Ohio's Model Curriculum | Mathematics

with Instructional Supports

Algebra 2/Math 3 Course

Ohio | Department of Education

Mathematics Model Curriculum
with Instructional Supports
Algebra 2/Math 3 Course

TABLE OF CONTENTS

Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

Standards for Mathematical Practice—Algebra 2/Math 3

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.
Students apply their understanding of various functions to real-world problems. They approach complex mathematics problems and break them down into smaller problems, synthesizing the results when presenting solutions.

MP.2 Reason abstractly and quantitatively.
Students deepen their understanding of transformations of graphs by changing the form of rational function $f(x) = \frac{a(x)}{b(x)}$ where $a(x)$ and $b(x)$ represent polynomials and $b(x)$ is not 0, to reveal and interpret the key features of the function.

MP.3 Construct viable arguments and critique the reasoning of others.
Students continue to reason through the solution of an equation and justify their reasoning to their peers. Students defend their choice of a function when modeling a real-world situation.

MP.4 Model with mathematics.
Students apply their new mathematical understanding to real-world problems, making use of their expanding repertoire of functions in modeling. Students also discover mathematics through experimentation and by examining patterns in data from real-world contexts.

MP.5 Use appropriate tools strategically.
Students continue to use graphing technology to deepen their understanding of the behavior of polynomial, rational, square root, and trigonometric functions.

MP.6 Attend to precision.
Students make note of the precise definition of complex number, understanding that real numbers are a subset of complex numbers. They pay attention to units in real-world problems and use unit analysis as a method for verifying their answers.

MP.7 Look for and make use of structure.
Students see the operations of complex numbers as extensions of the operations for real numbers. They understand the periodicity of sine and cosine and use these functions to model periodic phenomena.

Continued on next page

Standards for Mathematical Practice

MP.8 Look for and express regularity in repeated reasoning.

Students observe a pattern that powers of the imaginary number i cycles through the same four outcomes, i, –1, $-i$ and 1, since i^4 = 1 and any power of i with an integer exponent that is a multiple of 4 has a value 1.

$i = i$	$i^5 = i$
$i^2 = -1$	$i^6 = -1$
$i^3 = -i$	$i^7 = -i$
$i^4 = 1$	$i^8 = 1$

Students use this observation to make a conjecture about any power of i.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

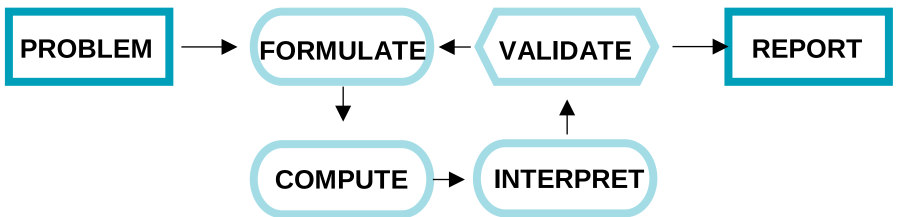
In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

Continued on next page

Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum

with Instructional Supports

Algebra 2/Math 3 Course

STANDARDS	MODEL CURRICULUM
Number and Quantity **THE REAL NUMBER SYSTEM** **Extend the properties of exponents to rational exponents.** **N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**Expectations for Learning** In previous grades, students learn and apply the properties of integer exponents to generate equivalent numerical expressions. Now students will discover that those properties hold for rational exponents as well. In later courses, they may use this understanding to explore exponential functions with continuous domains. **ESSENTIAL UNDERSTANDINGS** • Radicals can be expressed in terms of rational exponents. • The properties of integer exponents hold for rational exponents. • The "properties of radicals" can be derived from the definitions that relate radicals to rational exponents and the properties of exponents. **MATHEMATICAL THINKING** • Use properties of exponents flexibly. • Discern and use a pattern or structure. • Generalize concepts based on patterns. • Use formal reasoning with symbolic representation. **INSTRUCTIONAL FOCUS** • Rewrite expressions between radical form and rational exponent form. • Explain the definition of a radical using properties of rational exponents. • Use the properties of radicals to rewrite a radical in an equivalent form. • Add, subtract, multiply, and divide radicals. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Solve simple rational and radical equations in one variable (A.REI.2). • Analyze functions using different representations (F.IF.7-8). • Use the properties of exponents to transform expressions for exponential functions (A.SSE.3). • Build a function that models a relationship between two quantities (F.BF.1, 4). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM

Number and Quantity
THE REAL NUMBER SYSTEM
Use properties of rational and irrational numbers.
N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
	Expectations for Learning
	In previous courses, students perform arithmetic operations with rational and irrational numbers. Now they generalize the results of operations on rational and irrational numbers. The emphasis is on developing mathematical reasoning habits that can be used in numerous future situations.
	
	ESSENTIAL UNDERSTANDINGS
	• The sum or product of two rational numbers is rational.
	• The sum of a rational number and an irrational number is irrational.
	• The product of a nonzero rational number and an irrational number is irrational.
	
	MATHEMATICAL THINKING
	• Use informal reasoning.
	• Make and analyze mathematical conjectures.
	• Explain mathematical reasoning.
	
	INSTRUCTIONAL FOCUS
	• Explain why the following statements are always true and provide examples to illustrate the following:
	  o The sum or product of two rational numbers is rational.
	  o The sum of a rational number and an irrational number is irrational.
	  o The product of a nonzero rational number and an irrational number is irrational.
	
	Content Elaborations
	OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS
	• Algebra 2/Math 3, Number 2, pages 4-6
	
