinate plane, students should realize that when two ordered pairs differ only by signs, they are reflections across one or both of the axes. Using number lines to model negative numbers, prove the distance between opposites. This understanding behind the meaning of absolute value easily transfers to the creation and usage of four-quadrant coordinate grids. Make connections that distances between end points can be found by counting the distance between endpoints on the grid (carefully taking into consideration the scale). They can also take the sums or differences of the absolute values of *x* or *y*-coordinate depending on whether the points are in different or the same quadrants. Actual computation with negatives and positives is handled in Grade 7. Also, finding distances between points is limited to horizontal and vertical lines, since students will learn the Pythagorean Theorem in Grade 8. This cluster can be connected to 6.G.3.

EXAMPLE

   a. Plot the point A (3, -5) on a coordinate grid.
   b. Reflect point A across the *x*-axis, and label it B.
   - What are the coordinates of the reflection?
   - How do the reflection's coordinates compare to the original Point A?
   - How does this connect to the idea of opposite numbers?
   - What do you notice about the absolute values of the *x*- and *y*-coordinates of points A and B?
   - How does the absolute value of the coordinates of points A and B relate to its distance from the *x*-axis?
   c. Reflect point A across the y-axis and label it B.
   - What are the coordinates of the reflection?
   - How do the reflection's coordinates compare to the original Point A?
   - How does this connect to the idea of opposite numbers?
   - What do you notice about the absolute values of the *x*- and *y*-coordinates of points A and B?
   - How does the absolute value of the coordinates of points A and B relate to its distance from the *y*-axis?
   d. What could you do to Point A, so that both its *x*-coordinate and *y*-coordinate are opposite?

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

EXAMPLE
Chenglei wants to plant a rectangular garden in his backyard. He wants to place the following posts from the northwest corner of his house.
- Post A is 5 ft to the west and 6 ft north.
- Post B is 5 ft to the west and 12 ft north.
- Post C is 4 ft to the east and 12 ft north.
- Post D is 4 ft to the east and 6 ft north.
    a. Draw and label the points on a coordinate grid.
    b. Find the perimeter of the garden.
    c. Find the area of the garden.

Discussion: After students plot the points, they can find the length and the width of the rectangle. The easiest way to do so is by counting. Connect finding the horizontal and vertical lengths of a figure to finding the length of the distance between two points on a number line using absolute value. Discuss how using the absolute value differs depending on if the signs of the *x*-values (or *y*-values) are the same or if they are different. Discuss why you use only the *y*-values when finding the length of a vertical line and why you use only the *x*-values when finding the length of a horizontal line.

Students may incorrectly think that they can find the length of a diagonal line using the same methods. Have them draw a figure on grid paper and use a ruler to prove that their methodology does not work.

Instructional Tools/Resources
These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.
Manipulatives/Technology
- Grid paper
- Number lines
- Play money
- Check register
- Graphing utilities such as a graphing calculator, Desmos, or GeoGebra

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

Interpreting Integers
- How Much Did the Temperature Change in Boston? by Yummy Math is a task where students analyze a graph of temperatures in Boston.
- Deflategate by Yummy Math is a task where students apply negative number understanding and proportional reasoning to a controversial situation involving a deflated football.
- Extending the Number Line by Illustrative Mathematics is a task that introduces the need for negative numbers.

Absolute Value
- Weather Extremes by Yummy Math is a task where students apply absolute value concepts to find the difference in temperatures.
- Above and Below Sea Level by Illustrative Mathematics is a task that helps students interpret signed numbers using a magnitude and direction and to make sense of absolute value in a context.

Coordinate Plane
- Coordinate Plane Song by Numberock Math Songs is an animated video of a song on how to plot points in all four quadrants.
- The (Awesome) Coordinate Plane Activity by Nathan Kraft is a Desmos activity where students have to write coordinates to hit a target.
- Episode 36: The X and Y Files is an episode from the British TV series Math Mansion about plotting points on the coordinate plane.
- Episode 4: Ordering Positive and Negative Numbers is an episode from the British TV series Math Mansion about ordering positive and negative numbers.
- Plotter the Penguin by NCTM Illuminations is a game that moves a penguin around a coordinate grid using ordered pairs. *NCTM now requires a membership to view their lessons.*
- Finding Your Way Around by NCTM Illuminations is an activity where students plot and name points and use points to draw geometric figures. *NCTM now requires a membership to view their lessons.*
- Describe the Graph by NCTM Illuminations is a lesson that reviews plotting points on the coordinate graph. *NCTM now requires a membership to view their lessons.*
- Nome, Alaska by Illustrative Mathematics is a task where students solve real-world problems by interpreting and comparing points in the coordinate plane.

General Resources
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- Arizona's 6-High School Progression on the Number System is an informational document for teachers. This cluster is addressed on pages 7-8.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.NS.5-8)

Curriculum and Lessons from Other Sources

- EngageNY, Grade 6, Module 3, Topic A, Lesson 1: Positive and Negative Numbers on the Number Line—Opposite Direction and Value, Lesson 2: Real-World Positive and Negative Numbers and Zero, Lesson 3: Real-World Positive and Negative Numbers and Zero, Lesson 4: The Opposite of a Number, Lesson 5: The Opposite of a Number's Opposite, Lesson 6: Rational Numbers on the Number Line, Lesson 7: Ordering Integers and Other Rational Numbers, Lesson 8: Ordering Integers and Other Rational Numbers, Lesson 9: Comparing Integers and Other Rational Numbers, Lesson 10: Writing and Interpreting Inequality Statements Involving Rational Numbers, Lesson 11: Absolute Value—Magnitude and Distance, Lesson 12: The Relationship Between Absolute Value and Order, Lesson 13: Statements of Order in the Real World, Lesson 14: Ordered Pairs, Lesson 15: Locating Ordered Pairs on the Coordinate Plane, Lesson 16: Symmetry in the Coordinate Plane, Lesson 17: Drawing the Coordinate Plane and Points on the Plane, Lesson 18: Distance on the Coordinate Plane, Lesson 19: Problem Solving and the Coordinate Plane are lessons that pertain to this cluster.
- Illustrative Mathematics, Grade 6, Unit 7: Rational Numbers has many lessons that pertain to this cluster.
- Georgia's Standards of Excellence Curriculum Frameworks, Grade 6, Unit 7: Rational Numbers and their Opposites has many tasks that pertain to this cluster.

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STANDARDS	MODEL CURRICULUM (6.EE.1-4)
EXPRESSIONS AND EQUATIONS **Apply and extend previous understandings of arithmetic to algebraic expressions.** **6.EE.1** Write and evaluate numerical expressions involving whole　number exponents. **6.EE.2** Write, read, and evaluate expressions in which letters stand　for numbers. 　a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 − y.* 　b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.* *Continued on next page*	**Expectations for Learning** In prior grades, students developed strategies for writing and interpreting numerical expressions. In Grade 6, students apply and extend their previous understanding of arithmetic operations and notations to use algebraic order of operations when writing, interpreting, and finding equivalent algebraic expressions. Students will extend their knowledge of powers of ten to whole number exponents with other bases. In Grade 6, students start to use properties of operations to manipulate algebraic expressions to produce different, but equivalent expressions for different purposes. This will build the foundation for solving equations and inequalities with rational numbers and operations with scientific notation in grades seven and eight. **ESSENTIAL UNDERSTANDINGS** **Reading, Writing, and Evaluating Expressions** • A variable can represent an unknown value or set of values. • A factor can be a single entity or sum/difference of terms, e.g., 2(3 + 5) is two factors 2 and (3 + 5). • A term is a number, variable, product, or quotient in an expression. • Terms are separated by addition and/or subtraction signs within an expression. • A term is either a single number or variable, or numbers and variables multiplied together. • If a term consists of only variables, its coefficient is 1. • A constant is an explicit number whose value does not change. • A coefficient is the numerical factor of a term with a variable. • An expression is a variable or combination of variables, numbers, and symbols that represent a mathematical calculation. • An expression does not contain an equal sign. • Exponents represent repeated multiplication of the base. • Multiplication can be represented by algebraic notation such as parentheses, a raised dot, or with a coefficient and variable, e.g., 3*x*. • Division can be represented by a fraction bar. • Parentheses initiate an order when simplifying numerical expressions. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.EE.1-4)
6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **6.EE.4** Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	## Expectations for Learning, continued ### ESSENTIAL UNDERSTANDING, CONTINUED **Equivalent Expressions** • Equivalent expressions always have the same value. • Equivalent expressions can be generated using properties of operations (Distributive Property, Associative Property of Addition, Associative Property of Addition Multiplication, Commutative Property of Addition, Commutative Property of Multiplication, and Identity Property of Multiplication). ### MATHEMATICAL THINKING • Solve multi-step problems accurately. • Use precise mathematical vocabulary and symbols. • Compute accurately and efficiently with grade-level numbers. • Use different properties of operations flexibly. • Use reasoning with symbolic representations. ### INSTRUCTIONAL FOCUS *Note: Although, rote memorization of the names of the properties is not encouraged, it is expected for teachers to use formal language so that students gain familiarity and are able to recognize and apply the correct terminology.* **Reading, Writing, and Evaluating Expressions** • Identify parts of expressions using mathematical terms. • Evaluate an algebraic expression by substituting a given value for the variable. • Use algebraic order of operations, including whole number exponents to evaluate numerical expressions. • Evaluate formulas based on real-world problems where the variable can be a whole number, fraction, or a decimal. • Write expressions, including exponents, for mathematical and real-world situations. • Define the variable when writing expressions in real-world situations. • Translate expressions from word form to algebraic form. *Continued on next page*

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STANDARDS	MODEL CURRICULUM (6.EE.1-4)
	## Expectations for Learning, continued **INSTRUCTIONAL FOCUS, CONTINUED** **Equivalent Expressions** • Identify when two expressions are equivalent. • Substitute the same value into two or more expressions to determine their equality. • Generate equivalent expressions using properties of operations. • Differentiate between repeated addition and repeated multiplication of a single variable and the equivalent mathematical representation for each. ## Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40 • Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10 • Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19 **CONNECTIONS ACROSS STANDARDS** • Find the greatest common factor and use the distributive property (6.NS.4). • Use variables to represent numbers and write expressions (6.EE.6). • Apply the formulas for volume (6.G.2).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction *not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.*

The skills of reading, writing, and evaluating expressions are essential for future work with expressions and equations. The use of negatives and positives should mirror the level of introduction in Grade 6: The Number System; students are developing the concept and not generalizing operation rules.

> ***Standards for Mathematical Practice***
> *This cluster focuses on but is not limited to the following practices:*
> **MP.2** *Reason abstractly and quantitatively.*
> **MP.6** *Attend to precision.*
> **MP.7** *Look for and make use of structure.*
> **MP.8** *Look for and express regularity in repeated reasoning.*
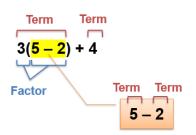
WRITING, READING, AND INTERPRETING EXPRESSIONS

An expression is a phrase in a sentence about a mathematical or real-life situation. In Grade 5 students move from viewing expressions as a sequence of operations to an object. Grade 6 furthers this understanding. For example, $3(5-2)$ can be viewed as a product of the two factors: 3 and $5-2$. However, the second factor $(5-2)$ can be viewed as a single entity or the difference of two terms.

Therefore 5 – 2 can be viewed as a single factor or as the difference of two terms.

Provide opportunities for students to
- Write expressions for numerical and real-world situations;
- Write multiple statements that represent a given algebraic expression. For example, the expression $x - 10$ could be written as "ten less than a number," "a number minus ten," "the temperature fell ten degrees," "I scored ten fewer points than my brother," etc;
- Read an algebraic expression and write their own statement or read a problem; and
- Write their own expression or even create their own problems and write the corresponding expression.

EXAMPLE

Write an expression to represent the change from $5 after buying candy.

Price of Candy ($)	0.55	1.49	2.19	3.35	m
Change from $5	$5-0.55$	$5-1.49$	$5-2.19$	$5-3.35$	$5-m$

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

The use of technology can assist in the exploration of the meaning of expressions. Many calculators will allow one to store a value for a variable and then use the variable in expressions. This enables the student to discover how the calculator deals with expressions like x^2, $5x$, xy, and $2(x + 5)$.

EXAMPLE

Kevin ran against Kate in race measured in feet. His speed can be described as $20a - 4$ where a is the number of seconds. What does 20 represent? What could 4 represent?

EXAMPLE

- If $h + g = 16$,
  then $h + g + 2 = ?$
- If $k - 12 = 16$,
  then $k - 13 = ?$
- If $m + n = 7$,
  then $m + n + p = ?$

Discussion: Although students can solve these by using guess and check strategies, draw attention to the structure of the expressions. Part **c.** might give some students difficulties as the solution is the expression $7 + p$.

Parts of an Expression

At Grade 6, the emphasis should not be placed on counting terms in an expression. This can cause confusion. For example, students can say $3(x + 2) + y$ has two terms, while the equivalent expression $3x + 3(2) + y$ has three terms. Although $(x + 2)$ is a factor with two terms inside a longer expression consisting of two terms, students do not need to have that deep of an understanding.

Use and encourage students to use proper mathematical vocabulary when discussing terms, factors, coefficients, etc. Link vocabulary to glossary.

EQUIVALENT EXPRESSIONS
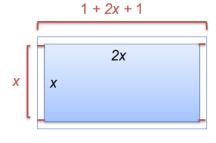
Provide opportunities for students to write equivalent expressions, both numerically and with variables. For example, given the expression $x + x + x + x + 4 \cdot 2$, students could write $2x + 2x + 8$ or some other equivalent expression. Make the connection to the equivalent form with the fewest terms of this expression as $4x + 8$. Because this is a foundational year for building the bridge between the concrete concepts of arithmetic and the abstract thinking of algebra, using hands-on materials (such as algebra tiles, counters, unifix cubes, "Hands on Algebra") to help students translate between concrete numerical representations and abstract symbolic representations is critical.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

EXAMPLE
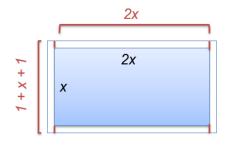
A pool is twice as long as it is wide. It is surrounded by a border of 1-foot square tiles.
- Draw a picture of the situation.
- Write an expression to show how many tiles are needed using pictures and/or words to explain your expression.
- Write two other equivalent expressions to show how many tiles are needed using pictures and/or words to explain your expressions.
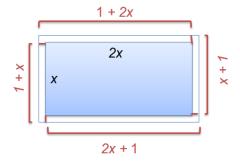

$(1+ 2x + 1) + x + (1 + 2x + 1) + x$

or

$2(1+ 2x + 1) + 2(x)$

$(1+ x + 1) + 2x + (1 + x + 1) + 2x$

or

$2(1+ x + 1) + 2(2x)$

$(1+ 2x)+ (x + 1) + (2x + 1) + (1+ x)$
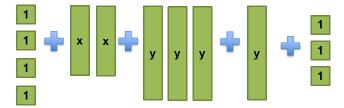
Combing Like Terms

It is important to use models to help illustrate combing like terms. This will prevent students from making errors such as $3x + x = 3x^2$ instead of $4x$. Algebra tiles and number lines are helpful models for combining like terms. In Grade 7, students will extend combining like terms to negative numbers. If using algebra tiles as a model, students may want to start with combining like terms using integers, and then move to combining like terms involving rational numbers without using algebra tiles.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)
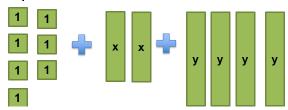
EXAMPLE

Create an equivalent expression with the least number of terms: 4 + 2x + 3y – x + y + 3
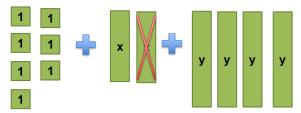
Step 1: Represent all your additions, and highlight your subtractions. (As you can see – x is highlighted above.)

Step 2: Combine tiles that are alike.

Step 3: Subtract by taking away.
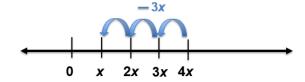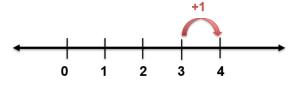
So we are left with 7 + *x* + 4*y*.

Discussion: Students should represent the problem with algebra tiles. The benefit of using the algebra tiles is that it connects to concepts of area. Explain that an *x*-tile is a 1 unit by *x*-unit long tile. A *y*-tile is a tile that is 1 unit by *y*-units long. In our example the *x*-tile is shorter than the *y*-tile, but that is not necessarily the case. Since students do not know how to add negative numbers, they actually take away tiles for subtracting. The example above just shows one way to handle subtraction. For example students could subtract before combining.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

EXAMPLE

Rewrite with the least number of terms: $4x + 3 - 3x + 3y + 1 - \frac{1}{2}y$

Discussion: The benefit of a number line model over the algebra tiles model is that it can show fractional values of variables. An arithmetic number line contains 0, and 1 is the unit of reference. An algebra number line contains 0, but x is the unit of reference. Each variable has its own number line with its own interval since variables typically can have different amounts. It would be useful to have a discussion about how although the x's seem smaller in the diagram, they could have a greater value or even the same value as y. They can apply the same strategies to add and subtract on a number line as they used in previous grades. Therefore, the solution is $x + 4 - 2\frac{1}{2}y$.

TIP! Students often get confused when they see a variable such as x without a coefficient. Make a connection to the Multiplicative Identity Property and have students write the "invisible" 1 in front of the variables without coefficient. For example, $x + 3x - 2$ can be written as $1x + 3x - 2$.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

Properties of Operations

Students should apply the properties of operations (associative, commutative, distributive, and identity) and equality found in Table 3 and Table 4 of Ohio's Learning Standards when rewriting expressions or solving equations (6.EE.7). Teachers should be using the correct terminology to justify steps when performing operations and solving equations. Although, Grade 6 students should not be required to know the formal names of the properties, they should be encouraged to recognize them and use them to justify their steps when solving equations. For example, students may say "change order" for Commutative Property or "rearranging groups" for Associative Property, and that is acceptable at this level, but teachers and/or classmates should reiterate the correct vocabulary during discussions. Students should not be assessed in situations where they have to recall the formal property names, but they should be able to recognize them. *Note: The Addition Property of Equality and the Subtraction Property of Equality can be used interchangeable since subtracting a number is the same as adding its opposite. The same is true for the Multiplication Property of Equality and the Division Property of Equality*
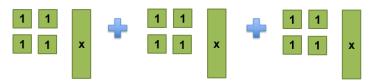
Distributive Property

Although students should be fluent using all the properties, special attention should be paid to the Distributive Property. Previously students have used area models to represent the Distributive Property, they can now extend this thinking to fractions, decimals, and variables. Students can use algebra tiles to develop the concepts and then move towards using the box method.
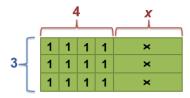
EXAMPLE

Rewrite 3(4 + x) using the distributive property and area model.

