Ohio's Model Curriculum | Mathematics

High School Conceptual Category: Geometry

Ohio | Department of Education

Mathematics Model Curriculum
High School Conceptual Category: Geometry

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018. *Note: The Instructional Supports section will be published by high school course only.*

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
Continued on next page

Standards for Mathematical Practice, continued

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Standards for Mathematical Practice, continued

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation ${}^{(y-2)}/_{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

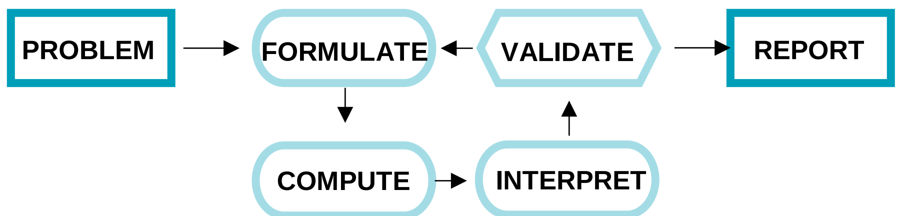
In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

Continued on next page

Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum
High School Conceptual Category: Geometry

STANDARDS	MODEL CURRICULUM
CONGRUENCE **Experiment with transformations in the plane.** **G.CO.1** Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length. **G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch. **G.CO.3** Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. **b.** Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes. *Continued on next page*	**Expectations for Learning-Geometry/Math 1** In middle school, students first learn about the basic rigid motions (translations, rotations, and reflections) and verify their properties experimentally. In this cluster, students formalize the notion of a transformation as a function from the plane to itself. Building on their hands-on work, students develop mathematical definitions of the basic rigid motions. These definitions serve as a logical basis for the theorems that students prove in Geometry. An important step in high school is to perform appropriate transformations and give precise descriptions of sequences of basic rigid motions that carry one figure onto another. Transformations provide language to be precise about symmetry; this is the first time students have encountered formal symmetry. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • A transformation is a function from the plane to itself; input and output values are points, not numbers. • Rigid motions are transformations that preserve distance and angle. • Some transformations preserve distance and angle measures, and some do not. • In order to perform a translation, a distance and a direction is required. • A rotation requires a center and an angle. • A reflection requires a line. • The symmetries of a figure are the transformations that carry the figure onto itself. *Continued on next page*

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**Expectations for Learning-Geometry/Math 1,continued** **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary and symbolic notations. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Recognize, apply, and justify mathematical concepts, terms, and their properties. **INSTRUCTIONAL FOCUS** • Know precise definitions of basic terms: ray, angle, circle, perpendicular line, parallel line, and line segment. • Develop and use appropriate geometric notation. • Formalize definitions of basic rigid motions (translations, rotations, and reflections). • Perform and identify transformations using a variety of tools. • Identify the symmetries shown in a figure (rotational and line symmetries). *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, page 4-5 • Math 1, Number 5, pages 9-10 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Understand congruence in terms of rigid motion (G.CO.6-8). • Prove and apply geometric theorems (G.CO.9). • Make formal geometric constructions (G.CO.12). • Justify the slope criteria for parallel and perpendicular lines (G.GPE.5). • Understand similarity in terms of similarity transformations (G.SRT.1-2). **Math 1** • Understand congruence in terms of rigid motion (G.CO.6-8). • Prove and apply geometric theorems (G.CO.9). • Make formal geometric constructions (G.CO.12). • Justify the slope criteria for parallel and perpendicular lines (G.GPE.5). • Reason quantitatively (N.Q.2-3).

STANDARDS	MODEL CURRICULUM
CONGRUENCE **Understand congruence in terms of rigid motions.** **G.CO.6** Use geometric descriptions of rigid motions[G] to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent[G]. **G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	**Expectations for Learning-Geometry/Math 1** In middle school, students understand congruence through a sequence of basic rigid motions (reflections, rotations, and translations). In this cluster, students will build on this knowledge to prove that two figures are congruent if there is a sequence of rigid motions carrying one onto the other. The triangle congruence criteria can then be established using the definition of congruence in terms of rigid motions. This is the time when students are first exposed to the criteria for triangle congruence; students should know and be able to use AAS, ASA, SAS, and SSS and understand that the criteria follow from rigid motions. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Two figures are defined to be congruent if one can be mapped onto the other by rigid motions. **MATHEMATICAL THINKING** • Explain mathematical thinking. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Represent concepts symbolically. • Use formal and informal reasoning. • Use accurate and precise mathematical vocabulary. **INSTRUCTIONAL FOCUS** • Use rigid transformations to determine if the figures are congruent • Given congruent triangles, describe the rigid transformations that map one triangle onto the other • Establish the criteria for triangle congruence (AAS, ASA, SAS, and SSS) in terms of rigid motions. • Know and be able to use triangle congruence (AAS, ASA, SAS, and SSS) in solving problems.

	Content Elaborations
	OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS • Geometry, Number 2, pages 4-5 • Math 1, Number 5, pages 9-10 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Experiment with transformations in the plane (G.CO.1-5). • Prove and apply theorems about triangles (G.CO.10). • Prove and apply theorems about parallelograms (G.CO.11). • Use coordinates to prove simple geometric theorems algebraically (G.GPE.4). • Prove and apply theorems involving similarity (G.SRT.5). **Math 1** • Experiment with transformations in the plane (G.CO.1-5). • Prove and apply theorems about triangles (G.CO.10). • Prove and apply theorems about parallelograms (G.CO.11). • Reason quantitatively (N.Q.2-3)

