



# Ohio

## Ohio's Model Curriculum | Mathematics with Instructional Supports

### Grade 8

# Mathematics Model Curriculum

## with Instructional Supports

### Grade 8

#### TABLE OF CONTENTS

INTRODUCTION	5
STANDARDS FOR MATHEMATICAL PRACTICE	6
<b>THE NUMBER SYSTEM (8.NS)</b>	<b>8</b>
<b>KNOW THAT THERE ARE NUMBERS THAT ARE NOT RATIONAL, AND APPROXIMATE THEM BY RATIONAL NUMBERS. (8.NS.1-2)</b>	<b>8</b>
EXPECTATIONS FOR LEARNING	8
CONTENT ELABORATIONS	10
INSTRUCTIONAL STRATEGIES	11
INSTRUCTIONAL TOOLS/RESOURCES	11
<b>EXPRESSIONS AND EQUATIONS (8.EE)</b>	<b>12</b>
<b>WORK WITH RADICALS AND INTEGER EXPONENTS. (8.EE.1-4)</b>	<b>12</b>
EXPECTATIONS FOR LEARNING	12
CONTENT ELABORATIONS	14
INSTRUCTIONAL STRATEGIES	15
INSTRUCTIONAL TOOLS/RESOURCES	15
<b>UNDERSTAND THE CONNECTIONS BETWEEN PROPORTIONAL RELATIONSHIPS, LINES, AND LINEAR EQUATIONS. (8.EE.5-6)</b>	<b>16</b>
EXPECTATIONS FOR LEARNING	16
CONTENT ELABORATIONS	17
INSTRUCTIONAL STRATEGIES	18
INSTRUCTIONAL TOOLS/RESOURCES	18

<b>EXPRESSIONS AND EQUATIONS, CONTINUED (8.EE)</b>	<b>19</b>
<b>ANALYZE AND SOLVE LINEAR EQUATIONS AND PAIRS OF SIMULTANEOUS LINEAR EQUATIONS. (8.EE.7-8)</b>	<b>19</b>
EXPECTATIONS FOR LEARNING	19
CONTENT ELABORATIONS	21
INSTRUCTIONAL STRATEGIES	22
INSTRUCTIONAL TOOLS/RESOURCES	22
<b>FUNCTIONS (8.F)</b>	<b>23</b>
<b>DEFINE, EVALUATE, AND COMPARE FUNCTIONS. (8.F.1-3)</b>	<b>23</b>
EXPECTATIONS FOR LEARNING	23
CONTENT ELABORATIONS	24
INSTRUCTIONAL STRATEGIES	25
INSTRUCTIONAL TOOLS/RESOURCES	25
<b>USE FUNCTIONS TO MODEL RELATIONSHIPS BETWEEN QUANTITIES. (8.F.4-5)</b>	<b>26</b>
EXPECTATIONS FOR LEARNING	26
CONTENT ELABORATIONS	28
INSTRUCTIONAL STRATEGIES	29
INSTRUCTIONAL TOOLS/RESOURCES	29
<b>GEOMETRY (8.G)</b>	<b>30</b>
<b>UNDERSTAND CONGRUENCE AND SIMILARITY USING PHYSICAL MODELS, TRANSPARENCIES, OR GEOMETRY SOFTWARE. (8.G.1-5)</b>	<b>30</b>
EXPECTATIONS FOR LEARNING	30
CONTENT ELABORATIONS	32
INSTRUCTIONAL STRATEGIES	33
INSTRUCTIONAL TOOLS/RESOURCES	33
<b>UNDERSTAND AND APPLY THE PYTHAGOREAN THEOREM. (8.G.6-8)</b>	<b>34</b>
EXPECTATIONS FOR LEARNING	34
CONTENT ELABORATIONS	36
INSTRUCTIONAL STRATEGIES	37
INSTRUCTIONAL TOOLS/RESOURCES	37

<b>GEOMETRY, CONTINUED (8.G)</b>	<b>38</b>
<hr/>	
<b>SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS INVOLVING VOLUME OF CYLINDERS, CONES, AND SPHERES. (8.G.9)</b>	<b>38</b>
EXPECTATIONS FOR LEARNING	38
CONTENT ELABORATIONS	39
INSTRUCTIONAL STRATEGIES	40
INSTRUCTIONAL TOOLS/RESOURCES	40
<b>STATISTICS AND PROBABILITY (8.SP)</b>	<b>41</b>
<hr/>	
<b>INVESTIGATE PATTERNS OF ASSOCIATION IN BIVARIATE DATA. (8.SP.1-4)</b>	<b>41</b>
EXPECTATIONS FOR LEARNING	41
CONTENT ELABORATIONS	43
INSTRUCTIONAL STRATEGIES	44
INSTRUCTIONAL TOOLS/RESOURCES	44
<b>ACKNOWLEDGMENTS</b>	<b>45</b>

## Introduction

### PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

### COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

**Expectations for Learning:** This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

**Content Elaborations:** This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

### COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

## Standards for Mathematical Practice—Grade 8

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

### **MP.1 Make sense of problems and persevere in solving them.**

In Grade 8, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”

### **MP.2 Reason abstractly and quantitatively.**

In Grade 8, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number(s) or variable(s) as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

### **MP.3 Construct viable arguments and critique the reasoning of others.**

In Grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.

### **MP.4 Model with mathematics.**

In Grade 8, students model problem situations symbolically, graphically, in tables, and contextually. Working with the new concept of a function, students learn that relationships between variable quantities in the real-world often satisfy a dependent relationship, in that one quantity determines the value of another. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context. Students should be encouraged to answer questions such as “What are some ways to represent the quantities?” or “How might it help to create a table, chart, graph, or \_\_\_?”