Method 1: Represent as 3 groups of (4 + x).

Method 2: Use an area model where the factors 3 and (4 + x) are the length and width.

The solution is 12 + 3x.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

EXAMPLE

Rewrite 15 + 6y using the Distributive Property and area model.
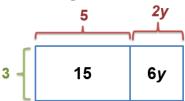
Step 1: Draw an area model.

15	6*y*

Step 2: Find a common factor and label it as its width.

3 { | 15 | 6*y* |

Step 3: Find the length of each box using division.

5   2*y*

3 { | 15 | 6*y* |

Therefore 15 + 6*y* = 3(5 + 2*y*)

EVALUATING EXPRESSIONS

Provide a variety of expressions and problem situations for students to practice and deepen their skills. They should understand that when evaluating an expression through substitution, like variables will have the same value. Start with simple expressions to evaluate and move to more complex expressions. Likewise start with simple whole numbers and move to fractions and decimal numbers. The substituted values should be positive rational numbers since students do not yet know how to do operations with negative numbers.

Provide expressions and formulas to students, along with values for the variables, so students can substitute values and evaluate the expression. Evaluate expressions using the order of operations with and without parentheses.

Students might think x represents a missing digit in a two-digit number instead of a missing value to be multiplied by the coefficient so that 4x means 41, or 42, or 43. In reality 4x means 4 times x or $x + x + x + x$, so evaluating 4x when $x = 5$ is 4 • 5 not 45. In addition, students need to realize that the x can be any number including a 2-digit, 3-digit, or 4-digit number, etc.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

ALGEBRAIC ORDER OF OPERATIONS

In earlier grades, students added grouping symbols ( ) to reduce ambiguity when solving equations. Now the focus is on using ( ) to denote terms in an expression or equation. Order of operations should be applied inside grouping symbols. Likewise, the division symbol (3 ÷ 5) was used in earlier grades and should now be replaced with a fraction bar ($\frac{3}{5}$). Less confusion will occur as students write algebraic expressions and equations if *x* represents only variables and not multiplication. The use of a dot (•) or parentheses between number terms is preferred.

Using the mnemonic PEMDAS or "Please Excuse My Dear Aunt Sally" causes many misconceptions and is highly discouraged. Students get the impression that multiplication must come before division, addition must come before subtraction, operations must be performed left to right, and that calculations in parenthesis must come first. Instead people may solve 5 – 9 + 3 – 5 + 9 by changing the order using the Associative and Additive Inverse properties (5 – 5) + (–9 + 9) + 3 instead of adding and subtracting from left to right.

The algebraic order of operations tells us how to interpret expressions but not necessarily how to evaluate them. For example, students can manipulate expressions using the properties of operations: 35 (100 – 1) can be thought of as 35(100) – 35(1). Research is pointing toward avoiding the mnemonic all together. Instead the algebraic order of operations can be viewed by the following criteria:

- ***Multiplication* must come before *Addition*!** This is really the only rule students need to remember. To be noted the reason that multiplication comes before addition is because multiplication is repeated addition.
- *Parentheses* are not an operation; they are grouping representations that state that the calculations must be performed separately not necessarily first.
- *Exponents* are just repeated *Multiplication*.
- *Division* can be viewed as a type of *Multiplication* since 3 ÷ 4 is the same as $3\left(\frac{1}{4}\right)$.
- *Subtraction* can be viewed as a type of *Addition* since they are inverse operations. (In Grade 7, students will learn that subtraction is the same as adding the opposite.)
- An expression can be rewritten using the Commutative or Associative Properties before calculating from left to right. In Grade 6, students may be only able to manipulate addition and not subtraction, but in Grade 7 they should be able to manipulate both since subtraction is just adding the opposite.

Therefore, PEMDAS can be replaced by *exponents first, then multiplications, then additions* or even more simply *multiplications, then additions* since exponents are really just repeated multiplications. Instead it would be helpful to rewrite division as multiplication. Then identify the terms and factors of the expression first. The terms can be grouped with parenthesis. This allows the structure of the expression become more apparent, so students can apply the appropriate properties of operation.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

EXAMPLE

Evaluate the expression: $4^3 + 5 \cdot 17 \cdot 23 \cdot 5$.

$4^3 + 5 \cdot 17 + 23 \cdot 5$	The terms are 4^3, $5 \cdot 17$, $23 \cdot 5$. ▪ 4^3 has 3 factors. ▪ $5 \cdot 17$ has 2 factors. ▪ $23 \cdot 5$ has 2 factors.
$4^3 + (5 \cdot 17) + (23 \cdot 5)$	Group the terms using parenthesis.
$4^3 + 5(17 + 23)$	Apply the Distributive Property.
$4^3 + 5(40)$	Multiply first.
$64 + 200 = 264$	Then add.

Discussion: In the example the student used parenthesis to group the multiplications together. Then he or she applied the Distributive Property. Then the student did the exponents last. This method is much more efficient and promotes number sense rather than having students memorize the mnemonic PEMDAS.

EXAMPLE

Evaluate the expression: $5 + 8 \cdot 4 \div 8 - 8 \div 2$.

Discussion: Although students could evaluate this using PEMDAS, they may be able to solve it mentally and perhaps more efficiently using the properties of operations. In this example the student rewrote the divisions as multiplications and used parenthesis to group the multiplications together. Then they used the Commutative Property, the Multiplicative Inverse Property, and the Additive Inverse Property instead of performing the operations from left to right.

$5 + 8 \cdot 4 \div 8 - 8 \div 2$	Rewrite division as multiplication.
$5 + 8 \cdot 4 \cdot \dfrac{1}{8} - 8 \cdot \dfrac{1}{2}$	The terms are 5, $8 \cdot 4 \cdot \dfrac{1}{8}$, $8 \cdot \dfrac{1}{2}$. ▪ 5 has 1 factor. ▪ $8 \cdot 4 \cdot \dfrac{1}{8}$ has 3 factors. ▪ $8 \cdot \dfrac{1}{2}$ has 2 factors. Group the terms using parenthesis.
$5 + \left(8 \cdot 4 \cdot \dfrac{1}{8}\right) - \left(8 \cdot \dfrac{1}{2}\right)$	Use the Commutative Property
$5 + \left(8 \cdot \dfrac{1}{8} \cdot 4\right) - \left(8 \cdot \dfrac{1}{2}\right)$	Use the Multiplicative Inverse Property.
$5 + 4 - \left(8 \cdot \dfrac{1}{2}\right)$	Multiply first.
$5 + 4 - 4$	Use the Additive Inverse Property instead of adding left to right.
5	

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

Include whole-number exponents, fractions, and decimals when writing expressions. Encourage students to show step-by-step thinking when rewriting an expression with the least number of terms. Provide opportunities for students to share their work and explain their thinking. This gives them opportunities to see that difference processes can maintain equivalence.

Exponents
In Grade 5 students are introduced to exponents using powers of ten. Now they are applying whole number exponents to any base. Just as a multiplication sign is used to represent repeated addition, an exponent is used to represent repeated multiplication. Bases include variables and rational numbers. They should use whole number exponents in numerical expressions

EXAMPLE
Evaluate $\left(\frac{2}{3}\right)^4 + 3$.

EXAMPLE
What is the difference between $5c$ and c^5?

Discussion: Emphasize the 5 in $5c$ is a factor and represents repeated addition, so $5c$ means c added 5 times or $c + c + c + c + c$. Whereas the exponent in c^5, represents repeated multiplication $c \cdot c \cdot c \cdot c \cdot c$.

Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example with numerical expressions, 4^3 does not equal $4 \cdot 3$ but rather $4 \cdot 4 \cdot 4$. For example, knowing the operations that are being referenced with notation like, x^3, $4x$, $3(x + 2y)$ is critical. The fact that x^3 means $x \cdot x \cdot x$, means x times x times x, not $3x$ or 3 times.

EXAMPLE
Evaluate $4^3 \cdot 2$.

Discussion: Some students may incorrectly think that they can multiply the 4 and the 2 first to get 8^3. Encourage students to write the power as repeated multiplication before solving: $4 \cdot 4 \cdot 4 \cdot 3$.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology
- Algebra tiles
- Number lines
- Algebra Tiles Interactive by Holt McDougal Online is an applet with Algebra tiles.

Equivalent Expressions
- Using Models to Identify Equivalent Expressions by Learnzillion is a video that uses area models to help reinforce the key concept that equivalent expressions have the same value. (Teachers need to create free account.)

Writing Expressions
- Fantasy Football by Yummy Math has students write expressions to calculate fantasy football points.
- The Nested Splats by Steve Wyborney has lessons that could be used for writing expressions or generating equivalent expressions.

Exponents
- Seven to the What?!? by Illustrative Mathematics has students look at patterns of powers to express regularity in repeated reasoning.
- Problem of the Month: Double Down by Inside Mathematics explores exponential growth.
- Djinni by GeoGebra is an applet that show the number of coins growing exponentially. It parallels nicely with Illustrative Mathematics task The Djinni's Offer.
- Sierpinski's Carpet by Illustrative Mathematics is a task that explores the usefulness of exponential notation.
- Exponent Exploration 1 and Exponent Exploration 2 by Illustrative Mathematics has students analyze exponential numerical expressions.

Order of Operations
- Order of Operations Bingo by NCTM Illuminations has students practice problems through a Bingo game. It includes common misconceptions for teacher reference. *NCTM now requires a membership to view their lessons.*
- Order of Operations by Michelle Schade from BetterLessonPlans has students work in stations to use the order of operations to solve numerical expressions.
- Who Had the Greatest NBA Season Ever? by Yummy Math has students work with order of operations, expressions, formulas, and the Distributive Property to calculate how players rank in their best season.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

Properties of Operations
- Extending the Distributive Property by CPalms is a lesson where students will build upon their arithmetic experiences with the Distributive Property to equate algebraic expressions through a series of questions related to real-world situations and the use of manipulatives.

Curriculum and Lessons from Other Sources
- EngageNY, Grade 6, Module 4, Topic A, Lesson 1: The Relationship of Addition and Subtraction, Lesson 2: The Relationship of Multiplication and Division, Lesson 3: The Relationship of Multiplication and Addition, Lesson 4: The Relationship of Division and Subtraction are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 4, Topic B, Lesson 5: Exponents, Lesson 6: The Order of Operations are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 4, Topic C, Lesson 7: Replacing Letters with Numbers, Lesson 8: Replacing Numbers with Letters are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 4, Topic D, Lesson 9: Writing Addition and Subtracting Expressions, Lesson 10: Writing and expanding Multiplication Expressions, Lesson 11: Factoring Expressions, Lesson 12: Distributing Expressions, Lessons 13: Writing Division Expressions, Lesson 14: Writing Division Expressions are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 4, Topic E, Lesson 15: Read Expressions in Which Letters Stand for Numbers, Lesson 16: Write Expressions in Which Letters Stand for Numbers, Lesson 17: Write Expressions in Which Letters Stand for Numbers are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 4, Topic F, Lesson 18: Writing and Evaluating Expressions—Addition and Subtractions, Lesson 19: Substituting to Evaluate Addition and Subtraction Expressions, Lesson 20: Writing and Evaluating Expressions—Multiplication and Division, Lesson 21: Writing and Evaluating Expressions—Multiplication and Addition, Lesson 22: Writing and Evaluating Expressions—Exponents are lessons that pertain to this cluster.
- Georgia Standards of Excellence Framework, Grade 6, Unit 3: Expressions has many tasks that align with this cluster.
- Illustrative Mathematics, Grade 6, Unit 6, Lesson 6: Write Expressions Where Letters Stand for Numbers, Lesson 8: Equal and Equivalent, Lesson 9: The Distributive Property, Part 1, Lesson 10: The Distributive Property, Part 2, Lesson 11: The Distributive Property, Part 3, Lesson 12: Meaning of Exponents, Lesson 13: Expressions with Exponents, Lesson 14: Evaluating Expressions with Exponents, Lesson 15: Equivalent Exponential Expressions are lessons that pertain to this cluster.

General Resources
- Arizona 6-8 Progression on Expressions and Equations is an informational text for teachers. This cluster is addressed on pages 4-6.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.1-4)

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STANDARDS	MODEL CURRICULUM (6.EE.5-8)
EXPRESSIONS AND EQUATIONS **Reason about and solve one-variable equations and inequalities.** **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	## Expectations for Learning In prior grades, students have used the inequality signs to compare numbers. This cluster introduces students to the use of a variable as an unknown quantity or set of quantities in expressions, equations, and inequalities. Students utilize their understanding of opposites and division of rational numbers to solve one-step equations. In Grade 7, students will expand on techniques to solve multi-step equations and inequalities. ### ESSENTIAL UNDERSTANDINGS **Equations** • A variable can represent an unknown value or a set of values. • An algebraic equation is a mathematical statement that says that two expressions are equal. • The same operation must be performed on both sides of an equation to maintain equivalence. • Addition and subtraction are inverse operations. • Multiplication and division are inverse operations. • A solution is a value that makes an equation or an inequality true. **Inequalities** • Inequalities can have infinitely many solutions. • Solutions to inequalities can be represented on number line diagrams. • Point c is not included in the graphical solution to $x > c$ or $x < c$; the number line diagram represents this with an open circle around point c. • All of the solutions to an inequality are represented with a shaded region (or an arrow) on a number line diagram. • The inequality $x > c$ is equivalent to $c < x$. ### MATHEMATICAL THINKING • Compute using strategies or models. • Determine reasonableness of results. • Use reasoning to represent a concept symbolically. • Recognize and use a pattern or structure.

STANDARDS	MODEL CURRICULUM (6.EE.5-8)
	## Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Define the variable in context when writing equations and inequalities. • Use substitution to identify solution(s) from a given set. • Determine whether a given value is a solution to an equation or inequality. • Write equations and inequalities to represent a context. **Equations** • Solve one-step equations of the form $x + p = q$, $x - p = q$, or $px = q$, where p, x, and q are all nonnegative rational numbers using models and algebraically. • Solve one-step equations of the form $\frac{x}{p} = q$ where x and q are all nonnegative rational numbers and p is a positive integer using models and algebraically. • Solve real-world problems by writing one-step equations. **Inequalities** • Explain the meaning of an inequality. • Determine if a single value is required as a solution, or if the solution allows for multiple solutions. • Write an inequality in the form of $x > c$ or $x < c$ to represent a constraint or condition in a real-world problem. • Represent solutions of inequalities of the form $x > c$ or $x < c$ on a number line diagram. ## Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40 • Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10 • Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19 **CONNECTIONS ACROSS STANDARDS** • Divide fractions by fractions (6.NS.1). • Recognize opposites of numbers (6.NS.6a). • Evaluate expressions (6.EE.2).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on underlined instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.2 Reason abstractly and quantitatively.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

AN EQUATION OR INEQUALITY AS A TRUE STATEMENT

The skill of solving an equation must be developed *conceptually* before it is developed *procedurally*. This means that students should be thinking about what numbers could possibly be a solution to the equation before solving the equation. For example, in the equation $x + 21 = 32$ students know that $21 + 9 = 30$ therefore the solution must be 2 more than 9 or 11, so $x = 11$.

EXAMPLE

Which numbers in the set $\{7, 0, \frac{1}{2}, 0.2, 4, 4.1, \frac{13}{3}, 100\}$ make the inequality true?

$12 < 3a$

Although solving equations in this cluster is limited to one-step equations and inequalities, students should be given more complicated linear equations when using substitution to determine whether numbers in a set make the equation true.

EXAMPLE

Which number(s) in the set $\{7, 0, \frac{1}{2}, 0.2, 4, 4.1, \frac{13}{3}, 100\}$ make the inequality true?

$3h - 1 > 2.$

EXAMPLE

Which number in the set $\{0, 18.6, 12, 24, 36, 6.5\}$ makes the equation true?

$18 = \frac{1}{2} b - 6$

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

EXAMPLE

a. Which number(s) in the set $\{\frac{1}{3}, \frac{2}{3}, 0, 1, 3, 9\}$ make the inequality true?

$3y > 2y$

b. Which number(s) make this inequality true?

$3y > 2y + 1$

Discussion: In this example students should discuss why any positive number (greater than 1) makes this inequality true*.*

LETTERS USED AS VARIABLES THAT VARY

Letters are used in mathematics in a variety of ways: labels (*m* for meters), constants (π), unknowns ($3x + 1 = 10$), universal statements ($a + b = b + a$), varying quantities ($y = 3x - 5$) , parameters such as *m* and *b* in $y = mx + b$, and quantities in a formula ($A = lw$). This may lead to confusion when students encounter letters that are used as variables.

In Grade 6 a variable is defined as a letter that represents—

- an unknown quantity(6.EE.6); or
- any number in a specified set (6.EE.6); or
- related quantities that change (covary) in relationship to one another (6.EE.9).

Many students incorrectly view the variable as a label; they have a hard time distinguishing the name of an object from the name of the attribute and from a quantity of measure. The term 0.77*b* could be understood to mean 0.77 bananas instead of the cost of *b* pounds of bananas at $0.77 per pound. Incorrectly viewing variables as labels may be a difficult misconception to undo, so it may be wise to avoid using mnemonic variables.

Have students compare expressions to understand how variables can be used along with the structure of an expression to create meaning.

EXAMPLE
Which is greater 2*x* or *x* + 2?

Discussion: Some students will say 2*x* because it is multiplication, but allow exploration to draw attention to the fact that it depends on the value of *x*. 2*x* is only greater if *x* is greater than 2. Provide the opportunity for students by trying different variables for *x* to notice that as *x* increases the difference between 2*x* and *x* + 2 increases; however, that only holds true as long as *x* > 2. Students may want to create a table of values to demonstrate this.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

EXAMPLE

State all the values for *m* which make the following statement true: $m + 4 = 4 + m$.

Discussion: Many students incorrectly think a variable can only be one number instead of a generalized number that can take on more than one value. Most students will go straight to trying to solve the equation instead of viewing the statement as a relationship. When students get stuck trying to solve the equation, allow for the exploration to draw attention to the structure of the expressions and how the left and the right side of the equation are equivalent. They should then be able to come to the conclusion by looking at the relationship that *m* can be any real number. Facilitate explorations so students can realize that solutions can be rational numbers such as fractions or even negative rational numbers.

