Ohio's Model Curriculum | Mathematics

with Instructional Supports

Geometry Course

Ohio | Department of Education

Mathematics Model Curriculum
with Instructional Supports
Geometry Course

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

Standards for Mathematical Practice—Geometry

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.
Students construct accurate diagrams of geometry problems to help make sense of them. They organize their work so that others can follow their reasoning, e.g., in proofs.

MP.2 Reason abstractly and quantitatively.
Students understand that the coordinate plane can be used to represent geometric shapes and transformations, and therefore they connect their understanding of number and algebra to geometry.

MP.3 Construct viable arguments and critique the reasoning of others.
Students use formal and informal proofs to verify, prove, and justify geometric theorems with respect to congruence and similarity. These proofs can include paragraph proofs, flow charts, coordinate proofs, two-column proofs, diagrams without words, indirect proofs, or the use of dynamic software.

MP.4 Model with mathematics.
Students apply their new mathematical understanding to real-world problems. They learn how transformational geometry and basic trigonometric functions can be used to model the physical world.

MP.5 Use appropriate tools strategically.
Students make use of visual tools for representing geometry, such as simple patty paper, transparencies, or dynamic geometry software.

MP.6 Attend to precision.
Students develop and use precise definitions of geometric terms. They verify that a particular shape has specific properties and justify the categorization of the shape, e.g., a rhombus versus a quadrilateral.

MP.7 Look for and make use of structure.
Students construct triangles in quadrilaterals or other shapes and use congruence criteria of triangles to justify results about those shapes.
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Standards for Mathematical Practice, continued

MP.8 Look for and express regularity in repeated reasoning.

Students explore rotations, reflections, and translations, noticing that some attributes of shapes (e.g., parallelism, congruency, orientation) remain the same. They develop properties of transformations by generalizing these observations.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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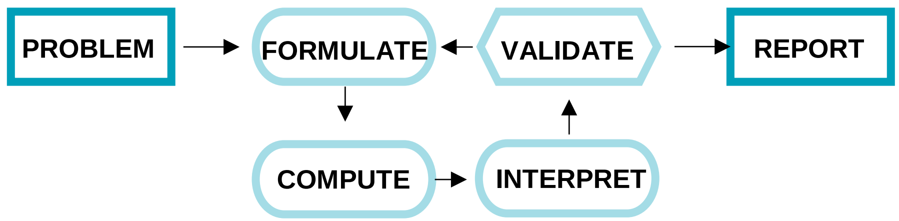
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum
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Geometry Course

STANDARDS	MODEL CURRICULUM
Geometry **CONGRUENCE** **Experiment with transformations in the plane.** **G.CO.1** Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length. **G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch. **G.CO.3** Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. *Continued on next page*	**Expectations for Learning** In middle school, students first learn about the basic rigid motions (translations, rotations, and reflections) and verify their properties experimentally. In this cluster, students formalize the notion of a transformation as a function from the plane to itself. Building on their hands-on work, students develop mathematical definitions of the basic rigid motions. These definitions serve as a logical basis for the theorems that students prove in Geometry. An important step in high school is to perform appropriate transformations and give precise descriptions of sequences of basic rigid motions that carry one figure onto another. Transformations provide language to be precise about symmetry; this is the first time students have encountered formal symmetry. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • A transformation is a function from the plane to itself; input and output values are points, not numbers. • Rigid motions are transformations that preserve distance and angle. • Some transformations preserve distance and angle measures, and some do not. • In order to perform a translation, a distance and a direction is required. • A rotation requires a center and an angle. • A reflection requires a line. • The symmetries of a figure are the transformations that carry the figure onto itself. *Continued on next page*

