Ohio's Model Curriculum | Mathematics

High School Conceptual Category: Functions

Ohio | Department of Education

Mathematics Model Curriculum
High School Conceptual Category: Functions

TABLE OF CONTENTS

Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018. *Note: The Instructional Supports section will be published by high school course only.*

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Standards for Mathematical Practice, continued

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Standards for Mathematical Practice, continued

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation ${}^{(y-2)}/_{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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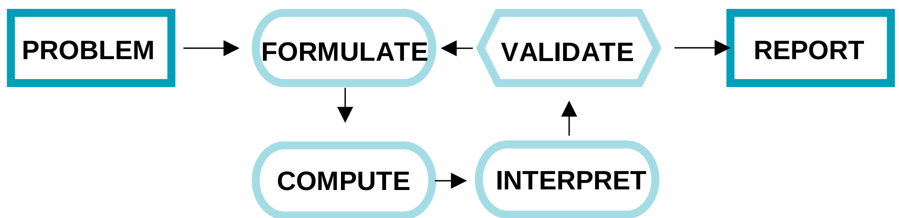
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Ohio | Department of Education

Mathematics Model Curriculum
High School Conceptual Category: Functions

STANDARDS	MODEL CURRICULUM
INTERPRETING FUNCTIONS **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f(x)*. **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.	**Expectations for Learning-Algebra 1/Math 1** In the eighth grade, students learn a semi-formal definition of a function and know that a function pairs an input value with an output value. Eighth grade students do not use function notation nor the terms domain and range. In this cluster, students will now expand their understanding of functions to include function notation and the terms domain and range. Also, students will evaluate and interpret functions, including sequences as functions. Distinguishing between relations and functions is not a primary focus. This cluster is the foundation for all future work with functions. **ESSENTIAL UNDERSTANDINGS** • Function notation illustrates a formal connection between inputs and outputs. • Functions can be tied to real-world scenarios given by tables, graphs, equations, or verbal descriptions. • Function notation *f(x)* is shorthand for the output of *f* when the input is *x*. • Function notation, *f(x),* is a new representation for students and is articulated as "*f* of *x*", and it is not related to multiplication. • Sequences are functions whose domain is a subset of the integers, paying careful attention to how a sequence is indexed. For example, the sequence may be indexed from 0 to *n*, from 1 to *n* – 1, or something else. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. *Continued on next page*

Expectations for Learning-Algebra 1/Math 1, continued

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to describe mathematical reasoning.
- Represent a concept symbolically.
- Determine reasonableness of results.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.

INSTRUCTIONAL FOCUS

- Make connections among different representations (tables, graphs, symbols, and verbal descriptions) of functions, focusing on linear, quadratic (A1 only), and exponential functions.
- Solve problems with functions represented in tables, graphs, symbols, and verbal descriptions.
- Explain function notation in a real-world context. For example, if $f(x)$ represents the height of particle at x seconds, then $f(1)$ represents the height of the particle at 1 second.
- Interpret number patterns as sequences and their graphs as discrete points. When the number pattern arises from a context, consider whether it is appropriate to "connect the dots."
- Use function notation to specify sequences, both explicitly and recursively. (Subscript notation is not required.)
- Relate linear functions to arithmetic sequences and relate exponential functions to geometric sequences.

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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1,Number 2, pages 5-7 • Math 1, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 1/Math 1** • Build a function that models a relationship between two quantities (F.BF.1a, 2). • Build new functions from existing functions (F.BF.4a). • Interpret expressions for functions in terms of the situation they model (F.LE.5). • Construct and compare linear, quadratic (A1), and exponential models, and solve problems (F.LE.2). • Represent and solve equations and inequalities graphically (A.REI.10).

STANDARDS	MODEL CURRICULUM
INTERPRETING FUNCTIONS **Interpret functions that arise in applications in terms of the context.** **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3) **a.** Focus on linear and exponential functions. (M1) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) *Continued on next page*	**Expectations for Learning-Algebra 1** In eighth grade, students model linear relations between two quantities; analyze graphs to determine where they are increasing and decreasing; and determine if relations are linear or non-linear. In this cluster, students interpret additional key features of the graphs and tables of linear, quadratic, and exponential functions only. They also determine the domain of a function by looking at a graph or table. In a real-life scenario students can find the restrictions on the domain. In Algebra 2, students extend identifying and interpreting key features of functions to include periodicity. Students also have to select appropriate functions that model the data presented. Average rate of change over a specific interval will also be included in Algebra 2. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, and then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem. • The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols. • Functions can have continuous or discrete domains. • A quadratic function is symmetrical about its axis of symmetry. **MATHEMATICAL THINKING** • Connect mathematical relationships to contextual scenarios. • Attend to meaning of quantities. • Determine reasonableness of results. *Continued on next page*

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
a. Focus on linear and exponential functions. (M1)
b. Focus on linear, quadratic, and exponential functions. (A1, M2)
c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)
	Expectations for Learning-Algebra 1, continued
	INSTRUCTIONAL FOCUS
	**Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*
	• For linear functions represented as tables, graphs, or verbal descriptions, interpret intercepts and rates of change in the contexts of the problem.
	• For exponential functions, interpret intercepts, growth/decay rates, and end behaviors in the contexts of the problems, given tables, graphs, and verbal descriptions.
	• For quadratic functions, interpret intercepts; maximum or minimum; symmetry; intervals of increase or decrease; and end behavior, given tables, graphs, and verbal descriptions.
	• Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required).
	• Determine whether to connect points on a graph based on the context (continuous vs. discrete domain).
	• Demonstrate understanding of domain in the context of a real-world problem.
	• Compare the key features of quadratic functions to linear and exponential functions. For example,
	  o Linear functions are either always increasing, decreasing, or constant.
	  o Exponential functions are either always increasing or decreasing.
	  o Quadratic functions increase to a maximum then decrease or decrease to a minimum then increase.
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Expectations for Learning-Math 1

In eighth grade, students model linear relations between two quantities; analyze graphs to determine where they are increasing and decreasing; and determine if relations are linear or non-linear.

In this cluster, students interpret additional key features of the graphs and tables of linear and exponential functions only. They also determine the domain of a function by looking at a graph or table. In a real-life scenario students can find the restrictions on the domain.

In Math 2, students apply these concepts to quadratic functions.

Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, and then identifying the key features of the graph and connecting the key features to the symbols.

ESSENTIAL UNDERSTANDINGS

- Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem.
- The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols.
- Functions can have continuous or discrete domains.

MATHEMATICAL THINKING

- Connect mathematical relationships to contextual scenarios.
- Attend to meaning of quantities.
- Determine reasonableness of results.