	CONNECTIONS ACROSS STANDARDS
	• Extend the properties of exponents to rational exponents (N.RN.1-2).
	• Perform arithmetic operations on polynomials (A.APR.1).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Number and Quantity **THE COMPLEX NUMBER SYSTEM** **Perform arithmetic operations with complex numbers.** **N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	**Expectations for Learning** In previous courses, students perform operations with polynomial and solve quadratic equations with real solutions. The expansion to the set of complex numbers allows students to find complex solution of the quadratic equations. Now in Algebra 2/Math 3 they extend this knowledge to recognize and rewrite complex numbers and expressions, using properties of polynomial operations. **ESSENTIAL UNDERSTANDINGS** • Complex numbers exist as a solution to a quadratic equation when taking the square root of a negative number. • Any number can be written as a complex number with a real and an imaginary component. • Complex numbers can be rewritten using polynomial operations (addition, subtraction, multiplication) and properties of exponents. • Associative, commutative, and distributive properties apply to rewriting complex numbers. **MATHEMATICAL THINKING** • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Generalize concepts based on patterns (repeated reasoning). • Use different properties of operations flexibly. **INSTRUCTIONAL FOCUS** • Write a complex number in the form *a* + *bi*, identifying the real and imaginary components. • Perform operations with complex numbers (addition, subtraction, multiplication). • Rewrite powers of imaginary numbers utilizing properties of exponents and the definition: $i^2 = -1$. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Solve quadratic equations with real coefficients that have complex solutions (N.CN.7). • Rewrite expressions involving radicals and rational exponents using the properties of exponents (N.RN.2). • Perform arithmetic operations on polynomials (A.APR.1). • (+) Extend polynomial identities to the complex numbers (N.CN.8).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Number and Quantity **THE COMPLEX NUMBER SYSTEM** **Use complex numbers in polynomial identities and equations.** **N.CN.7** Solve quadratic equations with real coefficients that have complex solutions. (+) **N.CN.8** Extend polynomial identities to the complex numbers*. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* (+) **N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	**Expectations for Learning** Previously, students solved quadratic equations with real solutions. Now they will be extending this knowledge to solve quadratic equations with complex solutions. Advanced students will extend polynomial identities to the complex numbers and will know and apply the Fundamental Theorem of Algebra. *Note: N.CN.8 and N.CN.9 are not required for all students but are intended for students who are pursuing advanced mathematics.* **ESSENTIAL UNDERSTANDINGS** • Complex numbers exist as a solution to a quadratic equation when taking the square root of a negative number. • (+) Polynomial identities extend to complex numbers. • (+) The Fundamental Theorem of Algebra holds for quadratic polynomials. **MATHEMATICAL THINKING** • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • (+) Use properties of operations flexibly. **INSTRUCTIONAL FOCUS** • Solve a quadratic equation with a complex solution. • (+) Perform operations with complex numbers (addition, subtraction, multiplication). • (+) Extend polynomial identities to the complex numbers. • (+) Know and apply the Fundamental Theorem of Algebra. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Perform arithmetic operations with complex numbers (N.CN.1-2). • Perform arithmetic operations on polynomials (A.APR.1). • (+) Understand the relationship between zeros and factors of polynomials (A.APR.2-3). • (+) Using polynomial identities to solve problems (A.APR.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **SEEING STRUCTURE IN EXPRESSIONS** **Interpret the structure of expressions.** **A.SSE.1**. Interpret expressions that represent a quantity in terms of its context. ★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, to factor* $3x(x - 5) + 2(x - 5)$*, students should recognize that the "*$x - 5$*" is common to both expressions being added, so it simplifies to* $(3x + 2)(x - 5)$*; or see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*	**Expectations for Learning** Students build expressions in grades K-5 with arithmetic operations. As they move into the middle grades and progress through high school, students build expressions with algebraic components, beginning with linear, exponential, and quadratic expressions. In later courses, students may build algebraic expressions with logarithmic expressions. In this cluster, they focus on interpreting the components of expressions with polynomial, rational, radical, and trigonometric expressions and their meaning in mathematical and real-world contexts. They also determine when rewriting or manipulating expressions is helpful in order to reveal different insights into a mathematical or real-world context. **ESSENTIAL UNDERSTANDINGS** • An expression is a collection of terms separated by addition or subtraction. • A term is a product of a number and a variable raised to an nonnegative integer exponent. • Components of an expression or expressions within an equation may have meaning in a mathematical context, e.g., $b^2 - 4ac$ in the quadratic formula indicates the number and nature of solutions to the equation. • Components of an expression may have meaning in a real-world context, e.g., $5{,}000(1.07)^t$ can be used to model compound interest formula with a 7% interest rate and $5,000 principal. • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. **MATHEMATICAL THINKING** • Attend to the meaning of quantities. • Use precise mathematical language. • Apply grade-level concepts, terms, and properties. • Look for and make use of structure. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Identify the components, such as terms, factors, or coefficients, of an expression and interpret their meaning in terms of a mathematical or real-world context. • Explain the meaning of each part of an expression, including linear, exponential, quadratic, polynomial, rational, radical, and trigonometric expressions, in a mathematical or real-world context. • Analyze an expression and recognize that it can be rewritten in different ways. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Write expressions in equivalent forms (A.SSE.4). • Rewrite rational expressions in different forms (A.APR.6). • Create equations in one or two variables (A.CED.1, A.CED.2). • Combine standard function types using arithmetic operations (F.BF.1b).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Algebra **SEEING STRUCTURE IN EXPRESSIONS** **Write expressions in equivalent forms to solve problems.** **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.* (+) **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★	**Expectations for Learning** In previous courses, students write arithmetic and geometric sequences both recursively and with a formula, connecting them with linear and exponential functions, respectively. In this cluster, (+) students derive the formula for the sum of finite geometric series. *Note: Standard A.SSE.4 is not required for all students, but is intended for students who are pursing advanced mathematics coursework.* **ESSENTIAL UNDERSTANDINGS** • Understanding the properties of exponents is essential for rewriting exponential expressions. • (+) Geometric series can model real-world phenomena, such as determining mortgage payments. **MATHEMATICAL THINKING** • Discern and use a pattern or a structure. • Solve real-world problems accurately. • Use different properties of operations flexibly. • (+) Generalize concepts based on patterns. **INSTRUCTIONAL FOCUS** • Rewrite exponential expressions by using properties of exponents. • (+) Derive the formula for the sum of a finite geometric series. • (+) Use the formula for the sum of a finite geometric series to solve problems in real-world contexts. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Rewrite expressions using rational exponent properties (N.RN.1-2). • Factors are used to find zeros of polynomials (A.APR.2-3). • Use properties of exponents to interpret expressions for exponential functions (F.IF.8b). • Graph exponential functions (F.IF.7f). • Exponential equations can be written as logarithmic equations (F.LE.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Algebra **ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS** **Perform arithmetic operations on polynomials.** **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **b.** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)	**Expectations for Learning** In previous courses, students simplify linear and quadratic expressions using the operations of addition, subtraction, and multiplication. In this cluster, extend these ideas to include polynomial expressions of higher degree. **ESSENTIAL UNDERSTANDINGS** • Polynomials form a system (like the integers) in which addition, subtraction, and multiplication always result in another polynomial, but sometimes division does not. **MATHEMATICAL THINKING** • Compute accurately and efficiently. • Use different properties of operations flexibly. • Recognize and apply mathematical concepts, terms, and their properties. **INSTRUCTIONAL FOCUS** • Add, subtract, and multiply polynomial expressions, focusing on those of degree greater than two. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Use properties of exponents (N.RN.2). • Interpret the structure of expressions (A.SSE.1). • Use polynomial operations to examine the relationship between zeros and factors of polynomials (A.APR.2-5).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS** **Understand the relationship between zeros and factors of polynomials.** **A.APR.2** Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$. In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. **A.APR.3** Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Expectations for Learning** In previous courses, students learn to factor quadratic expressions and divide integers using long division. In this cluster, students apply these concepts to understand and use the Remainder Theorem for polynomials. Students will also identify zeros of polynomial functions and use the zeros, along with concepts of end behavior and *y*-intercepts from F.IF.4 and F.IF.7, to sketch graphs of the functions. **ESSENTIAL UNDERSTANDINGS** • The division of polynomials is analogous to the division of integers, yielding quotients and remainders. • If a polynomial $p(x)$ is divided by $(x - a)$, the remainder is the constant $p(a)$. • $p(a) = 0$, if and only if $(x - a)$ is a factor of $p(x)$. • The real solutions of the polynomial equation $p(x) = 0$ are the zeros of the function $p(x)$, and are the x-intercepts of the graph $y = p(x)$. • The zeros of a polynomial function can help in constructing a rough sketch of its graph. • An nth degree polynomial function has at most n zeros. **MATHEMATICAL THINKING** • Solve multi-step mathematical problems accurately. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Divide polynomials using long division. Using synthetic division is discouraged until fluency in long division is achieved and the relationship between the two methods is understood.
- Rewrite a function as a sum of the remainder and the product of the quotient and divisor (If $p(x)$ is divided by $(x – a)$, then $p(x) = (x – a)q(x) + r$).
- Explain why when a polynomial $p(x)$ is divided by a linear polynomial $(x – a)$, the remainder, r, is equal to $p(a)$. Note the special case: When $(x – a)$ is a factor of $p(x)$, then $p(a) = 0$.
- Factor a polynomial expression to identify the zeros of the function it defines.
- Using the zeros, make a rough sketch of the graph of a polynomial function with consideration of end behavior, the y-intercept, and perhaps other points.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 2/Math 3, Number 2, pages 4-6

