



# Ohio

## Ohio's Model Curriculum | Mathematics with Instructional Supports

### Grade 6

# Mathematics Model Curriculum

## with Instructional Supports

### Grade 6

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## Introduction

### PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

### COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

**Expectations for Learning:** This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

**Content Elaborations:** This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

### COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

## Standards for Mathematical Practice—Grade 6

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as they engage with and master new and more advanced mathematical ideas across the grade levels.

### **MP.1 Make sense of problems and persevere in solving them.**

In Grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”. Students can explain the relationships between equations, verbal descriptions, tables, and graphs. Mathematically proficient students check their answers to problems using different methods.

### **MP.2 Reason abstractly and quantitatively.**

In Grade 6, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations or other meaningful moves. To reinforce students’ reasoning and understanding, teachers might ask, “How do you know?” or “What is the relationship of the quantities?”.

### **MP.3 Construct viable arguments and critique the reasoning of others.**

In Grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.

### **MP.4 Model with mathematics.**

In Grade 6, students model problem situations symbolically, graphically, in tables, contextually, and with drawings of quantities as needed. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students begin to explore covariance and represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate and apply them to a problem context. Students should be encouraged to answer questions such as “What are some ways to represent the quantities?” or “What formula might apply in this situation?”

## Standards for Mathematical Practice, continued

### **MP.5 Use appropriate tools strategically.**

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in Grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to create dot plots, histograms, and box plots to visually compare the center and variability of the data. Visual fraction models can be used to represent situations involving division of fractions. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures. Students should be encouraged to answer questions such as “What approach did you try first?” or “Why was it helpful to use?”

### **MP.6 Attend to precision.**

In Grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations, or inequalities. When using ratio reasoning in solving problems, students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. Students also learn to express numerical answers with an appropriate degree of precision when working with rational numbers in a situational problem. Teachers might ask, “What mathematical language, definitions, or properties can you use to explain \_\_\_?”

### **MP.7 Look for and make use of structure.**

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (e.g.,  $6 + 2n = 2(3 + n)$  by distributive property) and solve equations (e.g.,  $3c = 15$  so  $c = 5$  by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume. Teachers might ask, “What do you notice when \_\_\_?” or “What parts of the problem might you eliminate, simplify, or \_\_\_?”

### **MP.8 Look for and express regularity in repeated reasoning.**

In Grade 6, students use repeated reasoning to understand algorithms and make generalizations about patterns. Given multiple opportunities to solve and model problems, they may notice that  $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$  and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between covariance, rates, and representations showing the relationships between quantities. Students should be encouraged to answer questions such as, “How would we prove that \_\_\_?” or “How is this situation similar and/or different from other situations?”

# Mathematics Model Curriculum

## with Instructional Supports

### Grade 6

STANDARDS	MODEL CURRICULUM
<p><b>RATIO AND PROPORTIONAL RELATIONSHIPS</b>  <b>Understand ratio concepts and use ratio reasoning to solve problems.</b>  <b>6.RP.1</b> Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”  <b>6.RP.2</b> Understand the concept of a unit rate <math>\frac{a}{b}</math> associated with a ratio <math>a:b</math> with <math>b \neq 0</math>, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>\frac{3}{4}</math> cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”  <i>Continued on next page</i></p>	<p><b>Expectations for Learning</b>  The study of ratio and proportion extends student learning of multiplication, division, and measurement from previous grades. It is essential for students to make sense of quantities that involve proportional relationships within a context. This is the first interaction students have with ratios and ratio/rate reasoning including percents as ratios. The learning in this standard should be focused on developing an understanding of ratios and solving real-world problems through the use of visual models. In Grade 7, students recognize and represent proportional relationships and extend this reasoning to direct variation equations and more advanced percent problems.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Ratios</b></p> <ul style="list-style-type: none"> <li>• A ratio is a comparison used to describe relationships between two (or more) quantities.</li> <li>• The quantities in ratios may or may not have the same unit.</li> <li>• A ratio compares parts to parts or parts to a whole.</li> <li>• Fractions and percents are specific types of ratios which compares parts to a whole.</li> <li>• Ratios can be written in various forms, e.g., 3:1, 3 to 1, or <math>\frac{3}{1}</math>.</li> <li>• Rows (and columns) of ratio tables are multiples of each other.</li> <li>• Ratios are multiplicative relationships.</li> <li>• Ratios can be used to convert units of measure.</li> </ul> <p><i>Continued on next page</i></p>



- 6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams<sup>6</sup>, double number line diagrams<sup>6</sup>, or equations.
- Make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  - Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
  - Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means  $\frac{30}{100}$  times the quantity; solve problems involving finding the whole, given a part and the percent.
  - Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDINGS, CONTINUED

#### Rates

- A unit rate is a comparison of two quantities where the second quantity (denominator) is one.
- Equivalent ratios have the same unit rate.
- The division line in a rate can mean “for every 1,” “for each,” and “per.”

#### Percents

- Percents are used to compare part-to-whole relationships.
- Percents can be found using different sized wholes.
- Percents are out of a 100.
- A percent is a specific type of ratio, which can be represented as a fraction, with a denominator of 100.
- All percent problems involve a part and a whole (100) measured in some unit and the same part and whole measured in hundredths.
- Benchmark percents can be used to estimate and calculate other percents.