STANDARDS	MODEL CURRICULUM
CONGRUENCE **Prove geometric theorems both formally and informally using a variety of methods.** **G.CO.9** Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **G.CO.10** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **G.CO.11** Prove and apply theorems about parallelograms. *Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	**Expectations for Learning-Geometry/Math 1** In middle school, students informally define and apply the relationships of lines, angles, triangles, and parallelograms. For this cluster, students now develop conjectures and construct valid proofs about lines, angles, triangles, and parallelograms. They should begin with informal proof and work toward formal proof using a variety of methods including coordinate-based methods. Also, students should apply these relationships to real-world settings and to proofs. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). *Note: Although Math 1 students may use coordinate-based methods for proofs for all standards in this cluster, it is only required that they use coordinate based methods for G.CO.9. This is due to the movement of G.GPE.4 to Math 2.* **ESSENTIAL UNDERSTANDINGS** • The process of proof can vary from informal to formal reasoning. • A proof is a deductive argument that explains why a claim must be true. • Proof can rely on formal and informal language; there are many ways to justify a claim, not all of which rely on technical vocabulary. • Students should demonstrate a knowledge of the content listed in the standards and be able to apply those concepts in various problem solving settings. • Coordinate proof is a method that uses algebraic techniques to prove geometric theorems and properties. *Continued on next page*

Expectations for Learning-Geometry/Math 1, continued

MATHEMATICAL THINKING

- Explain mathematical thinking.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Represent concepts symbolically.
- Use formal and informal reasoning.
- Use accurate and precise mathematical vocabulary.
- Plan a solution pathway.
- Make and analyze mathematical conjectures.
- Solve real-world and mathematical problems accurately.
- Create a drawing, and add components as appropriate.

INSTRUCTIONAL FOCUS

- Form conjectures about geometric relationships and examine their validity, providing evidence to support or refute the claim.
- Using previously established facts about lines, angles, triangles, and parallelograms, construct a valid argument for why a conjecture is true or not true.
- Solve problems involving lines, angles, triangles, and parallelograms by applying theorems.

Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, page 4-5 • Math 1, Number 5, pages 9-10 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Experiment with transformations in the plane (G.CO.1, 3, 4). • Understand congruence in terms of rigid transformations (G.CO.6-8). • Use coordinates to prove simple geometric theorems algebraically (G.GPE.4-5). • Prove and apply theorems involving similarity (G.SRT.4, 5). **Math 1** • Experiment with transformations in the plane (G.CO.1, 3, 4). • Understand congruence in terms of rigid transformations (G.CO.6-8). • Use coordinates to prove simple geometric theorems algebraically (G.GPE.5). • Reason quantitatively (N.Q.1-3). • Create equations that describe relationships (A.CED.1-2). • Reason with equations by explaining steps (A.REI.1, 3).

STANDARDS	MODEL CURRICULUM
CONGRUENCE **Make geometric constructions.** **G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	**Expectations for Learning-Geometry/Math 1** In elementary and middle school, students learn to use measurement tools to informally draw geometric shapes with given conditions. In this cluster, students make formal and precise constructions using a variety of tools, and they understand the geometric relationships upon which the constructions are based. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Construction is a process of reasoning that does not use a scale and does not use measurement. • Simple constructions can be used to develop an understanding of mathematical relationships. **MATHEMATICAL THINKING** • Make sound decisions about using tools. • Strategically use technology to deepen understanding. • Plan a pathway to complete constructions. • Determine accuracy of results. • Create a drawing and add components as appropriate. **INSTRUCTIONAL FOCUS** • Distinguish between a rough sketch, a careful drawing with measurements, and a construction with compass and straightedge. • Use a variety of geometric tools to make precise constructions. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, pages 4-5 • Math 1, Number 5, pages 9-10 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Experiment with transformations in the plane (G.CO.1, 5). • Understand and apply theorems about circles (G.C.3, (+) 4). • Prove and apply geometric theorems (G.CO.9-11). • Prove and apply theorems involving similarity (G.SRT.4-5). **Math 1** • Experiment with transformations in the plane (G.CO.1, 5). • Understand and apply theorems about circles (G.C.3, (+) 4). • Prove and apply geometric theorems (G.CO.9-11). • Reason quantitatively (N.Q.1-3).

STANDARDS	MODEL CURRICULUM
CONGRUENCE **Classify and analyze geometric figures.** **G.CO.14** Classify two-dimensional figures in a hierarchy based on properties	**Expectations for Learning-Geometry/Math 1** In elementary school, students learn to classify two-dimensional figures based on their properties. In middle school, students focus on drawing quadrilaterals and triangles with given conditions. Now in high school, they learn to analyze and relate categories of two-dimensional shapes explicitly based on their properties. Based on analysis of properties, students create hierarchies for two-dimensional figures. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • There is a distinction between the definition of a figure and its properties, e.g., side lengths, angles, parallel/perpendicular sides, diagonals, symmetry. • Figures may be categorized in different ways based on their properties. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to describe geometric relationships. • Make connections between terms and properties. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Generalize concepts based on patterns. • Use formal reasoning. *Continued on next page*

Expectations for Learning-Geometry/Math 1, continued

INSTRUCTIONAL FOCUS

- Explain the difference between the definition of a figure and its properties.
- Know precise definitions of special polygons, e.g., rhombus, parallelogram, rectangle, square, kite, trapezoid, isosceles trapezoid, equilateral triangle, isosceles triangle, and regular polygon.
- Compare and contrast definitions of quadrilaterals, including both definitions of trapezoids.
- Know and apply properties of special polygons and use them to classify figures.
- Explain the relationships among special quadrilaterals.
- Explain the relationships among special triangles.
- Create hierarchies in order to represent the relationship between pairs of figures and among several figures.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 2, pages 4-5
- Math 1, Number 5, pages 9-10

CONNECTIONS ACROSS STANDARDS

Geometry

- Prove and apply theorems about quadrilaterals and triangles (G.CO.10-11).
- Prove theorems algebraically about quadrilaterals and triangles using coordinates (G.GPE.4).
- Justify the slope criteria for parallel and perpendicular lines (G.GPE.5).
- Know precise definitions (G.CO.1).

Math 1

- Prove and apply theorems about quadrilaterals and triangles (G.CO.10-11).
- Justify the slope criteria for parallel and perpendicular lines (G.GPE.5).
- Know precise definitions (G.CO.1).