CONNECTIONS ACROSS STANDARDS

- Interpret the structure of expressions (A.SSE.1b).
- Perform arithmetic operations on polynomials (A.APR.1).
- Analyze functions using different representations (F.IF.7).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Algebra **ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS** **Use polynomial identities to solve problems.** **A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* (+) **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example by using coefficients determined for by Pascal's Triangle.* The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.	**Expectations for Learning** In previous courses, students examine writing an algebraic expression in different but equivalent forms. In this cluster, students prove polynomial identities based on numerical relationships. Students pursuing advanced mathematics courses (+) apply the Binomial Theorem and prove it by induction or a combinatorial argument. *Note: A.APR.5 is not required for all students, but is intended for students who are pursuing advanced mathematics.* **ESSENTIAL UNDERSTANDINGS** • When two expressions are equivalent, an equation relating the two is called an identity because it is true for all values of the variables. • To prove an algebraic identity means to show, using the properties of operations, that the equation is always true, for any values of the variables. • Recognize patterns in numerical relationships and be able to express these patterns as algebraic identities. • (+) Pascal's Triangle can be used to generate coefficients in the Binomial Theorem. **MATHEMATICAL THINKING** • Discern and use a pattern or structure. • Generalize concepts based on patterns. • Explain mathematical reasoning. • Use informal reasoning. **INSTRUCTIONAL FOCUS** • Produce algebraic proofs for various polynomial identities. • Represent numerical relationships using identities. • (+) Apply the Binomial Theorem. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Interpret the structure of expressions (A.SSE.2). • Perform arithmetic operations on polynomials (A.APR.1). • Create equations that describe numbers or relationships (A.CED.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Algebra **ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS** **Rewrite rational expressions.** **A.APR.6** Rewrite simple rational expressions[G] in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (+) **A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	**Expectations for Learning** In previous courses, students learn properties of and operations with rational numbers. They also learn to factor quadratic expressions. In A.APR.2, students use polynomial division to think about zeros of a polynomial function. In this cluster, students use polynomial division to rewrite rational expressions, which are quotients of polynomial expressions, in other forms. Students pursuing advanced mathematics courses (+) will add, subtract, multiply, and divide rational expressions. **ESSENTIAL UNDERSTANDINGS** • The result of polynomial division can be written as an single expression showing both the quotient and the remainder, in the same way that the improper fraction can be written in the form of a mixed number, for example, 14 divided by 3, or $\frac{14}{3}$, results in the quotient 4 remainder 2 or $4 + \frac{2}{3}$ which is equivalent to $4\frac{2}{3}$. • The properties of operations on rational numbers hold for rational expressions. In other words, rational expressions are fractions, and the arithmetic is the same. • (+) Rational expressions are closed under addition, subtraction, multiplication and division, excluding division by the zero polynomial. **MATHEMATICAL THINKING** • Make sound decisions about using tools. • Use technology strategically to deepen understanding. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. **INSTRUCTIONAL FOCUS** • Rewrite simple rational expressions (denominators with linear polynomials and numerators with at most quadratic polynomials) as equivalent expressions using inspection (e.g., for monomial denominators) or long division. • (+) Add, subtract, multiply, and divide (with nonzero denominators) rational expressions. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Graph rational functions (F.IF.7g). • Build new functions from existing functions (F.BF.3). • Know and apply the Remainder Theorem (A.APR.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Algebra **CREATING EQUATIONS** **Create equations that describe numbers or relationships.** **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ **c.** Extend to include more complicated function situations with the option to solve with technology. (A2, M3) **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **c.** Extend to include more complicated function situations with the option to graph with technology. (A2, M3) *Continued on next page*	**Expectations for Learning** In previous courses, students create equations and inequalities, focusing on linear, exponential, and quadratic equations. They also rearrange formulas to highlight a particular variable. In this course, students extend these skills to more complicated situations. **ESSENTIAL UNDERSTANDINGS** • Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts. • The graphical solution of a system of equations or inequalities is the intersection of the graphs of the equations or inequalities. • Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given. • A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations. **MATHEMATICAL THINKING** • Create a model to make sense of a problem. • Represent the concept symbolically. • Plan a solution pathway. • Determine the reasonableness of results. • Consider mathematical units and scale when graphing. **INSTRUCTIONAL FOCUS** • Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship. • For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales. • Identify the constraints implied by the scenario, and represent them with equations or inequalities. • Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario. • Solve formulas for a given variable.

<table>
<tr>
<td>

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1)

a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★

d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

</td>
<td>

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 2/Math 3, Number 4, pages 8-10

CONNECTIONS ACROSS STANDARDS

- Model and interpret the relationship between two quantities (F.IF.4).
- Rewrite expressions involving radicals and rational exponents (N.RN.2).
- Use the structure of an expression to identify ways to rewrite it (A.SSE.2).
- Write a function that describes a relationship between two quantities (F.BF.1b,c).
- Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.4).

</td>
</tr>
</table>

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM

Algebra
REASONING WITH EQUATIONS AND INEQUALITIES
Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
	Expectations for Learning
	In Algebra 1/Math 1, students solve simple equations and justify the steps in their solution process. In this cluster, students solve simple rational and radical equations using a variety a methods and explain why extraneous solutions may arise.
	
	ESSENTIAL UNDERSTANDINGS
	• To solve a rational equation, a helpful strategy is to multiply both sides of the equation by a common denominator of the rational expressions.
	  o This strategy may introduce extraneous solutions; thus, potential solutions must be checked in the original equation.
	• To solve an equation with square root expressions, a helpful strategy is to square both sides after a square root expression is isolated on one side of the equation if possible.
	  o This strategy may introduce extraneous solutions; thus, potential solutions must be checked in the original equation.
	
	MATHEMATICAL THINKING
	• Solve math equations accurately.
	• Determine reasonableness of results.
	• Use technology strategically to deepen understanding.
	