b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes. **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	**Expectations for Learning, continued** **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary and symbolic notations. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Recognize, apply, and justify mathematical concepts, terms, and their properties. **INSTRUCTIONAL FOCUS** • Know precise definitions of basic terms: ray, angle, circle, perpendicular line, parallel line, and line segment. • Develop and use appropriate geometric notation. • Formalize definitions of basic rigid motions (translations, rotations, and reflections). • Perform and identify transformations using a variety of tools. • Identify the symmetries shown in a figure (rotational and line symmetries). **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, page 4-5 **CONNECTIONS ACROSS STANDARDS** • Understand congruence in terms of rigid motion (G.CO.6-8). • Prove and apply geometric theorems (G.CO.9). • Make formal geometric constructions (G.CO.12). • Justify the slope criteria for parallel and perpendicular lines (G.GPE.5). • Understand similarity in terms of similarity transformations (G.SRT.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

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<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
<tr><td>
<h2>Geometry</h2>
CONGRUENCE
Understand congruence in terms of rigid motions.
G.CO.6 Use geometric descriptions of rigid motions[G] to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent[G].
G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
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<h2>Expectations for Learning</h2>
In middle school, students understand congruence through a sequence of basic rigid motions (reflections, rotations, and translations). In this cluster, students will build on this knowledge to prove that two figures are congruent if there is a sequence of rigid motions carrying one onto the other. The triangle congruence criteria can then be established using the definition of congruence in terms of rigid motions. This is the time when students are first exposed to the criteria for triangle congruence; students should know and be able to use AAS, ASA, SAS, and SSS and understand that the criteria follow from rigid motions.

The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction).

ESSENTIAL UNDERSTANDINGS
Two figures are defined to be congruent if one can be mapped onto the other by rigid motions.
MATHEMATICAL THINKING
Explain mathematical thinking.
Recognize, apply, and justify mathematical concepts, terms, and their properties.
Represent concepts symbolically.
Use formal and informal reasoning.
Use accurate and precise mathematical vocabulary.
INSTRUCTIONAL FOCUS
Use rigid transformations to determine if the figures are congruent
Given congruent triangles, describe the rigid transformations that map one triangle onto the other
Establish the criteria for triangle congruence (AAS, ASA, SAS, and SSS) in terms of rigid motions.
Know and be able to use triangle congruence (AAS, ASA, SAS, and SSS) in solving problems.
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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Experiment with transformations in the plane (G.CO.1-5). • Prove and apply theorems about triangles (G.CO.10). • Prove and apply theorems about parallelograms (G.CO.11). • Use coordinates to prove simple geometric theorems algebraically (G.GPE.4). • Prove and apply theorems involving similarity (G.SRT.5).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **CONGRUENCE** **Prove geometric theorems both formally and informally using a variety of methods.** **G.CO.9** Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **G.CO.10** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* *Continued on next page*	**Expectations for Learning** In middle school, students informally define and apply the relationships of lines, angles, triangles, and parallelograms. For this cluster, students now develop conjectures and construct valid proofs about lines, angles, triangles, and parallelograms. They should begin with informal proof and work toward formal proof using a variety of methods including coordinate-based methods. Also, students should apply these relationships to real-world settings and to proofs. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • The process of proof can vary from informal to formal reasoning. • A proof is a deductive argument that explains why a claim must be true. • Proof can rely on formal and informal language; there are many ways to justify a claim, not all of which rely on technical vocabulary. • Students should demonstrate a knowledge of the content listed in the standards and be able to apply those concepts in various problem solving settings. **MATHEMATICAL THINKING** • Explain mathematical thinking. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Represent concepts symbolically. • Use formal and informal reasoning. • Use accurate and precise mathematical vocabulary. • Plan a solution pathway. • Make and analyze mathematical conjectures. • Solve real-world and mathematical problems accurately. • Create a drawing and add components as appropriate. *Continued on next page*