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Expectations for Learning-Math 1, continued

INSTRUCTIONAL FOCUS

Remember, in this course, for exponential functions, assessments should focus on integer exponents only.

- For linear functions, represented as tables, graphs, or verbal descriptions, interpret intercepts and rates of change in the contexts of the problems, given tables, graphs, and verbal descriptions.
- For exponential functions, interpret intercepts, growth/decay rates, and end behaviors in the contexts of the problems, given tables, graphs, and verbal descriptions.
- Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required).
- Determine whether to connect points on a graph based on the context (continuous vs. discrete domain).
- Demonstrate understanding of domain in the context of a real-world problem.

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	Expectations for Learning-Math 2 Working with linear and exponential functions in Math 1, students interpret key features of graphs and tables. They also determine the domain of a function by looking at a graph or table. In a real-life scenario, students can find the restrictions on the domain. In this cluster, students apply these concepts to quadratic functions and compare them to the prior learning of linear and exponential functions from Math 1. In Math 3, students extend identifying and interpreting key features of functions to include periodicity. Students also have to select appropriate functions that model the data presented. Average rate of change over a specific interval will also be included in Math 3. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, and then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem. • The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols. • A quadratic function is symmetrical about its axis of symmetry. • Functions can have continuous or discrete domains. **MATHEMATICAL THINKING** • Connect mathematical relationships to contextual scenarios. • Attend to meaning of quantities. • Determine reasonableness of results. *Continued on next page*

Expectations for Learning-Math 2, continued

INSTRUCTIONAL FOCUS

**Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients..*

- For quadratic functions, interpret intercepts; maximum or minimum; symmetry; intervals of increase or decrease; and end behavior given tables, graphs, and verbal descriptions.
- Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required).
- Determine whether to connect points on a graph based on the context (continuous vs. discrete domain).
- Demonstrate understanding of domain in the context of a real-world problem.
- Compare the key features of quadratic functions to linear and exponential functions. For example,
    - Linear functions are either always increasing, decreasing, or constant.
    - Exponential functions are either always increasing or decreasing.
    - Quadratic functions increase to a maximum then decrease or decrease to a minimum then increase.

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Expectations for Learning-Algebra 2/Math 3

Working with linear, quadratic, and exponential functions in Algebra1/Math 2, students interpreted key features of graphs and tables. Students determined the domain and understand its limitations by looking at a graph, table, and a given real-life scenario.

In this cluster, students extend key features to include periodicity, polynomials of degree greater than 2, and others given by graphs, tables, symbols, or verbal descriptions. Students select appropriate functions that model the situation presented. Average rate of change over a specific interval is included in this cluster.

In future courses, some students will extend their understanding of average rate of change to instantaneous rate of change.

Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem, and making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, then identifying the key features of the graph and connecting the key features to the symbols.

ESSENTIAL UNDERSTANDINGS

- Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem.
- The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols.
- The context of a situation can suggest the shape of the graph and can suggest the function family that models relationships identified in the situation.
- Linear functions have a constant rate of change.
- Since non-linear functions do NOT have a constant rate of change, average rate of change is used the describe how the rate of change varies from interval to interval.
- The average rate of change of a function over an interval is finding the slope of a linear function that goes through the same two points.

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Expectations for Learning-Algebra 2/Math 3, continued

MATHEMATICAL THINKING

- Connect mathematical relationships to contextual scenarios.
- Attend to meaning of quantities.
- Determine reasonableness of results.

INSTRUCTIONAL FOCUS

- Use key features (including periodicity) of graphs, tables, and contexts to select an appropriate function family for modeling purposes.
- Utilize graphs to compare and understand behaviors and features of various types of functions.
- Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required).
- Demonstrate understanding of domain in the context of a real-world problem.
- Given a function graphically, estimate the average rate of change over a specified interval.
- Given a function symbolically, compute the average rate of change over a specified interval.
- Given a function represented as a table, compute the average rate of change over a specified interval.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 2, pages 5-7
- Algebra 1, Number 5, pages 11-12
- Math 1, Number 2, pages 4-6
- Math 2, Number 3, pages 6-7
- Algebra 2/Math 3, Number 4, pages 8-10

Continued on next page

Content Elaborations, continued

CONNECTIONS ACROSS STANDARDS

Algebra 1

- Create equations that describe numbers or relationships (A.CED.2a, b).
- Represent and solve equations and equations and inequalities graphically (A.REI.10).
- Understand the concept of a function, and use function notation (F.IF.1-3).
- Graph linear functions and indicate intercepts (F.IF.7a).
- Graph quadratic functions and indicate maxima and minima (F.IF.7b).
- Graph simple exponential functions, indicating intercepts, and end behavior (F.IF.7e).
- Interpret expressions for functions in terms of the situation they model (F.LE.5).
- Analyze functions using different representations (F.IF.9b).
- Interpret linear models (S.ID.7).

Math 1

- Create equations that describe numbers or relationships (A.CED.2a, 3).
- Represent and solve equations and equations and inequalities graphically (A.REI.10).
- Understand the concept of a function, and use function notation (F.IF.1-3).
- Graph linear functions and indicate intercepts (F.IF.7a).
- Graph simple exponential functions, indicating intercepts, and end behavior (F.IF.7e).
- Interpret expressions for functions in terms of the situation they model (F.LE.5).
- Interpret linear models (S.ID.7).

Math 2

- Create equations that describe numbers or relationships (A.CED.2b).
- Graph quadratic functions and indicate maxima and minima (F.IF.7b).
- Analyze functions using different representations (F.IF.9b).

	Content Elaborations, continued
	CONNECTIONS ACROSS STANDARDS, CONTINUED **Algebra 2/Math 3** • Create equations that describe numbers or relationships (A.CED.2c). • Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions (F.IF.7c). • Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior (F.IF.7d). • Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude (F.IF.7f). • (+) Graph rational functions, identifying zeros and asymptotes, when factoring is reasonable, and indicating end behavior (F.IF.7g). • Analyze functions using different representations (F.IF.9). • Model periodic phenomena with trigonometric functions (F.TF.5). • Fit a function to the data; use functions fitted to data to solve problems in the context of the data (S.ID.6a).

STANDARDS	MODEL CURRICULUM

STANDARDS

INTERPRETING FUNCTIONS

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

- **a.** Graph linear functions and indicate intercepts. (A1, M1)
- **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2)
- **c.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3)
- **d.** Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3)
- **e.** Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1)
- **f.** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline[G], and amplitude. (A2, M3)

Continued on next page

MODEL CURRICULUM

Expectations for Learning-Algebra 1

In eighth grade, students graph and write linear functions, but their knowledge of key features of functions is limited to slope and *y*-intercept. They are exposed to non-linear functions and can distinguish between linear and non-linear functions.