### MATHEMATICAL THINKING

- Use precise mathematical language to describe mathematical reasoning.
- Solve real-world problems accurately.
- Pay attention to and make sense of quantities.
- Make and modify a model to represent mathematical thinking.
- Recognize and use a pattern and structure to solve problems.
- Consider mathematical units in a problem.

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## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

#### Ratios and Rates

- Use ratio or rate language to describe the relationship between two quantities.
- Distinguish between part-to-part, part-to-whole, and whole-to-part comparisons.
- Write a ratio symbolically to describe the relationship between two quantities.
- Recognize ratios as multiplicative relationships.
- Use models to solve problems involving ratios and unit rates such as ratio tables, tape diagrams, and double number lines.
- Identify or create equivalent ratios.
- Identify and write unit rates.
- Use ratio reasoning to solve a variety of real-world problems.
- Apply ratio reasoning to convert measurement units within the same system.
- Solve real-life problems involving measurement units that need to be converted.
- Make tables of equivalent ratios relating quantities with whole number measurements to do the following:
  - Find missing values;
  - Plot pairs of values in the first quadrant of the coordinate plane;
  - Compare ratios; and
  - Develop the concept of proportion without solving proportions explicitly.

#### Percents

- Represent percents using models, such as 100 grids, tape diagrams, and double number lines.
- Use ratio reasoning to relate a percent of a quantity as a rate per 100.
- Use benchmark percents (1%, 5%, 10%, 20%, 25%, 50%, and 100%) to compute other percents of a given whole number both mentally and with a model.
- Identify the part, whole, and/or percent in a real-world or mathematical problem.
- Use a model to find the percent when given the part and whole.
- Use a model to find the whole when given the part and percent.
- Solve real-world percent problems using a model.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 1, page 36](#)
- [Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5](#)
- [Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7](#)
- [Ohio's K-8 Learning Progressions, Ratios and Proportional Relationships, page 15](#)

### CONNECTIONS ACROSS STANDARDS

- Use variables to represent two quantities in a real-world problem that change in relation to one another (6.EE.9).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>THE NUMBER SYSTEM</b>  <b>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</b></p> <p><b>6.NS.1</b> Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models<sup>g</sup> and equations to represent the problem. <i>For example, create a story context for <math>(\frac{2}{3}) \div (\frac{3}{4})</math> and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that <math>(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}</math> because <math>\frac{3}{4}</math> of <math>\frac{8}{9}</math> is <math>\frac{2}{3}</math>. (In general, <math>(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}</math>.)</i>  <i>How much chocolate will each person get if 3 people share <math>\frac{1}{2}</math> pound of chocolate equally? How many <math>\frac{3}{4}</math> cup servings are in <math>\frac{2}{3}</math> of a cup of yogurt? How wide is a rectangular strip of land with length <math>\frac{3}{4}</math> mi and area <math>\frac{1}{2}</math> square mi?</i></p>	<p><b>Expectations for Learning</b></p> <p>In previous grades, students divided unit fractions by whole numbers and divided whole numbers by unit fractions. This is the first time students divide fractions by fractions. The learning in this standard should be focused on interpreting and computing quotients using visual models (not an algorithm) and to develop an understanding of the division of fractions and solving real-world problems. In future grades, students will extend this understanding to computation with rational numbers.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>• There are two meanings of division: partitive and measurement.</li> <li>• Partitive problems are sharing problems and rate problems. (Rate problems are not always partitive problems.)</li> <li>• Measurement problems are repeated subtraction or equal groups.</li> <li>• There is a relationship between multiplication and division that can be seen using visual models.</li> </ul> <p><b>MATHEMATICAL THINKING</b></p> <ul style="list-style-type: none"> <li>• Draw a picture or create a model to make sense of mathematical and real-world problems.</li> <li>• Compute using strategies or models.</li> <li>• Interpret and explain a model to solve mathematical and real-world problems.</li> <li>• Use a pattern or structure.</li> <li>• Compute accurately and efficiently with grade-level numbers.</li> </ul> <p><b>INSTRUCTIONAL FOCUS</b></p> <p><b>Visual Models</b></p> <ul style="list-style-type: none"> <li>• Recognize and interpret a visual model for division of a fraction by a fraction.</li> <li>• Create and use a visual model for division of a fraction by a fraction.</li> <li>• Divide a fraction by a fraction using visual models.</li> <li>• Use visual models to show the relationship between the multiplication and division of fractions.</li> </ul> <p><i>Continued on next page</i></p>

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS, CONTINUED

#### Equations

- Create a story context to demonstrate understanding of dividing a fraction by a fraction.
- Use numerical equations to show the relationship between the multiplication and division of fractions.
- Use a numerical equation to represent a real-world problem involving the division of a fraction by a fraction.
- Explore patterns and visual models of dividing a fraction by a fraction to discover the relationship between multiplication and division to explain that

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

#### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38](#)
- [Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5](#)
- [Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7](#)
- [Ohio's K-8 Learning Progressions, The Number System, pages 16-17](#)