G.CO.11 Prove and apply theorems about parallelograms. *Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** • Form conjectures about geometric relationships and examine their validity, providing evidence to support or refute the claim. • Using previously established facts about lines, angles, triangles, and parallelograms, construct a valid argument for why a conjecture is true or not true. • Solve problems involving lines, angles, triangles, and parallelograms by applying theorems. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, page 4-5 **CONNECTIONS ACROSS STANDARDS** • Experiment with transformations in the plane (G.CO.1, 3, 4). • Understand congruence in terms of rigid transformations (G.CO.6-8). • Use coordinates to prove simple geometric theorems algebraically (G.GPE.4-5). • Prove and apply theorems involving similarity (G.SRT.4,5).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **CONGRUENCE** **Make geometric constructions.** **G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	**Expectations for Learning** In elementary and middle school, students learn to use measurement tools to informally draw geometric shapes with given conditions. In this cluster, students make formal and precise constructions using a variety of tools, and they understand the geometric relationships upon which the constructions are based. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Construction is a process of reasoning that does not use a scale and does not use measurement. • Simple constructions can be used to develop an understanding of mathematical relationships. **MATHEMATICAL THINKING** • Make sound decisions about using tools. • Strategically use technology to deepen understanding. • Plan a pathway to complete constructions. • Determine accuracy of results. • Create a drawing and add components as appropriate. **INSTRUCTIONAL FOCUS** • Distinguish between a rough sketch, a careful drawing with measurements, and a construction with compass and straightedge. • Use a variety of geometric tools to make precise constructions. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Experiment with transformations in the plane (G.CO.1, 5). • Understand and apply theorems about circles (G.C.3, (+)4). • Prove and apply geometric theorems (G.CO.9-11). • Prove and apply theorems involving similarity (G.SRT.4-5).

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STANDARDS	MODEL CURRICULUM
Geometry **CONGRUENCE** **Classify and analyze geometric figures.** **G.CO.14** Classify two-dimensional figures in a hierarchy based on properties	**Expectations for Learning** In elementary school, students learn to classify two-dimensional figures based on their properties. In middle school, students focus on drawing quadrilaterals and triangles with given conditions. Now in high school, they learn to analyze and relate categories of two-dimensional shapes explicitly based on their properties. Based on analysis of properties, students create hierarchies for two-dimensional figures. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • There is a distinction between the definition of a figure and its properties, e.g., side lengths, angles, parallel/perpendicular sides, diagonals, symmetry. • Figures may be categorized in different ways based on their properties. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to describe geometric relationships. • Make connections between terms and properties. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Generalize concepts based on patterns. • Use formal reasoning. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Explain the difference between the definition of a figure and its properties. • Know precise definitions of special polygons, e.g., rhombus, parallelogram, rectangle, square, kite, trapezoid, isosceles trapezoid, equilateral triangle, isosceles triangle, and regular polygon. • Compare and contrast definitions of quadrilaterals, including both definitions of trapezoids. • Know and apply properties of special polygons and use them to classify figures. • Explain the relationships among special quadrilaterals. • Explain the relationships among special triangles. • Create hierarchies in order to represent the relationship between pairs of figures and among several figures. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Prove and apply theorems about quadrilaterals and triangles (G.CO.10-11). • Prove theorems algebraically about quadrilaterals and triangles using coordinates (G.GPE.4). • Justify the slope criteria for parallel and perpendicular lines (G.GPE.5). • Know precise definitions (G.CO.1).

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Geometry

SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

Understand similarity in terms of similarity transformations.

G.SRT.1 Verify experimentally the properties of dilations[G] given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations[G] to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

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Expectations for Learning

The standards in this cluster make more precise the informal notion of "same shape, different size." In middle school, students represent proportional relationships within and between similar figures; create scale drawings; describe the effect of dilations on two-dimensional figures; and understand similarity transformations as a sequence of basic rigid motions and dilations. In this cluster, students verify the properties (given center and scale factor) of dilations and use those properties to establish the AA criterion for triangles. They also explore the relationships among corresponding parts of similar figures.

The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction).