In this cluster, students graph linear, quadratic, and exponential functions given a symbolic representation and indicate intercepts and end behavior. They compare linear, quadratic, and exponential functions given various representations.

In Algebra 2, students graph polynomial, square root, cube root, trigonometric, piecewise-defined, (+) rational, and (+) logarithmic functions. They identify and interpret key features (as applicable) including intercepts, end behavior, period, midline, amplitude, symmetry, asymptotes, maxima/minima, and zeros.

Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, and then identifying the key features of the graph and connecting the key features to the symbols.

ESSENTIAL UNDERSTANDINGS

- The graph of a linear function shows intercepts and rate of change.
- The graph of an exponential function shows the *y*-intercept and end behaviors.
- The graph of a quadratic function shows intercepts and maximum or minimum.
- Function families have commonalities in shapes and features of their graphs.
- The factored form of a quadratic function reveals the zeros of the function (i.e., the *x*-intercepts of the graph); the vertex form of a quadratic function reveals the maximum or minimum of the function; the standard form of a quadratic function reveals the *y*-intercept of the graph.
- Different representations (graphs, tables, symbols, verbal descriptions) illuminate key features of functions and can be used to compare different functions.
- More generally, writing a function in different ways can reveal different features of the graph of a function.

(+) **g.** Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (A2, M3)

(+) **h.** Graph logarithmic functions, indicating intercepts and end behavior.

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)

i. Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2)

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change*[G] *in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay.* (A2, M3)

i. Focus on exponential functions evaluated at integer inputs. (A1, M2)

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Expectations for Learning-Algebra 1, continued

MATHEMATICAL THINKING

- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Analyze a mathematical model.

INSTRUCTIONAL FOCUS

**Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*

- Given symbolic representations of linear, quadratic, and exponential functions, create accurate graphs showing all key features.
- Identify the key features of the graph of a quadratic function by factoring, using the quadratic formula, or completing the square
- Compare and contrast linear, quadratic, and exponential functions given by graphs, tables, symbols, or verbal descriptions.
- Determine the zeros of a quadratic function by factoring, using the quadratic formula, or completing the square.
- Use different forms of quadratic functions (standard form, vertex form, factored form) to reveal different features.
- Explore the relationship of the symbolic representation of a function and its graph by adjusting parameters.

Continued on next page

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **a.** Focus on linear and exponential functions. (M1) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)	**Expectations for Learning-Math 1** In eighth grade, students graph and write linear functions, but their knowledge of key features of functions is limited to slope and *y*-intercept. They are exposed to non-linear functions and can distinguish between linear and non-linear functions. In this cluster, students graph linear and exponential functions given a symbolic representation and indicate intercepts and end behavior. They compare linear and exponential functions given various representations. In Math 2, students graph quadratics and indicate key features. They will compare linear, quadratic, and exponential functions given various representations. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • The graph of a linear function shows intercepts and rate of change. • The graph of an exponential function shows the *y*-intercept and end behaviors. • Function families have commonalities in shapes and features of their graphs. • Different representations (graphs, tables, symbols, verbal descriptions) illuminate key features of functions and can be used to compare different functions. • More generally, writing a function in different ways can reveal different features of the graph of a function. **MATHEMATICAL THINKING** • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Analyze a mathematical model. *Continued on next page*

	Expectations for Learning-Math 1, continued
	INSTRUCTIONAL FOCUS **Remember, in this course, for exponential functions, assessments should focus on integer exponents only.* • Given symbolic representations of linear and exponential functions, create accurate graphs showing all key features. • Compare and contrast linear and exponential functions given by graphs, tables, symbols, or verbal descriptions. *Continued on next page*

Expectations for Learning-Math 2

In Math 1, students graph linear and exponential functions given a symbolic representation and indicate intercepts and end behavior. They compare linear and exponential functions given various representations.

In this cluster, students graph quadratic functions and indicate key features such as maxima/minima. They compare linear, quadratic, and exponential functions given various representations.

In Math 3, students graph polynomial, square root, cube root, trigonometric, piecewise-defined, (+) rational, and (+) logarithmic functions. Students identify and interpret key features (as applicable) including intercepts, end behavior, period, midline, amplitude, symmetry, asymptotes, maxima/minima, and zeros.

Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, then identifying the key features of the graph and connecting the key features to the symbols.

ESSENTIAL UNDERSTANDINGS

- The graph of a quadratic function shows intercepts and maximum or minimum.
- The factored form of a quadratic function reveals the zeros of the function (i.e., the x-intercepts of the graph); the vertex form of a quadratic function reveals the maximum or minimum of the function; the standard form of a quadratic function reveals the y-intercept of the graph.
- Different representations (graphs, tables, symbols, verbal descriptions) illuminate key features of functions and can be used to compare different functions.
- More generally, writing a function in different ways can reveal different features of the graph of a function.

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Expectations for Learning-Math 2, continued

MATHEMATICAL THINKING

- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Analyze a mathematical model.

INSTRUCTIONAL FOCUS

**Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*

- Given symbolic representations of quadratic functions, create accurate graphs showing all key features.
- Compare and contrast linear, quadratic, and exponential functions given by graphs, tables, symbols, or verbal descriptions.
- Determine the zeros of a quadratic function by factoring, using the quadratic formula, or completing the square.
- Use different forms of quadratic functions (standard form, vertex form, factored form) to reveal different features.
- Explore the relationship of the symbolic representation of a function and its graph by adjusting parameters.
- Identify the key features of the graph of a quadratic function by factoring, using the quadratic formula, or completing the square.