#### CONNECTIONS ACROSS STANDARDS

- Write expressions and equations to solve real-world problems involving non-negative rational numbers (6.EE.7).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

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STANDARDS	MODEL CURRICULUM
<p><b>THE NUMBER SYSTEM</b>  <b>Compute fluently with multi-digit numbers and find common factors and multiples.</b></p> <p><b>6.NS.2</b> Fluently<sup>G</sup> divide multi-digit numbers using a standard algorithm<sup>G</sup>.</p> <p><b>6.NS.3</b> Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.</p> <p><b>6.NS.4</b> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. <i>For example, express <math>36 + 8</math> as <math>4(9 + 2)</math>.</i></p>	<p><b>Expectations for Learning</b></p> <p>Place value has been a major emphasis in the elementary standards. In grade 6, students become fluent in the use of a standard division algorithm, continuing to use their understanding of place value to describe what they are doing. This standard is the end of this progression to address students' understanding of place value. Previously students have had exposure to addition, subtraction, multiplication, and division of whole numbers. Students also have had prior exposure to addition and subtraction of decimals and multiplication and division of whole numbers by decimals and decimals by whole numbers. The learning in these standards should be focused on multiplying and dividing decimals by decimals and developing fluency with grade-level decimals.</p> <p>Students have had experience with common multiples and common factors in earlier grades in relation to fractions. The learning in this standard should focus on finding and using least common multiple and greatest common factor to solve mathematical and real-world problems. Composing and decomposing numbers efficiently is a precursor to operating with algebraic expressions.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Whole Number and Decimal Operations</b></p> <ul style="list-style-type: none"> <li>• There are several acceptable standard algorithms for operations involving addition, subtraction, multiplication, and division.</li> <li>• When adding and subtracting, tenths are added/subtracted to tenths, and hundredths are added/subtracted to hundredths.</li> <li>• When adding and subtracting decimals, line up the decimal point to align the place values of the numbers.</li> <li>• A remainder can be expressed as a fraction or a decimal.</li> </ul> <p><b>Greatest Common Factors</b></p> <ul style="list-style-type: none"> <li>• Factors are the numbers being multiplied together.</li> <li>• Greatest Common Factor (GCF) is useful in expressing the numbers using the distributive property.</li> <li>• Composing and decomposing numbers can help solve routine mathematical and real-world problems.</li> </ul> <p><i>Continued on next page</i></p>



## Expectations for Learning, continued

### MATHEMATICAL THINKING

- Compute accurately and efficiently.
- Use different properties of operations flexibly.
- Determine the reasonableness of results.
- Attend to precision when performing mathematical operations.
- Solve multi-step problems accurately.

### INSTRUCTIONAL FOCUS

#### Whole Number and Decimal Operations

- Estimate sum, differences, products, or quotients before computing.
- Use estimation to determine the reasonableness of solutions.
- Add, subtract, multiply, and divide multi-digit whole numbers and decimals using a standard algorithm.
- Explain and justify the steps in a standard algorithm.
- Develop an understanding of and determine where to place the decimal point in products and quotients.
- Evaluate whether the location of the decimal point makes sense in a given context.
- Adjust the precision of the decimal to match the context.
- Apply and adapt a variety of appropriate strategies to solve routine and non-routine mathematical and real-world problems involving whole numbers and decimals.

#### Greatest Common Factor

- Find the least common multiple of two numbers less than or equal to 12.
- Find the greatest common factor of two whole numbers less than or equal to 100.
- Use the greatest common factor and distributive property to rewrite the sum of two whole numbers 1-100.

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### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38](#)
- [Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5](#)
- [Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7](#)
- [Ohio's K-8 Learning Progressions, The Number System, pages 16-17](#)

### CONNECTIONS ACROSS STANDARDS

- Apply the distributive property to algebraic expressions (6.EE.3).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

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**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>THE NUMBER SYSTEM</b>  <b>Apply and extend previous understandings of numbers to the system of rational numbers.</b></p> <p><b>6.NS.5</b> Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p> <p><b>6.NS.6</b> Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-(-3) = 3</math>, and that 0 is its own opposite.</p> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. In previous grades, students worked with positive fractions, decimals, and whole numbers on the number line and in quadrant one of the coordinate plane. In sixth grade, students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (e.g., thermometer) which facilitates the movement from number lines to coordinate grids. Students begin graphing in all four quadrants of the coordinate plane. They will explore absolute value, opposite numbers, comparing numbers, and ordering rational numbers using the number line or coordinate plane. This will lead to operations with integers in future grades.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Rational Numbers</b></p> <ul style="list-style-type: none"> <li>• A number line can show magnitude (quantity) and direction.</li> <li>• Negative numbers are to the left of zero on a horizontal number line and below zero on a vertical number line.</li> <li>• A number and its opposite are the same distance from zero on a number line.</li> <li>• Zero is its own opposite.</li> <li>• The negative sign means the “opposite of,” so <math>-p</math> is the opposite of <math>p</math>.</li> <li>• The set of integers consists of positive whole numbers, their opposites, and 0.</li> <li>• The opposite of the opposite of the number is the number itself.</li> <li>• Although fractions and decimals can be negative, they are not necessarily integers, e.g., <math>-\frac{3}{5}</math> and <math>-3.2</math> are not integers, but <math>-\frac{4}{1}</math> and <math>-3.0</math> are integers.</li> <li>• In a fraction the negative sign can be written in the numerator, the denominator, or out front, e.g., <math>\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}</math>.</li> <li>• As the magnitude of a negative number increases (moves to the left or downward on a number line), the value of the number decreases.</li> <li>• The absolute value of a number is the distance from zero.</li> <li>• The absolute value of zero is zero.</li> </ul> <p><i>Continued on next page</i></p>