ESSENTIAL UNDERSTANDINGS

- A dilation requires a center and a scale factor.
- A similarity transformation often requires a sequence of basic rigid motions, in addition to a dilation.
- A scale factor is a ratio corresponding lengths between figures.
- A similarity transformation with a scale factor of 1 is a special case, which is a congruence transformation.
- While the definition of similarity applies to polygons, it also applies to non-polygonal shapes, e.g., circles, parabolas, etc.
- The AA criterion is equivalent to the AAA criterion because the angle sum in a triangle is 180 degrees.
- The AA criterion and the AAA criterion apply only to triangles.

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to represent geometric relationships.
- Use formal reasoning with symbolic representation.
- Determine reasonableness of results.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Make connections between terms and properties.

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Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Use basic rigid motions and dilations to map similar figures onto one another.
- Given a figure, carry out a similarity transformation, and then verify its properties.
- Know the precise definitions of dilation and similarity.
- Identify center and scale factor of a dilation.
- Determine the scale factor.
- Establish that triangles with two pairs of corresponding congruent angles are similar.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 3, page 6

CONNECTIONS ACROSS STANDARDS

- Prove theorems involving similarity (G.SRT.4-5).
- Solve problems involving right triangles trigonometry (G.SRT.6-8).
- Apply the understanding that all circles are similar (G.C.1).
- Represent transformations in the plane (G.CO.2).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.6).
- Apply geometric concepts in modeling situations (G.MG.1-3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** **Prove and apply theorems both formally and informally involving similarity using a variety of methods.** **G.SRT.4** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.	**Expectations for Learning** In middle school, students draw, construct, and describe geometric figures; use informal arguments to establish facts about similar triangles; and explain a proof of the Pythagorean Theorem and its converse. In this cluster, students prove theorems and solve problems involving similarity of triangles. They will also solve problems by applying these theorems to geometric figures that can be decomposed into triangles. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstraction) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • The altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. • A line parallel to the side of a triangle makes similar triangles and divides the other two side lengths proportionally. • Two right triangles are similar if they have another congruent angle. • Polygons can be divided into congruent and/or similar triangles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Use formal reasoning with symbolic representation. • Make conjectures. • Plan a solution pathway. • Justify relationships in geometric figures. • Determine reasonableness of results. • Create a drawing and add components as appropriate. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Form conjectures and construct a valid argument for why the conjecture is true or not true, both formally and informally.
- Recognize when polygons are divided into congruent and/or similar triangles.
- Justify relationships in geometric figures that can be decomposed into triangles.
- Solve problems using triangle congruence and similarity.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 3, page 6

CONNECTIONS ACROSS STANDARDS

- Understand similarity (G.SRT.1-3).
- Define trigonometric ratios, and solve problems involving right triangles (G.SRT.6-8).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.5-6).
- Use coordinate geometry (G.GPE.4).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.6).
- Represent transformations in the plane (G.CO.2).
- Prove and apply geometric theorems (G.CO.9-10).
- Make geometric constructions (G.CO.12-13).
- Find arc lengths and areas of sectors of circles (G.C.5).
- Apply geometric concepts in modeling situations (G.MG.2-3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

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STANDARDS	MODEL CURRICULUM
Geometry **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** **Define trigonometric ratios, and solve problems involving right triangles.** **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.8** Solve problems involving right triangles.★ **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)	**Expectations for Learning** In middle school, students draw, construct, and describe geometric figures; use informal arguments to establish facts about similar triangles; and explain a proof of the Pythagorean Theorem and its converse. In this cluster, students use similarity to define trigonometric ratios and then solve problems using right triangles (excluding inverse trigonometric functions). The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Because right triangles with the same acute angle are similar, within-figure ratios are equal. Three of these possible ratios are named sine, cosine, and tangent. • The sine of an acute angle is equal to the cosine of its complement and vice versa. • Given an angle and a side length of a right triangle, the triangle can be solved, which means finding the missing sides and angles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Discern and use a pattern or structure. • Plan a solution pathway. • Justify relationships in geometric figures. • Determine reasonableness of results. • Create a drawing and add components as appropriate. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. • Connect mathematical relationships to real-world encounters. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Define trigonometric ratios for acute angles (sine, cosine, tangent).
- Explain and apply the relationship between sine and cosine of complementary angles.
- Solve problems involving right triangles (excluding inverses of trigonometric functions).
- Use the Pythagorean Theorem to explore exact trigonometric ratios for 30, 45, and 60 degree angles (fluency not required).
- Use triangle similarity criteria to define trigonometric ratios.
- Given the sine, cosine, or tangent of an angle, find other trigonometric ratios in the triangle.
- Solve mathematical and real-world problems given a side and an angle (or the sine, cosine, or tangent of an angle) of a right triangle.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 3, page 6