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	Expectations for Learning-Algebra 2/Math 3 In Algebra 1/Math 2, students graph linear, quadratic, and exponential functions given a symbolic representation and indicate intercepts and end behavior. They compare linear, quadratic, and exponential functions given various representations. In this cluster, students graph polynomial, square root, cube root, trigonometric, piecewise-defined, (+) rational, and (+) logarithmic functions. Students identify and interpret key features (as applicable) including intercepts, end behavior, period, midline, amplitude, symmetry, asymptotes, maxima/minima, and zeros. **ESSENTIAL UNDERSTANDINGS** • Different representations (graphs, tables, symbols, verbal descriptions) illuminate key features of functions and can be used to compare different functions. • For exponential functions, radicals can be used to make sense of the function values at non-integer input values. • Swapping the non-negative input and output values of the squaring function yields the square root function, so the graph of the square root function is half of a sideways parabola. • Trigonometric functions can be used to model periodic phenomena. • Piecewise-defined functions are used in real-world situations including bulk pricing, utility bills, income tax, etc. • A linear factor of a polynomial indicates a zero of that function. • The degree of a polynomial function indicates the maximum number of possible zeros. • Function families have commonalities in shapes and features of their graphs. • More complex functions can be graphed and analyzed as transformations of parent functions. • The factored form of a function reveals the zeros of the function (i.e., the *x*-intercepts of the graph); the vertex form of a quadratic function reveals the maximum or minimum of the function; the standard form of a quadratic function reveals the *y*-intercept of the graph. • More generally, writing a function in different ways can reveal different features of the graph of a function. *Continued on next page*

Expectations for Learning-Algebra 2/Math 3, continued

MATHEMATICAL THINKING

- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Analyze a mathematical model.

INSTRUCTIONAL FOCUS

- Given symbolic representations of various functions, create accurate graphs showing all key features, with and without technology. While fluency is still expected for linear, exponential, and quadratic functions, for other functions given symbolically, the focus should be on predicting the shape of the graph.
- Explore the relationship of the symbolic representation of a function and its graph by adjusting parameters.
- Identify the key features of the graph of a quadratic function by factoring, using the quadratic formula, or completing the square, including functions in which the leading coefficient is not equal to 1.
- Use factoring and long division to find zeros of polynomial functions.
- Use and interpret non-integer exponents in exponential functions.
- Use different symbolic forms of a function to reveal different features of its graph.
- Compare and contrast various functions given by graphs, tables, symbols, or verbal descriptions.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 2, pages 5-7
- Algebra 1, Number 5, 11-12
- Math 1, Number 2, pages 4-6
- Math 2, Number 3, pages 6-7
- Algebra 2/Math 3, Number 4, pages 8-10

Continued on next page

Content Elaborations, continued

CONNECTIONS ACROSS STANDARDS

Algebra 1

- Interpret functions that arise in applications in terms of the context (F.IF.4).
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) (A.REI.10).
- Construct and compare linear, quadratics, and exponential models and solve problems (F.LE.1-2).
- Interpret expressions for functions in terms of the situation they model (F.LE.5).
- Solve quadratic equations in one variable (A.REI.4).
- Write expressions in equivalent forms to solve problems (A.SSE.3).
- Build a function that models a relationship between two quantities (F.BF.1).

Math 1

- Interpret functions that arise in applications in terms of the context (F.IF.4).
- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) (A.REI.10).
- Construct and compare linear and exponential models and solve problems (F.LE.1-2).
- Interpret expressions for functions in terms of the situation they model (F.LE.5).

Math 2

- Interpret functions that arise in applications in terms of the context (F.IF.4)
- Solve quadratic equations in one variable (A.REI.4).
- Write expressions in equivalent forms to solve problems (A.SSE.3).
- Build a function that models a relationship between two quantities (F.BF.1).

Continued on next page

	Content Elaborations, continued
	CONNECTIONS ACROSS STANDARDS, CONTINUED **Algebra 2/Math 3** • Understand the relationship between zeros and factors of polynomials (A.APR.2-3). • Rewrite rational expressions (A.APR.6). • Create equations that describe numbers or relationships (A.CED.2-4). • Build a function that models a relationship between two quantities (F.BF.1). • Expand the properties of exponents to rational exponents (N.RN.1-2). • Use properties of rational and irrational numbers (N.RN.3). • Construct and compare linear, quadratic, and exponential models and solve problems (F.LE.4).

STANDARDS	MODEL CURRICULUM
BUILDING FUNCTIONS **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from context. **i.** Focus on linear and exponential functions. (A1, M1) **ii.** Focus on situations that exhibit quadratic or exponential relationships. (A1, M2) **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3) (+) **c.** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	**Expectations for Learning-Algebra 1** In the eighth grade, students create functions to model relationships between two quantities. In this cluster, students write linear, exponential, and quadratic functions symbolically given the relationship between two quantities. Relationships between quantities could be given as tables, graphs, or within a context. Students also write explicit and recursive rules for arithmetic and geometric sequences. In Algebra 2, students build functions from other functions allowing students to model more complex situations. This includes combining functions of various types using arithmetic operations or (+) composition. **ESSENTIAL UNDERSTANDINGS** • Functions can be written as explicit expressions, recursive processes, and in other ways. • An arithmetic sequence (informally, an addition pattern) has a starting term and a common difference between terms. • A geometric sequence (informally, a multiplication pattern) has a starting term and a common ratio between terms. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. • Some sequences can be defined recursively or explicitly, while others cannot be defined by a formula. • The relationships between quantities can be modeled with functions that are linear, exponential, quadratic, or none of these. **MATHEMATICAL THINKING** • Make and modify a model to represent mathematical thinking. • Discern and use a pattern or structure. *Continued on next page*

Expectations for Learning-Algebra 1, continued

INSTRUCTIONAL FOCUS

- Model relationships with linear functions, which may be arithmetic sequences, using tables, graphs, symbols, and words within a context.
- Model relationships with exponential functions, which may be geometric sequences, using tables, graphs, symbols, and words within a context.
- Model relationships with quadratic functions using tables, graphs, symbols, and words within a context.
- Model relationships that are not linear, exponential, or quadratic using tables, graphs, symbols, and words within a context.

Expectations for Learning-Math 1

In the eighth grade, students create functions to model relationships between two quantities. In this cluster, students write linear and exponential functions symbolically given the relationship between two quantities. Relationships between quantities could be given as tables, graphs, or within a context. Students also write explicit and recursive rules for arithmetic and geometric sequences. In Math 2, students focus on situations that exhibit exponential or quadratic relationships.

ESSENTIAL UNDERSTANDINGS

- Functions can be written as explicit expressions, recursive processes, and in other ways.
- An arithmetic sequence (informally, an addition pattern) has a starting term and a common difference between terms.
- A geometric sequence (informally, a multiplication pattern) has a starting term and a common ratio between terms.
- An arithmetic sequence is a linear function, and a geometric sequence is an exponential function.
- Some sequences can be defined recursively or explicitly, while others cannot be defined by a formula.
- The relationships between quantities can be modeled with functions that are linear, exponential, or neither of these.

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Expectations for Learning-Math 1, continued

MATHEMATICAL THINKING

- Make and modify a model to represent mathematical thinking.
- Discern and use a pattern or structure.