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

**6.NS.7** Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret  $-3 > -7$  as a statement that  $-3$  is located to the right of  $-7$  on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write  $-3^{\circ}\text{C} > -7^{\circ}\text{C}$  to express the fact that  $-3^{\circ}\text{C}$  is warmer than  $-7^{\circ}\text{C}$ .*

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## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDING, CONTINUED

#### Coordinate Plane

- The coordinate plane is a plane formed by the intersection of a horizontal number line with a vertical number line.
- The x-axis is the horizontal number line in a coordinate plane.
- The y-axis is the vertical number line in a coordinate plane.
- In an ordered pair  $(x, y)$ ,  $x$  represents the horizontal position and  $y$  represents the vertical position in the coordinate plane.
- The ordered pair gives a precise location in the coordinate plane.
- Quadrants are numbered counter-clockwise with the top-right quadrant being Quadrant I.
- Coordinate points can have fractional or decimal units.

### MATHEMATICAL THINKING

- Draw or use a picture, model, or graph to make sense of a problem.
- Apply mathematical vocabulary to describe real-world contexts.
- Represent a concept symbolically.
- Pay attention to and make sense of quantities.
- Communicate mathematical ideas.

### INSTRUCTIONAL FOCUS

#### Rational Numbers

- Find and position positive and negative rational numbers on a horizontal or vertical number line.
- Use positive and negative numbers to represent quantities on a number line and/or in real-world contexts.
- Explain the meaning of zero and opposite numbers in a real-world situation.
- Use number lines to compare and order positive and negative rational numbers in the same form and in different forms.
- Interpret and use the absolute value of quantities in real-world situations.

*Continued on next page*

- c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of  $-30$  dollars, write  $|-30| = 30$  to describe the size of the debt in dollars.*
- d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than  $-30$  dollars represents a debt greater than 30 dollars.*

**6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS, CONTINUED

#### Coordinate Plane

- Graph ordered pairs in all four quadrants.
- Name an ordered pair given a point on a graph.
- Represent real-world and mathematical problems by graphing points in all four quadrants.
- Determine the distance between two points with the same  $x$  or  $y$  coordinates by counting, using absolute value, or other strategies.
- Reflect an ordered pair across the  $x$ -axis, to discover that in the resulting image, the  $x$ -coordinate remains the same and the  $y$ -coordinate is its opposite, i.e.,  $(x, y)$  and  $(x, -y)$ .
- Reflect an ordered pair across the  $y$ -axis, to discover that in the resulting image, the  $y$ -coordinate remains the same the  $x$ -coordinate is its opposite, i.e.,  $(x, y)$  and  $(-x, y)$ .

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 2, pages 37-38](#)
- [Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5](#)
- [Ohio's K-8 Learning Progressions, Number and Operations--Fractions, pages 6-7](#)
- [Ohio's K-8 Learning Progressions, The Number System, pages 16-17](#)

### CONNECTIONS ACROSS STANDARDS

- Write and graph on a number line inequalities of the form  $x > c$  or  $x < c$  (6.EE.8).
- Draw a polygon in the coordinate plane and determine lengths of (horizontal and vertical) segments in the coordinate plane (6.G.3).

## INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

### Instructional Strategies

*This section is under revision and will be published in 2018.*

### Instructional Tools/Resources

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>EXPRESSIONS AND EQUATIONS</b>  <b>Apply and extend previous understandings of arithmetic to algebraic expressions.</b></p> <p><b>6.EE.1</b> Write and evaluate numerical expressions involving whole number exponents.</p> <p><b>6.EE.2</b> Write, read, and evaluate expressions in which letters stand for numbers.</p> <p><b>a.</b> Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract <math>y</math> from 5” as <math>5 - y</math>.</i></p> <p><b>b.</b> Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression <math>2(8 + 7)</math> as a product of two factors; view <math>(8 + 7)</math> as both a single entity and a sum of two terms.</i></p> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>In prior grades, students developed strategies for writing and interpreting numerical expressions. In Grade 6, students apply and extend their previous understanding of arithmetic operations and notations to use algebraic order of operations when writing, interpreting, and finding equivalent algebraic expressions. Students will extend their knowledge of powers of ten to whole number exponents with other bases. In Grade 6, students start to use properties of operations to manipulate algebraic expressions to produce different, but equivalent expressions for different purposes. This will build the foundation for solving equations and inequalities with rational numbers and operations with scientific notation in grades seven and eight.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Reading, Writing, and Evaluating Expressions</b></p> <ul style="list-style-type: none"> <li>• A variable can represent an unknown value or set of values.</li> <li>• A factor can be a single entity or sum/difference of terms, e.g., <math>2(3 + 5)</math> is two factors 2 and <math>(3 + 5)</math>.</li> <li>• A term is a number, variable, product, or quotient in an expression.</li> <li>• Terms are separated by addition and/or subtraction signs within an expression.</li> <li>• A term is either a single number or variable, or numbers and variables multiplied together.</li> <li>• If a term consists of only variables, its coefficient is 1.</li> <li>• A constant is an explicit number whose value does not change.</li> <li>• A coefficient is the numerical factor of a term with a variable.</li> <li>• An expression is a variable or combination of variables, numbers, and symbols that represent a mathematical calculation.</li> <li>• An expression does not contain an equal sign.</li> <li>• Exponents represent repeated multiplication of the base.</li> <li>• Multiplication can be represented by algebraic notation such as parentheses, a raised dot, or with a coefficient and variable, e.g., <math>3x</math>.</li> <li>• Division can be represented by a fraction bar.</li> <li>• Parentheses initiate an order when simplifying numerical expressions.</li> </ul> <p><i>Continued on next page</i></p>