*Continued on next page*

## Standards for Math Practice, continued

### **MP.5 Use appropriate tools strategically.**

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in Grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the between the angles created by a transversal that intersects parallel lines. Teachers might ask, “What approach are you considering?” or “Why was it helpful to use \_\_\_\_?”

### **MP.6 Attend to precision.**

In Grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Teachers might ask, “What mathematical language, definitions, or properties can you use to explain \_\_\_\_?”

### **MP.7 Look for and make use of structure.**

Students routinely seek patterns or structures to model and solve problems. In Grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

### **MP.8 Look for and express regularity in repeated reasoning.**

In Grade 8, students use repeated reasoning to understand the slope formula and to make sense of rational and irrational numbers. Through multiple opportunities to model linear relationships, they notice that the slope of the graph of the linear relationship and the rate of change of the associated function are the same. For example, as students repeatedly check whether points are on the line with a slope of 3 that goes through the point (1, 2), they might abstract the equation of the line in the form  $\frac{y-2}{x-1} = 3$ . Students should be encouraged to answer questions such as “How would we prove that \_\_\_\_?” or “How is this situation like and different from other situations using these operations?”

# Mathematics Model Curriculum

## with Instructional Supports

### Grade 8

STANDARDS	MODEL CURRICULUM
<p><b>THE NUMBER SYSTEM</b>  <b>Know that there are numbers that are not rational, and approximate them by rational numbers.</b></p> <p><b>8.NS.1</b> Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.</p> <p><b>8.NS.2</b> Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., <math>\pi^2</math>. <i>For example, by truncating the decimal expansion of <math>\sqrt{2}</math>, show that <math>\sqrt{2}</math> is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.</i></p>	<p><b>Expectations for Learning</b></p> <p>In previous grades, students learned to use long division to convert fractions to decimals. The resulting decimals either terminated or repeated. Until this point, their exploration of the real number system was limited to rational numbers. In eighth grade, students will extend their knowledge of real numbers to include irrational numbers whose decimal representation is non-repeating and non-terminating. In future grades, this leads to the understanding that there are numbers that are not real such as imaginary numbers and computations with radicals.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Real Numbers</b></p> <ul style="list-style-type: none"> <li>• Every real number can be classified as repeating, terminating, or non-repeating, non-terminating.</li> <li>• Real numbers are either rational or irrational.</li> <li>• A rational number is any number that can be written as the quotient or fraction of two integers, <math>\frac{p}{q}</math>, where <math>p</math> is the numerator and <math>q</math> is the non-zero denominator.</li> <li>• Rational numbers when written as a decimal expansion are repeating or terminating.</li> <li>• Irrational numbers when written as a decimal expansion are non-repeating and non-terminating.</li> <li>• A number is classified by its simplest form, e.g., <math>\sqrt{25}</math> is rational because 5 is rational.</li> </ul> <p><i>Continued on next page</i></p>



**Expectations for Learning, continued****ESSENTIAL UNDERSTANDINGS, CONTINUED****Real Numbers, continued**

- Square roots may be negative, e.g., Both 6 and  $-6$  are square roots of 36, and  $\sqrt{36}$  means only the positive square root whereas  $-\sqrt{36}$  means only the negative square root.
- A negative sign cannot be inside a square root number at this grade level.
- The roots of perfect squares and perfect cubes of whole numbers are rational.
- Square roots of whole numbers that are not perfect squares are irrational.
- Cube roots of whole numbers that are not perfect cubes are irrational.
- Given two distinct numbers, it is possible to find both a rational and an irrational number between them.

**MATHEMATICAL THINKING**

- Use precise mathematical language.
- Determine reasonableness of results.
- Generalize concepts based on patterns.
- Compute accurately and efficiently.

**INSTRUCTIONAL FOCUS**

- Identify if a real number is rational or irrational.
- Convert rational numbers from fractions to their decimal expansions.
- Discover that a rational number in simplest fractional form has a terminating decimal only if the prime factorization of its denominator contains only 2s and 5s.
- Approximate the value of irrational numbers.
- Explain how to get more precise approximations of irrational numbers.
- Approximate the location of irrational numbers on a number line.
- Compare and order rational and irrational numbers.

*Continued on next page*

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 4, page 55](#)
- [Ohio's K-8 Learning Progressions, The Number System, pages 16-17](#)

### CONNECTIONS ACROSS STANDARDS

- Know that  $\sqrt{2}$  is irrational (8.EE.2).
- Side lengths of right triangles can be irrational (8.G.6-8).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>EXPRESSIONS AND EQUATIONS</b>  <b>Work with radicals and integer exponents.</b></p> <p><b>8.EE.1</b> Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, <math>3^2 \times 3^5 = 3^3 = 1/3^3 = 1/27</math>.</i></p> <p><b>8.EE.2</b> Use square root and cube root symbols to represent solutions to equations of the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that <math>\sqrt{2}</math> is irrational.</p> <p><b>8.EE.3</b> Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. <i>For example, estimate the population of the United States as <math>3 \times 10^8</math>; and the population of the world as <math>7 \times 10^9</math>; and determine that the world population is more than 20 times larger.</i></p> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>In previous grades students were exposed to writing expressions with whole number exponents. In this cluster students should use patterns to extend their knowledge of exponents to include zero and negative powers. They will develop an initial understanding of the properties of exponents in numerical operations. Students are not expected to know or memorize the algorithmic rules for exponents but rather derive the rules using patterns. In high school, students will be using the properties of exponents more formally with algebraic expressions. In grade eight, students will also develop an understanding of square and cube roots and their symbols to solve equations.</p> <p>In previous grades students were exposed to understandings of powers of ten. In this cluster, students will write very small and very large numbers in scientific notation. They will use this knowledge to perform calculations and comparisons using scientific notation.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Exponents</b></p> <p><i>Note: In 8<sup>th</sup> Grade all exponents are integers, so all notes refer to cases using integer exponents.</i></p> <ul style="list-style-type: none"> <li>• <math>x^0 = 1</math>, when <math>x \neq 0</math>.</li> <li>• <math>x^n = \frac{1}{x^n}</math>, when <math>x \neq 0</math>.</li> <li>• A coefficient is different than an exponent. For example, <math>n^2</math> is different than <math>2n</math> because <math>n^2</math> means <math>n \times n</math> and <math>2n</math> means <math>2 \times n</math>, and <math>n^3</math> is different than <math>3n</math> because <math>n^3</math> means <math>n \times n \times n</math> and <math>3n</math> means <math>3 \times n</math>.</li> <li>• A positive exponent denotes repeated multiplication of the base.</li> <li>• A negative exponent denotes repeated division of the base.</li> <li>• A negative exponent does not change the sign of the base.</li> </ul> <p><i>Continued on next page</i></p>