EXAMPLE
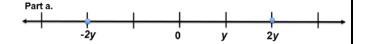
When does $n + b = b + c$? *Always*, *Sometimes*, or *Never*? Explain.
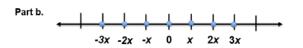
Discussion: Students often incorrectly think that the variables *n* and *c* must be different numbers. Through exploration and discussion allow students to reason through and discover that it is possible for *n* and *c* to both be the same number.
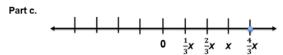
Another strategy to help students make a connection between numbers and variables is by connecting variables to the number line. In an arithmetic number line 0 and 1 are used as units of reference, so 0 and *x* can be used as points of reference in an algebraic number line. Drawing connections between the two number lines can be helpful in creating students understanding. The benefits of using number lines is that students can relate fractions and variables.

EXAMPLE
- Plot 2*y* and its opposite on a number line.
- Label 0, *x*, 2*x*, 3*x*, −*x*, −2*x*, and −3*x* on a number line.
- Elsa ran *x* distance along the track. Anna ran $\frac{4}{3}$ of Elsa's distance. Represent Anna's distance on a number line.

Part a.

Part b.

Part c.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

WRITING EQUATIONS AND INEQUALITIES

The process of translating between mathematical phrases and symbolic notation will also assist students in the writing of equations or inequalities for a situation. This process should go both ways; students should also be able to write a mathematical phrase for an equation. Additionally, the writing of equations from a situation or story does not come naturally for many students. A strategy for assisting with this is to give students an equation and ask them to come up with the situation or story that the equation could be modeling.

Provide multiple situations in which students must determine if a single value is required as a solution, or if the situation allows for multiple solutions. This creates the need for both equations (single solution for the situation) and inequalities (multiple solutions for the situation). Solutions to equations should not require using the rules for operations with negative numbers since the conceptual understanding of negatives and positives is only being introduced in Grade 6.

It may be helpful to compare arithmetic and algebraic solutions to simple word problems.

EXAMPLE

Nathan bought 3 candy bars for $1.62. How much did each candy bar cost?

Arithmetic Solution

$$1.62 \div 3 = \$0.54$$

Algebraic Solution

1. Define the variable.

   $n = cost\ of\ a\ candy\ bar$

2. Write an equation.

   $3n = 1.62$

3. Solve the equation.

   $$\frac{3n}{3} = \frac{1.62}{3}$$

   $n = \$0.54$

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)
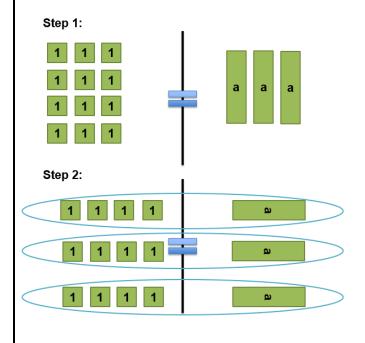
Writing and Solving One-Step Equations

A variety of concrete models such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. Students should move from concrete models, to pictures, and then equations. The focus in Grade 6 should be on inverse operations, so the discussion around the models should always go back to inverse operations.
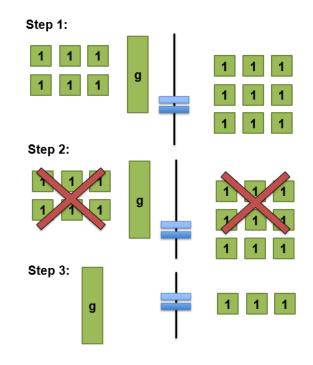
EXAMPLE

Solve 12 = 3a using Algebra tiles.

EXAMPLE

Solve 6 + g = 9 using Algebra tiles.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

EXAMPLE

Solve $k - 2 = 5$.

Step 1: Represent the total.
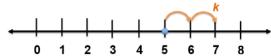
Step 2: Think what's the opposite of subtracting two?
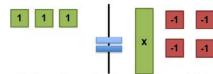
Step 3: Find k.

EXAMPLE
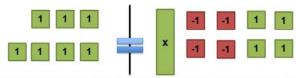
Solve $3 = x - 4$ using Algebra tiles.

Step 1:
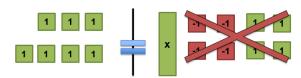
Note: Notice subtraction is represented with different colored tiles.

Step 2:
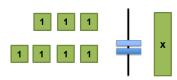
Note: Students should realize that when you combine a negative 4 and a positive 4, it makes zero. There should be a discussion about a zero pair which will be more formally introduced in 7.NS.1. Notice that all problems in 6th grade are contrived to avoid integer operations.

Step 3:
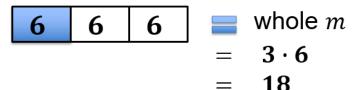
Step 4:

Note: The solution is 7 = x. Notice that the variable can be on the right side of the equation as well as the left. Students should have practice with both situations.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

EXAMPLE

Solve $\frac{m}{5} = 2$ using a model.

Step 1: Represent the problem.

Step 2: Divide the problem into 5 units.

Step 3: *Think: if each box is 2, what is the whole?*

Have students make connections between their model and using the inverse of division--multiplication.

Solving Equations with Fractions

Some studies have shown that a student's fractional knowledge correlates with their ability to write equations. Therefore, students need practice with one-step equations with fractions using manipulatives and diagrams instead of just using inverse operations in order to give the opportunity to create understanding. This will help alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used.

EXAMPLE

a. $\frac{1}{3}m = 6$

Ask students "$\frac{1}{3}$ of what number (quantity) is 6?"

whole m

Since $\frac{1}{3}$ is 6, students may put 6 in the one of the three boxes.

6   whole m

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

They should have the understanding that all fractional parts are the same size, so a 6 would go in each of the boxes. Therefore the whole or m is 18 since there are three 6s in the whole. *Note: This is an appropriate place to review 3.NF.1, 3.G.2, and 4.NF.4.*

6	6	6	whole m

$$= 3 \cdot 6$$
$$= 18$$

b. It should be contrasted side-by-side with a problem such as $\frac{1}{3} \cdot 6 = n$. *Note: Students should do many of the first type before contrasting side-by-side with the second type.*

			whole 6

In this case students know that the whole is 6. Therefore, they have to divide 6 into three equal groups.

n	n	n	whole 6

They may put a 2 in each box as shown below or they may use tally marks or manipulatives to divide up the 6. They can now see that one of the three boxes or $\frac{1}{3}$ of 6 is 2.

2	2	2	whole 6

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

EXAMPLE

a. $\frac{3}{4}h = 36$
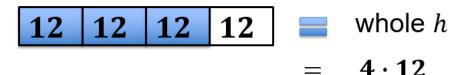
Ask students "$\frac{3}{4}$ of what number (quantity) is 36?"

whole h
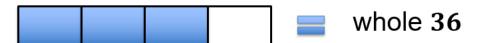
Since $\frac{3}{4}$ is 36, 36 needs to be divided up into 3 equal sized quantities. Students may write 12 in each box or physically divide it using tally marks or manipulatives.

| 12 | 12 | 12 | |

whole h

Since all the parts are equal, the fourth part must be 12 as well, so the total is 4 parts of 12 which is 48.

| 12 | 12 | 12 | 12 |

whole h

$$= 4 \cdot 12$$
$$= 48$$

b. It should be contrasted side-by-side with the problem $\frac{3}{4} \cdot 36 = h$.

whole 36

In order to get equal size pieces, 36 has to be divided into 4 sections, which means 9 goes into each box. Students may do this with numbers or tally marks or using manipulatives. Three groups of the four is 3 groups of 9 or $3 \cdot 9$ which equals 27, so h is 27.

| 9 | 9 | 9 | 9 |

whole 36

Grade 6

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)
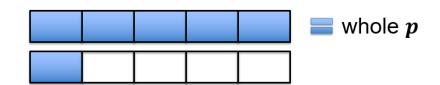
EXAMPLE
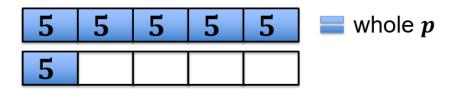
a. $\frac{6}{5}p = 30$

Ask students "$\frac{6}{5}$ of what number (quantity) is 30?"

Students need to come to the understanding by either remembering from earlier grades or discovery that the whole is $\frac{5}{5}$, so $\frac{6}{5}$ is 1 whole plus an additional $\frac{1}{5}$.

whole p

Students need to "struggle" and discover that they need to divide the 30 into 6 equal sized parts, which means 5 goes into each grey box.

whole p

Then students should be able to see that one whole is 5 groups of 5 or 5 · 5 which is 25, so *p, the whole,* is 25.

b. This should be contrasted side-by-side with the problem $\frac{6}{5} \cdot 30 = p$.

Since the whole is 30, students need to divide the 30 into 5 equal sections, so 6 goes into each part. Again, the student may use tally marks or manipulatives.

whole 30

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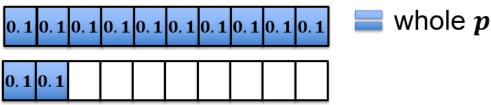
Since all parts in a fraction are equal in size and quantity the additional $\frac{1}{5}$ is also **6**, and since there are **6** parts of **6**, **6 · 6** is **36**, so $\frac{6}{5}$ of **30** is **36**. Students should discover or come to the understanding that the answer needs to be greater than the original whole.

After solving many of these types of equations using fraction models, students should look for patterns and start to discover that to solve an equation where the coefficient is a fraction, they divide by the numerator and multiply by the denominator.

TIP! It may be helpful to connect solving fractional equations using models with fractions to equivalent equations represented with fraction decimals. For example, $1.2p = 30$.

TIP! In addition to becoming fluent using fractions with equations, students should also come to the following understandings:

- When to use a decimal instead of a fraction or vice versa.
- Whether concepts in whole number settings apply to fractions
- Whether fractions can be used (or beneficial to use) if not explicit in the problem
- Whether fraction concepts apply to ratios.

From: Johanning, D. (2008). Learning to use fractions: Examining middle school students' emerging fraction literacy

WRITING AND REPRESENTING INEQUALITIES
Have students write inequalities to represent real-world problems. Writing out inequalities in words is a strategy for assisting students' conceptual understanding.

EXAMPLE
Use an inequality to represent the temperature of an ice cube.

Have students use inequalities that represent a constraint or condition on an equation or situation.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

EXAMPLE

Dominique has 4 pieces of candy and her brother gives her a certain amount each day. After a certain number of days, she has 34 pieces of candy.

a. Define the variable.

b. Write an equation to represent how many pieces of candy Dominique has on any given day?

c. Write an inequality that represents a constraint on the situation?

Students should be given examples with inequalities where the variable is on both sides of the inequality sign, and they should also practice writing equivalent inequalities such as $c > 9$ and $9 < c$. Working fluidly with the variable on both sides will allow students to have a better transition to compound inequalities in Algebra.

When graphing inequalities on a number line, do not tell students the arrow points in the direction of the inequality sign because this creates a misconception that is difficult to combat in later grades. Instead ask the student when graphing, which numbers make the inequality true. Have students plot those numbers on the line, and then draw an arrow in that direction. Discuss why there is an open circle on the number line when graphing inequalities.

When working with inequalities, provide situations in which the solution is not limited to the set of positive whole numbers but includes rational numbers. This is a good way to practice fractional numbers and introduce negative numbers. Students need to be aware that numbers less than zero could be part of a solution set for a situation. In sixth grade students are only working with less than (<) and greater than (>) inequalities. The circle is open to show that the solution can approach that value, but not include or equal that value. *Note: Examples should not include less than or equal to ($\leq$) or greater than or equal to ($\geq$).*
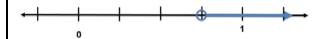
EXAMPLE

Graph $6.2 < g$ on a number line.

Discussion: Have the students ask themselves which numbers make the inequality true. (Notice that the variable is on the right side of the inequality this time.) The students should recognize that the *g* is greater than 6. Have them name numbers. Encourage decimal fractions and fractions. Ask the students if 6 is greater than 6.2. Ask about numbers really close to 6.2 such as 6.19999999 and 6.0000001. Discuss why there are infinite solution to inequalities. Discuss why a > or < sign is an open circle when representing an inequality on a number line.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

Guide students to look at the structure and mathematical reasoning for what a solution could be, not just rotely applying "inverse operations."

EXAMPLE
Write a word problem for the following inequality.

Instructional Tools/Resources
These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology
- Algebra tiles
- Number lines
- SMART Board's new tools for solving equations
- Virtual Algebra Tiles is an applet from Michigan Virtual University that allows students to use algebra tiles. This applet allows for positive and negative representation of the tiles.
- CPM Tiles is an applet from the CPM Educational Program that allows students to use algebra tiles. The benefit of this applet is that students can change the dimensionality of *x* and *y*. However, it is limited by not allowing for a negative representation of the tiles.

Equations
- Pan Balance by NCTM Illuminations is an introductory lesson using shapes and balancing scale to solve equations.
- One-Step Equations Soccer Game by Math-Play.com is an an online game to practice solving one-step equations (includes fractions and decimals).
- One-Step Equations Basketball Game by Math-Play.com is an an online game to practice solving one-step equations (includes fractions and decimals).
- Olympic Rankings by YummyMath has students consider ranking methods for metal counts at the 2008 summer Olympics. Students can create and analyze formulas.
- Interpreting Equations by Mathematics Assessment Project has students connect algebraic equations to real-life situations.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

Inequalities
- Inequalities on the Number Line is a Desmos activity where students explore linear inequalities and make connections among multiple representations.
- Height Requirements from Illustrative Mathematics is a task where students write constraints from a real-world context.

Curriculum and Lessons from Other Sources
- Georgia Standards of Excellence Framework, Grade 6, Unit 4: One-Step Equations and Inequalities has several tasks that pertain to this cluster. They can be found on pages 15-25, 55-66.
- EngageNY, Grade 6, Module 4, Topic G, Lesson 23: True or False Number Sentences, Lesson 24: True or False Number Sentences, Lesson 25: Finding Solutions to Make Equations True, Lesson 26: One-Step Equations—Addition and Subtraction, Lesson 27: One-Step Equations—Multiplication and Division, Lesson 28: Two-Step Problems—All Operations, Lesson 29: Multi-Step Problems—All Operations are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 4, Topic H, Lesson 30, One-Step Problems in the Real World, and Lesson 33: From Equations to Inequalities are lessons that pertain to this cluster.
- Illustrative Mathematics, Grade 6, Unit 6: Expressions and Equations, Lesson 1: Tape Diagrams and Equations, Lesson 2: Truth and Equations, Lesson 3: Staying in Balance, Lesson 4: Practice solving Equations and Representing Situations with Equations, Lesson 5: A New Way to Interpret a over b, Lesson 7: Revisit Percentages are lessons that pertain to cluster.

General Resources
- Arizona Progressions 6-8 Expressions and Equations is an informational document for teachers. This cluster is addressed on pages 6 and 7 under the heading Reason about and solve one-variable equations and inequalities.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

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- Akgün, L. & Özdemir, M. (2006). Students' understanding of the variable as general number and unknown: A case study. *The Teaching of Mathematics*, *9(1),* 45-51.
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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.5-8)

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STANDARDS	MODEL CURRICULUM (6.EE.9)
EXPRESSIONS AND EQUATIONS **Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*	## Expectations for Learning In prior grades variables are used as unknowns. In this cluster students will extend their understanding of variables to include the relationship between dependent and independent variables. Students use two variable equations to express relationships between two quantities that vary together. Students also understand that these relationships can be expressed as a table, graph, and/or equation. These initial understandings of the relationship between dependent and independent variables provide the introductory foundations for work with linear functions in 8th grade. ### ESSENTIAL UNDERSTANDINGS • Expressions on both sides of the equal sign have the same value. • The value of the dependent variable is determined by the value of the independent variable. • The relationship between two quantities can be represented as a table, graph, and/or equation. ### MATHEMATICAL THINKING • Represent real-world problems mathematically. • Attend to precision in recording mathematical statements. • Use accurate mathematical vocabulary to describe mathematical reasoning. • Recognize and use a pattern or structure. • Use informal reasoning. ### INSTRUCTIONAL FOCUS • Define variables in context using appropriate units. • Select appropriate independent and dependent variables based on the context of the problem. • Analyze the relationship between the dependent and independent variables using graphs and tables. • Relate an equation to a table and/or graph. • Write a one-step equation representing the relationship between the independent and dependent variable from a table, graph, or real-world context. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.EE.9)
	## Content Elaborations • [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40](#) • [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#) **CONNECTIONS ACROSS STANDARDS** • Relate equivalent ratios to graphs and tables (6.RP.3a). • Use variables to represent unknown quantities and write expressions (6.EE.2, 6.EE.6). • Solve real-world and mathematical problems using a coordinate plane (6.NS.8). • Solve real-world and mathematical problems by writing and solving equations (6.EE.7).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.9)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on <u>instruction</u> not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

> ***Standards for Mathematical Practice***
> *This cluster focuses on but is not limited to the following practices:*
> ***MP.2*** *Reason abstractly and quantitatively.*
> ***MP.6*** *Attend to precision.*
> ***MP.7*** *Look for and make use of structure.*
> ***MP.8*** *Look for and express regularity in repeated reasoning.*

INDEPENDENT AND DEPENDENT VARIABLES

In this cluster a variable takes on a new definition. Now students learn that two variables can change (covary) in relationship with each other. These quantities can take infinite numerical variables. The idea of covariance sets the foundation for later work in functions. The input (usually denoted by x) is the independent variable. The output (usually denoted by y) is the dependent variable. The dependent variable "depends" on the independent variable.

input	output
x-value	y-value
independent variable	dependent variable

TIP! The x or y variables does not vary, but the quantities that replace the variables vary.

Provide multiple situations for the student to analyze and determine what unknown is dependent on the other components. For example, how far I travel is dependent on the time and rate that I am traveling. How many gumballs I get out of a machine depends on how much money I put in.

EXAMPLE
Identify the independent and dependent variable in the following situations.
- The amount of water and the size of a fish tank
- The time it takes to travel 4 miles
- The amount of money to buy candy bars
- The number of apple pies and the amount of apples picked

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.9)

Using Multiple Representations
The goal is to help students connect the concepts together. This cluster connects equivalent ratios, graphing, and equations. It should be taught parallel with equivalent ratios in 6.RP.3 and graphing in 6.NS.8. It also builds on analyzing simple arithmetic and geometric patterns from 5.OA.3.
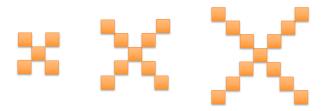
Students should use multiple representations to explore how one variable changes in relation to the other. They need to be able to translate freely among the story, words (mathematical phrases), models, tables, graphs, and equations. In addition students need to be flexible enough to start with any of the representations and use the initial representation to develop the others. Graphs, tables and equations are ways to show that the value of the independent variable can change thus changing the dependent variable.

🧠 Students may misunderstand what the graph represents in context. For example, that moving up or down on a graph does not necessarily mean that a person is moving up or down, it could mean that a person is walking faster or slower.

EXAMPLE
Compare the number of squares in the sequence.