c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = \frac{1}{2}$ .*

**6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression  $3(2 + x)$  to produce the equivalent expression  $6 + 3x$ ; apply the distributive property to the expression  $24x + 18y$  to produce the equivalent expression  $6(4x + 3y)$ ; apply properties of operations to  $y + y + y$  to produce the equivalent expression  $3y$ .*

**6.EE.4** Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. *For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.*

## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDING, CONTINUED

#### Equivalent Expressions

- Equivalent expressions always have the same value.
- Equivalent expressions can be generated using properties of operations (Distributive Property, Associative Property of Addition, Associative Property of Addition Multiplication, Commutative Property of Addition, Commutative Property of Multiplication, and Identity Property of Multiplication).

### MATHEMATICAL THINKING

- Solve multi-step problems accurately.
- Use precise mathematical vocabulary and symbols.
- Compute accurately and efficiently with grade-level numbers.
- Use different properties of operations flexibly.
- Use reasoning with symbolic representations.

### INSTRUCTIONAL FOCUS

*Note: Although, rote memorization of the names of the properties is not encouraged, it is expected for teachers to use formal language so that students gain familiarity and are able to recognize and apply the correct terminology.*

#### Reading, Writing, and Evaluating Expressions

- Identify parts of expressions using mathematical terms.
- Evaluate an algebraic expression by substituting a given value for the variable.
- Use algebraic order of operations, including whole number exponents to evaluate numerical expressions.
- Evaluate formulas based on real-world problems where the variable can be a whole number, fraction, or a decimal.
- Write expressions, including exponents, for mathematical and real-world situations.
- Define the variable when writing expressions in real-world situations.
- Translate expressions from word form to algebraic form.

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## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS, CONTINUED

#### Equivalent Expressions

- Identify when two expressions are equivalent.
- Substitute the same value into two or more expressions to determine their equality.
- Generate equivalent expressions using properties of operations.
- Differentiate between repeated addition and repeated multiplication of a single variable and the equivalent mathematical representation for each.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40](#)
- [Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

### CONNECTIONS ACROSS STANDARDS

- Find the greatest common factor and use the distributive property (6.NS.4).
- Use variables to represent numbers and write expressions (6.EE.6).
- Apply the formulas for volume (6.G.2).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>EXPRESSIONS AND EQUATIONS</b>  <b>Reason about and solve one-variable equations and inequalities.</b></p> <p><b>6.EE.5</b> Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p> <p><b>6.EE.6</b> Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p><b>6.EE.7</b> Solve real-world and mathematical problems by writing and solving equations of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math>, and <math>x</math> are all nonnegative rational numbers.</p> <p><b>6.EE.8</b> Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form <math>x &gt; c</math> or <math>x &lt; c</math> have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p>	<p><b>Expectations for Learning</b></p> <p>In prior grades, students have used the inequality signs to compare numbers. This cluster introduces students to the use of a variable as an unknown quantity or set of quantities in expressions, equations, and inequalities. Students utilize their understanding of opposites and division of rational numbers to solve one-step equations. In Grade 7, students will expand on techniques to solve multi-step equations and inequalities.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Equations</b></p> <ul style="list-style-type: none"> <li>• A variable can represent an unknown value or a set of values.</li> <li>• An algebraic equation is a mathematical statement that says that two expressions are equal.</li> <li>• The same operation must be performed on both sides of an equation to maintain equivalence.</li> <li>• Addition and subtraction are inverse operations.</li> <li>• Multiplication and division are inverse operations.</li> <li>• A solution is a value that makes an equation or an inequality true.</li> </ul> <p><b>Inequalities</b></p> <ul style="list-style-type: none"> <li>• Inequalities can have infinitely many solutions.</li> <li>• Solutions to inequalities can be represented on number line diagrams.</li> <li>• Point <math>c</math> is not included in the graphical solution to <math>x &gt; c</math> or <math>x &lt; c</math>; the number line diagram represents this with an open circle around point <math>c</math>.</li> <li>• All of the solutions to an inequality are represented with a shaded region (or an arrow) on a number line diagram.</li> <li>• The inequality <math>x &gt; c</math> is equivalent to <math>c &lt; x</math>.</li> </ul> <p><b>MATHEMATICAL THINKING</b></p> <ul style="list-style-type: none"> <li>• Compute using strategies or models.</li> <li>• Determine reasonableness of results.</li> <li>• Use reasoning to represent a concept symbolically.</li> <li>• Recognize and use a pattern or structure.</li> </ul>

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

- Define the variable in context when writing equations and inequalities.
- Use substitution to identify solution(s) from a given set.
- Determine whether a given value is a solution to an equation or inequality.
- Write equations and inequalities to represent a context.