CONNECTIONS ACROSS STANDARDS

- Understand similarity (G.SRT.1-3).
- Prove and apply theorems involving similarity (G.SRT.4-5).
- Apply geometric concepts in modeling situations (G.MG.1-3).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **CIRCLES** **Understand and apply theorems about circles.** **G.C.1** Prove that all circles are similar using transformational arguments. **G.C.2** Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G.C.3** Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle. (+) **G.C.4** Construct a tangent line from a point outside a given circle to the circle.	**Expectations for Learning** In middle school, students have worked with measurements of circles such as circumference and area. In this cluster, students extend their understanding of similarity to circles. Students solve problems using the relationships among the arcs and angles created by radii, chords, secants, and tangents. They will also construct inscribed and circumscribed circles of a triangle. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • All circles are similar because one circle can be translated so that its center maps onto the center of the other and then dilated about the common center by the ratio of the radii. • The measure of an arc is equal to the measure of its corresponding central angle. • The measure of an inscribed angle is half the measure of its corresponding central angle. • Inscribed angles on a diameter of a circle are right angles (special case of inscribed angles). • A tangent is perpendicular to the radius at the point of tangency. • A secant is a line that intersects a circle at exactly two points. • A circumscribed angle is created by two tangents to the same circle from the same point outside the circle. • The center of the circumscribed circle is the point of concurrency of the perpendicular bisectors because it is equidistant from the vertices of the triangle. • The center of the inscribed circle is the point of concurrency of the angle bisectors because it is equidistant from the sides of the triangle. • While all triangles can be inscribed in a circle, a quadrilateral can be inscribed in a circle if and only if the opposite angles in the quadrilateral are supplementary. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary.
- Make connections between concepts, terms, and properties.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Solve mathematical and real-world problems accurately.
- Determine reasonableness of results.
- Consider mathematical units involved in a problem.
- Make sound decisions about using tools.

INSTRUCTIONAL FOCUS

- Use transformational arguments to prove that all circles are similar.
- Given a diagram, identify radii, chords, secants, and tangents, and the arcs and angles formed by them.
- Solve mathematical and real-world problems involving angles and arcs formed by radii, chords, secants, and tangents.
- Construct the angle bisectors of a triangle to locate the incenter, and then construct the inscribed circle.
- Construct the perpendicular bisectors of a triangle to locate the circumcenter, and then construct the circumscribed circle.
- Provide an informal argument for why the opposite angles of an inscribed quadrilateral are supplementary based on the arcs the angles intercept and their corresponding central angles.
- Solve problems using the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.
- (+) Construct a tangent line from a point outside a given circle to the circle.

Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 5, page 8 **CONNECTIONS ACROSS STANDARDS** • Experiment with transformations in the plane (G.CO.1). • Make geometric constructions (G.CO.12). • Understand similarity in terms of similarity transformations (G.SRT.2). • Find arc length and areas of sectors of circles (G.C.5).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Geometry **CIRCLES** **Find arc lengths and areas of sectors of circles.** **G.C.5** Find arc lengths and areas of sectors of circles. **a.** Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. **b.** Derive the formula for the area of a sector, and use it to solve problems.	**Expectations for Learning** In middle school, students are limited to working with measurements of circles such as circumference and area. This cluster spans Geometry/ Mathematics 2 and Algebra 2/ Mathematics 3. In Geometry/Mathematics 2, students are using part-to-whole proportional reasoning to find arc lengths and sector areas, in which the arc or central angle is measured in degrees. In Algebra 2/ Mathematics 3, students derive and use formulas relating degree and radian measure. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). *Note: Since in Algebra 1 students focus on quadratics with leading coefficients of 1 with occasional uses of other simple coefficients, geometry standards should only apply to equations where the squared terms have a coefficient of 1 or occasionally other simple leading coefficients.* **ESSENTIAL UNDERSTANDINGS** • A central angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. • The measure of an arc is equal to the measure of the corresponding central angle and is expressed in degrees, while the length of an arc is expressed in units of linear measure. • The arc length is a part of the circumference of a circle. • The ratio of the central angle to 360 degrees is equal to the ratio of the length of the arc to the circumference of the circle. • The sector area is a part of the area of a circle. • The ratio of the central angle to 360 degrees is equal to the ratio of the area of the sector to the area of the circle. • Because all circles are similar, if the radius of the circle is scaled by k, the corresponding arc length is multiplied by k and the sector area is multiplied by k^2. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Consider mathematical units involved in a problem.
- Make connections between concepts and terms.
- Generalize concepts based on patterns.
- Use proportional reasoning (part to whole).
- Draw a picture to make sense of a problem.
- Solve real-world and mathematical problems accurately.
- Plan a solution pathway.
- Attend to the meaning of quantities.

INSTRUCTIONAL FOCUS

- Develop understanding of the formulas for arc length and area of a sector. through derivation.
- Solve problems using arc lengths and areas of sectors of circles.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 5, page 8

CONNECTIONS ACROSS STANDARDS

- Understand and apply theorems about circles (G.C.1-2).
- Experiment with transformations in the plane (G.CO.1).
- Explain volume formulas, and use them to solve problems (G.GMD.1).
- Understand similarity in terms of similarity transformations (G.SRT.2).
- Understand the relationships between lengths, areas, and volumes (G.GMD.5-6).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS** **Translate between the geometric description and the equation for a conic section.** **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**Expectations for Learning** In middle school, students use the Pythagorean Theorem to find distances between points within the coordinate system. In the high school algebra, students complete the square to solve quadratic equations. In this cluster, students derive the equation of a circle using the Pythagorean Theorem. They also complete the square to find the center and radius of a circle. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • The equation of a circle relates a fixed center, a fixed radius, and a set of variable points, which are the points on the circle. • Just as the distance formula is an application of the Pythagorean Theorem, so is the equation of a circle. **MATHEMATICAL THINKING** • Use precise mathematical language. • Discern and use a pattern or structure. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Justify relationships in geometric figures. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. **INSTRUCTIONAL FOCUS** • Use the Pythagorean Theorem to derive the equation of a circle. • Given the equation of a circle that is not in standard form, find the center and radius of the circle by completing the square. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 5, page 8 **CONNECTIONS ACROSS STANDARDS** • Prove that all circles are similar (G.C.1). • Prove theorems about triangles (G.SRT.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Geometry **EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS** **Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.** **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2) **G.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point. **G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	**Expectations for Learning** In middle school, students find the distance between two points in a coordinate system; work with linear functions; solve linear equations; and apply the Pythagorean Theorem in the coordinate system. In addition, they use square root symbols to represent solutions to equations and they evaluate square roots of rational numbers. In this cluster, students use the coordinate system to justify slope criteria for parallel and perpendicular lines; partition line segments proportionally; and compute perimeters and areas of geometric figures. These strategies are used for proof of geometric relationships and properties. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • Coordinate proof is a method that uses algebraic techniques to prove geometric theorems and properties. • Properties of geometric figures, especially special quadrilaterals, can be proven on a coordinate plane using lengths of segments, slopes of lines, and equations of lines. • Coordinate proof can be used to prove that figures are congruent or similar. • The slopes of parallel lines are equal, and the product of the slopes of perpendicular lines is –1, except for horizontal and vertical lines. • Partitioning a line segment into a given ratio is an application of similar triangles. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to represent geometric relationships.
- Make connections between terms and formulas.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Compute using strategies or models.
- Use formal reasoning with symbolic representation.
- Determine reasonableness of results.
- Solve multi-step problems accurately.
- Discern and use a pattern or structure.