STANDARDS	MODEL CURRICULUM
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY **Understand similarity in terms of similarity transformations.** **G.SRT.1** Verify experimentally the properties of dilations[G] given by a center and a scale factor: **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations[G] to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**Expectations for Learning-Geometry/Math 2** The standards in this cluster make more precise the informal notion of "same shape, different size." In middle school, students represent proportional relationships within and between similar figures; create scale drawings; describe the effect of dilations on two-dimensional figures; and understand similarity transformations as a sequence of basic rigid motions and dilations. In this cluster, students verify the properties (given center and scale factor) of dilations and use those properties to establish the AA criterion for triangles. They also explore the relationships among corresponding parts of similar figures. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • A dilation requires a center and a scale factor. • A similarity transformation often requires a sequence of basic rigid motions, in addition to a dilation. • A scale factor is a ratio corresponding lengths between figures. • A similarity transformation with a scale factor of 1 is a special case, which is a congruence transformation. • While the definition of similarity applies to polygons, it also applies to non-polygonal shapes, e.g., circles, parabolas, etc. • The AA criterion is equivalent to the AAA criterion because the angle sum in a triangle is 180 degrees. • The AA criterion and the AAA criterion apply only to triangles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Use formal reasoning with symbolic representation. • Determine reasonableness of results. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between terms and properties. *Continued on next page*

Expectations for Learning-Geometry/Math 2, continued

INSTRUCTIONAL FOCUS

- Use basic rigid motions and dilations to map similar figures onto one another.
- Given a figure, carry out a similarity transformation, and then verify its properties.
- Know the precise definitions of dilation and similarity.
- Identify center and scale factor of a dilation.
- Determine the scale factor.
- Establish that triangles with two pairs of corresponding congruent angles are similar.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 3, page 6
- Math 2, Number 4, page 8

CONNECTIONS ACROSS STANDARDS

Geometry

- Prove theorems involving similarity (G.SRT.4-5).
- Solve problems involving right triangles trigonometry (G.SRT.6-8).
- Apply the understanding that all circles are similar (G.C.1).
- Represent transformations in the plane (G.CO.2).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.6).
- Apply geometric concepts in modeling situations (G.MG.1-3).

Math 2

- Prove theorems involving similarity (G.SRT.4-5).
- Apply right triangles trigonometry (G.SRT.6-8).
- Apply the understanding that all circles are similar (G.C.1).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.6).
- Apply geometric concepts in modeling situations (G.MG.1-3).

STANDARDS	MODEL CURRICULUM
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY **Prove and apply theorems both formally and informally involving similarity using a variety of methods.** **G.SRT.4** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.	**Expectations for Learning-Geometry/Math 2** In middle school, students draw, construct, and describe geometric figures; use informal arguments to establish facts about similar triangles; and explain a proof of the Pythagorean Theorem and its converse. In this cluster, students prove theorems and solve problems involving similarity of triangles. They will also solve problems by applying these theorems to geometric figures that can be decomposed into triangles. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstraction) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • The altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. • A line parallel to the side of a triangle makes similar triangles and divides the other two side lengths proportionally. • Two right triangles are similar if they have another congruent angle. • Polygons can be divided into congruent and/or similar triangles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Use formal reasoning with symbolic representation. • Make conjectures. • Plan a solution pathway. • Justify relationships in geometric figures. • Determine reasonableness of results. • Create a drawing and add components as appropriate. *Continued on next page*

Expectations for Learning-Geometry/Math 2, continued

INSTRUCTIONAL FOCUS

- Form conjectures and construct a valid argument for why the conjecture is true or not true, both formally and informally.
- Recognize when polygons are divided into congruent and/or similar triangles.
- Justify relationships in geometric figures that can be decomposed into triangles.
- Solve problems using triangle congruence and similarity.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 3, page 6
- Math 2, Number 4, page 8

CONNECTIONS ACROSS STANDARDS

Geometry

- Understand similarity (G.SRT.1-3).
- Define trigonometric ratios, and solve problems involving right triangles (G.SRT.6-8).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.5-6).
- Use coordinate geometry (G.GPE.4).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.6).
- Represent transformations in the plane (G.CO.2).
- Prove and apply geometric theorems (G.CO.9-10).
- Make geometric constructions (G.CO.12-13).
- Find arc lengths and areas of sectors of circles (G.C.5).
- Apply geometric concepts in modeling situations (G.MG.2-3).

Continued on next page

	Content Elaborations, continued
	CONNECTIONS ACROSS STANDARDS, CONTINUED **Math 2** • Understand similarity (G.SRT.1-3). • Define trigonometric ratios, and solve problems involving right triangles (G.SRT.6-8). • Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.5-6). • Use coordinate geometry (G.GPE.4). • Use coordinates to prove simple geometric theorems algebraically (G.GPE.6). • Find arc lengths and areas of sectors of circles (G.C.5). • Apply geometric concepts in modeling situations (G.MG.2-3). • Write equations in one variable, and use them to solve problems (A.CED.1). • Solve quadratic equations in one variable (A.REI.4).

STANDARDS	MODEL CURRICULUM
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY **Define trigonometric ratios, and solve problems involving right triangles.** **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.8** Solve problems involving right triangles.★ **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2) (+) **b.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ (A2, M3)	**Expectations for Learning-Geometry/Math 2** In middle school, students draw, construct, and describe geometric figures; use informal arguments to establish facts about similar triangles; and explain a proof of the Pythagorean Theorem and its converse. In this cluster, students use similarity to define trigonometric ratios and then solve problems using right triangles (excluding inverse trigonometric functions). The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Because right triangles with the same acute angle are similar, within-figure ratios are equal. Three of these possible ratios are named sine, cosine, and tangent. • The sine of an acute angle is equal to the cosine of its complement and vice versa. • Given an angle and a side length of a right triangle, the triangle can be solved, which means finding the missing sides and angles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Discern and use a pattern or structure. • Plan a solution pathway. • Justify relationships in geometric figures. • Determine reasonableness of results. • Create a drawing and add components as appropriate. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. • Connect mathematical relationships to real-world encounters. *Continued on next page*