### Equations

- Solve one-step equations of the form  $x + p = q$ ,  $x - p = q$ , or  $px = q$ , where  $p$ ,  $x$ , and  $q$  are all nonnegative rational numbers using models and algebraically.
- Solve one-step equations of the form  $\frac{x}{p} = q$  where  $x$  and  $q$  are all nonnegative rational numbers and  $p$  is a positive integer using models and algebraically.
- Solve real-world problems by writing one-step equations.

### Inequalities

- Explain the meaning of an inequality.
- Determine if a single value is required as a solution, or if the solution allows for multiple solutions.
- Write an inequality in the form of  $x > c$  or  $x < c$  to represent a constraint or condition in a real-world problem.
- Represent solutions of inequalities of the form  $x > c$  or  $x < c$  on a number line diagram.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40](#)
- [Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

### CONNECTIONS ACROSS STANDARDS

- Divide fractions by fractions (6.NS.1).
- Recognize opposites of numbers (6.NS.6a).
- Evaluate expressions (6.EE.2).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>EXPRESSIONS AND EQUATIONS</b>  <b>Represent and analyze quantitative relationships between dependent and independent variables.</b></p> <p><b>6.EE.9</b> Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time.</i></p>	<p><b>Expectations for Learning</b></p> <p>In prior grades variables are used as unknowns. In this cluster students will extend their understanding of variables to include the relationship between dependent and independent variables. Students use two variable equations to express relationships between two quantities that vary together. Students also understand that these relationships can be expressed as a table, graph, and/or equation. These initial understandings of the relationship between dependent and independent variables provide the introductory foundations for work with linear functions in 8<sup>th</sup> grade.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>• Expressions on both sides of the equal sign have the same value.</li> <li>• The value of the dependent variable is determined by the value of the independent variable.</li> <li>• The relationship between two quantities can be represented as a table, graph, and/or equation.</li> </ul> <p><b>MATHEMATICAL THINKING</b></p> <ul style="list-style-type: none"> <li>• Represent real-world problems mathematically.</li> <li>• Attend to precision in recording mathematical statements.</li> <li>• Use accurate mathematical vocabulary to describe mathematical reasoning.</li> <li>• Recognize and use a pattern or structure.</li> <li>• Use informal reasoning.</li> </ul> <p><b>INSTRUCTIONAL FOCUS</b></p> <ul style="list-style-type: none"> <li>• Define variables in context using appropriate units.</li> <li>• Select appropriate independent and dependent variables based on the context of the problem.</li> <li>• Analyze the relationship between the dependent and independent variables using graphs and tables.</li> <li>• Relate an equation to a table and/or graph.</li> <li>• Write a one-step equation representing the relationship between the independent and dependent variable from a table, graph, or real-world context.</li> </ul> <p><i>Continued on next page</i></p>

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 3, pages 39-40](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

### CONNECTIONS ACROSS STANDARDS

- Relate equivalent ratios to graphs and tables (6.RP.3a).
- Use variables to represent unknown quantities and write expressions (6.EE.2, 6.EE.6).
- Solve real-world and mathematical problems using a coordinate plane (6.NS.8).
- Solve real-world and mathematical problems by writing and solving equations (6.EE.7).



## INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

### Instructional Strategies

*This section is under revision and will be published in 2018.*

### Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
<p><b>GEOMETRY</b>  <b>Solve real-world and mathematical problems involving area, surface area, and volume.</b></p> <p><b>6.G.1</b> Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.</p> <p><b>6.G.2</b> Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas <math>V = \ell \cdot w \cdot h</math> and <math>V = B \cdot h</math> to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p> <p><b>6.G.3</b> Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>This cluster extends students' understanding of area of rectangles in previous grades to an understanding of area of other polygons. In Grade 5, students discovered the connection between filling a right rectangular prism with unit cubes and formulas for finding volume. These formulas include <math>V = \ell \cdot w \cdot h</math> and <math>V = B \cdot h</math>. In Grade 6, students will extend this understanding and apply it to fractional edge lengths both through discovery and using the formula. It is the first time that students explore methods for finding the area of triangles and special quadrilaterals. Using models and real-world contexts students will develop strategies and formulas to find areas of various polygons. They will find surface areas of three-dimensional figures composed of rectangular and/or triangular faces including right prisms, pyramids, and other solids using nets. Students will compute accurately and efficiently with grade-level numbers including whole numbers, fractions, and decimals to solve surface area and volume problems. This will be the foundation for future work in geometric measurement for three-dimensional figures.</p> <p>The student understanding of this cluster aligns with a van Hiele Level 1 (Analysis).</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Area</b></p> <ul style="list-style-type: none"> <li>• Any side of a triangle can be a base.</li> <li>• The height of a polygon is a perpendicular line segment drawn from a vertex to the opposite side (base) or its extension.</li> <li>• The area of a triangle is half the area of a parallelogram with the same base and height.</li> <li>• Any polygon can be composed or decomposed into known figures to determine area.</li> <li>• The total area of a two-dimensional composite shape is the sum of the areas of all its parts.</li> </ul> <p><i>Continued on next page</i></p>