INSTRUCTIONAL FOCUS

- Know and use the distance formula.
- Use coordinates to prove simple geometric theorems algebraically.
- Verify geometric relationships algebraically.
- Justify the slope criteria for parallel and perpendicular lines.
- Write equations of parallel and perpendicular lines.
- Solve geometric problems using the slopes of parallel and perpendicular lines.
- Partition a line segment given a ratio.
- Use coordinates to compute perimeters and areas.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 4, page 7
- Geometry, Number 5, page 8

CONNECTIONS ACROSS STANDARDS

- Know precise definitions (G.CO.1).
- Prove geometric theorems (G.CO.9-10).
- Understand and apply theorems about circles (G.C.1-4).
- Prove theorems involving similarity (G.SRT.4-5).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6).
- Apply geometric concepts in modeling situations (G.MG.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
Geometry **GEOMETRIC MEASUREMENT AND DIMENSION** **Explain volume formulas, and use them to solve problems.** **G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	**Expectations for Learning** In middle school, students use established circumference, area, and volume formulas for two- and three-dimensional figures. Instead of using area and volume formulas rotely, students in this cluster give informal justifications for these formulas and use them to solve problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • A three-dimensional solid can be viewed as a stack of layers. • If all of the layers of a three-dimensional solid have the same area, then the volume is the area of the base times the height. • The volume remains unchanged when layers parallel to the base in a three-dimensional solid are shifted. • A cone's volume is $\frac{1}{3}$ of the volume of a cylinder if their base areas are equal and their heights are congruent. • A pyramid's volume is $\frac{1}{3}$ of the volume of a prism if their base areas are equal and their heights are congruent. • Volume, like area, is additive, so to find the volume of a composite figure, cut the figure into pieces of known volume and add or subtract as appropriate. • The cross sections of a cylinder are circles of equal area. • The cross sections of a prism are congruent to the base, so therefore the areas are equal. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Make and modify a model to represent mathematical thinking. • Attend to meaning of quantities. • Consider mathematical units involved in a problem. • Solve real-world and mathematical problems accurately. • Determine reasonableness of results. • Use informal reasoning.