**8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.

## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDINGS, CONTINUED

#### Roots

- The equation  $x^2 = p$  has two solutions for  $x$ :  $\sqrt{p}$  and  $-\sqrt{p}$ . For example in describing the solutions to  $x^2 = 36$ , students can write  $x = \pm\sqrt{36} = \pm 6$ .
- The  $\sqrt{p}$  is defined to be the positive solution to the equation  $x^2 = p$ . For example it is **not** correct to say that  $\sqrt{36} = \pm 6$  instead it should be  $\sqrt{36} = 6$ .
- $x^3 = p$  has only one solution for  $x$ :  $\sqrt[3]{p}$ .
- The square root of 2 is irrational.

#### Scientific Notation

- Scientific notation is a mathematical expression written as a decimal number greater than or equal to one but less than 10 multiplied by a power of ten, e.g.  $3.1 \times 10^4$ .
- A number expressed in scientific notation that has a negative exponent is between negative one and positive one.
- A number expressed in scientific notation that has a positive exponent is greater than one or less than negative one.
- Powers of ten can be used to compare numbers written in scientific notation.

### MATHEMATICAL THINKING

- Compute accurately and efficiently with grade-level numbers.
- Pay attention to and make sense of quantities.
- Recognize and use a pattern or structure.
- Use technology to deepen understanding.
- Represent a concept symbolically.

*Continued on next page*

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

#### Exponents

- Use patterns and structure to develop an understanding of the properties of exponents.
- Create equivalent numerical expressions using the properties of integer exponents including negative and zero exponents.

#### Roots

- Solve  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Show solutions with square and cube root symbols.
- Evaluate square roots of perfect squares and cube roots of perfect cubes.

#### Scientific Notation

- Identify numbers in scientific notation.
- Convert numbers from scientific notation to decimal notation or from decimal notation to scientific notation.
- Compare numbers in scientific notation.
- Perform operations on numbers where both scientific notation and decimal notation are used.
- Interpret how different technology devices show scientific notation.
- Choose appropriately sized units when dealing with very large or very small quantities.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 4, page 55](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

### CONNECTIONS ACROSS STANDARDS

- Define and approximate irrational numbers (8.NS.1).
- Estimate irrational numbers (8.NS.2).
- Apply the Pythagorean Theorem (8.G.6-8).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>EXPRESSIONS AND EQUATIONS</b>  <b>Understand the connections between proportional relationships, lines, and linear equations.</b></p> <p><b>8.EE.5</b> Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i></p> <p><b>8.EE.6</b> Use similar triangles to explain why the slope <math>m</math> is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation <math>y = mx</math> for a line through the origin and the equation <math>y = mx + b</math> for a line intercepting the vertical axis at <math>b</math>.</p>	<p><b>Expectations for Learning</b></p> <p>In previous grades students were exposed to unit rates and the constant of proportionality. In this cluster students will extend their vocabulary to include rate of change, rise over run, and slope. Students will understand that these are equivalent to the unit rate and constant of proportionality and is represented by the variable <math>m</math>. Students will use their knowledge of similar triangles to derive the slope of a line. Students will be introduced to the concept of the <math>y</math>-intercept. They will construct equations in the form of <math>y = mx</math> or <math>y = mx + b</math> to describe the relationship between two variables. These linear relationships provide the initial foundation for an understanding of functions.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>• The slope is a constant ratio between the rise and the run for any two points on a line.</li> <li>• A graph of a proportional relationship is a line that passes through the origin.</li> <li>• Only the slope, <math>m</math>, of the equation <math>y = mx</math> represents a proportional relationship.</li> <li>• Slope is represented by <math>m</math> in the equation <math>y = mx</math> or <math>y = mx + b</math>.</li> <li>• Corresponding angles in similar right triangles are equal.</li> <li>• Corresponding sides of similar triangles are proportional.</li> <li>• A line in the form <math>y = mx</math> and intersects the origin.</li> <li>• A line in the form <math>y = mx + b</math> intersects the <math>y</math>-axis at <math>(0, b)</math> with <math>b</math> being the <math>y</math>-intercept. <i>Note: A linear function has neither a slope nor a <math>y</math>-intercept. But the graph of a linear function has both.</i></li> <li>• A relationship between two variables can be represented as a graph, table, equation, or verbal description.</li> </ul> <p><b>MATHEMATICAL THINKING</b></p> <ul style="list-style-type: none"> <li>• Use accurate mathematical vocabulary to describe mathematical reasoning.</li> <li>• Represent a concept symbolically.</li> <li>• Solve real-world problems accurately.</li> <li>• Compute using strategies or models.</li> <li>• Make connections between concepts, terms, and properties within the grade level and with previous grade levels.</li> </ul>



## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

- Graph proportional relationships, identifying the constant of proportionality (unit rate) as the slope of the line.
- Compare different representations of a proportional relationship (graph, table, equation, verbal description).
- Use similar right triangles to derive the slope of the line.
- Determine the relationship between the two variables by writing an equation in the form  $y = mx$  or  $y = mx + b$ .
- Compare the equations  $y = mx$  and  $y = mx + b$ .