	Expectations for Learning-Geometry/Math 2, continued **INSTRUCTIONAL FOCUS** • Define trigonometric ratios for acute angles (sine, cosine, tangent). • Explain and apply the relationship between sine and cosine of complementary angles. • Solve problems involving right triangles (excluding inverses of trigonometric functions). • Use the Pythagorean Theorem to explore exact trigonometric ratios for 30, 45, and 60 degree angles (fluency not required). • Use triangle similarity criteria to define trigonometric ratios. • Given the sine, cosine, or tangent of an angle, find other trigonometric ratios in the triangle. • Solve mathematical and real-world problems given a side and an angle (or the sine, cosine, or tangent of an angle) of a right triangle. *Continued on next page*

	Expectations for Learning-Algebra 2/Math 3 In Geometry/Math 2, students define trigonometric ratios and solve problems using right triangles. In this cluster, students solve problems using right triangles including inverse trigonometric functions. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Inverse trigonometric functions yield an angle when given a trigonometric ratio. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Discern and use a pattern or structure. • Plan a solution pathway. • Determine reasonableness of results. • Create a drawing and add components as appropriate. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. • Connect mathematical relationships to real-world encounters. **INSTRUCTIONAL FOCUS** • Given two sides of a right triangle, use the Pythagorean Theorem and inverse trigonometric functions to solve mathematical and real-world problems. *Continued on next page*

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 3, page 6
- Math 2, Number 4, page 8
- Algebra 2/Math 3, Number 3, page 7

CONNECTIONS ACROSS STANDARDS

Geometry

- Understand similarity (G.SRT.1-3).
- Prove and apply theorems involving similarity (G.SRT.4-5).
- Apply geometric concepts in modeling situations (G.MG.1-3).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.4).

Math 2

- Understand similarity (G.SRT.1-3).
- Prove and apply theorems involving similarity (G.SRT.4-5).
- Apply geometric concepts in modeling situations (G.MG.1-3).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.4).
- Write equations in one variable, and use them to solve problems (A.CED.1).
- Solve quadratic equations in one variable (A.REI.4).

Algebra 2/Math 3

- (+) Apply trigonometry to general triangles (G.SRT.9-11).
- Extend the domain of trigonometric functions using the unit circle (F.TF.1-2).
- Model periodic phenomena with trigonometric functions (F.TF.5).
- Prove and apply trigonometric identities (F.TF.8).
- Interpret key features of trigonometric functions (F.IF.4).

STANDARDS	MODEL CURRICULUM
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY **Apply trigonometry to general triangles.** (+) **G.SRT.9** Derive the formula $A = {}^1/_2\, ab$ sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (+) **G.SRT.10** Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems. (+) **G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.	**Expectations for Learning-Algebra 2/Math 3** In Geometry/Math 2, students define trigonometric ratios and solve problems using right triangles. In this cluster, students solve problems using Laws of Sines and Cosines; explain the proof of the Laws of Sines and Cosines; and derive the area formula $A = \frac{1}{2}ab\ \sin C$. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • The Law of Sines and Law of Cosines can be used to solve non-right triangles. **MATHEMATICAL THINKING** • Discern and use a pattern or structure. • Plan a solution pathway. • Determine reasonableness of results. • Create a drawing and add components as appropriate. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. • Connect mathematical relationships to real-world encounters. **INSTRUCTIONAL FOCUS** • Solve problems involving any triangle. • Explain proofs of the Laws of Sines and Cosines. • Use the Laws of Sines and Cosines to solve problems. • Derive the area formula ($A = \frac{1}{2}ab\ \sin C$) by the use of an auxiliary line. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 3, page 7 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/ Math 3** • Extend the domain of trigonometric functions using the unit circle (F.TF.1-2). • Model periodic phenomena with trigonometric functions (F.TF.5). • Prove and apply trigonometric identities (F.TF.8). • Interpret key features of trigonometric functions (F.IF.4). • Use polynomial identities to solve problems (A.APR.4). • Create equations that describe numbers or relationships (A.CED.1-4). • Understand solving equations as a process of reasoning and explain the reasoning (A.REI.2).

STANDARDS	MODEL CURRICULUM
CIRCLES **Understand and apply theorems about circles.** **G.C.1** Prove that all circles are similar using transformational arguments. **G.C.2** Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G.C.3** Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle. (+) **G.C.4** Construct a tangent line from a point outside a given circle to the circle.	**Expectations for Learning-Geometry** In middle school, students have worked with measurements of circles such as circumference and area. In this cluster, students extend their understanding of similarity to circles. Students solve problems using the relationships among the arcs and angles created by radii, chords, secants, and tangents. They will also construct inscribed and circumscribed circles of a triangle. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • All circles are similar because one circle can be translated so that its center maps onto the center of the other and then is dilated about the common center by the ratio of the radii. • The measure of an arc is equal to the measure of its corresponding central angle. • The measure of an inscribed angle is half the measure of its corresponding central angle. • Inscribed angles on a diameter of a circle are right angles (special case of inscribed angles). • A tangent is perpendicular to the radius at the point of tangency. • A secant is a line that intersects a circle at exactly two points. • A circumscribed angle is created by two tangents to the same circle from the same point outside the circle. • The center of the circumscribed circle is the point of concurrency of the perpendicular bisectors because it is equidistant from the vertices of the triangle. • The center of the inscribed circle is the point of concurrency of the angle bisectors because it is equidistant from the sides of the triangle. • While all triangles can be inscribed in a circle, a quadrilateral can be inscribed in a circle if and only if the opposite angles in the quadrilateral are supplementary. *Continued on next page*

Expectations for Learning-Geometry, continued

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary.
- Make connections between concepts, terms, and properties.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Solve mathematical and real-world problems accurately.
- Determine reasonableness of results.
- Consider mathematical units involved in a problem.
- Make sound decisions about using tools.