	INSTRUCTIONAL FOCUS
	• Solve simple rational and radical equations in one variable.
	• Explain why and where in the solution process the extraneous solution arose.
	Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Rewrite expressions involving radicals and rational exponents (N.RN.2). • Rewrite simple rational expressions (A.APR.6). • Interpret functions that arise in applications in terms of the context (F.IF.4). • Graph rational functions (F.IF.7g). • Build new functions from existing functions (F.BF.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **REASONING WITH EQUATIONS AND INEQUALITIES** **Solve systems of equations.** **A.REI.6** Solve systems of linear equations algebraically and graphically. **b.** Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)	**Expectations for Learning** In previous courses, students solve systems of equations in two variables algebraically and graphically. In this cluster, students solve systems of equations in three variables algebraically. For students planning to take advanced mathematics courses, topics for extension include matrices and vectors. **ESSENTIAL UNDERSTANDINGS** • A system of linear equations in three variables can have no solutions, one solution, or infinitely many solutions. • The solution to a system of linear equations in three variables is the set of values that satisfies all three equations. **MATHEMATICAL THINKING** • Use informal reasoning with symbolic representation. • Solve multi-step problems accurately. • Plan a solution pathway. **INSTRUCTIONAL FOCUS** • Solve a system of linear equations in three variables algebraically. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 5-6 **CONNECTIONS ACROSS STANDARDS** • Rearrange formulas (A.CED.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **REASONING WITH EQUATIONS AND INEQUALITIES** **Represent and solve equations and inequalities graphically.** **A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.	**Expectations for Learning** In previous courses, students use graphs to solve equations with linear, quadratic, and simple exponential expressions. In this cluster, students will similarly study the relationship between the graph and solutions of equations with rational, radical, absolute value, polynomial, exponential, and trigonometric expressions. **ESSENTIAL UNDERSTANDINGS** • An equation in one variable can be rewritten as a system of two equations in two variables, by thinking of each side of the equation as a function, i.e., writing y = left hand side and y = right hand side. o The approximate solution(s) to an equation in one variable is the *x*-value(s) of the intersection(s) of the graphs of the two functions. o Two-variable graphical and numerical (tabular) techniques to solve an equation with one variable always work and are particularly useful when algebraic methods are not applicable, e.g., $\sin x = x^2$. **MATHEMATICAL THINKING** • Use technology strategically to deepen understanding. • Plan a solution pathway. • Create a model to make sense of a problem. **INSTRUCTIONAL FOCUS** • Rewrite a one-variable equation as two separate functions and use the *x*-coordinate of their intersection point to determine the solution of the original equation. • Approximate intersections of graphs of two equations using technology, tables of values, or successive approximations. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** • Solve simple rational and radical equations (A.REI.2). • Construct a rough graph of a polynomial function (A.APR.3). • Graph functions expressed symbolically (F.IF.7). • Evaluate exponential functions using technology (F.LE.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Functions **INTERPRETING FUNCTIONS** **Interpret functions that arise in applications in terms of the context.** **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3) **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **c.** Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3) **F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)	**Expectations for Learning** Working with linear, quadratic, and exponential functions in Algebra1/Math 2, students interpreted key features of graphs and tables. Students determined the domain and understand its limitations by looking at a graph, table, and a given real-life scenario. In this cluster, students extend key features to include periodicity, polynomials of degree greater than 2, and others given by graphs, tables, symbols, or verbal descriptions. Students select appropriate functions that model the situation presented. Average rate of change over a specific interval is included in this cluster. In future courses, some students will extend their understanding of average rate of change to instantaneous rate of change. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem, and making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem. • The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols. • The context of a situation can suggest the shape of the graph and can suggest the function family that models relationships identified in the situation. • Linear functions have a constant rate of change. • Since non-linear functions do NOT have a constant rate of change, average rate of change is used the describe how the rate of change varies from interval to interval. • The average rate of change of a function over an interval is finding the slope of a linear function that goes through the same two points. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Connect mathematical relationships to contextual scenarios.
- Attend to meaning of quantities.
- Determine reasonableness of results.

INSTRUCTIONAL FOCUS

- Use key features (including periodicity) of graphs, tables, and contexts to select an appropriate function family for modeling purposes.
- Utilize graphs to compare and understand behaviors and features of various types of functions.
- Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required).
- Demonstrate understanding of domain in the context of a real-world problem.
- Given a function graphically, estimate the average rate of change over a specified interval.
- Given a function symbolically, compute the average rate of change over a specified interval.
- Given a function represented as a table, compute the average rate of change over a specified interval.

Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 4, pages 8-10 **CONNECTIONS ACROSS STANDARDS** • Create equations that describe numbers or relationships (A.CED.2c). • Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions (F.IF.7c). • Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior (F.IF.7d). • Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude (F.IF.7f). • (+) Graph rational functions, identifying zeros and asymptotes, when factoring is reasonable, and indicating end behavior (F.IF.7g). • Analyze functions using different representations (F.IF.9). • Model periodic phenomena with trigonometric functions (F.TF.5). • Fit a function to the data; use functions fitted to data to solve problems in the context of the data (S.ID.6a).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
<tr><td>

Functions

INTERPRETING FUNCTIONS

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3)

d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3)

f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline[G], and amplitude. (A2, M3)

(+) **g.** Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (A2, M3)

(+) **h.** Graph logarithmic functions, indicating intercepts and end behavior. (A2, M3)

Continued on next page

</td><td>

Expectations for Learning

In Algebra 1/Math 2, students graph linear, quadratic, and exponential functions given a symbolic representation and indicate intercepts and end behavior. They compare linear, quadratic, and exponential functions given various representations.

In this cluster, students graph polynomial, square root, cube root, trigonometric, piecewise-defined, rational (+), and logarithmic (+) functions. Students identify and interpret key features (as applicable) including intercepts, end behavior, period, midline, amplitude, symmetry, asymptotes, maxima/minima, and zeros.

ESSENTIAL UNDERSTANDINGS

- Different representations (graphs, tables, symbols, verbal descriptions) illuminate key features of functions and can be used to compare different functions.
- For exponential functions, radicals can be used to make sense of the function values at non-integer input values.
- Swapping the non-negative input and output values of the squaring function yields the square root function, so the graph of the square root function is half of a sideways parabola.
- Trigonometric functions can be used to model periodic phenomena.
- Piecewise-defined functions are used in real-world situations including bulk pricing, utility bills, income tax, etc.
- A linear factor of a polynomial indicates a zero of that function.
- The degree of a polynomial function indicates the maximum number of possible zeros.
- Function families have commonalities in shapes and features of their graphs.
- More complex functions can be graphed and analyzed as transformations of parent functions.
- The factored form of a quadratic function reveals the zeros of the function (i.e., the *x*-intercepts of the graph); the vertex form of a quadratic function reveals the maximum or minimum of the function; the standard form of a quadratic function reveals the *y*-intercept of the graph.
- More generally, writing a function in different ways can reveal different features of the graph of a function.