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, page 50-51](#)
- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

### CONNECTIONS ACROSS STANDARDS

- Describe the effect of dilations (8.G.3).
- Use informal arguments to establish facts about the angle sum and exterior angles of triangles and angles created when parallel lines are cut by a transversal (8.G.5).
- Compare properties of two functions each represented in a different way (8.F.2).
- Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line (8.F.3).
- Construct a function to model a linear relationship between two quantities (8.F.4)
- Determine the rate of change and initial value from a table or graph (8.F.4).
- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting slope and  $y$ -intercept (8.SP.4).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>EXPRESSIONS AND EQUATIONS</b>  <b>Analyze and solve linear equations and pairs of simultaneous linear equations.</b></p> <p><b>8.EE.7</b> Solve linear equations in one variable.</p> <p>a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</p> <p>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p> <p><b>8.EE.8</b> Analyze and solve pairs of simultaneous linear equations graphically.</p> <p>a. Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously.</p> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>In prior grades, students learn to solve and graph linear equations algebraically and through real-world examples. In this cluster, students will continue to solve and graph linear equations that include rational coefficients, variables on both sides, and distributive property. Students will solve pairs of linear equations through graphing and simple inspection. They will determine if a pair of linear equations in two variables has no solutions, one solution, or infinitely many solutions. This cluster sets the foundation to solving systems of equations in high school Algebra.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Linear Equations</b></p> <ul style="list-style-type: none"> <li>Linear equations can have no solutions, one solution, or infinitely many solutions.</li> <li>Linear equations are solved by using inverse operations.</li> <li>Linear equations that are equivalent to <math>x = a</math> have one solution.</li> <li>Linear equations that are equivalent to <math>a = a</math> have infinitely many solutions.</li> <li>Linear equations that are equivalent to <math>a = b</math> have no solutions.</li> </ul> <p><b>Pairs of Linear Equations</b></p> <ul style="list-style-type: none"> <li>Pairs of linear equations can have no solutions, one solution, or infinitely many solutions.</li> <li>Pairs of linear equations in two variables that intersect at one point have one solution.</li> <li>Pairs of linear equations in two variables that are parallel have no solutions.</li> <li>Pairs of linear equations in two variables that have all points in common have infinitely many solutions.</li> <li>The solution(s) to a pair of linear equations in two variables make both equations true.</li> <li>A solution to a pair of linear equations in two variables is often written as an ordered pair.</li> </ul> <p><i>Continued on next page</i></p>

- b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example,  $3x + 2y = 5$  and  $3x + 2y = 6$  have no solution because  $3x + 2y$  cannot simultaneously be 5 and 6.*
- c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (Limit solutions to those that can be addressed by graphing.)

## Expectations for Learning, continued

### MATHEMATICAL THINKING

- Create a model to make sense of a problem.
- Determine reasonableness of results.
- Apply grade-level concepts, terms, and properties.
- Recognize and use a pattern or structure.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.

### INSTRUCTIONAL FOCUS

#### Linear Equations

- Solve multi-step equations with rational coefficients that may include variables on both sides, distributive property, and collecting like terms.
- Determine whether a linear equation has no solutions, one solution, or infinitely many solutions.
- Graph the solution to a linear equation on a number line.

#### Pairs of Linear Equations

- Determine whether a pair of linear equations in two variables has no solutions, one solution, or infinitely many solutions.
- Find or estimate which points are solutions to a pair of linear equations in two variables.
- Solve pairs of linear equations in two variables graphically and by simple inspection.
- Solve real-world problems using pairs of linear equations in two variables that can be addressed graphically.
- Utilize different scales when graphing pairs of linear equations in two variables.

*Continued on next page*

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, pages 50-51](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

### CONNECTIONS ACROSS STANDARDS

- Use the equation of a linear model to solve problems in the context of bivariate measurement data (8.SP.3).
- Graph proportional relationships, and compare proportional relationships represented in different ways (8.EE.5).
- Use similar triangles to explain slope (8.EE.6).
- Understand that a function is a rule that assigns to each input exactly one output (8.F.1).
- Compare the properties of two functions (8.F.2).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>FUNCTIONS</b>  <b>Define, evaluate, and compare functions.</b></p> <p><b>8.F.1</b> Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.</p> <p><b>8.F.2</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</i></p> <p><b>8.F.3</b> Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. <i>For example, the function <math>A = s^2</math> giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.</i></p>	<p><b>Expectations for Learning</b></p> <p>In prior grades students have interpreted and analyzed proportions using graphs, tables, and equations. The focus for this cluster is linear equations, those in the form <math>y = mx + b</math>. Students will relate their understanding of the constant of proportionality to slope/rate of change. Students should be able to distinguish between linear and non-linear functions in different representations such as equations, graphs, and tables. These initial understandings provide the foundations for concepts that will be developed in high school math such as domain, range, function notation, as well as non-linear functions such as quadratic and exponential functions.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>• A function is a rule that assigns each input exactly one output.</li> <li>• The graph of a function is a set of ordered pairs consisting of an input and a corresponding output.</li> <li>• Functions can be represented as an equation, graph, table, and verbal description.</li> <li>• Properties of graphs of linear functions include slope/rate of change, y-intercept/initial value, x-intercept, where the slope is increasing, constant, or decreasing.</li> <li>• A vertical line has an undefined slope, where <math>y</math> is not a function of <math>x</math>.</li> <li>• A graph of a linear function is a non-vertical straight line.</li> <li>• A non-linear function is a function whose graph is not a straight line.</li> <li>• A table represents a linear function when constant differences between input values produce constant difference between output values.</li> <li>• Linear functions have a constant rate of change.</li> <li>• Some functions are not continuous.</li> </ul> <p><b>MATHEMATICAL THINKING</b></p> <ul style="list-style-type: none"> <li>• Generalize concepts based on patterns.</li> <li>• Recognize and use a pattern or structure.</li> <li>• Use precise mathematical language.</li> <li>• Represent a concept symbolically.</li> </ul> <p><i>Continued on next page</i></p>