**6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDING, CONTINUED

#### Surface Area

- The surface area of a three-dimensional figure is made up of the sums of the areas of its faces.
- A net is a composite two-dimensional shape of a three-dimensional object used to find the surface area.
- Surface area of a three-dimensional figure includes faces that are visible and not visible from a given viewpoint.

#### Coordinates

- The area and side lengths of polygons can be found by plotting coordinates in a coordinate plane.
- If both  $x$ -coordinates of a line segment are the same, a vertical line segment is formed and the length can be determined.
- If both  $y$ -coordinates of a line segment are the same, a horizontal line segment is formed and the length can be determined.

#### Volume

- Rectangular prisms may have edge lengths that are fractions.
- Volume of a rectangular prism can be determined using the formulas and/or by packing it with unit cubes of the appropriate unit fraction edge lengths.

### MATHEMATICAL THINKING

- Draw a picture or create a model to represent mathematical thinking.
- Compute accurately and efficiently with grade-level numbers.
- Consider mathematical units involved in a problem.
- Use and analyze structure.

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**Expectations for Learning, continued****INSTRUCTIONAL FOCUS****Area**

- Draw a picture or create a model to compose and decompose polygons to find the area of triangles, special quadrilaterals, and polygons.
- Explore, develop, and apply area formulas for parallelograms and triangles.
- Apply techniques for finding area in the context of solving real-world and mathematical problems.
- Use appropriate units to label area problems.

**Surface Area**

- Draw nets made up of rectangles and triangles to represent three-dimensional figures, e.g., cubes, rectangular prisms, triangular prisms, and pyramids, where a face or faces are not visible.
- Use nets to solve real-world and mathematical surface area problems.
- Use appropriate units to label surface area problems.

**Coordinate Plane**

- Draw polygons in the coordinate plane given coordinates for the vertices.
- Find the length of a side of a polygon formed by points with the same first or second coordinate.
- Find the area and perimeter of geometric figures on a coordinate plane.

**Volume**

- Find the volume of a right rectangular prism with fractional edge lengths.
- Apply formulas to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- Use appropriate units to label volume problems.

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### Content Elaborations

- [Ohio's K-8 Critical Area of Focus, Grade 6, Number 5, page 42](#)
- [Ohio's K-8 Learning Progressions, K-5 Geometry, page 11](#)
- [Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14](#)
- [Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21](#)

### CONNECTIONS ACROSS STANDARDS

- Use horizontal and vertical distance to find lengths of sides of polygons in the coordinate plane (6.NS.8).

## INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

### Instructional Strategies

*This section is under revision and will be published in 2018.*

### Instructional Tools/Resources

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>STATISTICS AND PROBABILITY</b>  <b>Develop understanding of statistical problem solving.</b></p> <p><b>6.SP.1</b> Develop statistical reasoning by using the GAISE model:</p> <ol style="list-style-type: none"> <li>Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. <i>For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because of the variability in students’ ages.</i> (GAISE Model, step 1)</li> <li>Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)</li> <li>Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)</li> <li>Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)</li> </ol> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>Students will be introduced to and develop a conceptual understanding of the four steps of the GAISE model for statistical problem solving, which will be used throughout high school. The focus of these standards is to recognize and understand the process of the GAISE model, with focus on steps 1 and 2, Level A. In Level A, teachers pose questions and students distinguish between questions that would have a statistical answer with variability and a fixed answer. With the aid of a teacher, students conduct a census or simple experiment. They begin to understand individual to individual variability and to describe the idea of distribution. Students are able to make inferences for their own classroom, but acknowledge the differences or limitations when making generalizations for a larger group.</p> <p>The application of steps 3 and 4 will be addressed in 6.SP.4-5. Students begin to think and reason statistically by first recognizing and formulating a statistical question as one that can be answered by collecting data. They learn that the data collected to answer a statistical question have a distribution that is often summarized in terms of center, variability, and shape.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>Statistics is the name for the science of collecting, analyzing, and interpreting data.</li> <li>The GAISE model (outlined in 6.SP.1) is used to analyze and interpret data and has 4 steps: Formulate the Question; Collect Data to Answer the Question; Analyze the Data; and Interpret Results.</li> <li>Data are not just numbers; they are numbers generated with respect to a particular context and situation.</li> <li>There are two types of data: categorical and numerical.</li> <li>Categorical data are sorted into groups and categories.</li> <li>Numerical data are measurable.</li> <li>A statistical question anticipates a response that varies, from one individual to the next, and this variability is described in terms of spread and overall shape.</li> <li>A distribution shows all values of data and how often they occur.</li> <li>A set of data has a distribution which can be described by its center, spread, and overall shape.</li> <li>The measure of variation describes how data values vary with a single number.</li> </ul>

- 6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- 6.SP.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

## Expectations for Learning, continued

### MATHEMATICAL THINKING

- Make sense of statistical problems.
- Formally explain mathematical reasoning.
- Use of formal, precise mathematical language.
- Pay attention to and make sense of quantities.