INSTRUCTIONAL FOCUS

- Use transformational arguments to prove that all circles are similar.
- Given a diagram, identify radii, chords, secants, and tangents, and the arcs and angles formed by them.
- Solve mathematical and real-world problems involving angles and arcs formed by radii, chords, secants, and tangents.
- Construct the angle bisectors of a triangle to locate the incenter, and then construct the inscribed circle.
- Construct the perpendicular bisectors of a triangle to locate the circumcenter, and then construct the circumscribed circle.
- Provide an informal argument for why the opposite angles of an inscribed quadrilateral are supplementary based on the arcs the angles intercept and their corresponding central angles.
- Solve problems using the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.
- (+) Construct a tangent line from a point outside a given circle to the circle.

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	Expectations for Learning-Math 1 In middle school, students have worked with measurements of circles such as circumference and area. In this cluster, students solve problems using the relationships among the arcs and angles created by radii, chords, secants, and tangents. They will also construct inscribed and circumscribed circles of a triangle. In Math 2, students extend their understanding of similarity to circles. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • The measure of an arc is equal to the measure of its corresponding central angle. • The measure of an inscribed angle is half the measure of its corresponding central angle. • Inscribed angles on a diameter of a circle are right angles (special case of inscribed angles). • A tangent is perpendicular to the radius at the point of tangency. • A secant is a line that intersects a circle at exactly two points • A circumscribed angle is created by two tangents to the same circle from the same point outside the circle. • The center of the circumscribed circle is the point of concurrency of the perpendicular bisectors because it is equidistant from the vertices of the triangle. • The center of the inscribed circle is the point of concurrency of the angle bisectors because it is equidistant from the sides of the triangle. • While all triangles can be inscribed in a circle, a quadrilateral can be inscribed in a circle if and only if the opposite angles in the quadrilateral are supplementary. *Continued on next page*

Expectations for Learning-Math 1, continued

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary.
- Make connections between concepts, terms, and properties.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Solve mathematical and real-world problems accurately.
- Determine reasonableness of results.
- Consider mathematical units involved in a problem.
- Make sound decisions about using tools.

INSTRUCTIONAL FOCUS

- Given a diagram, identify radii, chords, secants, and tangents, and the arcs and angles formed by them.
- Solve mathematical and real-world problems involving angles and arcs formed by radii, chords, secants, and tangents.
- Construct the angle bisectors of a triangle to locate the incenter, and then construct the inscribed circle.
- Construct the perpendicular bisectors of a triangle to locate the circumcenter, and then construct the circumscribed circle.
- Provide an informal argument for why the opposite angles of an inscribed quadrilateral are supplementary based on the arcs the angles intercept and their corresponding central angles.
- Solve problems using the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.
- (+) Construct a tangent line from a point outside a given circle to the circle.

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	Expectations for Learning-Math 2 In middle school, students have worked with measurements of circles such as circumference and area. In Math 1, students solve problems using the relationships among the arcs and angles created by radii, chords, and tangents. They construct inscribed and circumscribed circles of a triangle. In this course, they extend their understanding of similarity to circles. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • All circles are similar as follows: translate one circle so that its center maps onto the center of the other, and then dilate about the common center by the ratio of the radii. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary. • Make connections between concepts, terms, and properties. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Solve mathematical and real-world problems accurately. • Determine reasonableness of results. • Consider mathematical units involved in a problem. • Make sound decisions about using tools. **INSTRUCTIONAL FOCUS** • Use transformational arguments to prove that all circles are similar. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 5, page 8 • Math 1, Number 7, page 12 • Math 2, Number 5, page 9 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Experiment with transformations in the plane (G.CO.1). • Make geometric constructions (G.CO.12). • Understand similarity in terms of similarity transformations (G.SRT.2). • Find arc length and areas of sectors of circles (G.C.5). **Math 1** • Experiment with transformations in the plane (G.CO.1). • Make geometric constructions (G.CO.12). • Reason quantitatively, and use units to solve problems (N.Q.1-3). **Math 2** • Understand similarity in terms of similarity transformations (G.SRT.2). • Find arc length and areas of sectors of circles (G.C.5).

STANDARDS	MODEL CURRICULUM
CIRCLES **Find arc lengths and areas of sectors of circles.** **G.C.5** Find arc lengths and areas of sectors of circles. **a.** Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. **b.** Derive the formula for the area of a sector, and use it to solve problems. **G.C.6** Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)	**Expectations for Learning-Geometry/Math 2** In middle school, students are limited to working with measurements of circles such as circumference and area. This cluster spans Geometry/ Math 2 and Algebra 2/ Math 3. In Geometry/Math 2, students are using part-to-whole proportional reasoning to find arc lengths and sector areas, in which the arc or central angle is measured in degrees. In Algebra 2/ Math 3, students derive and use formulas relating degree and radian measure. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). *Note: Since in Algebra 1/Math 2 students focus on quadratics with leading coefficients of 1 with occasional uses of other simple coefficients, geometry standards should only apply to equations where the squared terms have a coefficient of 1 or occasionally other simple leading coefficients.* **ESSENTIAL UNDERSTANDINGS** • A central angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. • The measure of an arc is equal to the measure of the corresponding central angle and is expressed in degrees, while the length of an arc is expressed in units of linear measure. • The arc length is a part of the circumference of a circle. • The ratio of the central angle to 360 degrees is equal to the ratio of the length of the arc to the circumference of the circle. • The sector area is a part of the area of a circle. • The ratio of the central angle to 360 degrees is equal to the ratio of the area of the sector to the area of the circle. • Because all circles are similar, if the radius of the circle is scaled by k, the corresponding arc length is multiplied by k and the sector area is multiplied by k^2. *Continued on next page*

Expectations for Learning-Geometry/Math 2, continued

MATHEMATICAL THINKING

- Consider mathematical units involved in a problem.
- Make connections between concepts and terms.
- Generalize concepts based on patterns.
- Use proportional reasoning (part to whole).
- Draw a picture to make sense of a problem.
- Solve real-world and mathematical problems accurately.
- Plan a solution pathway.
- Attend to the meaning of quantities.