</td></tr>
</table>

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) **b.** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change*[G] *in functions such as* $y = (1.02)^t$, *and* $y = (0.97)^t$ *and classify them as representing exponential growth or decay.* (A2, M3) **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)	**Expectations for Learning, continued** **MATHEMATICAL THINKING** • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Analyze a mathematical model. **INSTRUCTIONAL FOCUS** • Given symbolic representations of various functions, create accurate graphs showing all key features, with and without technology. While fluency is still expected for linear, exponential, and quadratic functions, for other functions given symbolically, the focus should be on predicting the shape of the graph. • Explore the relationship of the symbolic representation of a function and its graph by adjusting parameters. • Identify the key features of the graph of a quadratic function by factoring, using the quadratic formula, or completing the square, including functions in which the leading coefficient is not equal to 1. • Use factoring and long division to find zeros of polynomial functions. • Use and interpret non-integer exponents in exponential functions. • Use different symbolic forms of a function to reveal different features of its graph. • Compare and contrast various functions given by graphs, tables, symbols, or verbal descriptions. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 4, pages 8-10 **CONNECTIONS ACROSS STANDARDS** • Understand the relationship between zeros and factors of polynomials (A.APR.2-3). • Rewrite rational expressions (A.APR.6). • Create equations that describe numbers or relationships (A.CED.2-4). • Build a function that models a relationship between two quantities (F.BF.1). • Expand the properties of exponents to rational exponents (N.RN.1-2). • Use properties of rational and irrational numbers (N.RN.3). • Construct and compare linear, quadratic, and exponential models and solve problems (F.LE.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Functions **BUILDING FUNCTIONS** **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3)	**Expectations for Learning** In prior math courses, students write linear, quadratic, and exponential functions. They also write explicit and recursive rules for arithmetic and geometric sequences. In this cluster, students build functions from other functions to model more complex situations. This includes combining functions of various types using arithmetic operations or (+) function composition. **ESSENTIAL UNDERSTANDINGS** • Known functions can be combined using arithmetic operations or (+) function composition, yielding new functions. • Addition or subtraction by a constant function can be interpreted as a vertical shift. • Multiplication or division by a non-zero constant function can be interpreted as a vertical stretch/shrink. **MATHEMATICAL THINKING** • Make and modify a model to represent mathematical thinking. • Discern and use a pattern or structure. **INSTRUCTIONAL FOCUS** • Demonstrate the effects of arithmetic operations on known functions: Choose an *x*-value; find the two *y*-values; do the operation; and then repeat with several other x-values. • For addition and subtraction of functions, compare the heights of the functions to their sum or difference. • Select appropriate functions and arithmetic operations on those functions to model situations. • (+) Compose functions algebraically and for the purpose of modeling. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 4, pages 9-10 **CONNECTIONS ACROSS STANDARDS** • Create equations that describe numbers or relationships (A.CED.2). • Fit a function to the data; use functions fitted to data to solve problems in the context of the data (S.ID.6a). • Interpret linear models (S.ID.7). • Analyze functions using different representations (F.IF.8). • Perform arithmetic operations on polynomials (A.APR.1b).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
<tr><td>

Functions

BUILDING FUNCTIONS

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)

F.BF.4 Find inverse functions.

(+) **b.** Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)

(+) **c**. Verify by composition that one function is the inverse of another. (A2, M3)

(+) **d.** Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)

</td><td>

Expectations for Learning

In prior math courses, students transform graphs of quadratic functions, but did not utilize function notation for describing these transformations.

In Algebra 2/Math 3, students perform all types of transformations for various function families, using function notation. They recognize even and odd functions, and (+) work extensively with inverses of functions.

ESSENTIAL UNDERSTANDINGS

- Transformations of graphs of functions include shifts, reflections, and stretches/shrinks.
- Some functions can be characterized as even or odd, indicating symmetry.
- Transformations of the form $f(x – k)$ are horizontal shifts.
- Transformations of the form $f(x) + k$ are vertical shifts.
- Transformations of the form $f(kx)$ are horizontal stretches/shrinks when $k > 0$.
- Transformations of the form $kf(x)$ are vertical stretches/shrinks when $k > 0$.
- Transformations of the form $–f(x)$ are reflections across the x-axis.
- Transformations of the form $f(–x)$ are reflections across the y-axis.

MATHEMATICAL THINKING

- Explain mathematical reasoning.
- Use technology strategically to deepen understanding.
- Discern and use a pattern or structure.

INSTRUCTIONAL FOCUS

- Transform a variety of functions, including simple rational, radical, and exponential functions, and interpret the transformations geometrically.
- Use function notation to express transformations, such as $f(x) = x^2$, $g(x) = f(x–h)$.
- Recognize even and odd functions from symmetry in their graphs and from their algebraic expressions.
- Generalize the effects of transformations across function families.
- (+) Find inverses of functions algebraically, graphically, or from a table.

</td></tr>
</table>

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 4, pages 8-10 **CONNECTIONS ACROSS STANDARDS** • Analyze functions using different representations (F.IF.7c, d, f, and g). • (+) Compose functions (F.BF.1c). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Functions **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★	**Expectations for Learning** Students study exponential functions with integer domains in Algebra 1 or in Math 1 and Math 2. Earlier in this course, students develop an understanding of rational exponents in order to talk about exponential functions with a domain that is the real numbers. Based on these understandings, students in Algebra 2/Math 3 focus on logarithms as numbers that are solutions to exponential equations. Logarithms as functions and the laws of logarithms are recommended for higher-level courses. **ESSENTIAL UNDERSTANDINGS** • Logarithms are exponents; they are numerical solutions to exponential equations, such as $10^x = 30$. **MATHEMATICAL THINKING** • Represent a concept symbolically. • Make and modify a model to represent mathematical thinking. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. **INSTRUCTIONAL FOCUS** • Solve exponential equations with an unknown exponent by translating between exponential and logarithmic forms, with the support of tables and graphs, interpreting the solution in context. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 4, pages 8-10 **CONNECTIONS ACROSS STANDARDS** • Interpret functions that arise in applications in terms of the context (F.IF.4). • Analyze functions using different representations (F.IF.9). • Interpret the structure of expressions (A.SSE.3c). • Build new functions from existing functions (F.BF.3). • Extend the properties of exponents to rational exponents (N.RN.1-2). • Use properties of rational and irrational numbers (N.NR.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Functions **TRIGONOMETRIC FUNCTIONS** **Extend the domain of trigonometric functions using the unit circle.** **F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	**Expectations for Learning** In Geometry/Math 2, students use the right-triangle definitions of sine and cosine and solve problems involving right triangles. Students compute lengths of arcs based on similarity. In Algebra 2/Math 3, students are introduced to the concept of radians. They extend their understanding of right triangle trigonometry to circular trigonometry. In future courses, students may be expected to be fluent with trigonometric functions of special angles, and they may also expected to be fluent in converting between degrees and radians. **Note:** Students taking Algebra 2 before Geometry, will not have experience with trigonometric ratios, arc length, or circumference. **ESSENTIAL UNDERSTANDINGS** • Radians can be interpreted as lengths of arcs on the unit circle. • Angles, measured in degrees or radians, can be any real number, i.e., angles can be negative, greater than 360 degrees, or 2π radians. • An angle determines a point on the unit circle. The sine of the angle is the *y*-coordinate of the point, and the cosine of the angle is the *x*-coordinate of the point. • The coordinates of a point on the unit circle (not on an axis) represent the lengths of the legs of a reference right triangle, where the signs of the *x* and *y* coordinates of that point indicate which quadrant the angle lies in. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Make and modify a model to represent mathematical thinking. • Consider mathematical units involved in a problem. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Generalize concepts based on patterns (repeated reasoning). *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

** The focus should be on sine and cosine functions, with some exposure to the tangent function. In advanced mathematics, students use all six trigonometric functions. Students should be using radians and degrees to develop fluency between them both.*

- Given an angle measure, identify the coordinates of a point on the unit circle as follows:
  - With a vertex located at the origin and the initial side of the angle is the positive half of the x-axis, the angle indicates the amount of counterclockwise turning to determine the terminal side of the angle. Likewise, the clockwise turning determines the terminal side of the negative angle.
  - The terminal side of the angle intersects the unit circle at a point.
  - The *x*-coordinate of this point is called the cosine of the angle, and the *y*-coordinate of this point is called the sine of the angle.
- From a point on the unit circle, draw reference right triangles to connect back to right triangle trigonometry.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 2/Math 3, Number 3, page 7