- Define your variables and indicate which is the independent and which is dependent variable.
- Make a table.
- Predict what the 10th figure would be.
- Write an equation to show the relationship between the two variables.
- Graph the points on your table.
- Explain how the graph and the equation are connected.

🧠 Students often have a difficult time understanding that the table, the graph, and the equation all represent the same relationship.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.9)

It is important for students to identify the two quantities that are being compared and then choose variables to represent the quantities and finally define what each variable means in the context of the problem.

EXAMPLE
Yolanda walks 7 meters in 5 seconds.
- Identify the two quantities being compared and determine which is independent and which is dependent.
- Define your variables.
- Make a table of values that shows how far she will walk in 1 minute.
- Make a graph using your table of values.
- Write an equation modeling the situation.
- Explain how the graph and the equation are connected.

Discussion: Notice this example is the same example illustrated in 6.RP.1-3. The only difference is that it has a discussion on independent and dependent variables. As the year progresses students' tables of values should move toward more multiplicative reasoning. When appropriate, draw attention to how the unit rate in the equation is connected to the graph.

WRITING EQUATIONS WITH TWO VARIABLES
Throughout the expressions and equations domain in Grade 6, students need to have an understanding of how the expressions or equations relate to situations presented, as well as the process of solving them.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.9)

EXAMPLE

Matt's book is three times longer than Julie's book. Write an equation to describe the situation. Don't forget to define the variable.

Discussion: Students typically have difficulty writing these types of equations. Research suggests that the reversals are more than just careless errors but true misconceptions. The errors seem resilient through high school and beyond, so students need practice of this type or problems surrounded by rich discussion. Many students view the variable as a label (hence the reason of avoiding using the letter's M and J for variables) and/or think that the equal sign indicates a comparison or association instead of stating equivalence. Understanding the relationship between the quantities and the variable is key to representing this problem. It is imperative that students define their variables for this situation. Encourage students to draw pictures and use tables before attempting to write the equations. Notice that either the length of Julie's book or the length of Matt's book could be the independent variable in this situation. A different equation will result depending on how students define the independent variable and for an equation to correctly represent the situation it must match how the variables are defined. Once students are able to arrive at the correct equation, draw attention to the fact that in order for an equation to be "balanced" the larger operation ($\times 3$) has to be performed the smaller variable (length of Julie's book) or the smaller operation ($\times \frac{1}{3}$) has to be performed on the larger variable (length of Matt's book).

Length of Matt's Book

How to make equal?

Length of Julie's Book

Situation 1: Length of Matt's Book is Independent Variable

Length of Matt's Book = x
Length of Julie's Book = y

x	y
3	1
6	2
9	3

Equation: $y = \frac{1}{3}x$

Situation 2: Length of Juile's Book is Independent Variable

Length of Julie's Book = x
Length of Matt's Book = y

x	y
1	3
2	6
3	9

Equation: $y = 3x$

Equations with Fractions

Multiplying unknowns and variables by fractions is essential for solving equations and understanding functions. According to research using fractions as multipliers on unknown quantities is a significant challenge for students. In order to be successful in later mathematics, students need to develop reciprocal reasoning. For example, if $y = \frac{3}{5}x$ then $x = \frac{5}{3}y$.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.9)

Building on the previous example surrounding the length of Matt's and Julie's book, connect the equation $y = \frac{1}{3}x$ (where the length of Matt's book was defined as the independent variable) with the equivalent equations $3y = x$ (keeping the independent variable as the length of Matt's book.)

Once students are able to reason reciprocally using equations involving whole numbers and their inverses, move towards equations using inverses that are reciprocals to rational numbers beyond whole numbers.

EXAMPLE
Monica earned some money babysitting. She earned $\frac{3}{5}$ of what Ellen earned.

- Draw a picture or use a table to represent the situation.
- Define your variables.
- Write an equation to represent the situation.
- Test your equation with numbers to make sure its correct.
- Without redefining your variables write an equivalent equation.
- Test your equation with numbers to make sure its correct.
- What do you notice about the two equations?

Instructional Tools/Resources
These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology
- Graphic organizers as tools for connecting various representations
- Technology (such as Desmos, computer apps, hand held technology) to collect data, create tables and graphs

Independent and Dependent Variables
- Identify Independent and Dependent Variables by Learn Zillion is a lesson that explains independent and dependent variables.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.9)

Covariance
- Chocolate Bar Sales by Illustrative Mathematics has students use different representations to analyze the relationship between two quantities and to solve a real-world problem.
- Families of Triangles by Illustrative Mathematics is a task to introduce students to the idea of a relationship between two quantities by using a geometric context.
- Interpreting Equations by Mathematics Assessment Project has students address misconceptions concerning the meaning of variables in equations.

Curriculum and Lessons from Other Sources
- Illustrative Mathematics, Grade 6, Unit 6: Expressions and Equations, Lesson 16: Two Related Quantities, Part 1, Lesson 17: Two Related Quantities, Part 2, Lesson 18: More Relationships are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 4, Topic H, Lesson 31: Problems in Mathematical Terms and Lesson 32: Multi-Step Problems in the Real World are lessons that pertain to this cluster.

General Resources
- Arizona 6-8 Progression on Expressions and Equations is an informational document for teachers. This cluster is addressed on page 7.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

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- Akgün, L. & Özdemir, M. (2006) Students' understanding of the variable as general number and unknown: A case study. *The Teaching of Mathematics*, *9(1),* 45-51.
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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.EE.9)

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STANDARDS	MODEL CURRICULUM (6.G.1-4)
GEOMETRY **Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.1** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. **6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	## Expectations for Learning This cluster extends students' understanding of area of rectangles in previous grades to an understanding of area of other polygons. In Grade 5, students discovered the connection between filling a right rectangular prism with unit cubes and formulas for finding volume. These formulas include $V = \ell \cdot w \cdot h$ and $V = B \cdot h$. In Grade 6, students will extend this understanding and apply it to fractional edge lengths both through discovery and using the formula. It is the first time that students explore methods for finding the area of triangles and special quadrilaterals. Using models and real-world contexts students will develop strategies and formulas to find areas of various polygons. They will find surface areas of three-dimensional figures composed of rectangular and/or triangular faces including right prisms, pyramids, and other solids using nets. Students will compute accurately and efficiently with grade-level numbers including whole numbers, fractions, and decimals to solve surface area and volume problems. This will be the foundation for future work in geometric measurement for three-dimensional figures. The student understanding of this cluster aligns with a van Hiele Level 1 (Analysis). ### ESSENTIAL UNDERSTANDINGS **Area** • Any side of a triangle can be a base. • The height of a polygon is a perpendicular line segment drawn from a vertex to the opposite side (base) or its extension. • The area of a triangle is half the area of a parallelogram with the same base and height. • Any polygon can be composed or decomposed into known figures to determine area. • The total area of a two-dimensional composite shape is the sum of the areas of all its parts. *Continued on next page*

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STANDARDS	MODEL CURRICULUM (6.G.1-4)
	Expectations for Learning, continued **ESSENTIAL UNDERSTANDING, CONTINUED** **Surface Area** • The surface area of a three-dimensional figure is made up of the sums of the areas of its faces. • A net is a composite two-dimensional shape of a three-dimensional object used to find the surface area. • Surface area of a three-dimensional figure includes faces that are visible and not visible from a given viewpoint. **Coordinates** • The area and side lengths of polygons can be found by plotting coordinates in a coordinate plane. • If both x-coordinates of a line segment are the same, a vertical line segment is formed and the length can be determined. • If both y-coordinates of a line segment are the same, a horizontal line segment is formed and the length can be determined. **Volume** • Rectangular prisms may have edge lengths that are fractions. • Volume of a rectangular prism can be determined using the formulas and/or by packing it with unit cubes of the appropriate unit fraction edge lengths. **MATHEMATICAL THINKING** • Draw a picture or create a model to represent mathematical thinking. • Compute accurately and efficiently with grade-level numbers. • Consider mathematical units involved in a problem. • Use and analyze structure. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.G.1-4)
	### Expectations for Learning, continued **INSTRUCTIONAL FOCUS** **Area** • Draw a picture or create a model to compose and decompose polygons to find the area of triangles, special quadrilaterals, and polygons. • Explore, develop, and apply area formulas for parallelograms and triangles. • Apply techniques for finding area in the context of solving real-world and mathematical problems. • Use appropriate units to label area problems. **Surface Area** • Draw nets made up of rectangles and triangles to represent three-dimensional figures, e.g., cubes, rectangular prisms, triangular prisms, and pyramids, where a face or faces are not visible. • Use nets to solve real-world and mathematical surface area problems. • Use appropriate units to label surface area problems. **Coordinate Plane** • Draw polygons in the coordinate plane given coordinates for the vertices. • Find the length of a side of a polygon formed by points with the same first or second coordinate. • Find the area and perimeter of geometric figures on a coordinate plane. **Volume** • Find the volume of a right rectangular prism with fractional edge lengths. • Apply formulas to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. • Use appropriate units to label volume problems. *Continued on next page*

STANDARDS	MODEL CURRICULUM (6.G.1-4)
	## Content Elaborations
	- Ohio's K-8 Critical Area of Focus, Grade 6, Number 5, page 42
	- Ohio's K-8 Learning Progressions, K-5 Geometry, page 11
	- Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14
	- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21
	CONNECTIONS ACROSS STANDARDS
	- Use horizontal and vertical distance to find lengths of sides of polygons in the coordinate plane (6.NS.8).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on <u>instruction</u> not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Understanding area and volume requires students to understand the following:
- What area and volume are, i.e., covering and filling;
- How they are measured (unit squares versus unit cubes);
- How to meaningfully enumerate rectangular two-dimensional and three-dimensional arrays of squares and cubes;
- That if two figures can be mapped on top of each other than they have the same area;
- As a figure is decomposed and recomposed, the area and volume remain the same (conserved).
- That specific numerical operations can be used to calculate area and volume of specific figures; and
- That these numerical processes can be generalized using algebra.

Some difficulties that students might encounter are generalizing their concept of area and volume to—
- fractional units;
- curved and irregular shapes;
- large spaces such as house or rooms; and/or
- decomposing figures into non-square and non-cube units.

When exploring area and volume it is very important for students to continue to physically manipulate materials and create drawings in order to make connections to the symbolic and more abstract aspects of geometry. The National Assessment of Educational Progress (NAEP) indicates that students do not have a good understanding of formulas. Students need to understand the concepts behind the formulas rather than just being given numbers to substitute into the formulas.

Students should use real-world contexts to solve area, surface area, and volume problems. They should use appropriate notation when labeling shapes.

> **Standards for Mathematical Practice**
> *This cluster focuses on but is not limited to the following practices:*
> **MP.1** *Make sense of problems and persevere in solving them.*
> **MP.3** *Construct viable arguments and critique the reasoning of others.*
> **MP.4** *Model with mathematics.*
> **MP.5** *Use appropriate tools strategically.*

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

TIP!   To enable the students to discover the formula for the areas of a parallelograms, rectangles, and triangles, have them compose and decompose rectangles. They should discover that the formula for the area of a rectangle *base ✗ height* also applies to a parallelogram.

Students often incorrectly think that when sides of a square or a cube double, so does its area and volume. Have students draw a square, double the side-lengths of the square, draw the new square beside the original one, and discover the difference in area. Discuss how that when the side-length doubles, the area increases by 2^2 (or 4) because each of the 2-dimensions doubles. Have students build a cube, double the side-length of a cube, build the new cube, and discover the difference in volume. Discuss how that when the side-length doubles, the volume increases by 2^3 (or 8) because each of the three-dimensions doubles.

VAN HIELE CONNECTION

Level 1 (Analysis) can be characterized by the student doing some or all of the following:
- comparing length, area, or volume by manipulating and matching parts;
- visually comparing shapes by composing/decomposing;
- visualizing structured iteration (repeating over and over) of length, area, or volume units;
- organizing area and volume units into (2D, 3D) array structure without gaps or overlaps;
- using a single unit, row, or layer repeatedly (iterating) to correctly measure or construct length, area, or volume respectively;
- determining measurement without having to show every unit instead of using only numbers (no visible units or repeated units); and/or
- creating composite units, columns, rows, or layers to find length, area, or volume.

See the van Hiele pdf on ODE's website for more information about van Hiele levels.

UNITS

Although unit squares and unit cubes are the most common units used in area and volume problem, units can be anything including sheets of paper or boxes. These are oftentimes the type of units used in real-life area and volume problems. It might be helpful to extend students' thinking by exposing them to problems with different units besides unit squares. For example, have students measure the area table using a piece of paper as a unit, and then have them measure the same table using standard units such as square feet or square inches. Make the connection that although, the area is represented with different numbers and units, the area itself remains the same.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Discuss with students why square units are used to measure area and cube units are used to measure volume. Make a connection between area being two-dimensional (length × width) and volume being three-dimensional (length × width × height).

AREA

Area is the two-dimensional space inside a region. It can be thought of as "covering" a figure. Focus on what area means conceptually and move students away from the formulaic definition of length *times* width which will no longer hold true when students are required to find the area of shapes besides rectangles. In Grade 4 students explored finding the perimeter and area of rectangles. In Grade 6 Students should come with the following understandings about area:

- Area "covers" and perimeter "surrounds."
- The area of a figure is greater than 0.
- A unit square is a square whose side lengths are 1.
- Two identical shapes have equal area.
- The area of a unit square is 1 square unit.
- Area is additive. (For example a composite figure composed of two shapes is the sum of both shapes.)
- The area of a rectangle is the product of the length of its sides.

See 4.MD.1-3 for scaffolding ideas.

Students may believe that the orientation of a figure changes the figure. In Grade 6, some students still struggle with recognizing common figures in different orientations. For example, a square rotated 45° is no longer seen as a square and instead is incorrectly called a diamond (A diamond is a non-mathematical name for a rhombus.) This impacts students' ability to decompose composite figures and to appropriately apply formulas for area. Providing multiple orientations of objects within classroom examples and work is essential for students to overcome this misconception.

TIP! Students tend to do better in area problems when using squares to fill an area instead of rulers. They often have trouble relating the number of squares to the length of its sides. Use a grid as an area ruler to help make the connection between the length of the figure and the number of squares on its side. Give them opportunities to calculate area and volume by measuring to improve their measuring skills. For example, a grid printed on an overhead transparency or tracing paper can be used. Eventually the middle of the grid can be erased.

A common error is that the students will count the corner squares twice when filling an area with squares, because this square is "counted twice" when counting side-length measurements. Guide students to keep track of the square units that they have counted.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Area in common usage can refer to a region or portion of a surface, but in mathematics it refers to the amount of the surface in a region.

TIP!

Polygons in the Coordinate Plane

The coordinate plane can help students see movement and change in both geometry and algebra. Given the coordinates for vertices, students will draw polygons in the coordinate plane. They will determine the lengths of vertical and/or horizontal sides of the polygon by subtraction or counting. Students should discover that for a vertical line, the *x*-coordinates are the same and for a horizontal line the *y*-coordinates are the same. Include shapes whose bases are not parallel to the *x*-axis. Some activities that can be done on the coordinate plane are as follows:

- Give students coordinates of the vertices of a shape and have students make inferences about the shape before plotting it.
- Describe a polygon by giving all the coordinates of the vertices except one.
- Given the coordinates of a shape, give new coordinates of a shape that is identical in shape.
- Create a shape given a vertex and dimensions. How many other identical shapes can you create with the same criteria?

Taken from Pugalee D, et.al, 2002, Navigating Through Geometry 6-8

In addition, they will find the area and perimeter of figures in the coordinate plane. To further develop the concept of area, it may be helpful to have students estimate the area of irregular figures on the coordinate plane by counting the squares and partial squares.

Using the coordinate plane, they should be able to justify properties for shapes such as special quadrilaterals or triangles, e.g., Finding that the opposite side lengths of a parallelogram are the same.

Conservation of Area

To build the concept of conservation of area, have students do activities where the area is cut up and rearranged. Tangrams can also be useful for exploring this concept. Have a discussion how pieces can have the same size (area) but different shape.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)
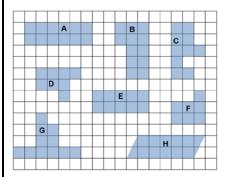
EXAMPLE
- Draw 6 identical rectangles on grid paper and cut them out.
- Now cut each rectangle into different pieces one at a time keeping the rectangles pieces together.
- Rearrange the pieces of each rectangle into a different shape, and glue them on a piece of paper.
- Is one shape bigger than the other? Explain.
- Did one shape take more paper to make? Explain.
- Does the way you cut the rectangle and rearrange the pieces effect the area of the shape? Explain.

Taken from Van DeWalle, J., et al. 2010, Elementary and Middle School Mathematics: Teaching Developmentally

EXAMPLE

- Which figures have the same area?
- Explain why their areas are the same without counting squares.

Discussion: Through discussion draw attention to the fact, that all the figures with identical areas are really just Rectangle A decomposed and then recomposed. Draw special attention to Figure H which is a parallelogram.

Parallelogram
It may be helpful to redefine the formula for the area of rectangle as *base × height* instead of *length × width*. This will allow students to generalize their understanding to other figures and solids besides rectangles.

There are a variety of meanings for length. It can be defined as the linear magnitude of anything measured end-to-end or it could mean the greatest dimension of an object. Length can also be measured in different directions.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

EXAMPLE
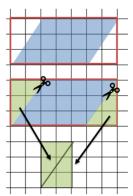
Draw 4 different parallelograms on grid paper and find each parallelogram's area by using what you know about rectangles. Feel free to cut out the shapes if needed.

Discussion: Have students show their various methods for finding the area of a parallelogram. (Student may not use triangles.) Ask students how they can find the area without cutting and decomposing. Encourage the connection of the formula for area of a rectangle to the formula for the area of a parallelogram. Draw attention to the fact the side of the rectangle is also its height because it is a rectangle, so *base × height* is a better formula to use since it applies to parallelograms, rectangles, and rhombuses. Discuss how any side of a parallelogram can be the base since a shape can rotate, but the height must be perpendicular to the base.

Triangles
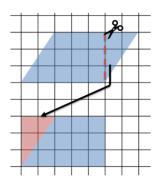
Students discover the formula for the area of a triangle by decomposing squares, rectangles, or parallelograms. They may overlay a rectangle on top of the triangle, or they may decompose/recompose the triangle.

Method 1: Subtraction

- Draws a rectangle around the parallelogram
- Finds the area of the rectangle (7 x 3) = 21
- Find the area of a rectangle composed by the triangles (2 x 3) = 6
- Subtracts the little rectangle from the big rectangle (21 - 6) = 15 square units.