INSTRUCTIONAL FOCUS

- Develop understanding of the formulas for arc length and area of a sector through derivation.
- Solve problems using arc lengths and areas of sectors of circles.

Continued on next page

	Expectations for Learning-Algebra 2/Math 3 In middle school, students are limited to working with measurements of circles such as circumference and area. This cluster spans Geometry/Math 2 and Algebra 2/Math 3. In Geometry/Math 2, students are using part-to-whole proportional reasoning to find arc lengths and sector areas, in which the arc or central angle is measured in degrees. In Algebra 2/Math 3, students derive and use formulas relating degree and radian measure. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Measures of angles and of arcs can be expressed in degrees or radians. • Converting between degrees and radians is an application of proportional reasoning. **MATHEMATICAL THINKING** • Consider mathematical units involved in a problem. • Make connections between concepts and terms. • Generalize concepts based on patterns. • Use proportional reasoning (part-to-whole). • Draw a picture to make sense of a problem. • Solve real-world and mathematical problems accurately. • Plan a solution pathway. • Attend to the meaning of quantities. **INSTRUCTIONAL FOCUS** • Define radian measure. • Derive the formulas that relate degrees and radians. • Convert between radians and degrees. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 5, page 8 • Math 2, Number 5, page 9 • Algebra 2/Math 3, Number 3, page 7 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Understand and apply theorems about circles (G.C.1-2). • Experiment with transformations in the plane (G.CO.1). • Explain volume formulas, and use them to solve problems (G.GMD.1). • Understand similarity in terms of similarity transformations (G.SRT.2). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). **Math 2** • Understand and apply theorems about circles (G.C.1). • Explain volume formulas, and use them to solve problems (G.GMD.1). • Understand similarity in terms of similarity transformations (G.SRT.2). • Create equations that describe numbers or relationships (A.CED.1, 2, 4). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). **Algebra 2/Math 3** • Extend the domain of trigonometric functions using the unit circle (F.TF.1-2). • Model periodic phenomena with trigonometric functions (F.TF.5). • (+) Extend the domain of trigonometric functions using the unit circle (F.TF.3). • Create equations that describe numbers or relationships (A.CED.1, 2, 4).

STANDARDS	MODEL CURRICULUM
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS **Translate between the geometric description and the equation for a conic section.** **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (+) **G.GPE.2** Derive the equation of a parabola given a focus and directrix. (+) **G.GPE.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	**Expectations for Learning-Geometry/Math 2** In middle school, students use the Pythagorean Theorem to find distances between points within the coordinate system. In the high school, students complete the square to solve quadratic equations. In this cluster, students derive the equation of a circle using the Pythagorean Theorem. They also complete the square to find the center and radius of a circle. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • The equation of a circle relates a fixed center, a fixed radius, and a set of variable points, which are the points on the circle. • Just as the distance formula is an application of the Pythagorean Theorem, so is the equation of a circle. **MATHEMATICAL THINKING** • Use precise mathematical language. • Discern and use a pattern or structure. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Justify relationships in geometric figures. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. **INSTRUCTIONAL FOCUS** • Use the Pythagorean Theorem to derive the equation of a circle. • Given the equation of a circle that is not in standard form, find the center and radius of the circle by completing the square. *Continued on next page*

Expectations for Learning-Fourth Year Course

In the high school, students derive the equation of a circle using the Pythagorean Theorem and develop equation solving strategies to support manipulating quadratic equations. Now in this cluster, students use the geometric definitions of a parabola, ellipse, and hyperbola to derive their equations.

The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction).

ESSENTIAL UNDERSTANDINGS

- (+) Circles, parabolas, ellipses, and hyperbolas are called conic sections because they are two-dimensional cross sections of cones in three dimensions.

MATHEMATICAL THINKING

- Use precise mathematical language.
- Discern and use a pattern or structure.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Justify relationships in geometric figures.
- Use technology strategically to deepen understanding.
- Solve routine and straightforward problems accurately.

INSTRUCTIONAL FOCUS

- (+) Derive the equation of a parabola, an ellipse, and a hyperbola.

Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 5, page 8 • Math 2, Number 5, page 9 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Prove that all circles are similar (G.C.1). • Prove theorems about triangles (G.SRT.4). **Math 2** • Solve equations and inequalities in one variable (A.REI.4). • Prove that all circles are similar (G.C.1). • Prove theorems about triangles (G.SRT.4). • Write equations in two variables, and use them to solve problems (A.CED.2). • Solve quadratic equations in one variable (A.REI.4). • Solve systems of linear and quadratic equations (A.REI.7).

STANDARDS	MODEL CURRICULUM
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS **Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.** **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2) **G.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. **G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	**Expectations for Learning-Geometry** In middle school, students find the distance between two points in a coordinate system; work with linear functions; solve linear equations; and apply the Pythagorean Theorem in the coordinate system. In addition, they use square root symbols to represent solutions to equations, and they evaluate square roots of rational numbers. In this cluster, students use the coordinate system to justify slope criteria for parallel and perpendicular lines; partition line segments proportionally; and compute perimeters and areas of geometric figures. These strategies are used for proof of geometric relationships and properties. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • Coordinate proof is a method that uses algebraic techniques to prove geometric theorems and properties. • Properties of geometric figures, especially special quadrilaterals, can be proven on a coordinate plane using lengths of segments, slopes of lines, and equations of lines. • Coordinate proof can be used to prove that figures are congruent or similar. • The slopes of parallel lines are equal, and the product of the slopes of perpendicular lines is –1. • Partitioning a line segment into a given ratio is an application of similar triangles. *Continued on next page*

	Expectations for Learning-Geometry, continued **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Make connections between terms and formulas. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Compute using strategies or models. • Use formal reasoning with symbolic representation. • Determine reasonableness of results. • Solve multi-step problems accurately. • Discern and use a pattern or structure. **INSTRUCTIONAL FOCUS** • Know and use the distance formula. • Use coordinates to prove simple geometric theorems algebraically. • Verify geometric relationships algebraically. • Justify the slope criteria for parallel and perpendicular lines. • Write equations of parallel and perpendicular lines. • Solve geometric problems using the slopes of parallel and perpendicular lines. • Partition a line segment given a ratio. • Use coordinates to compute perimeters and areas. *Continued on next page*