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

- Determine if a table, graph, equation, or a verbal description represents a linear or nonlinear function.
- Identify a set of input and output values for a function.
- Compare properties of two functions in different representations.
- Identify functions that are linear and nonlinear.
- Give examples of functions that are nonlinear.
- Interpret the slope/rate of change and y-intercept/initial value of a linear function.
- Complete tables to show a relationship that is a function.
- Determine if it is reasonable to “connect the points” on a graph based on the context.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 2, page 52](#)
- [Ohio's K-8 Learning Progressions, Functions, page 20](#)

### CONNECTIONS ACROSS STANDARDS

- Graph proportional relationships (8.EE.5).
- Derive the equation  $y = mx + b$  (8.EE.6).
- Sketch a graph from a verbal description (8.F.5).
- Use an equation of linear model to solve problems (8.SP.3).



**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>FUNCTIONS</b>  <b>Use functions to model relationships between quantities.</b></p> <p><b>8.F.4</b> Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two <math>(x, y)</math> values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p><b>8.F.5</b> Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>	<p><b>Expectations for Learning</b></p> <p>Students have experienced interpreting and translating linear functions in different representations. The focus for this cluster is modeling, constructing, and analyzing linear relationships and qualitative graphs. Students should be able to describe properties of linear relationships. These initial understandings provide the foundations for key features that are developed in high school math such as domain, range, function notation, and other types of functions.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Rate of Change</b></p> <ul style="list-style-type: none"> <li>• The rate of change of a function is the slope of its graph.</li> <li>• The slope is a ratio of the change in <math>y</math>-values compared to the change in <math>x</math>-values between two points.</li> <li>• A vertical line has an undefined slope, where <math>y</math> is not a function of <math>x</math>.</li> <li>• The graph of a horizontal line has a slope of zero, i.e., <math>m = 0</math>, which is a constant rate of change.</li> <li>• The graph of a line is increasing if it rises from left to right. The slope is positive, i.e., <math>m &gt; 0</math>.</li> <li>• The graph of a line is decreasing if it falls from left to right. The slope is negative, i.e., <math>m &lt; 0</math>.</li> <li>• The absolute value of slope (<math>m</math>) is related to the steepness of a graph of a line.</li> </ul> <p><b>Describing and Graphing Functions</b></p> <ul style="list-style-type: none"> <li>• Properties of graphs of linear functions include the following: slope/rate of change; <math>y</math>-intercept/initial value, where the slope is increasing, constant, and decreasing.</li> <li>• Qualitative graphs represent essential elements of a situation in graph form.</li> <li>• Qualitative graphs do not necessarily need to have numerical values.</li> <li>• Linear functions can show proportional relationships <math>y = mx</math> or non-proportional relationships <math>y = mx + b</math> where <math>b \neq 0</math>.</li> <li>• The scale of an axis must have equal intervals.</li> <li>• The <math>x</math>-axis and <math>y</math>-axis do not have to have the same scale.</li> </ul> <p><i>Continued on next page</i></p>

## Expectations for Learning, continued

### MATHEMATICAL THINKING

- Generalize concepts based on patterns.
- Recognize and use a pattern or structure
- Use precise mathematical language.
- Represent a concept symbolically.
- Explain mathematical reasoning.

### INSTRUCTIONAL FOCUS

#### Describing and Graphing Functions

- Construct functions in different representations such as equations, graphs, tables, and verbal descriptions.
- Identify and analyze whether a function is linear or non-linear.
- Sketch a qualitative graph of a function from a verbal description, e.g., increasing, constant, decreasing, linear, not linear.
- Analyze a qualitative graph with and without numerical context.
- Given a real-world situation, sketch a graph to model the relationship that may include vertical and horizontal lines.
- Given a graph of a situation, write a description of the situation that may include vertical and horizontal lines.
- Describe the relationship between the inputs and outputs of a function in a qualitative way.
- Choose an appropriate scale for each axis; scales should not be limited to 1.
- Investigate and recognize that a line that increases from left to right has a positive slope and a line that decreases from left to right has a negative slope.
- Investigate and recognize that as the absolute value of the slope increases the line of the graph gets steeper.
- Investigate and recognize that the absolute value of a slope greater than one is a steeper line and less than one is a flatter line.
- Recognize that in a table the  $y$ -intercept is the  $y$ -value when  $x$  is equal to 0.
- Recognize that in a table the  $x$ -intercept is the  $x$ -value when  $y$  is equal to 0.
- Explain why the slope of a horizontal line is zero and a vertical line is undefined.

*Continued on next page*

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS, CONTINUED

#### Rate of Change

- Determine and interpret the rate of change and initial value of a linear function in context (equation, verbal description, and/or table).
- Determine and interpret, and when appropriate approximate, the rate of change and initial value from a graph.
- Find the slope of a line given two ordered pairs, a table, or a graph.

#### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 2, page 52](#)
- [Ohio's K-8 Learning Progressions, Functions, page 20](#)

### CONNECTIONS ACROSS STANDARDS

- Graph proportional relationships (8.EE.5).
- Derive the equation  $y = mx + b$  (8.EE.6).
- Interpret the equations of  $y = mx + b$  (8.F.3).
- Use the equation of a linear model to solve problems (8.SP.3).
- Describe the effect of dilations on two-dimensional figures using coordinates (8.G.3).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>GEOMETRY</b>  <b>Understand congruence and similarity using physical models, transparencies, or geometry software.</b></p> <p><b>8.G.1</b> Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates).</p> <p>a. Lines are taken to lines, and line segments are taken to line segments of the same length.</p> <p>b. Angles are taken to angles of the same measure.</p> <p>c. Parallel lines are taken to parallel lines.</p> <p><b>8.G.2</b> Understand that a two-dimensional figure is congruent<sup>G</sup> to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.)</p> <p><b>8.G.3</b> Describe the effect of dilations<sup>G</sup>, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>This is the first experience students have with transformations with and without coordinates. Students need many opportunities to explore transformations (rotations, reflections, translations, and dilations) of figures to verify their properties. They should discover that rotations, reflections, and translations preserve angle measures and side lengths. In contrast, they should discover that dilations only preserve angle measure whereas corresponding side lengths are proportional.</p> <p>Students should have many opportunities to explore and discover angle relationships in this cluster using transformations. In particular, students should explore interior and exterior angles of a triangle and the angles formed by parallel lines cut by a transversal. Students justify their findings through informal arguments involving transformations. Proofs of these informal arguments will be a component of high school mathematics.</p> <p>The student understanding of this cluster aligns with a van Hiele Level 2 (Informal Deduction/Abstraction).</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Angle Relationships</b></p> <ul style="list-style-type: none"> <li>• Parallel lines cut by a transversal create relationships, either congruent or supplementary, between pairs of angles.</li> <li>• The sum of the measure of the interior angles of a triangle is 180 degrees.</li> <li>• Any exterior angle of a triangle is congruent to the sum of the measures of the two remote interior angles of the triangle.</li> <li>• If two angles in one triangle are congruent to two angles in another, then the triangles are similar.</li> </ul> <p><i>Continued on next page</i></p>

**8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)

**8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDINGS, CONTINUED

#### Transformations

- Identify corresponding sides and angles of transformed figures.
- Reflections, rotations, and translations preserve angle measures and side lengths.
- Two figures are congruent if there is a sequence of reflections, rotations, and translations that maps one figure precisely to the other.
- Reflections and rotations change location and orientation.
- Translations change only location.
- Reflections require a line of reflection.
- Rotations require a point of rotation, a degree of rotation, and a direction of rotation.
- Translations require distance and direction.
- Dilations preserve angle measures while corresponding side lengths are proportional.
- Dilations require a center of dilation and a scale factor.
- A sequence of transformations, including a dilation that transforms one figure to another, results in figures that are similar.

#### MATHEMATICAL THINKING

- Explain mathematical reasoning.
- Use mathematical vocabulary.
- Use informal reasoning.
- Make and modify a model to represent mathematical thinking.

*Continued on next page*

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

#### Transformations

- Use transformation vocabulary and notation, e.g., image, pre-image,  $A$ ,  $A'$ .
- Use a variety of resources including physical models, transparencies, or geometry software to explore transformations to verify their properties using examples both with and without coordinates.
  - Discover that rotations, reflections, and translations preserve angle measures and side lengths.
  - Discover that dilations only preserve angle measure whereas corresponding side lengths are proportional.
- Explore the relationships between the coordinates of figures before and after transformations.
- Describe the effect of transformations on two-dimensional figures using coordinates.
- Given two similar figures, describe a sequence of transformations that demonstrates the similarity between them using examples both with and without coordinates.
- Investigate the sum of the interior angles of a triangle using physical models or geometry software.

#### Angle Relationships

- Explore the angle relationship between an exterior angle and the two remote interior angles of the triangle.
- Investigate relationships of pairs of angles created by parallel lines cut by a transversal using transformations.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54](#)
- [Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21](#)

### CONNECTIONS ACROSS STANDARDS

- Use similar triangles to explain why the slope,  $m$ , is the slope between any two distinct point on a non-vertical line (8.EE.6).



**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>GEOMETRY</b>  <b>Understand and apply the Pythagorean Theorem.</b>  <b>8.G.6</b> Analyze and justify an informal proof of the Pythagorean Theorem and its converse.  <b>8.G.7</b> Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two- and three-dimensions.  <b>8.G.8</b> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>	<p><b>Expectations for Learning</b></p> <p>This is the introduction to the Pythagorean Theorem. It is the first time students are exposed to a conditional as a logical structure in relation to the Pythagorean Theorem and its converse.</p> <p>Using manipulatives and drawings, students understand the Pythagorean Theorem and its converse. Students will learn in this cluster that in a right triangle the sum of the squares of the legs is equal to the square of the hypotenuse. It is imperative that students are fluent in applying the Pythagorean Theorem and its converse. These ideas will be developed in high school to include trigonometry and analytical geometry. The use of the converse of the Pythagorean theorem to determine whether a triangle is acute or obtuse is also explored in high school geometry. Informal proofs are introduced in eighth grade; formal proofs will begin in high school. Students will be able to apply the Pythagorean Theorem to the coordinate plane, in three-dimensional representations, and other real-world applications.</p> <p>The student understanding of this cluster aligns with a van Hiele Level 2. (Informal Deduction/Abstraction).</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>• Side lengths need not be represented by rational numbers.</li> <li>• The Pythagorean Theorem states that in a right triangle the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.</li> <li>• The Pythagorean Theorem is represented symbolically by <math>(leg\ a)^2 + (leg\ b)^2 = hypotenuse^2</math></li> <li>• The Pythagorean Theorem only applies to right triangles.</li> <li>• The hypotenuse is the longest side of a right triangle and opposite the right angle.</li> <li>• The legs of a right triangle are perpendicular.</li> <li>• The distance between two non-vertical or non-horizontal points in the coordinate plane can be determined by creating a right triangle with vertical and horizontal legs and applying the Pythagorean Theorem.</li> </ul> <p><i>Continued on next page</i></p>

## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDINGS, CONTINUED

- Converse is the reverse order of a hypothesis.
- The converse of the Pythagorean Theorem states that if the sum of the squares of the legs is equal to the square of the hypotenuse then the triangle is a right triangle
- The converse is used to determine whether or not a triangle is a right triangle
- The converse of the Pythagorean Theorem works because of the uniqueness of a triangle.