INSTRUCTIONAL FOCUS

- Model relationships with linear functions, which may be arithmetic sequences using tables, graphs, symbols, and words in context.
- Model relationships with exponential functions, which may be geometric sequences using tables, graphs, symbols, and words in context.
- Model relationships that are not linear or exponential using tables, graphs, symbols, and words in context.

Expectations for Learning-Math 2

In Math 1, students write linear and exponential functions symbolically given the relationship between two quantities. They also write explicit and recursive rules for arithmetic and geometric sequences.

In Math 2, students focus on quadratic situations and compare to linear and exponential relationships. Students also continue to work with sequences.

In Math 3, students build functions from other functions allowing students to model more complex situations. This includes combining functions of various types using arithmetic operations or (+) composition.

ESSENTIAL UNDERSTANDINGS

- Functions (including quadratic functions) can be written as explicit expressions, recursive processes, and in other ways.
- Some sequences may be defined recursively or explicitly while others cannot be defined by a formula.
- The relationships between quantities can be modeled with functions that are linear, exponential, quadratic, or none of these.

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Expectations for Learning-Math 2, continued

MATHEMATICAL THINKING

- Make and modify a model to represent mathematical thinking.
- Discern and use a pattern or structure.

INSTRUCTIONAL FOCUS

- Model relationships with linear, exponential, and quadratic functions using tables, graphs, symbols, and words in context.
- Model relationships that are not linear, exponential, or quadratic using tables, graphs, symbols, and words in context.

Expectations for Learning-Algebra 2/Math 3

In prior math courses, students write linear, quadratic, and exponential functions. They also write explicit and recursive rules for arithmetic and geometric sequences.

In this cluster, students build functions from other functions to model more complex situations. This includes combining functions of various types using arithmetic operations or (+) function composition.

ESSENTIAL UNDERSTANDINGS

- Known functions can be combined using arithmetic operations or (+) function composition, yielding new functions.
- Addition or subtraction by a constant function can be interpreted as a vertical shift.
- Multiplication or division by a non-zero constant function can be interpreted as a vertical stretch/shrink.

MATHEMATICAL THINKING

- Make and modify a model to represent mathematical thinking.
- Discern and use a pattern or structure.

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	Expectations for Learning-Algebra 2/Math 3, continued **INSTRUCTIONAL FOCUS** • Demonstrate the effects of arithmetic operations on known functions: Choose an *x*-value; find the two *y*-values; do the operation; and then repeat with several other x-values. • For addition and subtraction of functions, compare the heights of the functions to their sum or difference. • Select appropriate functions and arithmetic operations on those functions to model situations. • (+) Compose functions algebraically and for the purpose of modeling. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 2, pages 5-7 • Algebra 1, Number 5, pages 11-12 • Math 1, Number 2, pages 4-6 • Math 2, Number 3, pages 6-7 • Algebra 2/Math 3, Number 4, pages 9-10 **CONNECTIONS ACROSS STANDARDS** **Algebra 1** • Create equations that describe numbers or relationships (A.CED.2). • Fit a linear function for a scatterplot that suggests a linear association (S.ID.6c). • Interpret linear models (S.ID.7). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.1). • Analyze functions using different representations (F.IF.8). *Continued on next page*

	Content Elaborations, continued **CONNECTIONS ACROSS STANDARDS, CONTINUED** **Math 1** • Create equations that describe numbers or relationships (A.CED.2). • Fit a linear function for a scatterplot that suggests a linear association (S.ID.6c). • Interpret linear models (S.ID.7). • Construct and compare linear and exponential models, and solve problems (F.LE.1). **Math 2** • Create equations that describe numbers or relationships (A.CED.2). • Analyze functions using different representations (F.IF.8). **Algebra 2/Math 3** • Create equations that describe numbers or relationships (A.CED.2). • Fit a function to the data; use functions fitted to data to solve problems in the context of the data (S.ID.6a). • Interpret linear models (S.ID.7). • Analyze functions using different representations (F.IF.8). • Perform arithmetic operations on polynomials (A.APR.1b).

STANDARDS	MODEL CURRICULUM

BUILDING FUNCTIONS
Build new functions from existing functions.
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)
a. Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)
F.BF.4 Find inverse functions.
a. Informally determine the input of a function when the output is known. (A1, M1)
(+) **b.** Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)
(+) **c.** Verify by composition that one function is the inverse of another. (A2, M3)
(+) **d.** Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)
(+) **e.** Produce an invertible function from a non-invertible function by restricting the domain.
Continued on next page
	Expectations for Learning-Algebra 1
	In eighth grade, students learn that functions map inputs to outputs. In this cluster, students informally reverse this to find the input of a function when the output is known. In later classes, (+) some students more fully develop the concepts, procedures, and notation for inverses of functions.
	
	In eighth grade, students attend to slope and intercepts for graphs of linear functions, without explicit attention to transformations of the graphs. In this cluster, students transform graphs of quadratic functions. Transformations of quadratic functions can be interpreted conveniently by observing the effect on the vertex and whether the parabola opens up or down. Students do not perform transformations of the form *f(kx).* In Algebra 2, students perform all types of transformations for various function families and recognize even and odd functions.
	ESSENTIAL UNDERSTANDINGS
	• Sometimes the input of a function can be found when the output is given.
	• Vertical and horizontal transformations of $y = x^2$ are as follows:
	  o horizontal shift: $g(x) = (x - h)^2$;
	  o vertical stretch/shrink: $g(x) = ax^2$ when $a > 0$;
	  o vertical shift: $g(x) = x^2 + k$;
	  o reflection across the x-axis: $g(x) = -x^2$; and
	  o a combination of transformations: $g(x) = a(x - h)^2 + k$.
	
	MATHEMATICAL THINKING
	• Explain mathematical reasoning.
	
	INSTRUCTIONAL FOCUS
	** Limit to situations where inverse values are unique. Exclude formal notation; exclude finding the inverse algebraically; exclude switching x and y; exclude reflecting about the line $y = x$. Transformations occur in the quadratic expression rather than inside the function notation.*
	• Use graphs and tables to find the input value of a function when given an output, and interpret the values in context.
	• Transform graphs of quadratic functions and interpret the transformations geometrically.