	Expectations for Learning-Math 1 In middle school, students find the distance between two points in a coordinate system; work with linear functions; solve linear equations; and apply the Pythagorean Theorem in the coordinate system. In addition, they use square root symbols to represent solutions to equations, and they evaluate square roots of rational numbers. In this cluster, students use the coordinate system to justify slope criteria for parallel and perpendicular lines and compute perimeters and areas of geometric figures. In Math 2, these strategies are used for proof of geometric relationships and properties and partitioning line segments. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • Coordinate proof is a method that uses algebraic techniques to prove geometric theorems and properties. • The slopes of parallel lines are equal, and the product of the slopes of perpendicular lines is -1. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Make connections between terms and formulas. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Compute using strategies or models. • Determine reasonableness of results. • Solve multi-step problems accurately. • Discern and use a pattern or structure. *Continued on next page*

Expectations for Learning-Math 1, continued

INSTRUCTIONAL FOCUS

- Know and use the distance formula.
- Justify the slope criteria for parallel and perpendicular lines.
- Write equations of parallel and perpendicular lines.
- Solve geometric problems using the slopes of parallel and perpendicular lines.
- Use coordinates to compute perimeters and areas.

Expectations for Learning–Math 2

In middle school, students find the distance between two points in a coordinate system; work with linear functions; solve linear equations; and apply the Pythagorean Theorem in the coordinate system. In addition, they used square root symbols to represent solutions to equations, and they evaluate square roots of rational numbers. In Math 1, students use the coordinate system to justify slope criteria for parallel and perpendicular lines and compute perimeters and areas of geometric figures. In this course, these strategies are used for proof of geometric relationships with respect to properties of figures and partitioning line segments.

The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction).

Note: In Math 2, revisit G.CO.10-11 with respect to coordinate and transformational proofs. Since G.GPE.4 is no longer placed in Math 1, students may not have had exposure to these types of proofs for that content.

ESSENTIAL UNDERSTANDINGS

- Coordinate proof is a method that uses algebraic techniques to prove geometric theorems and properties.
- Properties of geometric figures, especially special quadrilaterals, can be proven on a coordinate plane using lengths of segments, slopes of lines, and equations of lines.
- Coordinate proof can be used to prove that figures are congruent or similar.
- Partitioning a line segment into a given ratio is an application of similar triangles.

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Expectations for Learning-Math 2, continued

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to represent geometric relationships.
- Make connections between terms and formulas.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Compute using strategies or models.
- Use formal reasoning with symbolic representation.
- Determine reasonableness of results.
- Solve multi-step problems accurately.
- Discern and use a pattern or structure.

INSTRUCTIONAL FOCUS

- Know and use the distance formula.
- Use coordinates to prove simple geometric theorems algebraically.
- Verify geometric relationships algebraically.
- Partition a line segment given a ratio.

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Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 4, page 7
- Geometry, Number 5, page 8
- Math 1, Number 6, page 11
- Math 2, Number 4, page 8
- Math 2, Number 5, page 9

CONNECTIONS ACROSS STANDARDS

Geometry

- Know precise definitions (G.CO.1).
- Prove geometric theorems (G.CO.9-10).
- Understand and apply theorems about circles (G.C.1-4).
- Prove theorems involving similarity (G.SRT.4-5).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6).
- Apply geometric concepts in modeling situations (G.MG.3).

Math 1

- Know precise definitions (G.CO.1).
- Prove geometric theorems (G.CO.9-10).
- Understand and apply theorems about circles (G.C.2-4).
- Represent and solve equations graphically (A.REI.10).
- Create equations that describe numbers or relationships (A.CED.2, 4).
- Relate parallel and perpendicular lines as a system of equations (A.REI.5).

Math 2

- Understand and apply theorems about circles (G.C.1, 5).
- Prove theorems involving similarity (G.SRT.4-5).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6).
- Create equations that describe numbers or relationships (A.CED.4).
- Represent and solve equations graphically (A.REI.4).
- Apply geometric concepts in modeling situations (G.MG.3).

STANDARDS	MODEL CURRICULUM
GEOMETRIC MEASUREMENT AND DIMENSION **Explain volume formulas, and use them to solve problems.** **G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* (+) **G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	**Expectations for Learning-Geometry/Math 2** In middle school, students use established circumference, area, and volume formulas for two- and three-dimensional figures. Instead of using area and volume formulas rotely, students in this cluster give informal justifications for these formulas and use them to solve problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • A three-dimensional solid can be viewed as a stack of layers. • If all of the layers of a three-dimensional solid have the same area, then the volume is the area of the base times the height. • The volume remains unchanged when layers parallel to the base in a three-dimensional solid are shifted. • A cone's volume is $\frac{1}{3}$ of the volume of a cylinder if their base areas are equal and their heights are congruent. • A pyramid's volume is $\frac{1}{3}$ of the volume of a prism if their base areas are equal and their heights are congruent. • Volume, like area, is additive, so to find the volume of a composite figure, cut the figure into pieces of known volume, and add or subtract as appropriate. • The cross sections of a cylinder are circles of equal area. • The cross sections of a prism are congruent to the base, so therefore the areas are equal. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Make and modify a model to represent mathematical thinking. • Attend to meaning of quantities. • Consider mathematical units involved in a problem. • Solve real-world and mathematical problems accurately. • Determine reasonableness of results. • Use informal reasoning.

Expectations for Learning-Geometry/Math 2, continued

INSTRUCTIONAL FOCUS

- Explain and justify the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone.
- Apply volume formulas in real-world and mathematical problems.
- (+) Informally apply Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 3, page 6
- Math 2, Number 6, page 10

CONNECTIONS ACROSS STANDARDS

Geometry

- Experiment with transformations in the plane (G.CO.1).
- Understand and apply theorems about circles (G.C.2, 5).
- Understand the relationships between lengths, areas, and volumes (G.GMD.5-6).
- Apply geometric concepts in modeling situations (G.MG.1-3).