Method 2: Decomposing/Recomposing
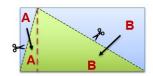
- Decomposes by cutting a triangle off
- Recomposes to make a rectangle
- Finds the area of a rectangle (5 x 3) = 15 square units

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

EXAMPLE
Take a sheet of paper and draw and color in a triangle so that the triangle's base shares one of the bases of the rectangular sheet of paper and so that the vertex of the triangle is on the opposite base of the rectangle. Cut out the triangle, and compare the area of the triangle to the original rectangle. (It may be helpful to start by using right triangles).

Discussion: Through exploration and discussion students should come to the understanding that it takes two triangles to make up the original rectangle, so the area of a triangle is $\frac{1}{2}$ of a rectangle with the same base and height.

EXAMPLE
- Draw 3 pairs of identical triangles on grid paper. Use different types or triangles (right, obtuse, isosceles, etc.)
- Use what you know about rectangles and/or parallelograms to find the area of a triangle. You may use scissors to cut out the shape if you wish.

Discussion: Students should come to the conclusion that two identical triangles make up a rectangle, so they can find the area of a rectangle and take $\frac{1}{2}$ of it. Remind students that dividing by 2 is the same thing as multiply by $\frac{1}{2}$.

Trapezoid
There are at least ten different methods for discovering the formula for the area of a trapezoid using triangles and parallelograms. Have students use what they know to figure out their own method. Their formulas may not be the same as the "official formula", but their formulas could be used as discussion points about equivalent expressions or transforming expressions using the properties of operations to connect it to $A = \frac{1}{2}(b_1 + b_2)h$.
Note: Students do not need to memorize the formula for the area of a trapezoid, but they should be able to find the area using decomposition or recomposition techniques.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Some methods students might choose to use are the following:
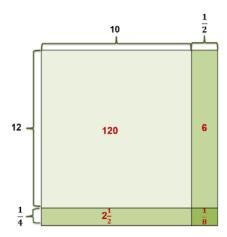
- Doubling the trapezoids to form a parallelogram;
- Halving the trapezoid and then rotating it to form a parallelogram;
- Creating two triangles by drawing a diagonal;
- Drawing a parallelogram inside the trapezoid resulting in a parallelogram and triangle;
- Creating a parallel line to the bases joining the midpoint of the nonparallel sides with a line and using that as the "average" measure of the two bases; or
- Decomposing the trapezoid into a rectangle and one or two triangles.

AREA WITH FRACTIONAL EDGE LENGTHS

Have students compute area of fractional edge lengths by applying the Distributive Property.

EXAMPLE
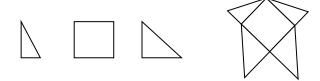
Mario wants to paint an accent wall that is $10\frac{1}{2}$ feet by $12\frac{1}{4}$ feet. Calculate the area of the wall.

Discussion: There are many ways a student can approach the task. In some cases, when appropriate, students can convert the fractions to decimals and calculate the total area by multiplying base times height. However, it might be beneficial in order to strengthen fractional computations to have students solve it both using the Distributive Property and with other fraction computational strategies.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)
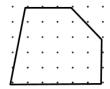
COMPOSITE FIGURES
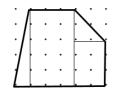
Create composite shapes by rearranging rectangles and triangles, and determine the area of the new figure. This process will reinforce the concept that composite shapes are created by joining together other shapes and that the total area of the two-dimensional composite shape is the sum of the areas of all its parts.
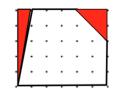
Students may also decompose the figure into shapes for which they can determine the area. Once the shape is decomposed, the area of each part can be determined and the sum of the area of each part is the total area.

One strategy is to place a composite shape on grid or dot paper (or overlay a transparency with grids or dots on the shape). This aids in the decomposition of a shape into its foundational parts. Some students may see the larger rectangle and subtract the negative space instead of decomposing the shape into smaller known shapes. This is also an effective strategy.
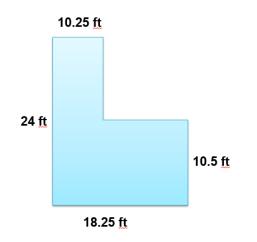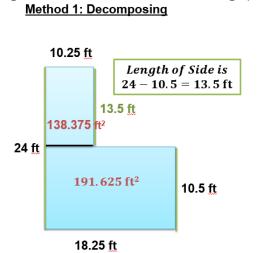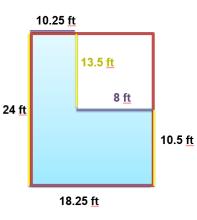
Some figures in composite shapes may have missing lengths. One method to help students find the missing lengths is to highlight the parallel sides with a highlighter. Other students may also consider the larger rectangle that covers the entire figure, and then subtract the shapes that are not included in the original composite shape to determine the area.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

EXAMPLE

Kwan needs to paint the ceiling in his living room. Find the area of the ceiling. (The picture is not drawn to scale.)

Method 1: Decomposing

Method 2: Subtraction

10.25 ft

24 ft

10.5 ft

18.25 ft

10.25 ft

Length of Side is
$24 - 10.5 = 13.5$ ft

13.5 ft

138.375 ft²

24 ft

191.625 ft²

10.5 ft

18.25 ft

Area of Blue Figure is
$138.375 + 191.625 = 330$ ft².

10.25 ft

13.5 ft

24 ft

8 ft

10.5 ft

18.25 ft

Area of Red Rectangle is
$18.25(24) = 438$ ft²

Lenth of Yellow Side is
$24 - 10.5 = 13.5$ ft.
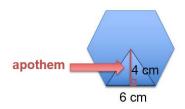
Length of Purple side is
$18.25 - 10.25 = 8$ ft.

Area of Missing Rectangle is
$13.5(8) = 108$ ft².
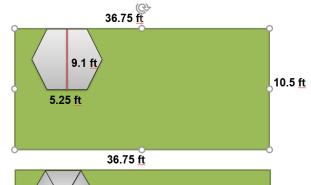
Area of Blue Figure is
$438 - 108 = 330$ ft².

Students should solve real-world composite shape problems. They should be able to find the area of regular polygons given a side length and an apothem using triangles. (Students do not need to know the word *apothem*.)

apothem

4 cm

6 cm

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

EXAMPLE

36.75 ft

9.1 ft

5.25 ft

10.5 ft

36.75 ft

4.55 ft

5.25 ft

10.5 ft

Jason wants to pour concrete into a patio that is shaped like a regular hexagon where each side length equals 5.25 feet and the full length of the patio is 9.1 feet. He also wants to lay grass seed on the lawn surrounding he patio. His lawn is 36.75 feet by 10.5 feet. (The drawing is not drawn to scale.)

- What is the area of the patio?
- What is the area of the grass.

Discussion: The student can draw a lawn with a hexagon inside and label all the parts. Then the student can then draw 6 triangles inside the hexagon. Since the full length of the patio is 9.1 feet, the apothem is 4.55 feet which is the height of the triangle. The student can then find the area of one triangle: $\frac{1}{2} \cdot 5.25 \cdot 4.55 = 11.94375$ ft². Next, the student could multiply that number by 6 to get the area of the patio which is 71.6625 ft². The area of the rectangle could be found by multiplying length times width: $36.75 \cdot 10.5 = 385.875$. Finally, the student could subtract the area of the patio from the area of the grass to get the total area of the grass: $385.875 - 71.6625 \approx 314.21$ ft².

Area and Perimeter

Students should be doing problems that include perimeter. They should also be relating area to perimeter. As shapes get more complex, the formula for perimeter ($P = 2l + 2w$) is no longer applicable. They should think of perimeter as the distance that "surrounds" a figure.
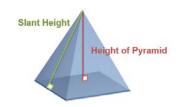
Students often incorrectly think that when perimeter increases, so does area. Give student problems to confront this misconception.

Using string for the perimeter and cut squares for the area, have students find how many different rectangles they can make with a given perimeter. Then have them compare the areas of these rectangles, connecting this with the learning of area of rectangles. This may stump them a little because they may be expecting the same area since the perimeter is the same. Tying this activity to factors and prime numbers could also be beneficial.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

DRAWING AND CONSTRUCTING THREE-DIMENSION FIGURES

To build spatial reasoning have students build three-dimensional shapes out of Polydrons, Geofix shapes, coffee stirrers, spaghetti noodles, toothpicks, straws, gumdrops, or marshmallows. Draw attention to the different properties such as parallel and perpendicular lines, symmetry, etc. Students should also become familiar with vocabulary surrounding geometric solids including the difference between the height and the slant height in a pyramid. Also draw attention to the fact that the word "base" usually refers to the length of a base in a two-dimensional shape and that the word "Base" refers to the area of the Base in a three-dimensional solid.
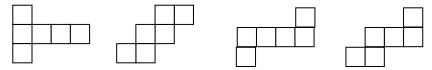
Drawing figures is essential to the development of spatial reasoning. Teach students to draw three-dimensional objects such as prisms and pyramids. See Model Curriculum 7.G.1-3 for more information on how to draw three-dimensional figures.

SURFACE AREA

Introduce surface area as the idea of "wrapping." Have students build prisms using unit cubes. Then have them count the number of squares on each face to connect the idea of area to surface area. Remind them that they also must count the bottom of the figure.

Exploring possible nets could be done by taking apart (unfolding) three-dimensional objects such as Kleenex boxes. This process is foundational for the study of surface area of prisms. Have students cut apart the faces and rearrange them to illustrate that there are many different nets for the same object.

Both the composition and decomposition of rectangular prisms should be explored Understanding that there are multiple nets for the same object may be difficult for some to visualize; provide concrete examples of nets for the object.. The understanding that there may be multiple nets that create a cube may be challenging. For example the following are a few of the possible nets that will create a cube.
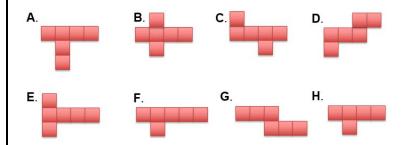
Another activity is to have students create and draw as many different nets as possible (11) using 6 squares that will fold up to a cube. Have the students cut the nets out and fold them into cubes to check their work.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

EXAMPLE

Which of the following nets does not form a cube?

A.

B.

C.

D.

E.

F.

G.

H.

Discussion: Shapes A., F., and H. do not form cubes. Have students cut out and fold the shapes to build their spatial reasoning skills. Allow them to discover and share their findings, orally, and in writing, about nets that do not work and explain why they do not work. For example, they may eliminate nets that do not have enough faces such as H. and eliminate nets that would fold over themselves such as A. and F.

TIP!   An introductory activity is to have students cut out squares of white, thick construction paper (8.5-inch by 8.5-inch squares are typically more readily available, but 12-inch by 12-inch squares have the advantage of allowing students to understand square feet and cube feet). Have the students, using crayons, write their name on one of the squares, have them depict their favorite subject on one square, their favorite sport on another one, their favorite food on yet another. . .etc., until all six squares are beautifully decorated and then tape the squares together to make a "cube of each child". Then the cubes can be used throughout the year to model volume, doubling side lengths etc. If the class is big enough, they may even be able to visually see a cubic yard.
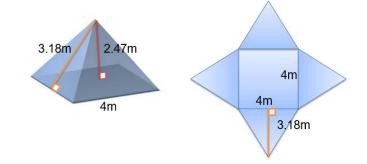
Building upon the understanding that a net is the two-dimensional representation of the object, students can apply the concept of area to find surface area. The surface area of a prism or pyramid is the sum of the areas for each face and other figures composed of rectangles and triangles. Students should find the surface area in the context of real-world problems.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

EXAMPLE

Kylie wants to find the amount of material used in a tent with a square base which includes the floor to make an identical tent for her brother. How much material is used in the tent?

Discussion: In Grade 6, students should find surface area by using nets. They should realize that the slant height on the pyramid is the height of the triangle. Then they could find the area of the square $4 \cdot 4 = 16\text{m}^2$ and find the area of one of the triangles $\frac{1}{2} \cdot 3.18 \cdot 4 = 6.36 \text{ m}^2$ and multiply 6.36 by 4 since there are 4 triangles to get $25.44 \ m^2$. Then they would need to add the base and the triangular sides:

$16 + 25.44 = 41.44 \text{ m}^2$ of fabric. Reinforce that multiplying by $\frac{1}{2}$ is the same as

dividing by 2, $\frac{3.18(4)}{2} = 6.36 \text{ m}^2$.

Students could role-play as a small business owner, where they sell items by posting them on a website. Tell them that they will need to ship a "mystery item" (e.g., egg, single potato chip, shoes). Provide students with characteristics (e.g., perishable, breakable, sharp edges, liquid, flammable, approximate dimensions) of the "mystery item." Then students can design a package to accommodate the "mystery item", identifying the area, surface area, and volume. Lead a discussion, or host a career speaker, to relate this skill to various career fields that require critical thinking, problem solving, and mathematic calculations (e.g., logistics, transportation, health).

EXAMPLE

Mary has 28 in^2 of paper to cover a box in order to decorate it. If she wants to use all the paper, what dimensions could the box have.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

VOLUME

Volume is the three-dimensional space inside a solid. It can be thought of as "filling" a solid. Focus on what volume means conceptually and move students away from the formulaic definition *length × width × height* which only holds true for rectangular prisms. Instead have the students think about layers, so they discover the formula *B × h* which is more inclusive. In Grade 5 students explored finding the volume of rectangular prisms with whole number lengths. Students should come to Grade 6 with the following understandings about volume:

- Volume "fills" and surface area "wraps."
- The volume of a figure is greater than 0.
- A unit cube is a cube whose side lengths are 1.
- The area of a unit cube is 1-cube unit.
- Two identical figures have equal area.
- Volume is additive. (For example a composite figure composed of two rectangular prisms is the sum of both prisms.)
- The volume of a prism is the product of the area of its base (B) and height of its side.

Students may still need additional help "seeing" arrays in three-dimensions. In order for a student to understand the location of a unit cube in an array of a prism, they must "see" the cube in a three-dimensional coordinate system consisting of rows and columns and layers. (Layers may be vertical or horizontal.) See 5.MD.3-5 for scaffolding ideas. With three-dimensional figures, students must be able to make inferences about what they cannot see on the other side of a solid based on what they can see. They need to infer and visualize that there are sometimes unit cubes in a solid that cannot be seen from any side.

Computing Volume with Fractional Edge Lengths

In Grade 6 students worked with filling prisms with unit cubes. It is a difficult concept for students to extend this visualization to fractional edge lengths. Before moving towards fractional edge lengths, it may be helpful to have students calculate volume with units other than the unit cube.

An essential understanding to this strategy is the volume of a rectangular prism does not change when the units used to measure the volume changes. Since the focus in Grade 6 is to use fractional lengths to measure, if the same object is measured using $\frac{1}{4}$-inch cubes and $\frac{1}{2}$-inch cubes, the volume will appear to be greater with the smaller unit. However, students need to understand that the value or the number of cubes it takes to fill the prism may be is greater, but the volume is the same. This same concept applies when comparing rectangular prisms as a unit to cube units or comparing cubic volumes using cubic centimeters to using cubic inches.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

EXAMPLE

A&J packing company needs to ship toys in packages that are 4 inches by 2 inches by 2 inches in a shipping box that is 16 inches by 12 inches by 12 inches.

- What is the volume of the shipping box using toy packages as units?
- Does the orientation of the packages affect how many packages can fit in the box?
- What is the volume of the shipping box in cubic inches?
- Is the volume of the box the same or different when using different units? Explain.

Discussion: Allow students to either build the packages using cubes or create a drawing of the situation. As students use different orientations in their models, discuss whether orientation affects the volume. Make a connection to the times when orientation could affect how many packages could fit in a box. For example if the packages do not go in evenly and there is empty space such as when packing a truck. Also, they should come to the understanding that although different units may be used (packages vs cubic inches) to measure the volume, the volume itself remains the same.

One method of accomplishing this standard is to fill prisms with cubes of fractional edge lengths (½ × ½ × ½ or ¼ × ¼ × ¼) to explore the relationship between the length of the cube unit and the number of cubic units needed. However, it may be difficult to obtain these types of cubes. Another method is to redefine the whole, so any cube can represent a fractional edge length.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)
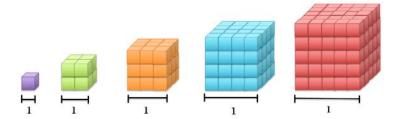
EXAMPLE

Part 1

Build various sized cubes. Label the edge lengths of each cube as 1. These are now unit cubes since they have a dimensions of $1 \times 1 \times 1$ even though they look different. Then fill out the table. *Note: the unit cube is made up of various small cubes called blocks.*

Number of blocks	Length of 1 block	Volume of 1 block	Write a multiplication expression describing the volume of 1 block.
1	1	1	$1 \cdot 1 \cdot 1$
8	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
27	$\frac{1}{3}$	$\frac{1}{27}$	$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$
64	$\frac{1}{4}$	$\frac{1}{64}$	$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$
125	$\frac{1}{5}$	$\frac{1}{125}$	$\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

Part 2

Using the "different" unit cubes you made as a reference, compute the volume of prisms with the following dimensions. Explain how you arrived at your answer using your unit cubes.

- $\frac{1}{2} \cdot \frac{2}{2} \cdot \frac{1}{2}$
- $\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$
- $\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$
- $\frac{1}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}$

e. $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}$

f. $\frac{1}{4} \cdot \frac{3}{4} \cdot \frac{1}{2}$

g. $\frac{1}{5} \cdot \frac{3}{5} \cdot \frac{2}{5}$

h. $\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5}$
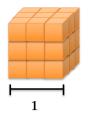
Illustration: $\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$
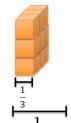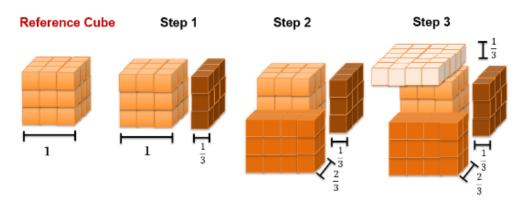

Reference Cube   **Step 1**   **Step 2**   **Step 3**
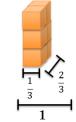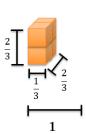
If there are 4 blocks each with the volume of $\frac{1}{27}$, then the volume is $4 \cdot \frac{1}{27}$ or $\frac{4}{27}$ cube units.