CONNECTIONS ACROSS STANDARDS

- Find arc lengths and areas of sectors of circles (G.C.6).
- (+) Apply trigonometry to general triangles (G.SRT.8b, 9, 10, 11).
- Model periodic phenomena with trigonometric functions (F.TF.5).
- Prove and apply trigonometric identities (F.TF.8).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Functions **TRIGONOMETRIC FUNCTIONS** **Model periodic phenomena with trigonometric functions.** **F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	**Expectations for Learning** Geometry/Math 2, students learn trigonometric ratios of right triangles. They study the right-triangle definitions of sine and cosine and apply their understanding to solve problems involving right triangles. In Algebra 2/Math 3, students are introduced to the concept of radians and extend their understanding of right triangle trigonometry to circular trigonometry. In this cluster, students apply their understanding of circular trigonometry to model periodic phenomena such as tides, electrical current, height of Ferris wheels, temperature, etc. The language in the standard states that students will "choose trig functions", but additionally, students must also choose the parameters for a trigonometric function that models real-world phenomena. In future courses, students will graph the six trigonometric functions by hand and/or using technology and will also identify periods and phase shifts by analyzing graphs and equations. **ESSENTIAL UNDERSTANDINGS** • Many real-world phenomena, including sound waves; oscillation on a spring; the motion of a pendulum; and phases of the moon, are cyclical and can be approximated by trigonometric functions. • The period is the horizontal length of one cycle, and it can be interpreted in terms of a horizontal stretch. • The equation of the midline is the average of the maximum and minimum values of the function, and it is a horizontal axis about which the graph of the function oscillates. • The amplitude is the distance between the midline and the maximum or minimum values of the function, and it can be interpreted in terms of a vertical stretch. **MATHEMATICAL THINKING** • Connect mathematical relationships and contexts. • Make and modify a model to represent mathematical thinking. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Given a real-world phenomenon, select an appropriate trigonometric function that models that phenomenon, understanding that sine and cosine functions will be used most frequently because of their tendency to model common real-world phenomena.
- Given a real-world phenomenon, identify the parameters (midline, amplitude, period) that describe that phenomenon.
- When phenomena are modeled with a sine ($f(x)$ = A sin(Bx + C) + D) or cosine ($f(x)$ = A cos(Bx + C) + D) function, students will choose the parameters A, B, C, and D that best model the scenario.
- Interpret amplitude and midline as vertical stretches and shifts of the graphs. Include some attention to period and frequency as horizontal stretches. Reserve phase shifts as horizontal shifts for advanced courses.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 2/Math 3, Number 3, page 7

CONNECTIONS ACROSS STANDARDS

- Extend the domain of trigonometric functions using the unit circle (F.TF.1-2, (+) 3-4).
- Analyze functions using different representations (F.IF.7f).
- Build new functions from existing functions (F.BF.3, (+) 4b-d).
- (+) Apply trigonometry to general triangles (G.SRT.8b, 9, 10, 11)

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Functions **TRIGONOMETRIC FUNCTIONS** **Prove and apply trigonometric identities.** **F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	**Expectations for Learning** In Geometry/Math 2, students learn trigonometric ratios of right triangles. They study the right-triangle definitions of sine and cosine and apply their understanding to solve problems involving right triangles. They solve right triangles and learn to write the equation of circles. Earlier in this course, students are introduced to the concept of radians. They extend their understanding of right triangle trigonometry to circular trigonometry. In this cluster, students apply their knowledge of the Pythagorean Theorem and circular trigonometry to prove this important identity. They also find values of trigonometric functions given the value of one trigonometric function and a quadrant. In future courses, students use their understanding of this trigonometric identity to generate other trigonometric identities and apply them. Students also learn and use inverse trigonometric functions to solve problems. **ESSENTIAL UNDERSTANDINGS** • The identity $\sin^2(\theta) + \cos^2(\theta) = 1$ is the Pythagorean Theorem applied to coordinates on the unit circle. • The value of a trigonometric function and a given quadrant are sufficient to find the values of the other trigonometric functions. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Make and analyze mathematical conjectures. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** **Students in this course do not need fluency with special angles. Students aiming for advanced courses should begin to work on fluency.* • Interpret $\sin^2(\theta) + \cos^2(\theta) = 1$, with an angle in standard position, as an equation for the unit circle, which is an application of the Pythagorean Theorem. • Check that the identity is true in all four quadrants and on the axes. • Given a trigonometric function and a quadrant, sketch a triangle to determine the values of other trigonometric functions. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 3, page 7 **CONNECTIONS ACROSS STANDARDS** • Extend the domain of trigonometric functions using the unit circle (F.TF.1-2, (+) 3-4). • (+) Apply trigonometry to general triangles (G.SRT.8b, 9, 10, 11).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Geometry **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** **Define trigonometric ratios, and solve problems involving right triangles.** **G.SRT.8** Solve problems involving right triangles.★ (+) **b.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ (A2, M3)	**Expectations for Learning** In Geometry/Math 2, students define trigonometric ratios and solve problems using right triangles. In this cluster, students solve problems using right triangles including inverse trigonometric functions. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Inverse trigonometric functions yield an angle when given a trigonometric ratio. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Discern and use a pattern or structure. • Plan a solution pathway. • Determine reasonableness of results. • Create a drawing and add components as appropriate. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. • Connect mathematical relationships to real-world encounters. **INSTRUCTIONAL FOCUS** • Given two sides of a right triangle, use the Pythagorean Theorem and inverse trigonometric functions to solve mathematical and real-world problems. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 3, page 7 **CONNECTIONS ACROSS STANDARDS** • (+) Apply trigonometry to general triangles (G.SRT.9-11). • Extend the domain of trigonometric functions using the unit circle (F.TF.1-2). • Model periodic phenomena with trigonometric functions (F.TF.5). • Prove and apply trigonometric identities (F.TF.8). • Interpret key features of trigonometric functions (F.IF.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** **Apply trigonometry to general triangles.** (+) **G.SRT.9** Derive the formula *A* = $^1/_2$ *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (+) **G.SRT.10** Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems. (+) **G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.	**Expectations for Learning** In Geometry/Math 2, students define trigonometric ratios and solve problems using right triangles. In this cluster, students solve problems using Laws of Sines and Cosines; explain the proof of the Laws of Sines and Cosines; and derive the area formula $A = \frac{1}{2}ab\ \sin C$. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • The Law of Sines and Law of Cosines can be used to solve non-right triangles. **MATHEMATICAL THINKING** • Discern and use a pattern or structure. • Plan a solution pathway. • Determine reasonableness of results. • Create a drawing and add components as appropriate. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. • Connect mathematical relationships to real-world encounters. **INSTRUCTIONAL FOCUS** • Solve problems involving any triangle. • Explain proofs of the Laws of Sines and Cosines. • Use the Laws of Sines and Cosines to solve problems. • Derive the area formula ($A = \frac{1}{2}ab\ \sin C$) by the use of an auxiliary line. *Continued on next page*