### MATHEMATICAL THINKING

- Use precise mathematical language
- Draw a picture or create a model to make sense of a problem
- Determine reasonableness of the results.
- Solve real-world and mathematical problems accurately.
- Describe mathematical reasoning using accurate mathematical vocabulary.

### INSTRUCTIONAL FOCUS

- Identify the legs and hypotenuse of a right triangle.
- Investigate and generalize the relationship between the areas of the squares created using the side lengths of a triangle.
- Generalize an informal justification of the Pythagorean Theorem and its converse.
- Use the Pythagorean Theorem to determine if a triangle is a right triangle.
- Use the Pythagorean Theorem to find a missing side length of a right triangle.
- Plot right triangles on a coordinate plane to investigate the relationship of side lengths.
- Recognize that the distance between two coordinate points can be the length of the hypotenuse of a right triangle.
- Apply the Pythagorean Theorem to find the distance between two points in the coordinate plane,
- Model real-world problems including lengths that cannot be directly measured.

*Continued on next page*

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54](#)
- [Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21](#)

### CONNECTIONS ACROSS STANDARDS

- Students will use rational and irrational numbers (8.NS.1, 2).
- Students will use square roots (8.EE.2).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
<p><b>GEOMETRY</b>  <b>Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</b>  <b>8.G.9</b> Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.</p>	<p><b>Expectations for Learning</b></p> <p>Students have previously worked with finding the volume of right prisms and the areas of circles. The students add to their knowledge of volume by solving problems involving cones, cylinders, and spheres. These shapes were previously introduced by their attributes in past grades.</p> <p>The student understanding of this cluster aligns with a van Hiele Level 1 (Analysis) and moves toward van Heile Level 2 (Informal Deduction/Abstraction).</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <ul style="list-style-type: none"> <li>• The bases of cones and cylinders are circles.</li> <li>• The net of a cylinder is a rectangle with 2 circles.</li> <li>• Cones and pyramids have one base.</li> <li>• The point of a cone and pyramid is called the apex.</li> <li>• The height of a pyramid or cone is the perpendicular distance from the apex to the (possibly extended) base.</li> <li>• The slant height of a pyramid or cone is the distance measured along the lateral face from the apex to the base.</li> <li>• The volume of a pyramid is <math>\frac{1}{3}</math> of the volume of a prism with congruent bases and heights.</li> <li>• The volume of a cone is <math>\frac{1}{3}</math> of the volume of a cylinder with congruent bases and heights.</li> </ul> <p><b>MATHEMATICAL THINKING</b></p> <ul style="list-style-type: none"> <li>• Consider mathematical units involved in the problem.</li> <li>• Solve real-world problems accurately.</li> <li>• Solve mathematical problems accurately.</li> <li>• Draw a picture or create a model to make sense of a problem.</li> <li>• See structure in equations.</li> </ul> <p><i>Continued on next page</i></p>

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

- Recognize the components of the formulas for the volume.
- Compare the formulas for volume.
- Given the volume, find the radius, diameter, height, or area of the base.
- Given the dimensions, find the volume.
- Differentiate between linear units, square units, and cubic units.
- Use volume to model real-world situations.
- Explore and make connections about the relationship between the volume of a cube and pyramid with the same base and height.
- Explore and make connections about the relationship between the volume of a cylinder and cone with the same base and height.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 3, pages 53-54](#)
- [Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21](#)

### CONNECTIONS ACROSS STANDARDS

- When given the volume, find the square or cube roots to determine the radius (8.EE.2).

**INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM****Instructional Strategies**

*This section is under revision and will be published in 2018.*

**Instructional Tools/Resources**

*This section is under revision and will be published in 2018.*



STANDARDS	MODEL CURRICULUM
<p><b>STATISTICS AND PROBABILITY</b>  <b>Investigate patterns of association in bivariate data.</b></p> <p><b>8.SP.1</b> Construct and interpret scatter plots for bivariate<sup>G</sup> measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4)</p> <p><b>8.SP.2</b> Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)</p> <p><b>8.SP.3</b> Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <i>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</i> (GAISE Model, steps 3 and 4)</p> <p><i>Continued on next page</i></p>	<p><b>Expectations for Learning</b></p> <p>Building on the study of statistics using univariate data (one variable) in Grades 6 and 7 and using the framework of the GAISE Model, students are now ready to study bivariate data (two variables). Students will extend their descriptions and understanding of variation to the graphical displays and numerical analysis of bivariate data. In high school, students build on their experience from the middle grades with data exploration and summarization; randomization as the basis of statistical inference; and simulation as a tool to understand statistical methods.</p> <p><b>ESSENTIAL UNDERSTANDINGS</b></p> <p><b>Quantitative (numerical) variables</b></p> <ul style="list-style-type: none"> <li>• Scatterplots are used for bivariate quantitative data.</li> <li>• When two variables are represented on a scatterplot, an association may exist.</li> <li>• An association between two variables can be seen in the pattern created by the data: <ul style="list-style-type: none"> <li>○ clusters;</li> <li>○ positive, negative, or no association; and/or</li> <li>○ linear or nonlinear association.</li> </ul> </li> <li>• Outliers are bivariate points that do not fit the trend.</li> <li>• When scatter plots suggest a linear association, a line can be informally fitted to the data.</li> <li>• Closeness of data points to the line can be judged visually.</li> <li>• When looking for a linear association, a line takes all of the points into consideration, and the prediction is based on an overall pattern rather than just one or two points.</li> <li>• The slope and y-intercept describe the linear association between two variables.</li> <li>• Linear functions can be used to describe contextual problems through the following: <ul style="list-style-type: none"> <li>○ interpreting slope and intercept; and/or</li> <li>○ making predictions.</li> </ul> </li> </ul> <p><i>Continued on next page</i></p>

**8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

## Expectations for Learning, continued

### ESSENTIAL UNDERSTANDINGS, CONTINUED

#### Categorical variables

- Two-way tables are used for bivariate categorical data.
- Two-way tables are used to display frequencies and relative frequencies.
- Relative frequencies can be used to describe possible associations.
- If row (or column) relative frequencies in the table are the same, there is little or no association.
- If row (or column) relative frequencies in the table are different, there is some evidence of association.