(+) **F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	## Expectations for Learning-Math 1 In eighth grade, students learn that functions map inputs to outputs. In this cluster, students informally reverse this to find the input of a function when the output is known. In later classes, (+) some students more fully develop the concepts, procedures, and notation for inverses of functions. **ESSENTIAL UNDERSTANDINGS** • Sometimes the input of a function can be found when the output is given. **MATHEMATICAL THINKING** • Explain mathematical reasoning. **INSTRUCTIONAL FOCUS** ** Limit to situations where inverse values are unique. Exclude formal notation; exclude finding the inverse algebraically; exclude switching x and y; exclude reflecting about the line* $y = x$. • Use graphs and tables to find the input value of a function when given an output, and interpret the values in context. *Continued on next page*

Expectations for Learning-Math 2

In eighth grade and Math 1, students attend to slope and intercepts for graphs of linear functions, without explicit attention to transformations of the graphs. In this cluster, students transform graphs of quadratic functions. Transformations of quadratic functions can be interpreted conveniently by observing the effect on the vertex and whether the parabola opens up or down. Students do not perform transformations of the form *f(kx).* In Math 3, students perform all types of transformations for various function families and recognize even and odd functions.

ESSENTIAL UNDERSTANDINGS

- Vertical and horizontal transformations of $y = x^2$ are as follows:
  - horizontal shift: $g(x) = (x - h)^2$
  - vertical stretch/shrink: $g(x) = ax^2$ when $a > 0$
  - vertical shift: $g(x) = x^2 + k$
  - reflection across the x-axis: $g(x) = -x^2$
  - a combination of transformations: $g(x) = a(x - h)^2 + k$

MATHEMATICAL THINKING

- Explain mathematical reasoning.

INSTRUCTIONAL FOCUS

**Transformations occur in the quadratic expression rather than inside the function notation.*

- Transform graphs of quadratic functions, and interpret the transformations geometrically.

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	Expectations for Learning-Algebra 2/Math 3 In prior math courses, students transform graphs of quadratic functions, but did not utilize function notation for describing these transformations. In Algebra 2/Math 3, students perform all types of transformations for various function families, using function notation. They recognize even and odd functions, and (+) work extensively with inverses of functions. **ESSENTIAL UNDERSTANDINGS** • Transformations of graphs of functions include shifts, reflections, and stretches/shrinks. • Some functions can be characterized as even or odd, indicating symmetry. • Transformations of the form $f(x-k)$ are horizontal shifts. • Transformations of the form $f(x)+k$ are vertical shifts. • Transformations of the form $f(kx)$ are horizontal stretches/shrinks when $k>0$. • Transformations of the form $kf(x)$ are vertical stretches/shrinks when $k>0$. • Transformations of the form $-f(x)$ are reflections across the x-axis. • Transformations of the form $f(-x)$ are reflections across the y-axis. **MATHEMATICAL THINKING** • Explain mathematical reasoning. • Use technology strategically to deepen understanding. • Discern and use a pattern or structure. **INSTRUCTIONAL FOCUS** • Transform a variety of functions, including simple rational, radical, and exponential functions, and interpret the transformations geometrically. • Use function notation to express transformations, such as $f(x) = x^2$, $g(x) = f(x-h)$. • Recognize even and odd functions from symmetry in their graphs and from their algebraic expressions. • Generalize the effects of transformations across function families. • (+) Find inverses of functions algebraically, graphically, or from a table. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 2, pages 5-7 • Algebra 1, Number 5, pages 11-12 • Math 1, Number 2, pages 4-6 • Math 2, Number 3, pages 6-7 • Algebra 2/Math 3, Number 4, pages 8-10 **CONNECTIONS ACROSS STANDARDS** **Math 1** • Understand the concept of a function and use function notation (F.IF.1-2). **Algebra 1** • Understand the concept of a function and use function notation (F.IF.1-2). • Analyze functions using different representations (F.IF.7a, b, and e). **Math 2** • Analyze functions using different representations (F.IF.7b). **Algebra 2/Math 3** • Analyze functions using different representations (F.IF.7c, d, f, and g). • (+) Compose functions (F.BF.1c). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.4).

STANDARDS	MODEL CURRICULUM
LINEAR, QUADRATIC, AND EXPONENTIAL MODELS **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2) *Continued on next page*	**Expectations for Learning-Algebra 1** In eighth grade, students interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. Students also see examples of non-linear functions and learn and apply the properties of integer exponents. In Algebra 1, students compare across linear, exponential, and quadratic functions. **ESSENTIAL UNDERSTANDINGS** • Linear functions have a constant additive change. • Exponential functions have a constant multiplicative change. • Linear and exponential functions both have initial values. • To highlight the constant growth/decay rate, *r*, often expressed as a percentage, exponential functions can be written in the form, $f(n) = a(1+r)^n$. • To highlight the growth/decay factor, *b*, exponential functions can be written in the form, $f(n) = a(b)^n$. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. • The phrase "eventually exceeds" (F.LE.3) directs the focus towards large values in the domain and consideration of the base and *y*-intercept of the exponential function and the leading coefficient of the linear or quadratic function. • For large domain values, the growth of linear and quadratic functions is dominated by the leading term. **MATHEMATICAL THINKING** • Represent a concept symbolically. • Make and modify a model to represent mathematical thinking. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. *Continued on next page*

F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★	Expectations for Learning-Algebra 1, continued **INSTRUCTIONAL FOCUS** • Aim toward a multifaceted understanding of additive versus multiplicative change across different representations. • For linear functions (arithmetic sequences), focus on the constant rate of change across the tables, graphs, contexts, and the explicit and recursive representations. • For exponential functions (geometric sequences), focus on the constant growth/decay rate (or factor) across the tables, graphs, contexts, and the explicit and recursive representations. • Use graphs, tables, and contexts to see that as the domain value increases, the values of an exponential function will eventually exceed the corresponding values of a linear or quadratic function. Expectations for Learning-Math 1 In eighth grade, students interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. Students also see examples of non-linear functions and learn and apply the properties of integer exponents. In Math 1, students focus on comparing linear and exponential functions. In Math 2, students compare across linear, exponential, and quadratic functions. **ESSENTIAL UNDERSTANDINGS** • Linear functions have a constant additive change. • Exponential functions have a constant multiplicative change. • Linear and exponential functions both have initial values. • To highlight the constant growth/decay rate, r, often expressed as a percentage, exponential functions can be written in the form, $f(n) = a(1 + r)^n$. • To highlight the growth/decay factor, b, exponential functions can be written in the form, $f(n) = a(b)^n$. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. *Continued on next page*

Expectations for Learning-Math 1, continued

MATHEMATICAL THINKING

- Represent a concept symbolically.
- Make and modify a model to represent mathematical thinking.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.

INSTRUCTIONAL FOCUS

- Aim toward a multifaceted understanding of additive versus multiplicative change across different representations.
- For linear functions (arithmetic sequences), focus on the constant rate of change across the tables, graphs, contexts, and the explicit and recursive representations.
- For exponential functions (geometric sequences), focus on the constant growth/decay rate (or factor) across the tables, graphs, contexts, and the explicit and recursive representations.