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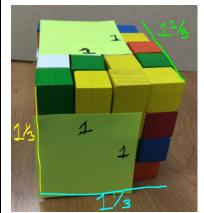# INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Part 3
Using one of your "different" unit cubes as a reference cube, build prisms with the following dimensions:

- $1\frac{1}{3} \cdot 1\frac{2}{3} \cdot 1\frac{1}{3}$

- $1\frac{3}{4} \cdot 1\frac{1}{4} \cdot 1\frac{3}{4}$

- $1\frac{1}{2} \cdot 1\frac{1}{2} \cdot 2$

- $1\frac{1}{5} \cdot \frac{2}{5} \cdot \frac{1}{5}$

Illustration: $1\frac{1}{3} \cdot 1\frac{2}{3} \cdot 1\frac{1}{3}$

Discussion: In the illustration, the reference cube is 1 whole. The fractional length of $\frac{1}{3}$, in the mixed number $1\frac{1}{3}$, creates a volume composed of 9 blocks that are $\frac{1}{27}$ cubic units per block, so the fractional length creates additional $\frac{9}{27}$ cubit units. The fraction width of $\frac{2}{3}$, in the mixed number $1\frac{2}{3}$, creates a volume composed of an additional 24 blocks that are $\frac{1}{27}$ cubic units per block, so the width is an additional $\frac{24}{27}$ cubic units. The fractional height of $\frac{1}{3}$, in the mixed number $1\frac{1}{3}$, creates a volume of 20 blocks that are $\frac{1}{27}$ cubic units per block, so the length is an additional $\frac{20}{27}$ cubic units. Therefore the volume of a cube with $1\frac{1}{3} \cdot 1\frac{2}{3} \cdot 1\frac{1}{3}$ dimensions is $1 + \frac{9}{27} + \frac{24}{27} + \frac{20}{27} = 2\frac{26}{27}$ cubic units. Students may not realize that they also have to fill in the corners when creating their figures. This is a nice connection to the Distributive Property and will help students understand scaling figures in high school. (Tip: It might be helpful to place a sticky note on the faces to keep track of your unit cube. They also can be applied at the end to make a connection between a figures units and dimensions.)
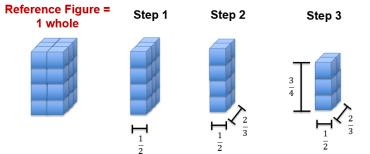
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Part 4

Using one of your "different" unit figures as a reference whole (not necessarily a cube). Build prisms with the following dimensions:

- $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{4}$
- $\frac{1}{5} \cdot \frac{2}{3} \cdot \frac{1}{3}$
- $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$
- $\frac{1}{4} \cdot 1\frac{1}{2} \cdot \frac{1}{3}$

Discussion: For Part 4, discuss with students that there can be a reference figure that is not necessarily a unit cube. They should come to the understanding that an easy reference figure to use is the one where the number of blocks equals the product of the denominators; although some students may be able to use LCD. Discuss what 1 block represents in their reference figure.

Illustration: $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$
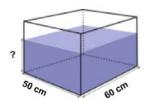
A prism with 24 blocks will be the easiest prism to represent the whole. Therefore, 1 block represents $\frac{1}{24}$ cube units. Since there are 6 blocks in the final figures the volume of a prism with the dimensions of $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$ is $6 \cdot \frac{1}{24} = \frac{1}{4}$.

Students should apply the concepts of volume to real-world problems.

EXAMPLE

A rectangular tank is 50 cm wide and 60 cm long. It can hold up to 126L of water when full. If Amy fills $\frac{2}{3}$ of the tank as shown, find the height of the water in centimeters. (Recall that 1L=1000 cm³.)

Taken from Computing Volume Progression 3 by Illustrative Mathematics. See Instructional Resources/Tools for the rest of the progression.

Grade 6

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Comparing Volume and Surface Area
Students should explore relationships between surface area and volume. Emphasize that surface area "wraps" and volume "fills."

EXAMPLE
Part 1
- Build 2 different prisms using 64 unit cubes. (That means they must have at least one different dimension regardless of orientation.)
- Calculate the surface area of your prisms.
- Explain how the volume can be the same, yet the surface areas can still be different.

Part 2
- Using graph paper, create two boxes that have a surface area of 64 cm^2. (That means they must have at least one different dimension regardless of orientation.)
- Calculate the volume of each of your boxes.
- Explain how the surface area can be the same, yet the volume can still be different.

Instructional Tools/Resources
These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Squares
- Cubes
- Grid Paper
- Dot Paper
- Geometric Solids
- Transparencies
- Tangrams
- Geoboards
- Scissors, paper, tape
- Geometry software such as GeoGebra
- Isometric Drawing Tool from NCTM Illuminations is an applet that allows students to draw two-dimensional and three-dimension figures.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Area of two-Dimensional Figures
- Area Tools from NCTM Illuminations is an applet that allows students to explore how the relationship between the length of a base and the height and area of a figure (trapezoid, parallelogram, or triangle). *NCTM now requires a membership to view their lessons.*
- Area Formulas by NCTM Illuminations has 4 lessons where students explore the area of triangles, trapezoid, parallelograms, and irregular figures. *NCTM now requires a membership to view their lessons.*
- Find Areas by Composing and Decomposing Polygons by Jack Gittinger has several GeoGebra activities relating to the area of polygons.
- Exploring Quadrilateral Area with Geoboards is a Desmos activity where students use geoboards to find area of quadrilaterals.
- Finding Area of Polygons by Illustrative Mathematics is a task that has students explore the area of irregular figures.
- 24 Unit Squares by Illustrative Mathematics is an introductory task that has students explore the concept of area.
- Same Base and Height, Variation 1 and Same Base and Height, Variation 2 by Illustrative Mathematics is a task that has students explore the area of a triangle.
- Area Formulas by Edward Knote contains many GeoGebra files exploring area.
- Exploring Triangle Area with Geoboards is a Desmos activity where students explore the area of a triangle.

Polygons on the Coordinate Plane
- Polygons on the Coordinate Plane is a Desmos activity designed to plot coordinates to develop shapes and drag points to fit coordinates.
- Walking the Block by Illustrative Mathematics is a task that has students apply finding distances on a coordinate-plane to a real-world context.

Pentominoes
- Play Pentominoes by Scholastic is an interactive game that has students rotate and flip pentominoes to cover the area of a rectangle.

Surface Area
- Drilling Many Cubes by NRICHing Mathematics is a problem that helps students visualize three-dimensional prisms and applies concept of surface area.
- Designing 3D Products: Candy Cartons by Mathematics Assessment Project is a task where students use nets to explore surface area and volume.
- Do I Have Enough Wrapping Paper? by Yummy Math is a task where students have to determine if they have enough tissue paper to wrap a present.
- Wallpaper Decomposition by Illustrative Mathematics is a task that has students explore surface area in a real-world context.
- Surface Area by Annenberg Learner has an applet that folds up a cube.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

Surface Area, continued
- Exploring Surface Area, Volumes, and Nets by Learn Alberta is an applet where students can see the unfolding and folding of nets.
- Inside Out by Nrich Math is a problem about the surface area of a cube.

Volume
- Computing Volume Progression 1, Computing Volume Progression 2, Computing Volume Progression 3, Computing Volume Progression 4, by Illustrative Mathematics is a series of tasks that explore volume.
- Banana Bread by Illustrative Mathematics is a task that has students apply volume to a real-world context.

Curriculum and Lessons from Other Sources
- EngageNY, Grade 6, Module 5, Topic A, Lesson 1: The Area of Parallelograms Through Rectangle Facts, Lesson 2:The Area of Right Triangles, Lesson 3: The Area of Acute Triangles Using Height and Base, Lesson 4: The Area of all Triangles Using Height and Base, Lesson 5: The Area of Polygons Through Composition and Decomposition, Lesson 6: Area in the Real World are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 5, Topic B, Lesson 7: Distance on the Coordinate Plane, Lesson 8: Drawing Polygons in the Coordinate Plane, Lesson 9: Determining Perimeter and Area of Polygons in the Coordinate Plane, Lesson 10: Distance, Perimeter, and Area in the Real World are lessons that pertain to this cluster.
- EngagyNY, Grade 6, Module 5, Topic C, Lesson 11: Volume with Fractional Edge Lengths and Unit Cubes, Lesson 12: From Unit Cubes to the Formulas for Volume are lessons that pertain to this cluster. Note: Lessons 13 and 14 regarding volume are incorrectly placed in the next module
- EngageNY, Grade 6, Module 5, Topic D, Lesson 13: The Formula for Volume, Lesson 14: Volume in the Real World, Lesson 15: Representing Three-Dimensional Figures Using Nets, Lesson 16: Constructing Nets, Lesson 17: From Nets to Surface Area, Lesson 19: Surface Area and Volume in the Real World, Lesson 19a: Addendum Lesson for Modeling—Applying Surface Area and Volume to Aquariums are lessons that pertain to this cluster.
- Illustrative Mathematics, Grade 6, Unit 1: Area and Surface Area have many lessons that pertain to this cluster.
- Illustrative Mathematics, Grade 6, Unit 4: Dividing Fractions, Lesson 13: Rectangles with Fractional Lengths, Lesson 14: Fractional Lengths in Triangles and Prisms, Lesson 15: Volume of Prisms, Lesson 17: Fitting Boxes into Boxes are lessons that pertain to this cluster.
- Georgia Standards of Excellence Framework, Grade 6, Unit 5: Area and Volume has many tasks that pertain to this cluster.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

General Resources

- Arizona K-6 Progression on Geometry is an informational document for teachers. This cluster is addressed on pages 19-20.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- van Hiele Model of Geometric Thinking is a pdf created by ODE that summarized the van Hiele levels.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.G.1-4)

References, continued
- Wu, H. (2013) "Teaching Geometry according to the Common Core Standards" Retrieved from https://math.berkeley.edu/~wu/Progressions_Geometry.pdf
- Wu, H. (2001). "How to prepare students for algebra." Retrieved from: https://www.aft.org/sites/default/files/periodicals/algebra.pdf

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STANDARDS	MODEL CURRICULUM (6.SP.1-3)

STATISTICS AND PROBABILITY
Develop understanding of statistical problem solving.
6.SP.1 Develop statistical reasoning by using the GAISE model:
  a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1)
  b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)
  c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)
  d. Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
	Expectations for Learning
	Students will be introduced to and develop a conceptual understanding of the four steps of the GAISE model for statistical problem solving, which will be used throughout high school. The focus of these standards is to recognize and understand the process of the GAISE model, with focus on steps 1 and 2, Level A. In Level A, teachers pose questions and students distinguish between questions that would have a statistical answer with variability and a fixed answer. With the aid of a teacher, students conduct a census or simple experiment. They begin to understand individual to individual variability and to describe the idea of distribution. Students are able to make inferences for their own classroom, but acknowledge the differences or limitations when making generalizations for a larger group.
	
	The application of steps 3 and 4 will be addressed in 6.SP.4-5. Students begin to think and reason statistically by first recognizing and formulating a statistical question as one that can be answered by collecting data. They learn that the data collected to answer a statistical question have a distribution that is often summarized in terms of center, variability, and shape.
	
	ESSENTIAL UNDERSTANDINGS
	• Statistics is the name for the science of collecting, analyzing, and interpreting data.
	• The GAISE model (outlined in 6.SP.1) is used to analyze and interpret data and has 4 steps: Formulate the Question; Collect Data to Answer the Question; Analyze the Data; and Interpret Results.
	• Data are not just numbers; they are numbers generated with respect to a particular context and situation.
	• There are two types of data: categorical and numerical.
	• Categorical data are sorted into groups and categories.
	• Numerical data are measurable.
	• A statistical question anticipates a response that varies, from one individual to the next, and this variability is described in terms of spread and overall shape.
	• A distribution shows all values of data and how often they occur.
	• A set of data has a distribution which can be described by its center, spread, and overall shape.
	• The measure of variation describes how data values vary with a single number.

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Expectations for Learning, continued
MATHEMATICAL THINKING
- Make sense of statistical problems.
- Formally explain mathematical reasoning.
- Use of formal, precise mathematical language.
- Pay attention to and make sense of quantities.

INSTRUCTIONAL FOCUS
Introduce the GAISE model
Step 1 – Formulate the Question:
- Recognize that a statistical question has variability.
- Distinguish between a statistical answer with variability and a fixed answer.

Step 2 - Collect Data to Answer the Question:
- Begin to design a collection method to answer a statistical question.
- Collect appropriate data from the following:
  - classroom census (survey) or
  - simple experiments.

Step 3 - Analyze the Data:
- Explain individual to individual variability; use a single classroom.
- Show that a distribution consists of an organized set of data with how often each data point occurs (frequency).
- Describe the center of a distribution by using mean, median, and/or mode.
- Describe the spread (variability) of a distribution by using range and/or interquartile range.
- Describe the overall shape of a distribution as being symmetric, skewed, or uniform. Identify features such as clusters, gaps, and outliers.
- Describe variation as how its data values vary with a single number.

Step 4 - Interpret Results:
- Draw conclusions from the analysis of the data collected.

Continued on next page.

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STANDARDS	MODEL CURRICULUM (6.SP.1-3)
	Content Elaborations 　• Ohio's K-8 Critical Areas of Focus, Grade 6, Number 4, page 41 　• Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23 　• GAISE Model, pages 14 – 15 　　　○ Focus of 6th grade is Level A, pages 23-35 **CONNECTIONS ACROSS STANDARDS** 　• Add, subtract, multiply, and divide multi-digit numbers (6.NS.2 – 3). 　• Connect to scientific method in Science standards.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Grade 6 is the introduction to the formal study of statistics for students using the GAISE model framework. The emphasis of 6.SP.1 is an introduction to the GAISE model and 6.SP.2-5 focuses on students using the GAISE model to answer teacher-posed questions. The GAISE model serves as a framework for pre-k-12 teachers that describes what is meant by a statistically literate high school graduate and provides steps and levels to achieve this goal. This framework helps demonstrate that statistics is an investigative problem-solving process immersed in a context and not a set of fancy tools and graphs and procedures for its own sake isolated from a context. It can thought of as the scientific method for statistics.

> *Standards for Mathematical Practice*
> *This cluster focuses on but is not limited to the following practices:*
> *MP.1 Make sense of problems and persevere in solving them.*
> *MP.2 Reason abstractly and quantitatively.*
> *MP.3 Construct viable arguments and critique the reasoning of others.*
> *MP.6 Attend to precision.*
> *MP.8 Look for and express regularity in repeated reasoning.*

GAISE MODEL FRAMEWORK (FOR EXAMPLES AT EACH STEP SEE PAGE 16-21 IN GAISE REPORT):

I. **Formulate Questions** - *Anticipating Variability—Making the Statistics Question Distinction*
- Clarify the problem at hand.
- Formulate one (or more) questions that can be answered with data.

II. **Collect Data** - *Acknowledging Variability—Designing for Differences*
- Design a plan to collect appropriate data.
- Employ the plan to collect the data.

III. **Analyze Data** - *Accounting of Variability—Using Distributions*
- Select appropriate graphical and numerical methods.
- Use these methods to analyze the data.

IV. **Interpret Results**- *Allowing for Variability—Looking beyond the Data*
- Interpret the analysis.
- Relate the interpretation to the original question.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

GAISE LEVELS
There are four steps and three levels in the GAISE model, and all four steps of the statistical process are used at all three levels A, B, C.

The depth of understanding and sophistication of methods used increases across the levels. Since this is the first time students will be introduced to the GAISE model, they will begin at Level A. Hands-on learning is predominant through every level of the GAISE model.
- o   In Level A the learning is more teacher driven (e.g., A class may collect data to answer teacher-provided questions about their classroom.)
- o   In Level B the learning becomes more student centered (e.g., A class may collect data to answer student-created questions about their school.)
- o   In Level C the learning is highly student-driven (e.g., A class may collect data to answer student-created questions about their community and analyze the relationship between data sets collected.)

STATISTICAL QUESTIONS
A statistical question has to result in an answer where the data varies not just in a single answer. For example, "How many pounds does my dog weigh?" is not a statistical question, but "What is the average weight of beagles?" is. In Grade 6, Level A, the teacher poses the questions (that are of interest to students), and students need to distinguish between a statistical and a non-statistical question. "What is a Statistical Question?" a lesson plan by the United States Census Bureau has an activity on identifying statistical questions.

EXAMPLE
Identify the statistical questions, and explain why they are statistical questions.
- How many people in our class have blue eyes?
- How many pets do each of my classmates have?
- Do you like pizza?
- What was the temperature outside our school today at noon?
- What proportion of students like cherry pie?
- What is the average temperature outside our school in January?
- How tall am I?

Discussion: Clarify that the use of "you" is singular. Discuss the impact on the statistical question of the "you" was the plural "you" or "you all."

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

COLLECTING DATA

In Grade 6, students are at Level A. At Level A, students should have some opportunities to collect data, but it is not necessary in every case. It is advised to use naturally occurring events in the classroom regarding data such has how many people on average buy lunch at school. When students collect data, the data should be limited to the classroom.

Students should also understand the difference between categorical and numerical data. Although teachers may want to use categorical data to help introduce the GAISE process, standards 6.SP.3-5 focus on numerical data.

Discuss with students the importance of experimental design when conducting simple experiments. A simple experiment consists of taking measurements on a particular condition or group. Highlight the importance of consistency on the controls of the experiment to gain more comparable data, which could be done directly or could be done indirectly leveraging students' competitive natures.

EXAMPLE

Have students do an experiment that implies winning in some way (such as measuring the distance of flinging frogs, the height of bouncing balls, or how fast a toy car will go down a hill) but do not give any guidance on how to collect the data. As each group collects data differently, some groups will have larger numerical data than the others. Other students will question the other groups' methodologies and claim their methodologies are "unfair." A teacher can capture these discussions and turn them into a teachable moment about why each group needs to perform the experiment in the same way to get comparable results.

Census at School is a good resource to obtain real-student data which students can work with in the classroom.

ANALYZING DATA

Students need multiple opportunities to look at data to determine if a question is statistical question. Numerical data should be analyzed using different tools, such as organized lists, box-plots, dot plots, histograms, and stem-and-leaf plots; categorical data should be analyzed using lists and bar graphs. After students have analyzed the data, it is important to think about what may have caused the data to look like it does. This should lead to a discussion on variability. Specifics with respect to analyzing data will be discussed more in-depth in the next cluster: 6.SP.4-5.

Measures of Center

While analyzing data students should begin to relate their informal knowledge of mean, median, and mode to understand that data can also be described by single numbers. A single value for each of the measures of center (mean, median, or mode) is used to summarize the data. The important purpose of the number is not the value itself, but the interpretation that could be made for the center of the data. Interpreting different measures of center for the same data set develops the understanding of how each measure sheds a different light on the data set. (See cluster 6.SP.4-5 for more information on measures of center.)

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

Mean and median are only appropriate for describing numerical variables not categorical variables. One cannot use frequency counts of categorical data to calculate the mean or median.

The frequency counts are the numerical summary for categorical data.

Students incorrectly assume that mean is always the best way to describe the center of data regardless of the context. Frequently provide students data where the median or mode is the best way to describe the data.