### INSTRUCTIONAL FOCUS

#### Introduce the GAISE model

##### **Step 1 – Formulate the Question:**

- Recognize that a statistical question has variability.
- Distinguish between a statistical answer with variability and a fixed answer.

##### **Step 2 - Collect Data to Answer the Question:**

- Begin to design a collection method to answer a statistical question.
- Collect appropriate data from the following:
  - classroom census (survey) or
  - simple experiments.

##### **Step 3 - Analyze the Data:**

- Explain individual to individual variability; use a single classroom.
- Show that a distribution consists of an organized set of data with how often each data point occurs (frequency).
- Describe the center of a distribution by using mean, median, and/or mode.
- Describe the spread (variability) of a distribution by using range and/or interquartile range.
- Describe the overall shape of a distribution as being symmetric, skewed, or uniform. Identify features such as clusters, gaps, and outliers.
- Describe variation as how its data values vary with a single number.

##### **Step 4 - Interpret Results:**

- Draw conclusions from the analysis of the data collected.



### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 4, page 41](#)
- [Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23](#)
- [GAISE Model, pages 14 – 15](#)
  - [Focus of 6<sup>th</sup> grade is Level A, pages 23-35](#)

### CONNECTIONS ACROSS STANDARDS

- Add, subtract, multiply, and divide multi-digit numbers (6.NS.2 – 3).
- Connect to scientific method in Science standards.

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>STATISTICS AND PROBABILITY</b>  <b>Summarize and describe distributions.</b>  <b>6.SP.4</b> Display numerical data in plots on a number line, including dot plots<sup>G</sup> (line plots), histograms, and box plots<sup>G</sup>. (GAISE Model, step 3)  <b>6.SP.5</b> Summarize numerical data sets in relation to their context.</p> <ol style="list-style-type: none"> <li>Report the number of observations.</li> <li>Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.</li> <li>Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range<sup>G</sup>) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution.</li> <li>Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.</li> </ol>	<p><b>Expectations for Learning</b>  For the first time, students will represent, analyze, and interpret data by creating and using dot plots, histograms, and box plots. Students extend their statistical knowledge using the GAISE Model to understand, represent, and discuss distribution, shape, center, and spread. These experiences help students to begin to develop an informal understanding of variability (spread). Understanding variability is essential for developing data sense.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>Distribution shows all values of data and how often they occur.</li> <li>Data can be represented in different ways to persuade people.</li> <li>Statistics change numbers into information.</li> <li>Dot plots are simple plots on a number line where each dot represents a piece of data in the data set.</li> <li>A histogram summarizes numerical data using intervals with frequencies.</li> <li>Boxplots display data in four equal groups (25% each) and are plotted horizontally or vertically on a number line.</li> <li>Quartiles are values that divide the data into four equal parts (quarters). The first quartile is the value at the 25<sup>th</sup> percentile. The third quartile is the value at the 75<sup>th</sup> percentile. The median of the set, the second quartile, is the value at the 50<sup>th</sup> percentile.</li> <li>Outliers are numbers that are really large or really small compared to the variation of most of the data.</li> <li>The mean, median, and mode are measures of location for describing the center of a numerical data set.</li> <li>Range is the measure of the total spread of the data, and interquartile range is the measure of spread between the lower quartile (Q1) and upper quartile (Q3).</li> </ul> <p><b>MATHEMATICAL THINKING</b></p> <ul style="list-style-type: none"> <li>Make sense of problems.</li> <li>Analyze and interpret graphs.</li> <li>Attend to accuracy in graphical displays</li> <li>Use precise mathematical language and vocabulary.</li> </ul>

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

- Create different visual models to represent a set of data.
- Find measures of variability (range, interquartile range) from graphical displays.
- Find measures of center (median, mean, and mode).
  - Interpret mean as equal (fair) share.
- Describe the shape of distributions: clusters, gaps, peaks, and/or outliers.
- Summarize the numerical data sets in relation to the context.
- Construct numerical one variable (univariate) visual models within the context:
  - dot plots/line plots;
  - histograms; and
  - box plots (box & whisker plots).
- Draw conclusions from the analysis of the data based on original question (GAISE Model, Step 4).

### **GAISE Model (Step 3) – Analyze the Data**

- Compare individual to individual.
- Compare individual to group.
  - Recognize variability (spread) within a group given a graphical display.
- Use specific properties (center, spread, shape) of distributions in context.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 6, Number 4, page 41](#)
- [Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23](#)
- [GAISE Model pages 14 – 15](#)
  - [Focus of 6<sup>th</sup> grade is Level A, pages 23-35](#)

### CONNECTIONS ACROSS STANDARDS

- Fluently add, subtract, multiply, and divide multi-digit decimals (6.NS.2-3).
- Understand the framework of GAISE Model (6.SP.1-3).

## INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

### Instructional Strategies

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### Instructional Tools/Resources

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