Math 2

- Understand and apply theorems about circles (G.C.5).
- Understand the relationships between lengths, areas, and volumes (G.GMD.5-6).
- Apply geometric concepts in modeling situations (G.MG.1-3).
- Create equations that describe numbers or relationships (A.CED.2, 4).

STANDARDS	MODEL CURRICULUM
GEOMETRIC MEASUREMENT AND DIMENSION **Visualize relationships between two-dimensional and three-dimensional objects.** **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**Expectations for Learning-Geometry/Math 2** In middle school, students identify cross-sections as a result of slicing right rectangular prisms and pyramids. In this cluster, which supports the previous cluster, students extend the identification of cross-sections to include other three-dimensional solids. Students will also identify three-dimensional objects created when a two-dimensional object is rotated about a line. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Two-dimensional figures can be used to understand three-dimensional solids. • A three-dimensional figure can be created by rotating a two-dimensional figure about a line. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Use technology strategically to deepen understanding. • Make connections between concepts, terms, and properties. **INSTRUCTIONAL FOCUS** • Identify two-dimensional cross-sections of three-dimensional objects. • Identify three-dimensional objects formed by rotations of two-dimensional objects. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 3, page 6 • Math 2, Number 6, page 10 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Experiment with transformations in the plane (G.CO.1, 5). • Understand congruence in terms of rigid motions (G.CO.6). • Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). • Apply geometric concepts in modeling situations (G.MG.1-3). • Classify and analyze geometric figures (G.CO.14). **Math 2** • Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). • Apply geometric concepts in modeling situations (G.MG.1-3).

STANDARDS	MODEL CURRICULUM
GEOMETRIC MEASUREMENT AND DIMENSION **Understand the relationships between lengths, areas, and volumes.** **G.GMD.5** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. **G.GMD.6** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.	**Expectations for Learning-Geometry/Math 2** In middle school, students solve problems involving two-dimensional similar figures and calculate the volumes of three-dimensional figures. In this cluster, students extend their knowledge of similarity to explore and understand how changes to length or angle measure in one figure will result in similar or non-similar figures. Students will also understand the effect that a scale factor has on the length, area, and volume of similar figures and use this relationship to solve problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Changes to the lengths and/or angle measures of a figure result in similar and non-similar figures. • When changes to a figure result in similar figures with a scale factor of k, the lengths of the resulting figures are a multiple of k. • When changes to a figure result in similar figures with a scale factor of k, the areas of the resulting figures are a multiple of k^2. • When changes to a figure result in similar figures with a scale factor of k, the volume of the resulting figures are a multiple of k^3. **MATHEMATICAL THINKING** • Use precise mathematical language. • Draw a picture or create a model to make sense of a problem. • Determine reasonableness of results. • Solve multi-step problems accurately. • Plan a solution pathway. • Solve mathematical and real-world problems accurately. • Consider mathematical units involved in a problem. • Attend to the meaning of quantities. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Generalize concepts based on patterns. *Continued on next page*

Expectations for Learning-Geometry/Math 2, continued

INSTRUCTIONAL FOCUS

- Classify objects as similar or non-similar when the lengths or angles of figures are changed.
- Explain the types of changes to a figure that result in similar and non-similar figures.
- Use geometry and algebra to explain how length, area, and volume are affected when scaling is applied.
- Solve problems involving length, area, and volume of figures under scaling.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 6, page 9
- Math 2, Number 6, page 10

CONNECTIONS ACROSS STANDARDS

Geometry

- Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3).
- Understand similarity in terms of similarity transformations (G.SRT.1-2).
- Apply geometric concepts in modeling situations (G.MG.2-3).

Math 2

- Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3).
- Understand similarity in terms of similarity transformations (G.SRT.1-2).
- Apply geometric concepts in modeling situations (G.MG.2-3).
- Create equations that describe numbers or relationships (A.CED.2, 4).
- Solve quadratic equations in one variable (A.REI.4).
- Build a function that models a relationship between two quantities (F.BF.1a).

STANDARDS	MODEL CURRICULUM
MODELING WITH GEOMETRY **Apply geometric concepts in modeling situations.** **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★ **G.MG.2** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★ **G.MG.3** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★	**Expectations for Learning-Geometry/Math 2** In middle school, students work with nets, area, and volume; use appropriate tools to represent situations; and solve real-life and mathematical problems. In this cluster, students make sense of the world around them by using geometric models and their properties to solve more sophisticated problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Composite figures can be analyzed by approximating them with traditional two- and three-dimensional figures. • Many real-life scenarios are related to length, area, and volume. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Make connections between terms and properties. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Use formal reasoning with symbolic representation. • Determine reasonableness of results. • Use proportional reasoning. • Plan a solution pathway. • Connect mathematical relationships to real-world encounters. • Draw a picture or create a model to represent a problem. **INSTRUCTIONAL FOCUS** • Use geometric shapes, their measures, and their properties to describe objects. • Identify useful quantities for modeling situations. • Apply concepts of density based on area and volume. • Solve design problems geometrically. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, page 5 • Geometry, Number 3, page 6 • Geometry, Number 5, page 8 • Math 2, Number 6, page 10 **CONNECTIONS ACROSS STANDARDS** **Geometry** • Solving problems involving right triangles (G.SRT.8). • Use volume formulas to solve problems (G.GMD.3-4, 6). • Use coordinates to compute perimeters and areas (G.GPE.7). **Math 2** • Solving problems involving right triangles (G.SRT.8). • Use volume formulas to solve problems (G.GMD.3-4, 6). • Model a relationship given a verbal description (F.IF.4). • Show key features of a function (F.IF.7). • Write a function that describes a relationship between two quantities (F.BF.1). • Interpret parts of an expression (A.SSE.1). • Write expressions in equivalent forms (A.SSE.3). • Create equations to describe relationships (A.CED.1-2, 4).

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