Students incorrectly think that the word "average" always refers to the mean. The mean refers to the arithmetic average; however, the term "average" can refer to the mean, median, or mode.

Variability

Analyzing data will help students begin to understand that responses to a statistical question will vary, and that this variability is described in terms of spread and overall shape. Example of a measure of spread are the range and interquartile range (IQR). Like the measures of center, a single number for measures of spread is used to summarize the data. Students also describe the shape of the distribution. Is the distributions symmetric or skewed (lopsided)? Are there any clusters? Are there any gaps? Are there any outliers? Why does the distribution take this shape?

EXAMPLE

Given measures of center and spread for a set of data:
- Use the values to describe the distribution in words.
- Use a similarity and difference graphic organizer to compare mean, median, and mode
- Use a similarity and difference graphic organizer to compare the range and interquartile range.

Discussion: These techniques may facilitate the understanding of the distinctions between measures of center and measures of spread.

Students may believe all graphical displays are symmetrical. Exposing students to graphs of various shapes will show this to be false.

Students confuse clustering and skewing. In addition students may think that when data are "skewed to the left" that most of the data is on the left. In fact, the tail of the data is on the left and most of the data are on the right.

Students should also be able to differentiate between variation and error. Teachers can use errors (mistakes) that occur in the classroom when collecting data and discuss how the impact of these errors impact the final results. Errors can sometimes create outliers, clusters, and gaps in the graphical representation of the data. Students should ask themselves if the outlier is legitimate or is it due to an error, and therefore should be discarded?

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

At Level A, students should understand the difference between measurement variability, natural variability, and induced variability.
- Measurement variability occurs when the measurement tool is imprecise or produces unreliable results.
- Natural variability is the idea that variability occurs in nature. People and things are different.
- Induced variability is something that occurs based on how an experiment or survey is designed or implemented. Sometimes it produces errors or unforeseen consequences, but oftentimes induced variability is helpful as it allows for control groups such as in a medical experiment.

Note: Students do not need to know these words, but have a basic understanding of the different causes of variability.

INTERPRETING RESULTS

The most important part of statistics is interpreting the results and writing a conclusion. A statistical conclusion has to be based on data. Once students make a conclusion, ask them "How do you know your conclusion is expected to be true?" "What evidence do you have to support your conclusion?" They can use numerical values or descriptions of graphs to back up their conclusion. Encourage students to avoid causation statements. At Level A, students make inferences only to their own classroom, but they may acknowledge that other classrooms might have different results.

Tell It Like It Is! by Statistics Education Web (STEW) from the American Statistical Association is a lesson on statistical conclusions.

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology
- Graphing calculator
- Desmos is a free online graphing utility and app.
- Census at School is an international classroom project that provides real data for classrooms. (Note: On a PC these data can be copied from their Excel spreadsheet right into Desmos.)

Statistical Questions
- Buttons: Statistical Questions by Illustrative Mathematics is a task where students identify which questions are statistical questions. The task also provides students with an opportunity to write a statistical question that pertains to the context.
- Statistical Questions by Illustrative Mathematics is a task where students discuss what makes a statistical question. This is an ideal place for a classroom discussion because answers are not always clear cut and there is a continuum going from questions that are clearly not statistical (Who was the King of France in 1716?) to questions that are definitely statistical (What is the average lifespan in the United States?).
- "What is a Statistical Question?" by the United States Census Bureau is a lesson plan where students identify statistical questions.
- Identifying Statistical Questions by Illustrative Mathematics is a task where students identify statistical questions.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

Collecting Data
- Clean Close Shave by Yummy Math is a lesson where students compare NFL QB rating of quarterbacks with and without facial hair. This task can be used to explore a number of math ideas such as sampling, causation versus correlation, misleading data, measures of central tendency, variability and data visualization.

Writing a Conclusion
- Tell It Like It Is! By David Edwards from STEW published in 2012 is a lesson that follows the GAISE model that helps students write a statistical conclusion.

Measures of Center and Distributions
- Describing Distributions by Illustrative Mathematics is a task where students are asked to describe data distributions in terms of center, spread, and overall shape and to also compare data distributions in terms of center and spread by selecting which of two distributions has a greater center and which has a greater spread. Students who can do this show an understanding of how center and variability in a data distribution is reflected in each of the three types of graphical displays (dot plots, histograms, and box plots).
- Is it Center or is it Variability? by Illustrative Mathematics is a task where students are challenged to think about whether they should be more interested in the center of the data distribution or in the spread of a data distribution in order to answer a given statistical question.
- Electoral College by Illustrative Mathematics is a task that relates to other disciplines (history, civics, current events, etc.). This task is intended to demonstrate that a graph can summarize a distribution as well as provide useful information about specific observations. With the table provided, the graph and values have context. The purpose of this task is to help students understand that a distribution can be described in terms of shape and center, and also to provide practice in selecting and calculating measures of center.

Curriculum and Lessons from Other Sources
- EngageNY, Grade 6, Module 6, Topic A, Lesson 1: Posing a Statistical Question is a lesson that pertains to this cluster.
- EngageNY, Grade 6, Module 6, Topic B, Lesson 6: Describing the Center of a Distribution Using the Mean, Lesson 8: Variability in a Data Distribution are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 6, Topic C, Lesson 12: Describing the Center of Distributing Using the Median is a lesson that pertains to this cluster.
- Illustrative Mathematics, Grade 6, Unit 8, Lesson 1: Got Data, Lesson 2: Statistical Questions, Lesson 9: Interpreting the Mean as Fair Share, Lesson 13: The Median of a Data Set, Lesson 14: Comparing Mean and Median are lessons that pertain to this cluster.
- Georgia Standards of Excellence Curriculum Frameworks, Grade 6, Unit 6: Statistics has many tasks that pertain to this cluster.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

General Resources

- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards. *Note: Ohio's Learning Standards 6.SP.1 is different than the Common Core Standards.*
- Arizona 6-8 Progression on Statistics and Probability is an informational document for teachers. This cluster is addressed on pages 2, 4-6.
  Page 2:
    o Overview, paragraph 1-2
    o Bullet points refer to GAISE model.
  Page 4:
    o Ohio's 6.SP.1 is different than what is listed in this document. Ohio's standard refers to the GAISE model.
    o Teachers formulate questions at this level.
    o Teachers could use graphs given on the right side of the page to discuss distributions.
    o 4.G.3 was deleted from Ohio's standards. This is a good time to discuss **symmetry** of graphs.
  Pages 5-6:
    o MAD is NOT 6th grade content. It is addressed in 7th grade in Ohio
- LOCUS is an NSF Funded project focused on developing assessments of statistical understanding. Teachers must create an account to access the assessments.
- Sources of Lesson Plans and Other Resources for Teaching Common Core Statistics and Probability Topics is a pdf has numerous links to resources to teach statistics.
- K-12 Statistics Education Resources is a collection of websites put together by the American Statistical Association for teachers.
- A Sequence of Activities for Developing Statistical Concepts by Christine Franklin & Gary Kader is an article published in The Statistics Teacher Network Number 68 Winter 2006. It has an overview of the GAISE model and its levels and it includes activities at each level.
- Statistics Teacher is an online journal published by the American Statistical Association (ASA) and NCTM. It includes articles, lesson plans, and professional development opportunities.
- Significance is a magazine that demonstrate the practical use of statistics and shows how statistics benefits society.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.1-3)

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- Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8, Statistics and Probability*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
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- Groth, R. & Bargagliotti, A. (August 2012). GAISEing into the Common Core of statistics. *Mathematics Teaching in the Middle School*, *18(1),* 38-45.
- Peters, S., Bennett, V., Young, M, and Watkins, J. (February 2016). A fair and balanced approach to the mean. *Mathematics Teaching in Middle School*, *21(6).* 364-372.

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STANDARDS	MODEL CURRICULUM (6.SP.4-5)

STATISTICS AND PROBABILITY
Summarize and describe distributions.
6.SP.4 Display numerical data in plots on a number line, including dot plots[G] (line plots), histograms, and box plots[G]. (GAISE Model, step 3)
6.SP.5 Summarize numerical data sets in relation to their context.
    a. Report the number of observations.
    b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.
    c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range[G]) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution.
    d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.
	## Expectations for Learning
	For the first time, students will represent, analyze, and interpret data by creating and using dot plots, histograms, and box plots. Students extend their statistical knowledge using the GAISE Model to understand, represent, and discuss distribution, shape, center, and spread. These experiences help students to begin to develop an informal understanding of variability (spread). Understanding variability is essential for developing data sense.
	
	ESSENTIAL UNDERSTANDINGS
	• Distribution shows all values of data and how often they occur.
	• Data can be represented in different ways to persuade people.
	• Statistics change numbers into information.
	• Dot plots are simple plots on a number line where each dot represents a piece of data in the data set.
	• A histogram summarizes numerical data using intervals with frequencies.
	• Boxplots display data in four equal groups (25% each) and are plotted horizontally or vertically on a number line.
	• Quartiles are values that divide the data into four equal parts (quarters). The first quartile is the value at the 25th percentile. The third quartile is the value at the 75th percentile. The median of the set, the second quartile, is the value at the 50th percentile.
	• Outliers are numbers that are really large or really small compared to the variation of most of the data.
	• The mean, median, and mode are measures of location for describing the center of a numerical data set.
	• Range is the measure of the total spread of the data, and interquartile range is the measure of spread between the lower quartile (Q1) and upper quartile (Q3).
	
	MATHEMATICAL THINKING
	• Make sense of problems.
	• Analyze and interpret graphs.
	• Attend to accuracy in graphical displays
	• Use precise mathematical language and vocabulary.

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STANDARDS	MODEL CURRICULUM (6.SP.4-5)

	## Expectations for Learning, continued
	INSTRUCTIONAL FOCUS
	• Create different visual models to represent a set of data.
	• Find measures of variability (range, interquartile range) from graphical displays.
	• Find measures of center (median, mean, and mode).
	    ○ Interpret mean as equal (fair) share.
	• Describe the shape of distributions: clusters, gaps, peaks, and/or outliers.
	• Summarize the numerical data sets in relation to the context.
	• Construct numerical one variable (univariate) visual models within the context:
	    ○ dot plots/line plots;
	    ○ histograms; and
	    ○ box plots (box & whisker plots).
	• Draw conclusions from the analysis of the data based on original question (GAISE Model, Step 4).
	GAISE Model (Step 3) – Analyze the Data
	• Compare individual to individual.
	• Compare individual to group.
	    ○ Recognize variability (spread) within a group given a graphical display.
	• Use specific properties (center, spread, shape) of distributions in context.
	
	## Content Elaborations
	• Ohio's K-8 Critical Areas of Focus, Grade 6, Number 4, page 41
	• Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23
	• GAISE Model pages 14 – 15
	    ▪ Focus of 6th grade is Level A, pages 23-35
	
	CONNECTIONS ACROSS STANDARDS
	• Fluently add, subtract, multiply, and divide multi- digit decimals (6.NS.2-3).
	• Understand the framework of GAISE Model (6.SP.1-3).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on <u>instruction</u> not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.3 *Construct viable arguments and critique the reasoning of others.*
MP.4 *Model with mathematics.*
MP.5 *Use appropriate tools strategically.*
MP.7 *Look for and make use of structure.*

In these standards students learn to organize data in appropriate representations such as histograms, box plots (box-and-whisker plots), dot plots, and stem-and-leaf plots.

MEASURES OF CENTER
This is the introduction to calculating measures of center: mean, median, and mode. Students find the mean, median, and mode from various graphical representations such as frequency tables, dot plots, or stem-and-leaf plots.

Mode
Students at Level A should recognize that the mode is a way to describe a "representative" or "typical" value for the distribution. Mode is a measure of center which describes the value(s) that occur most often. Mode is remembered as the "most" or most frequent entry in the set, not the largest value.

Median
The median is the "middle" of the data. It has the same number of data points (approximately half) above and below it. Initially use an odd amount of data points, so that students understand the median is the midpoint. Once students have grasped this idea, have students use an even number of data points where the midpoint is less obvious.

To find the median visually and kinesthetically, students could reorder the data in ascending or descending order, then place a finger on each end of the data and continue to move toward the center by the same increments until the fingers touch. This number is the median. (If there are an even number of data, the fingers will be touching two values. Take the mean of the two middle values as the median of the distribution.) Eventually they may move towards counting the number of data points and dividing by 2. Students may think that the median is always the number in the middle, but that middle number can only be determined after the data entries are arranged in ascending or descending order. It may be helpful to arrange the data in a vertical list instead of a horizontal list to prepare students to work with data in decimal forms as decimals may be difficult to read in a horizontal list.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

EXAMPLE

The number of points per game is shown for the Lions Girls' Basketball team. If the median number of points is 4 what could Lindsey and River have scored?

Name	Points
Shemera	5
Jade	14
Lauren	1
Anyssia	1
Rashauna	15
Gabby	3
Giana	0
MacKenzie	8
Kyana	6
Ming	2
Lindsey	
River	

Discussion: This is an open-ended problem with the goal of centering the data at 4. Both girls could have scored a 4, or one of them could score a 3 and the other a 6. Or one could have scored below a 3 and another above a 6.

Students sometimes incorrectly think that the median is halfway between the least and greatest values in the data set, but that is not necessarily the case. For example if the data set is 1, 1, 1, 1, and 5, the median is 1.

Sometimes students may incorrectly treat data points that have the same numerical value as one data point instead of counting each one separately.

Mean
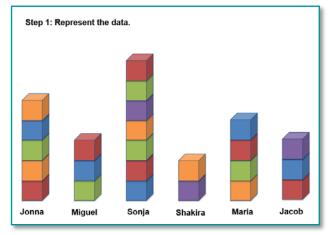
In Grade 6, mean is introduced as "fair share." It is the leveling out of data that can be connected to the unit rate. The concept of mean as "fair share" can be demonstrated visually and kinesthetically by using stacks of linking cubes or blocks. The blocks are redistributed among the towers so that all towers have the same number of blocks. It may be useful initially to present problems that work out as whole numbers, so students can use concrete models such as blocks. However, students should move towards finding the mean as a decimal or fraction, reinforcing the idea that the mean is a number representing the arithmetic average but not necessarily an actual number in the data set.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)
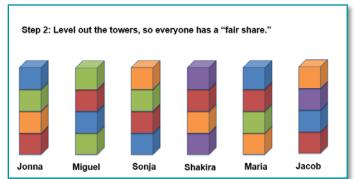
EXAMPLE

Jonna wanted to find out the mean number of family members each of her friends had. She found the following information:

- Jonna–5 people
- Miguel–3 people
- Sonja–7 people
- Shakira–2 people
- Maria–4 people
- Jakob–3 people

Step 1: Represent the data.

Step 2: Level out the towers, so everyone has a "fair share."
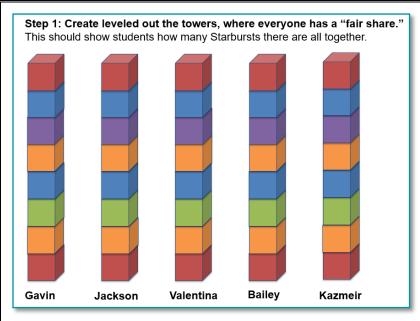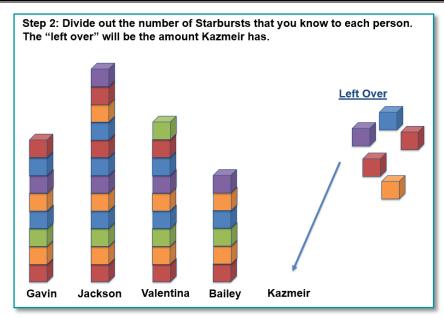
Discussion: After doing several sets using whole numbers move to problems where students get decimal or fractional answers. This can be easily demonstrated on graph paper as a part of a block. From this understanding students can come to the conclusion that adding up the numbers (family members) and then dividing them by the number of data points (friends) is a way to calculate the mean.

EXAMPLE

The kids were given a mean of 8 Starbursts for participating in the library contest. The number of Starbursts each child received is listed in the table. How many did Kazmeir get?

Name	# of Pieces
Gavin	8
Jackson	12
Valentina	9
Bailey	6
Kazmeir	

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

Step 1: Create leveled out the towers, where everyone has a "fair share." This should show students how many Starbursts there are all together.

Gavin   Jackson   Valentina   Bailey   Kazmeir

Step 2: Divide out the number of Starbursts that you know to each person. The "left over" will be the amount Kazmeir has.

Left Over

Gavin   Jackson   Valentina   Bailey   Kazmeir

Discussion: Similarly student can show the fair share approach by using grid paper. After doing several of these types of problems, move towards problems where the mean is represented as a fraction or decimal. Eventually move toward problems where the mean and all the data points except one are given, and students need to find the missing data point.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

EXAMPLE
Eight students scored an average of 84% on the most recent social studies test. What are their possible scores? Give at least three possible scenarios.

Discussion: The same mean can be represented by different data. The purpose of this problem is to show students there can be many possible combinations of numbers that when calculated together represent the mean. They can all have 84% or half could have 83% and the other half could have 85%. To extend this problem, allow students to suggest one or two of the students' test scores that do not balance out symmetrically. For example, one student could get a 52%.

Students should also come to the realization that an outlier can affect the mean significantly; whereas it may have only a slight effect on the median. Students should spend time on deciding whether the mean or median is a better descriptor of data. Usually the mean, mode, and median have different values, but sometimes those values can be the same. Give students examples where the measures of center are the same, and discuss the significance when that happens.

MEASURES OF VARIABILITY(SPREAD)
Variability can be described as looking at how data values vary with a single number. It includes looking at a graphical representation of data for symmetry, peaks, clusters, gaps, and any outliers in the data. Measures of variability at this level include the range and the interquartile range (IQR). Students should realize that the less the variability, the more precise the mean is in representing the data. Similarly, the greater the variability, the less precise the mean is in representing the data.

EXAMPLE
Two classes test grades are listed below.
Mrs. Gaylord's class: 87, 88, 84, 84, 83, 81, 86, 85, 85, 87, 86, 89, 88, 86, 84, 86, 87, 88, 85, 88, 85, 90
Mr. Reiley's class: 64, 90, 96, 75, 73, 82, 88, 93, 93, 57, 95, 71, 82, 87, 88, 90, 78, 96, 98, 98, 99, 99
   a. Find the mean of each class.
   b. Choose a type of graph to display the data. The type of graph should be the same for each class.
   c. How do the graphs compare?
   d. Whose class had the better scores? Explain.
   e. In which class is the mean a more accurate representation of a typical test grade. Explain.
Discussion: This example allows for a rich classroom discussion on comparing data distributions as the mean is the same for both classes. Students can debate which is better and why. Is it better to have a lot of high grades and a lot of low grade or is it better to have a lot of grades around 80? Personal opinion may factor in. Discuss that when there is greater variability, the less precise the mean is in representing the data. If there are multiple sections of the same class at a school, grades can be compared and a debate can ensue about which class did better. Students may become competitive and read the data differently in order to have the "best" grades. This can lead to a discussion about how statistics can be manipulated.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

DISPLAYING DATA
It is important that students display the same data using different representations and decide which display is most representative for the message they want to convey. They create graphs of data distributions, select an appropriate measure of center to describe a typical data value for a given data distribution, and also calculate and interpret an appropriate measure of variability based on the shape of the data distribution. (MP.4). Students need to use labels and scales for axes appropriately (MP.5).