#### MATHEMATICAL THINKING

- Represent concepts symbolically.
- Construct and modify models to represent mathematical thinking.
- Use informal reasoning.
- Attend to precision in justifying mathematical reasoning.

*Continued on next page*

## Expectations for Learning, continued

### INSTRUCTIONAL FOCUS

#### Quantitative

- Construct a scatterplot by choosing a scale, labeling axes, and plotting points.
- Describe a scatterplot in terms of clusters, gaps, and unusual data points (outliers).
- Describe a trend (linear, curved, positive, negative, strong association, weak association, no association).
- Sketch a trend line through the “center” of the points.
- Approximate the slope of the trend line.
- Write an equation of the trend line.
- Interpret the rate of change and intercept of the trend line in the context of the problem.
- Interpret data points in relationship to the trend line (above, below, or on the line).
- Answer questions and make predictions based on the trend line.
- Determine how a change in data changes the trend line.
- Measure the association between two quantitative variables using the Quadrant Count Ratio (QCR).

#### Categorical

- Construct a two-way frequency table.
- Calculate relative frequencies.
- Determine possible association between the two variables using row (or column) relative frequencies.

### Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 8, Number 1, pages 50-51](#)
- [Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23](#)
- [GAISE Model, pages 14 – 15](#)
  - [Focus of 7<sup>th</sup> grade is Level A – B, pages 22-59](#)

### CONNECTIONS ACROSS STANDARDS

- Apply concepts of linear functions (8.F.3-5).

## INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

### Instructional Strategies

*This section is under revision and will be published in 2018.*

### Instructional Tools/Resources

*This section is under revision and will be published in 2018.*

## Acknowledgments

### Wendy Arnold

Teacher, Lake Local Schools, NW

### Robert Batty

Curriculum Specialist/Coordinator,  
Darke County ESC, SW

### Dawn Bittner (WG)

Teacher,  
Cincinnati Public Schools, SW

### Annette Blue

Teacher, Hudson City Schools, NE

### Adam Brown

Teacher, Mathews Local Schools, NE

### Jeanne Cerniglia (AC)

Teacher,  
Southeast Local Schools, NE

### Hoyun Cho (WG)

Higher Education,  
Capital University, C

### Beth Drum

Teacher,  
Caldwell Exempted Village, SE

### Matthew Dunn

Teacher, Waverly City Schools, SE

### Lisa Floyd-Jefferson

Curriculum Specialist/Coordinator,  
Reynoldsburg City Schools, C

### Brittany Hammonds

Teacher,  
Greenfield Exempted Village, SW

### Michael Hernandez

Teacher,  
Manchester Local Schools, NE

### Michele Heron

Higher Education,  
Kent State University, NE

### Steve Hiner

Curriculum Specialist/Coordinator,  
Columbus City Schools, C

### Michael Huler

Curriculum Specialist/Coordinator,  
Westerville City Schools, C

### Melissa Jacobs

Curriculum Specialist/Coordinator,  
Toledo Public Schools, NW

### Niki Jelil

Teacher, Hamilton City Schools, SW

### Kamlesh Jindal

Retired Educator, SW

### Carole Katz

Curriculum Specialist/Coordinator,  
Beachwood City Schools, NE

### Heather Kidd

Curriculum Specialist/Coordinator,  
Northwest Local Schools, SW

### Melissa Kincaid

Consultant,  
Hamilton County ESC, SW

### Kathy Koepp

Curriculum Specialist/Coordinator,  
Elyria City Schools, NE

### Rita Leskovec

Teacher, Cleveland Metropolitan  
School District, NE

### Nikki Littleton

Curriculum Specialist/Coordinator,  
Warren City Schools, NE

### Denise Lutz

Administrator, South-Western City  
School District, C

### Alicia Mailhot

Teacher,  
Valley View Local Schools, SW

### Cindy McKinstry (WG)

Teacher, East Palestine Schools, NE

### Amanda Mickey

Teacher, Chillicothe City Schools, C

### Cindy Miller (WG)

Curriculum Specialist/Coordinator,  
Maysville Local Schools, SE

### Wendi Moorman

Curriculum Specialist/Coordinator,  
Mercer County ESC, NW

### Jerry Moreno

Higher Education,  
John Carroll University, NE

## Acknowledgements, continued

**Kristan Northcutt**

*Teacher, Dayton City Schools, SW*

**Melissa Novak**

*Teacher,  
Strongsville City Schools, NE*

**Robin Phillips**

*Teacher,  
Oak Hill Union Local Schools, SE*

**Rebecca Prelog**

*Teacher, Kent City Schools, NE*

**Christina Sherman**

*Consultant,  
Hamilton County ESC, SW*

**Jenni Stadtmiller**

*Curriculum Specialist/Coordinator,  
Hamilton County ESC, SW*

**Joyce Tatro Manes**

*Teacher, Adena Local Schools, C*

**Kelli Tebbe**

*Curriculum Specialist/Coordinator,  
Auglaize County ESC, NW*

**Betsy Thomas**

*Teacher, Lebanon City Schools, SW*

**Stephanie Wagoner**

*Curriculum Specialist/Coordinator,  
Brown County ESC, SW*

**Ruthie Ware**

*Curriculum Specialist/Coordinator,  
Muskingum Valley ESC, SE*

**Jessica Wills**

*Teacher, Hamilton Local Schools, C*

**Heather Wukelich**

*Curriculum Specialist/Coordinator,  
Austintown Local Schools, NE*

\*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.