Expectations for Learning-Math 2

In eighth grade, students interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. Students also see examples of non-linear functions and learn and apply the properties of integer exponents. In Math 1, students compare linear and exponential functions. In Math 2, students compare across linear, exponential, and quadratic functions.

ESSENTIAL UNDERSTANDINGS

- The phrase "eventually exceeds" (F.LE.3) directs the focus towards large values in the domain and consideration of the base and *y*-intercept of the exponential function and the leading coefficient of the linear or quadratic function.
- For large domain values, the growth of linear and quadratic functions is dominated by the leading term.

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Expectations for Learning-Math 2, continued

MATHEMATICAL THINKING

- Represent a concept symbolically.
- Make and modify a model to represent mathematical thinking.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.

INSTRUCTIONAL FOCUS

- Use graphs, tables, and contexts to see that as the domain value increases, the values of an exponential function will eventually exceed the corresponding values of a linear or quadratic function.

Expectations for Learning-Algebra 2/Math 3

Students study exponential functions with integer domains in Algebra 1 or in Math 1 and Math 2. Earlier in this course, students develop an understanding of rational exponents in order to talk about exponential functions with a domain that is the real numbers. Based on these understandings, students in Algebra 2/Math 3 focus on logarithms as numbers that are solutions to exponential equations. Logarithms as functions and the laws of logarithms are recommended for higher-level courses.

ESSENTIAL UNDERSTANDINGS

- Logarithms are exponents; they are numerical solutions to exponential equations, such as $10^x = 30$.

MATHEMATICAL THINKING

- Represent a concept symbolically.
- Make and modify a model to represent mathematical thinking.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.

INSTRUCTIONAL FOCUS

- Solve exponential equations with an unknown exponent by translating between exponential and logarithmic forms, with the support of tables and graphs, interpreting the solution in context.

Continued on next page

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 2, pages 5-7
- Algebra 1, Number 5, pages 11-12
- Math 1, Number 2, pages 4-6
- Math 2, Number 3, pages 6-7
- Algebra 2/Math 3, Number 4, pages 8-10

CONNECTIONS ACROSS STANDARDS

Algebra 1

- Build a function that models a relationship between two quantities (F.BF.1a, 2).
- Interpret functions that arise in applications in terms of the context (F.IF.4b, 5b).
- Analyze functions using different representations (F.IF.7a, b, and e).
- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6c).
- Interpret linear models (S.ID.7).
- Interpret the structure of expressions (A.SSE.1).
- Interpret the parameters in a linear or exponential function in terms of a context (F.LE.5).

Math 1

- Build a function that models a relationship between two quantities (F.BF.1a, 2).
- Interpret functions that arise in applications in terms of the context (F.IF.4a, 5a).
- Analyze functions using different representations (F.IF.7a, e).
- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6c).
- Interpret linear models (S.ID.7).
- Interpret the structure of expressions (A.SSE.1).
- Interpret the parameters in a linear or exponential function in terms of a context (F.LE.5).

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	Content Elaborations, continued **CONNECTIONS ACROSS STANDARDS, CONTINUED** **Math 2** • Build a function that models a relationship between two quantities (F.BF.1aii). • Interpret functions that arise in applications in terms of the context (F.IF.4b). • Analyze functions using different representations (F.IF.7b, 9b). **Algebra 2/Math 3** • Interpret functions that arise in applications in terms of the context (F.IF.4). • Analyze functions using different representations (F.IF.9). • Interpret the structure of expressions (A.SSE.3c). • Build new functions from existing functions (F.BF.3). • Extend the properties of exponents to rational exponents (N.RN.1-2). • Use properties of rational and irrational numbers (N.NR.3).

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
<tr><td>

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

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Expectations for Learning-Algebra 1/Math 1

This standard does not present new expectations for student learning. Rather, it emphasizes important habits to complement F.LE.1-3. In this cluster, students connect their understanding of the defining characteristics of linear functions (initial value and rate of change) to the defining characteristics of exponential functions (initial value and growth rate/growth factor) and by interpreting them in the context of a real-world problem.

ESSENTIAL UNDERSTANDINGS

- Linear functions have a constant additive change.
- Exponential functions have a constant multiplicative change.
- Linear and exponential functions both have initial values.
- To highlight the constant growth/decay rate, r, often expressed as a percentage, exponential functions can be written in the form, $f(n) = a(1 + r)^n$.
- To highlight the growth/decay factor, b, exponential functions can be written in the form, $f(n) = a(b)^n$.
- An arithmetic sequence is a linear function, and a geometric sequence is an exponential function.

MATHEMATICAL THINKING

- Connect mathematical relationships to contextual scenarios.
- Use accurate mathematical vocabulary to describe mathematical reasoning.
- Attend to meaning of quantities.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.

INSTRUCTIONAL FOCUS

- For linear functions (arithmetic sequences), focus on the constant rate of change across the tables, graphs, contexts, and the explicit and recursive representations.
- For exponential functions (geometric sequences), focus on the constant growth/decay rate (or factor) across the tables, graphs, contexts, and the explicit and recursive representations.

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	Content Elaborations
	OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS • Algebra 1, Number 2, pages 5-7 • Math 1, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 1/Math 1** • Build a function that models a relationship between two quantities (F.BF.1a, 2). • Interpret functions that arise in applications in terms of the context (F.IF.4-5). • Analyze functions using different representations (F.IF.7a, e). • Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6c). • Interpret linear models (S.ID.7). • Interpret the structure of expressions (A.SSE.1).

STANDARDS

TRIGONOMETRIC FUNCTIONS

Extend the domain of trigonometric functions using the unit circle.

F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

(+) **F.TF.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

(+) **F.TF.4** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

MODEL CURRICULUM

Expectations for Learning-Algebra 2/Math 3

In Geometry/Math 2, students use the right-triangle definitions of sine and cosine and solve problems involving right triangles. Students compute lengths of arcs based on similarity. In Algebra 2/Math 3, students are introduced to the concept of radians. They extend their understanding of right triangle trigonometry to circular trigonometry.

In future courses, students may be expected to be fluent with trigonometric functions of special angles, and they may also expected to be fluent in converting between degrees and radians.

Note: Students taking Algebra 2 before Geometry, will not have experience with trigonometric ratios, arc length, or circumference.