In step four of the GAISE model, students will analyze the data by comparing the different graphs of the same data which will help students develop understanding of the benefits of each type of representation.

TIP! Before graphing data, it may be useful to collect data using a frequency table.

Students might not be ready to create their own graphical representations, but students can use teacher provided graphs to generate rich discussions about variability, distribution, etc.

TIP!

TIP! Students could create different graphical representations of data and identify the best/most appropriate graphical display. Use this as an opportunity to have rich mathematical discussion student to teacher and student to student.

Dot Plots/Line Plots
A dot plot (line plot) is a graphical representation for data on one variable. Students may think that line plots and dot plots are different graphs but they are the same and the terms can be used interchangeably; one just has X's and the other has dots.
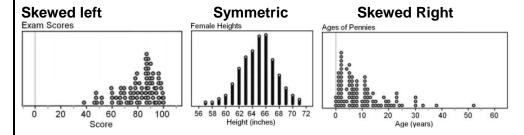
Students may incorrectly think that line plots are line graphs; they are not. Give them examples of both and discuss their differences to confront this misconception.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

To create a dot plot for numerical data, do the following:
- ❖ Set up a number line.
    - o Identify the max and min in order to choose an appropriate scale.
    - o Label the scale.
    - o Create a title for the dot plot.
- ❖ Plot data points on the graph.

A dot plot can help students see the shape of the data including but not limited to whether its symmetrical, skewed, or uniform. It can also help students visualize clusters, gaps, and outliers.

Skewed left   **Symmetric**   **Skewed Right**

Taken from Arizona 6-8 Progressions on Statistics and Probability, page 4.

To create a dot plot for categorical data, do the following:
- ❖ Draw a horizontal line and label it with categories.
- ❖ Plot data points on the graph.
- ❖ Give the dot plot a title.
- ❖ Label the vertical axis.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

Stem and Leaf Plots
A stem and leaf plot is useful for organizing data in numerical order. Although not explicitly mentioned in the standards, it is a useful strategy to organize data. It is important that the stem and leaf plot has a key. It also can give students a sense of the shape of the data.
 To create a stem-and-leaf plot, do the following:
   ❖ Break into intervals of 10 (or 100, 1,000, etc) .
   ❖ Create a T-chart.
   ❖ Place the tens digit (stem) on the left.
   ❖ Order the leaves from least to greatest for each stem.
   ❖ Place the ones digit for each of the tens (leaves) on the right.
   ❖ Create a key.
Note: Stems can be any digit or digits such as a 1 for a hundreds digit. That is why a key is important. Although 2 dots plots may look the same a key with $(3|4 = 34)$ *has very different data the key* $(3|4 = 340)$. *See Grade 7 for a double stem-and-leaf plot.*

Number of Push-ups

Stem	Leaves
1	1 3 4
2	0 2 3 6 7 9 9
3	2 3 4 4 5 7 8
4	0 1 2

Key: 3 | 4 = 3.4

Histograms
Large sets of data become impractical to display in stem and leaf graphs and dot plots. It is more practical to divide the data for large sets of data into intervals, and then display the data in a histogram.
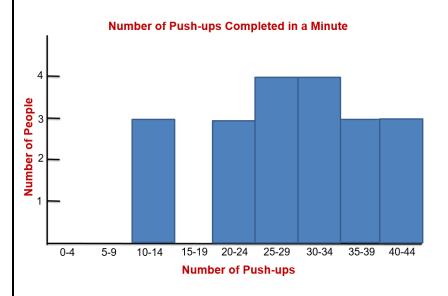
🧠 Students may incorrectly think bar graphs and histograms are the same, but bar graphs are used for categorical data and histograms are used for numerical data. In histograms—
   • The bars touch because where one interval ends the next one begins.
   • The vertical bars in a histogram act as "bins" rather than actual data points.
   • The height of the bar shows the combined frequency of all the values in the "bin."
   • Sometimes the whole interval is given and sometimes the interval is given just at the left-end value of the bar.
   • Students should also be introduced to the break symbol when a graphical display does not start at 0.
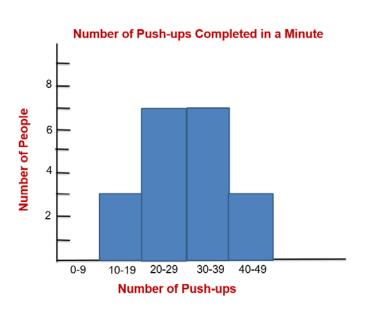
Students should have practice converting stem-and-leaf plots into histograms. Have students create several histograms with the same data but using different scales, and discuss how that impacts the message of the histogram.

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To create a histogram, do the following:
- ❖ Create a frequency table with equal intervals.
  - o Determine number of intervals.
  - o Organize the data with tallies and totals.
- ❖ Set up and label one axis with frequencies and the other axis with intervals.
- ❖ Create a title for the histogram.
- ❖ Use the data from the frequency table to make bars.

Note: There are also two types of histograms: Frequency and Relative Frequency Histograms. A Relative Frequency Histogram has the frequency represented in terms of its relative frequency using a percent (or its equivalent decimal representation). Grade 6 is not responsible for content with respect to Relative Frequency Histograms.

Box Plots (Box & Whisker Plots)
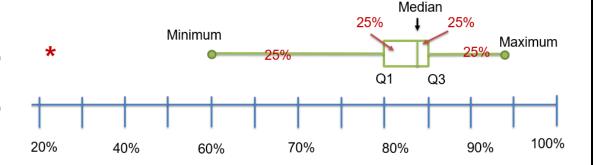
One of the most useful graphical displays for comparing two distributions is the box plot, but the focus in Grade 6 is displaying the data of only one data set using box plots. Grade 7 will go deeper into the comparison of two different data sets. However, doing some comparisons of graphs will help students solidify the purpose of the box plot.

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The box plot is based on the division of the ordered data into four groups with approximately the same number of data points in each group. One way we describe the groups is with the Five Number Summary which includes the minimum value, the first quartile, the median (the second quartile), the third quartile, and the maximum. The Five Number Summary is then turned into a graphical display where a box encompasses the second and third quartiles. Each quartile has approximately 25% of the data points in it.

The range shows the overall variability (spread) whereas the interquartile range (IQR) shows the spread between the first and third quartiles (the box). The IQR can be found by finding the difference between the third and first quartile. At this level, outliers are just "eyeballed", they will be formally calculated in later grades/course. The outliers are detached from whiskers and labeled with an asterisk.
To create a box plot, do the following:

❖ Order the data from least to greatest, e.g., organized list, table, stem-leaf plot.
  1. Identify the minimum (lower extreme).
  2. Identify the maximum (upper extreme).
  3. Find the median (second quartile or Q2).
  4. Find the median of the lower half (excluding the median of the distribution) and label it as the first quartile (Q1).
  5. Find the median of the upper half (excluding the median of the distribution) and label it as the third quartile (Q3).
❖ Draw a number line and determine the scale.
❖ Label the graph with a title.
❖ Draw vertical lines through the median, Q1, and Q3, and then connect horizontal lines to form a rectangle (Box).
❖ Draw a horizontal line to connect the midooint of the edge of the box to the lower extreme (min) and another to connect the box to the upper extreme (max). This creates the whiskers.
❖ Use an asterisk to identify any possible outliers and do not connect the asterisk to the whisker.

Unit 1 Test Grades

Summarizing Data
When summarizing data, students should be able to describe what a "typical" piece of data represents and connect this concept to measures of center. Students should not only determine measures of center and measures of variability, but also use these numbers to answer the statistical question asked in the context of the problem. The distribution should be described in terms of its center (mean, median, mode), spread (range, interquartile range), and shape (symmetry, skewness, clusters, gaps, outliers).

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Continue to have students connect contextual situations to data to describe the data set in words prior to computation. Therefore, determining the measures of variability and measures of center mathematically need to follow the development of the conceptual understanding. Students should experience data which reveal both different and identical values for each of the measures. They need opportunities to explore how changing a part of the data may change the measures of center and measure of variability. Also, by discussing their findings, students will solidify understanding of the meanings of the measures of center and measures of variability and what each of the measures do and do not tell about a set of data. This should all lead to a better understanding of their usage. Use this as an opportunity to have rich, mathematical discussions, student to teacher and student to student.

Provide activities that require students to sketch a graphical representation based upon given measures of center and variability and a context. This will help create connections between the measures and real-life situations.
- Explain what was investigated (statistical question).
- Explain what was collected.
    - How is it measured?
    - Units of measure?

Different Data Displays

Different graphical displays of data highlight different information. Each graphical display has advantages and disadvantages.
- Dot Plots are used for small sets of data.
- Box plots and histograms can be used for large sets of data.
- Dot plots can be used with both numerical and categorical data.
- The mean, median, or mode can be calculated from the dot plot.
- Dot plots show clusters, gaps, and outliers in a data set.
- Histograms are useful for large sets of numerical data, but not useful for small sets of data.
- It is difficult to find measures of central tendency when data are displayed in a histogram.
- A histogram allows one to see shape, spread, gaps, and clusters.
- Changing the intervals of a histogram can send a different message about the data set, which could be an advantage or a disadvantage.
- Box plots are useful for summarizing large sets of numerical data.
- Box plots are a useful way to illustrate data sets where the median is the most appropriate measure of center; however, the mean and mode cannot be identified.
- Box plots allow one to visualize symmetry, skewness, and outliers.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (6.SP.4-5)

Instructional Tools/Resources
These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology
- Graphing calculator
- Desmos is a free online graphing utility and app.
- Census at School is an international classroom project that provides real data for classrooms. (Note: On a PC data can be copied from their Excel spreadsheet right into Desmos.)
- Excel

Measures of Center
- Statistics and Probability—Mean as a Fair Share by Texas Instruments is a lesson using TI technologies to explore mean as a fair share.
- What Does It Mean? by Virginia Department of Education is a lesson that uses linking cubes to explore the mean as a fair share.
- Suzi's Company by Mathematics Assessment Project is a task where students help Suzi figure out the cost of annual salaries for her company and check some statistics about rates of pay.
- Mean, Median, and Range by Open Middle has students create a set of whole numbers that have the same mean, median, and range.
- Fantastic Beasts – What did it cost to make? by Yummy Math is an activity where students use the average cost of making a Harry Potter movie to estimate the cost of creating the "Beasts" movie. The task is open, in that it asks students to analyze central tendency, using either median, mode or mean. Which is the best predictor of the cost of making this new movie? As an extension, students make box plots and analyze the interquartile range to identify outliers.
- How Many Wins is Lebron Worth? by Yummy Math is an activity where students read graphs and use measures of central tendency as they compare the records of the Miami Heat and Cleveland Cavaliers with and without Lebron.
- Does it pay to get educated? by Yummy Math is an activity where students compare the median earnings by various education levels. Students compare the earnings of a non-high school graduate with workers who graduated high school and/or went on to complete higher education degrees.
- Typical Super Bowl Scores by Yummy Math is an activity where students study historical Super Bowl data to reflect on mean, median, and mode and ranges of both losing scores and winning scores.
How Much Does a Lego Cost? by Yummy Math is a 3-act task with an activity guide. Show students the act one video. What questions do they have? How much could the Star Wars kit cost?
- Seven People Went Fishing by Victoria's Department of Education and Training is a task where students explore the mean and median with multiple entry points.

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Measures of Center, continued
- Problem of the Month-Through the Grapevine, Level C by Inside Mathematics is a task that challenges a student to use measures of center to describe variations in data and decide which one best describes the data set. Students use their measures to make predictions about scaling the size of the box of raisins using proportional reasoning.
- Problem of the Month-Pick a Pocket, Level C by Inside Mathematics is a task that challenges a student to calculate measures of center from their line plot and find the range of the data. Students decide which measure best fits the data to make a prediction and then try to design a new set of data to give the same mean and median as the class set.

Dot Plots
- Creating Dot Plots by Khan Academy has 4 questions where students interactively create dot plots.
- Dot Plots by MathisFun has examples and explanations for creating dot plots.

Box Plots
- Interpreting Box Plots-Data On Camping and Backpacking Goods by the United States Census Bureau is a lesson where students create box plots.
- Using NBA Statistics for Box and Whisker Plots by NCTM Illuminations is a lesson where students use information from NBA statistics to make and compare box and whisker plots. The data provided in the lesson come from the NBA, but you could apply the lesson to data from the WNBA or any other sports teams or leagues for which player statistics are available. *NCTM now requires a membership to view their lessons.*
- Height of Students in our Class by NCTM Illuminations is a lesson that has students creating box-and-whisker plots with an extension of finding measures of center and creating a stem and leaf plot. *NCTM now requires a membership to view their lessons.*
- Comparing Test Scores by Illustrative Mathematics is a task where students have to create box plots and then critically compare the center and spread to make conclusions.

Histograms
- Histograms by MathisFun has examples and explanations for creating histograms.
- Creating and Interpreting Histograms—Age Distributions of Householders in the United States by the United States Census Bureau is a lesson where students create, compare, and interpret histograms.
- Create a Histogram in Excel is a tutorial on how to make a histogram using Excel.
- Histogram by Shodor is an interactive tool to assist in creating histograms.
- Managing Data with Histograms by Scholastic is a lesson where students learn how a histogram can reveal frequency distribution in the context of a flood.
- There is a Difference: Histograms vs Bar Graphs by Carol A. Marinas from NCTM Illuminations is a lesson where students explore the differences between bar graphs and histograms. *NCTM now requires a membership to view their lessons.*

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Histograms, continued
- Peak of Hurricane Season by Yummy Math is an activity where students read about hurricane classifications and calculate the possible increases in destruction from various scaled events. Estimation, data analysis with graphs to study trends, damage expense, average frequency, and histograms are all part of this lesson.
- Frequency Distributions—Hispanic or Latino Population Percentages in 50 States and the District of Columbia by the U.S. Census Bureau is a lesson where students create frequency tables and histograms that summarize and display data within a context.

Stem-and-Leaf Plots
- Wet Heads by PBS Learning is lesson where students create stem and leaf plots and back-to-back stem and leaf plots to display data collected from an investigative activity.

Multiple Representations
- Interpreting Dot and Box Plots--How Has the U.S. House of Representative Changed Over Time? by the United States Census Bureau is a lesson where students create frequency tables, dot plots, and box plots.
- Understanding Distributions of Data—Pet Food Manufacturing by the United States Census Bureau is a lesson where students analyze dot plots, box plots, and histograms.
- Which is Better … Original Movies or their Sequels? by Yummy Math is a task where students see what they can conclude from various types of graphs and consider what size random sampling of movies and their sequels is an adequate, representative population of movies. This is a very open ended activity that will allow students to conduct data analysis using measures of central tendency & variability, box plots, histograms etc.
- Describing Distributions by Illustrative Mathematics is a task where students describe data distributions in terms of center, spread, and overall shape.

Curriculum and Lessons from Other Sources
- EngageNY, Grade 6, Module 6, Topic A, Lesson 2: Displaying a Data Distribution, Lesson 3: Creating a Dot Plot, Lesson 4, Creating a Histogram are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 6, Topic B Lesson 6: Describing the Center of Distribution Using the Mean and Lesson 8: Variability in a Data Distribution are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 6, Topic C, Lesson 12: Describing the Center of a Distribution Using the Median, Lesson 13: Describing Variability Using the Interquartile Range, Lesson 14: Summarizing a Distribution Using a Box Plot, Lesson 15: More Practice with Box Plots, Lesson 16: Understanding Box Plots are lessons that pertain to this cluster.
- EngageNY, Grade 6, Module 6, Topic C, Lesson 17: Developing Statistical Project, Lesson 18: Connecting Graphical Representations and Numerical Summaries, Lesson 19: Comparing Data Distributions, Lesson 22: Presenting a Summary of a Statistical Project are lessons that pertain to this cluster.

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Curriculum and Lessons from Other Sources, continued
- Illustrative Mathematics, Grade 6, Unit 8, Lesson 3: Representing Data Graphically, Lesson 4: Dot Plots, Lesson 5: Using Dot Plots to Answer Statistical Questions, Lesson 6: Histograms, Lesson 7: Using Histograms to Answer Statistical Questions, Lesson 8: Describing Distributions on Histograms, Lesson 9: Interpreting the Mean as a Fair Share, Lesson 13: The Median of a Data Set, Lesson 14: Comparing Mean and Median, Lesson 15: Quartiles and Interquartile Range, Lesson 16: Box Plots, Lesson 17: Using Box Plots are lessons that pertain to this cluster.
- Georgia Standards of Excellence Curriculum Frameworks, Grade 6, Unit 6: Statistics has many tasks that pertain to this cluster.

General Resources
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards. *Note: Ohio's Learning Standards 7.SP.1 does not include random sampling as indicated on 7.SP.1*
- Arizona 6-8 Progression on Statistics and Probability is an informational document for teachers. This cluster is addressed on pages 2, 4-6.
  Page 2:
    - Overview, paragraph 1-2
    - Bullet points refer to GAISE model.
  Page 4:
    - Ohio's 6.SP.1 is different than what is listed in this document. Ohio's standard refers to the GAISE model.
    - Teachers formulate questions at this level.
    - Teachers could use graphs given on the right side of the page to discuss distributions.
    - Former standard 4.G.3 about symmetry was deleted from Ohio's standards in 2017. This is a good time to discuss **symmetry** of graphs.
  Pages 5-6:
    - MAD is NOT 6th grade content. It is addressed in 7th grade in Ohio.
- LOCUS is an NSF Funded project focused on developing assessments of statistical understanding. Teachers must create an account to access the assessments.
- K-12 Statistics Education Resources is a collection of websites put together by the American Statistical Association for teachers.
- A Sequence of Activities for Developing Statistical Concepts by Christine Franklin & Gary Kader is an article published in *The Statistics Teacher Network,* Number 68, Winter 2006. It has an overview of the GAISE model and its levels and it includes activities at each level.
- Statistics Teacher is an online journal published by the American Statistical Association (ASA) and NCTM. It includes articles, lesson plans, and professional development opportunities.
- Significance is a magazine that demonstrate the practical use of statistics and shows how statistics benefits society.

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