	Content Elaborations • Algebra 2/Math 3, Number 3, page 7 **CONNECTIONS ACROSS STANDARDS** • Extend the domain of trigonometric functions using the unit circle (F.TF.1-2). • Model periodic phenomena with trigonometric functions (F.TF.5). • Prove and apply trigonometric identities (F.TF.8). • Interpret key features of trigonometric functions (F.IF.4). • Use polynomial identities to solve problems (A.APR.4). • Create equations that describe numbers or relationships (A.CED.1-4). • Understand solving equations as a process of reasoning and explain the reasoning (A.REI.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Geometry **CIRCLES** **Find arc lengths and areas of sectors of circles.** **G.C.6** Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)	**Expectations for Learning** In middle school, students are limited to working with measurements of circles such as circumference and area. This cluster spans Geometry/Math 2 and Algebra 2/Math 3. In Geometry/Math 2, students are using part-to-whole proportional reasoning to find arc lengths and sector areas, in which the arc or central angle is measured in degrees. In Algebra 2/Math 3, students derive and use formulas relating degree and radian measure. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Measures of angles and of arcs can be expressed in degrees or radians. • Converting between degrees and radians is an application of proportional reasoning. **MATHEMATICAL THINKING** • Consider mathematical units involved in a problem. • Make connections between concepts and terms. • Generalize concepts based on patterns. • Use proportional reasoning (part-to-whole). • Draw a picture to make sense of a problem. • Solve real-world and mathematical problems accurately. • Plan a solution pathway. • Attend to the meaning of quantities. **INSTRUCTIONAL FOCUS** • Define radian measure. • Derive the formulas that relate degrees and radians. • Convert between radians and degrees. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 3, page 7 **CONNECTIONS ACROSS STANDARDS** • Extend the domain of trigonometric functions using the unit circle (F.TF.1-2). • Model periodic phenomena with trigonometric functions (F.TF.5). • (+) Extend the domain of trigonometric functions using the unit circle (F.TF.3). • Create equations that describe numbers or relationships (A.CED.1, 2, 4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Statistics and Probability **INTERPRETING CATEGORICAL AND QUANTITATIVE DATA** **Summarize, represent, and interpret data on a single count or measurement variable.** **S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★	**Expectations for Learning** In Algebra 1/Math 1, student build upon the GAISE Model that was introduced in Middle School. They compare the center and spread of two different data sets using mean absolute deviation and standard deviation. In Algebra 2/Math 3**,** students extend their knowledge of mean and standard deviation to normal distributions. The learning at this level is at the developmental Level C. See pages 99 and 102 for more information on Level C. **ESSENTIAL UNDERSTANDINGS** • Only some bell shaped curves are normal. • If a bell-shaped curve is normal, then 68% of the distribution is within one standard deviation of the mean; 95% is within two standard deviations of the mean; and 99.7% is within three standard deviations of the mean. This is known as the Empirical Rule. **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data. **INSTRUCTIONAL FOCUS** • By using the Empirical Rule, determine whether or not a distribution is normal (or approximately normal). • Estimate area using the 68-95-99.7% rule (Empirical Rule). • Estimate the area under a normal curve (population percentages) with tables and technology. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 1, page 3 **THE GAISE MODEL** • GAISE Model, pages 14 – 15 o Focus of the standard for Algebra 2/Math 3 in the cluster is Level C, pages 61-88 **CONNECTIONS ACROSS STANDARDS** • Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6). • Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.3-6).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Statistics and Probability **INTERPRETING CATEGORICAL AND QUANTITATIVE DATA** **Summarize, represent, and interpret data on two categorical and quantitative variables.** **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3) **b.** Informally assess the fit of a function by discussing residuals. (A2, M3)	**Expectations for Learning** For this cluster, the GAISE Model framework continues to be used: Formulating Questions; Collecting Data; Analyzing Data; and Interpreting Results. In Algebra 1/Math 1 students find the equation of a linear model, with and without technology using precise language to describe the relationship between variables. In Algebra 2/Math 3, concepts are now extended to quadratic and exponential functions. Up to this point students have only analyzed the fit of the model by looking at the closeness of the data points to a linear model. Now students are introduced to the idea of a residual, and they use them to informally assess the fit of the model. The learning at this level is at the developmental Level C. See pages 99 and 102 for more information on Level C. **ESSENTIAL UNDERSTANDINGS** • A linear, quadratic, or exponential function can be used as a model for association of two quantitative variables. • $\hat{y}$ is often used as the symbol for the predicted *y*-value for a given *x*-value. • A residual is the difference between the actual *y*-value and the *y*-value predicted by the chosen model $(y - \hat{y})$. **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data. • Accurately make computations using data. • Determine reasonableness of predictions. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** **Quantitative Data** • Reason about the context and the data to judge whether a linear, quadratic, or exponential model (or none of these) is appropriate. • Fit quadratic and exponential models to data using technology. • Use the chosen model to make contextual conclusions. • Discuss residual values to assess the appropriateness of a linear model. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 1, page 3 **THE GAISE MODEL** • GAISE Model, pages 14 – 15 o Focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88 **CONNECTIONS ACROSS STANDARDS** • Create equations that describe numbers or relationships (A.CED.2). • Build a function that models a relationship between two quantities (F.BF.1). • Interpret linear models (S.ID.9).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Statistics and Probability **INTERPRETING CATEGORICAL AND QUANTITATIVE DATA** **Interpret linear models.** **S.ID.9** Distinguish between correlation and causation.★	**Expectations for Learning** In Algebra 1/Math 1, students interpret the slope and intercept of a linear model. They also work with the correlation coefficient. In Algebra 2/Math 3, students are now introduced to and explore the distinction between correlation and causation. The learning of standard S.ID.9 is at developmental Level C. See pages 99 and 102 for more information on Level C. **ESSENTIAL UNDERSTANDINGS** • There are three main methods of data production in statistics: surveys of samples to estimate population parameters; randomized experiments to compare treatments and to show cause; and observational studies to indicate possible associations among variables. Students should understand the distinctions among these three and decide if appropriate inferences have been drawn. • Causation is a cause and effect relationship between two variables. • Correlation (i.e., strong correlation) does not imply causation. • Causation cannot be established after the research has been completed; it can only be established through well-designed experiments (not observational studies and not surveys). **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data. • Determine reasonableness of predictions. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Compare and contrast situations when changes in one variable cause changes in another.
- Compare and contrast when the direction of causation is not clear.
- Compare and contrast when changes in both variables are caused by something else.
- Use surveys of samples to estimate population parameters.
- Use randomized experiments to compare treatments and to show cause.
- Use observational studies to indicate possible associations among variables.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 2/Math 3, Number 1, page 3

THE GAISE MODEL

- GAISE Model, pages 14 – 15
    - The focus of S.ID.9 is at Level C for Algebra 2/Math 3, pages 61-85

CONNECTIONS ACROSS STANDARDS

- Summarize, represent, and interpret data in two categories and quantitative variables (S.ID.6).
- Interpret the structure of functions (F.IF.4).
- Build a function that models a relationship between two quantities (F.BF.1).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
<tr><td>

Statistics and Probability

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★

</td><td>

Expectations for Learning

This is students' first exposure to the necessity of randomness when sampling to make an inference about a population which is consistent with the GAISE Model (Step 4, Level C). In middle school, students are exposed to simulations (observed) and compare them to predicted (expected) outcomes based on probability. In this cluster, students decide if results are consistent with a given model by using simulations or computing probabilities.

Understanding the statistical concepts of GAISE model Level C enables a student to build on the foundations developed in Levels A and B. Although the previous levels are revisited, students are now expected to take prior learning to a deeper statistical nature. They are expected to draw on basic concepts from earlier work; extend the concept to cover a wider scope of investigatory issues; and develop a deeper understanding of inferential reasoning and its connection to probability. Students should be able to provide a more sophisticated interpretation that integrates the context and objectives of a study, and they should also be able to see limitations based on data.

ESSENTIAL UNDERSTANDINGS

- Random sampling guarantees that the sample chosen is representative of the population which ensures that the statistical conclusions will be valid.
- A random sample must be generated through a chance selection process.
- A statistic is generated from sample data to estimate the corresponding parameter for the entire population.
- A population parameter is a measure of some characteristic in the population such as the population proportion or the population mean.
- Experimental results are not a perfect match for theoretical models by the nature of variability.