ESSENTIAL UNDERSTANDINGS

- Radians can be interpreted as lengths of arcs on the unit circle.
- Angles, measured in degrees or radians, can be any real number, i.e., angles can be negative, greater than 360 degrees, or 2π radians.
- An angle determines a point on the unit circle. The sine of the angle is the y-coordinate of the point, and the cosine of the angle is the x-coordinate of the point.
- The coordinates of a point on the unit circle (not on an axis) represent the lengths of the legs of a reference right triangle, where the signs of the x and y coordinates of that point indicate which quadrant the angle lies in.

MATHEMATICAL THINKING

- Draw a picture or create a model to make sense of a problem.
- Make and modify a model to represent mathematical thinking.
- Consider mathematical units involved in a problem.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Generalize concepts based on patterns (repeated reasoning).

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	Expectations for Learning-Algebra 2/Math 3, continued
	INSTRUCTIONAL FOCUS
	** The focus should be on sine and cosine functions, with some exposure to the tangent function. In advanced mathematics, students use all six trigonometric functions. Students should be using radians and degrees to develop fluency between them both.*
	• Given an angle measure, identify the coordinates of a point on the unit circle as follows:
	    o With a vertex located at the origin and the initial side of the angle is the positive half of the x-axis, the angle indicates the amount of counterclockwise turning to determine the terminal side of the angle. Likewise, the clockwise turning determines the terminal side of the negative angle.
	    o The terminal side of the angle intersects the unit circle at a point.
	    o The *x*-coordinate of this point is called the cosine of the angle, and the *y*-coordinate of this point is called the sine of the angle.
	• From a point on the unit circle, draw reference right triangles to connect back to right triangle trigonometry.
	
	Content Elaborations
	OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS
	• Algebra 2/Math 3, Number 3, page 7
	
	CONNECTIONS ACROSS STANDARDS
	Algebra 2/Math 3
	• Find arc lengths and areas of sectors of circles (G.C.6).
	• (+) Apply trigonometry to general triangles (G.SRT.8b, 9, 10, 11)
	• Model periodic phenomena with trigonometric functions (F.TF.5).
	• Prove and apply trigonometric identities (F.TF.8).

STANDARDS	MODEL CURRICULUM
TRIGONOMETRIC FUNCTIONS **Model periodic phenomena with trigonometric functions.** **F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ (+) **F.TF.6** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. (+) **F.TF.7** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★	**Expectations for Learning-Algebra 2/Math 3** Geometry/Math 2, students learn trigonometric ratios of right triangles. They study the right-triangle definitions of sine and cosine and apply their understanding to solve problems involving right triangles. In Algebra 2/Math 3, students are introduced to the concept of radians and extend their understanding of right triangle trigonometry to circular trigonometry. In this cluster, students apply their understanding of circular trigonometry to model periodic phenomena such as tides, electrical current, height of Ferris wheels, temperature, etc. The language in the standard states that students will "choose trig functions", but additionally, students must also choose the parameters for a trigonometric function that models real-world phenomena. In future courses, students will graph the six trigonometric functions by hand and/or using technology and will also identify periods and phase shifts by analyzing graphs and equations. **ESSENTIAL UNDERSTANDINGS** • Many real-world phenomena, including sound waves; oscillation on a spring; the motion of a pendulum; and phases of the moon, are cyclical and can be approximated by trigonometric functions. • The period is the horizontal length of one cycle, and it can be interpreted in terms of a horizontal stretch. • The equation of the midline is the average of the maximum and minimum values of the function, and it is a horizontal axis about which the graph of the function oscillates. • The amplitude is the distance between the midline and the maximum or minimum values of the function, and it can be interpreted in terms of a vertical stretch. **MATHEMATICAL THINKING** • Connect mathematical relationships and contexts. • Make and modify a model to represent mathematical thinking. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Given a real-world phenomenon, select an appropriate trigonometric function that models that phenomenon, understanding that sine and cosine functions will be used most frequently because of their tendency to model common real-world phenomena.
- Given a real-world phenomenon, identify the parameters (midline, amplitude, period) that describe that phenomenon.
- When phenomena are modeled with a sine ($f(x)$ = A sin(Bx + C) + D) or cosine ($f(x)$ = A cos(Bx + C) + D) function, students will choose the parameters A, B, C, and D that best model the scenario.
- Interpret amplitude and midline as vertical stretches and shifts of the graphs. Include some attention to period and frequency as horizontal stretches. Reserve phase shifts as horizontal shifts for advanced courses.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 2/Math 3, Number 3, page 7

CONNECTIONS ACROSS STANDARDS

Algebra 2/Math 3

- Extend the domain of trigonometric functions using the unit circle (F.TF.1-2, (+) 3-4).
- Analyze functions using different representations (F.IF.7f).
- Build new functions from existing functions (F.BF.3, (+) 4b-d).
- (+) Apply trigonometry to general triangles (G.SRT.8b, 9, 10, 11)

STANDARDS	MODEL CURRICULUM
TRIGONOMETRIC FUNCTIONS **Prove and apply trigonometric identities.** **F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. (+) **F.TF.9** Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.	**Expectations for Learning-Algebra 2/Math 3** In Geometry/Math 2, students learn trigonometric ratios of right triangles. They study the right-triangle definitions of sine and cosine and apply their understanding to solve problems involving right triangles. They solve right triangles and learn to write the equation of circles. Earlier in this course, students are introduced to the concept of radians. They extend their understanding of right triangle trigonometry to circular trigonometry. In this cluster, students apply their knowledge of the Pythagorean Theorem and circular trigonometry to prove this important identity. They also find values of trigonometric functions given the value of one trigonometric function and a quadrant. In future courses, students use their understanding of this trigonometric identity to generate other trigonometric identities and apply them. Students also learn and use inverse trigonometric functions to solve problems. **ESSENTIAL UNDERSTANDINGS** • The identity $\sin^2(\theta) + \cos^2(\theta) = 1$ is the Pythagorean Theorem applied to coordinates on the unit circle. • The value of a trigonometric function and a given quadrant are sufficient to find the values of the other trigonometric functions. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Make and analyze mathematical conjectures. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** **Students in this course do not need fluency with special angles. Students aiming for advanced courses should begin to work on fluency.* • Interpret $\sin^2(\theta) + \cos^2(\theta) = 1$, with an angle in standard position, as an equation for the unit circle, which is an application of the Pythagorean Theorem. • Check that the identity is true in all four quadrants and on the axes. • Given a trigonometric function and a quadrant, sketch a triangle to determine the values of other trigonometric functions. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 3, page 7 **CONNECTIONS ACROSS STANDARDS** • Extend the domain of trigonometric functions using the unit circle (F.TF.1-2, (+) 3-4). • (+) Apply trigonometry to general triangles (G.SRT.8b, 9, 10, 11).

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