MATHEMATICAL THINKING

- Make connections between samples and their populations.
- Determine whether results are reasonable.
- Use precise mathematical language.

Continued on next page

</td></tr>
</table>

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Explain and interpret, within a context, the results of a data-generating process. • Explain, within a context, what a point on a dot plot from a simulation represents. • Decide if an outcome is unusual in a specified distribution. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 1, page 3 **THE GAISE MODEL** • GAISE Model, pages 14 – 15 o The focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88 **CONNECTIONS ACROSS STANDARDS** • Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.3-6). • (+) Develop probability models and expected values (S.MD.3-6).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Statistics and Probability **MAKING INFERENCES AND JUSTIFYING CONCLUSIONS** **Make inferences and justify conclusions from sample surveys, experiments, and observational studies.** **S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★ **S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★ **S.IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.★ **S.IC.6** Evaluate reports based on data.★	**Expectations for Learning** Previously, students have been informally introduced to data collection methods and bias. In this cluster, the concept of randomization is introduced in data collection methods. Students are also introduced to the concept of margin of error, and they begin to formalize the concept of statistical significance. **<u>The GAISE Model</u>** The GAISE Model is a framework for all statistical problem solving and should not be taught in isolation. For this cluster, the focus is on Steps 2, 3, and 4 at Level C. Students are building on the framework developed in earlier grades. Algebra 2/Math 3 students use more in-depth reasoning and a greater level of precision and complexity. **Step 1: Formulate the Question** • Students should be fluent in posing their own statistical question of interest. • Students should form questions to allow generalizations be made about a population **Step 2: Collect Data** • Students should purposefully design for differences through random selection or random assignment. • Students design samples through selection. • Students design experiments through randomization. **Step 3: Analyze Data** • Students understand and use global characteristics of distributions in analysis. • Students compare group to group using displays and measures of variability. • Students describe and quantify sampling error. **Step 4: Interpret Variability** • Students are able to look beyond the data in some contexts. • Students are able to generalize from a sample to population. • Students are aware of the effects of randomization on the results of experiments. • Students understand and distinguish between observational studies and experiments. *Continued on next page*

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS

- Surveys, observational studies, and experiments are different methods for data collection and each have their own advantages.
- Surveys and observational studies involve a researcher collecting information about a sample without imposing a treatment on subjects.
- A researcher should utilize a chance process to assign treatment groups in experiments.
- Causality can be established with well-designed experiments; surveys and observational studies cannot determine causality.
- The decision process based on sample data does not guarantee a correct answer to the underlying statistical question.
- The characteristics of distributions of sample statistics are simulation models for random sampling; it is predictable only if the sampling is random.
- The interval (observed statistic $\pm$ margin of error) should include the plausible values for the true population parameter.
- The results from an experiment are statistically significant if the differences between treatment groups are unlikely to have occurred by chance alone.

MATHEMATICAL THINKING

- Make sense of the structure of distributions.
- Use precise vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve statistical problems in real-world context.
- Determine whether results are reasonable.

Continued on next page

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Identify and choose when to use surveys, experiments, and observational studies.
- Explain the role of randomization as it relates to bias in surveys, experiments, and observational studies.
- Use a sample mean or sample proportion to estimate a population mean or population proportion.
- Explain how sampling variability relates to margin of error in the context of estimating population mean or proportion.
- Use simulation to estimate margin of error in the context of estimating population mean or population proportion.
- Develop an understanding of the definition of *statistical significance* as compared to everyday language *significance*; determine if the results of an experiment are statistically significant; draw conclusions in context based on a report.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 2/Math 3, Number 1, page 3

THE GAISE MODEL

- GAISE Model, pages 14 – 15
  - The focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88

CONNECTIONS ACROSS STANDARDS

- Understand and evaluate random processes (S.IC.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

Acknowledgements

Meghan Arnold
Teacher,
Miami East Local Schools, SW

Garry Barhorst
Teacher,
Clark-Shawnee Local Schools, SW

Alex Blohm
Teacher,
South-Western City Schools, C

Daniel Brahier
Higher Education,
Bowling Green State University, NW

Dana Butto
Curriculum Specialist/Coordinator,
Trumbull County ESC, NE

Margie Coleman (AC)
Teacher, Kings Local Schools, SW

Kathleen Cooey
Curriculum Specialist/Coordinator,
West Geauga Local Schools, NE

Elizabeth Cors
Teacher, Wooster City Schools, NE

Danielle Cummings
Teacher, Dayton City Schools, SW

Brett Doudican
Teacher, Dayton Early College, SW

Erika Draiss
Teacher, Celina City Schools, NW

Diane Farmer
Teacher,
Cleveland Catholic Dioceses, NE

Shari Fergusen
Teacher,
Tri-County North Local Schools, SW

Brad Findell (AC)
Higher Education,
Ohio State University, C

Ali Fleming (WG)
Teacher, Bexley City Schools, C

Amy Geiger
Teacher,
Greenon Local Schools, SW

Christian Hamman
Curriculum Specialist/Coordinator,
Medina City Schools, NE

Gary Herman (WG)
Curriculum Specialist/Coordinator,
Putnam County ESC, NW

Aryn Hinkle
Teacher,
Perrysburg Exempted Village, NW

Luci Howard
Teacher, London City Schools, C

Steven Ingrassia
Teacher, Newbury Local Schools, NE

Margaret (Peggy) Kasten (AC)
Retired Educator, C

Endora Kight Neal (WG)
Curriculum Specialist/Coordinator,
Cleveland Metropolitan School
District, NE

John Kramer
Teacher,
Flex High Charter School, C

Robert Krauss
Teacher, Manchester Local School
District, SW

Jill Madonia (WG)
Retired Educator,
Akron Public Schools, NE

Emily Meister
Teacher,
Grandview Heights Schools, C

Stephen Miller
Curriculum Specialist/Coordinator,
Summit County ESC, NE

Stephanie Minor
Teacher,
Worthington School District, C

Rachel Newell
Teacher,
Perrysburg Exempted Village, NW

Tara Nicely
Teacher, Chillicothe City Schools, C

Marla Norman
Teacher, Lebanon City Schools, SW

Acknowledgements, continued

Zachary Patterson
Teacher, Columbus City Schools, C

Steve Phelps
Teacher, Madeira City Schools, SW

Stephanie Putzier
Curriculum Specialist/Coordinator, Mahoning County ESC, NE

Scott Reveal
Teacher, Adams County Ohio Valley Schools, SW

Tess Rivero (WG)
Teacher, Bellbrook-Sugarcreek Local Schools, SW

Parameswaran Sivagurunathan
Teacher, Cleveland Metropolitan School District, NE

Melanie Stanley
Teacher, Gallia County Local Schools, SE

Patricia Talarczyk
Teacher, Mentor Exempted Village, NE

Heather VerStreate
Teacher, Mason City Schools, SW

Jennifer Walls (WG)
Teacher, Akron Public Schools, NE

Toby West
Curriculum Specialist/Coordinator, Delaware City Schools, C

Verdie Williams
Teacher, Pickaway Ross Joint Vocational School District, C

Richelle Zbinden
Teacher, Miami Valley Career Technology Center, SW

*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.