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Explain and justify the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. • Apply volume formulas in real-world and mathematical problems. ★ • (+) Informally apply Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 3, page 6 **CONNECTIONS ACROSS STANDARDS** • Experiment with transformations in the plane (G.CO.1). • Understand and apply theorems about circles (G.C.2, 5). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). • Apply geometric concepts in modeling situations (G.MG.1-3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **GEOMETRIC MEASUREMENT AND DIMENSION** **Visualize relationships between two-dimensional and three-dimensional objects.** **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**Expectations for Learning** In middle school, students identify cross-sections as a result of slicing right rectangular prisms and pyramids. In this cluster, which supports the previous cluster, students extend the identification of cross-sections to include other three-dimensional solids. Students will also identify three-dimensional objects created when a two-dimensional object is rotated about a line. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Two-dimensional figures can be used to understand three-dimensional solids. • A three-dimensional figure can be created by rotating a two-dimensional figure about a line. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Use technology strategically to deepen understanding. • Make connections between concepts, terms, and properties. **INSTRUCTIONAL FOCUS** • Identify two-dimensional cross-sections of three-dimensional objects. • Identify three-dimensional objects formed by rotations of two-dimensional objects. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 3, page 6 **CONNECTIONS ACROSS STANDARDS** • Experiment with transformations in the plane (G.CO.1, 5). • Understand congruence in terms of rigid motions (G.CO.6). • Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). • Apply geometric concepts in modeling situations (G.MG.1-3). • Classify and analyze geometric figures (G.CO.14).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **GEOMETRIC MEASUREMENT AND DIMENSION** **Understand the relationships between lengths, areas, and volumes.** **G.GMD.5** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. **G.GMD.6** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.	**Expectations for Learning** In middle school, students solve problems involving two-dimensional similar figures and calculate the volumes of three-dimensional figures. In this cluster, students extend their knowledge of similarity to explore and understand how changes to length or angle measure in one figure will result in similar or non-similar figures. Students will also understand the effect that a scale factor has on the length, area, and volume of similar figures and use this relationship to solve problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Changes to the lengths and/or angle measures of a figure result in similar and non-similar figures. • When changes to a figure result in similar figures with a scale factor of k, the lengths of the resulting figures are a multiple of k. • When changes to a figure result in similar figures with a scale factor of k, the areas of the resulting figures are a multiple of k^2. • When changes to a figure result in similar figures with a scale factor of k, the volume of the resulting figures are a multiple of k^3. **MATHEMATICAL THINKING** • Use precise mathematical language. • Draw a picture or create a model to make sense of a problem. • Determine reasonableness of results. • Solve multi-step problems accurately. • Plan a solution pathway. • Solve mathematical and real-world problems accurately. • Consider mathematical units involved in a problem. • Attend to the meaning of quantities. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Generalize concepts based on patterns. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Classify objects as similar or non-similar when the lengths or angles of figures are changed. • Explain the types of changes to a figure that result in similar and non-similar figures. • Use geometry and algebra to explain how length, area, and volume are affected when scaling is applied. • Solve problems involving length, area, and volume of figures under scaling. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 6, page 9 **CONNECTIONS ACROSS STANDARDS** • Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3). • Understand similarity in terms of similarity transformations (G.SRT.1-2). • Apply geometric concepts in modeling situations (G.MG.2-3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **MODELING WITH GEOMETRY** **Apply geometric concepts in modeling situations.** **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★ **G.MG.2** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★ **G.MG.3** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★	**Expectations for Learning** In middle school, students work with nets, area, and volume; use appropriate tools to represent situations; and solve real-life and mathematical problems. In this cluster, students make sense of the world around them by using geometric models and their properties to solve more sophisticated problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Composite figures can be analyzed by approximating them with traditional two- and three-dimensional figures. • Many real-life scenarios are related to length, area, and volume. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Make connections between terms and properties. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Use formal reasoning with symbolic representation. • Determine reasonableness of results. • Use proportional reasoning. • Plan a solution pathway. • Connect mathematical relationships to real-world encounters. • Draw a picture or create a model to represent a problem. **INSTRUCTIONAL FOCUS** • Use geometric shapes, their measures, and their properties to describe objects. • Identify useful quantities for modeling situations. • Apply concepts of density based on area and volume. • Solve design problems geometrically. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 2, page 5 • Geometry, Number 3, page 6 • Geometry, Number 5, page 8 **CONNECTIONS ACROSS STANDARDS** • Solving problems involving right triangles (G.SRT.8). • Use volume formulas to solve problems (G.GMD.3-4, 6). • Use coordinates to compute perimeters and areas (G.GPE.7).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS

Statistics and Probability

CONDITIONAL PROBABILITY AND RULES OF PROBABILITY

Understand independence and conditional probability, and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S.CP.3 Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

Continued on next page

MODEL CURRICULUM

Expectations for Learning

In middle school, students develop basic probability skills including probability as relative frequencies; probabilities of compound events; the development a uniform/non-uniform probability model; and the use of tree diagrams. Also students are introduced to two-way frequency tables in middle school. However, students' only prior exposure to the concept of independence was in S.ID.5 (Algebra 1/Math 1). This cluster focuses on the concept of independence between two categorical variables. It also focuses on the understanding of independence rather than symbolic notation and formulas. Fluency with independence is expected by the end of Geometry/Math 2.

ESSENTIAL UNDERSTANDINGS

- Approximations for the true probability of an event can be found by looking at the long-run relative frequency.
- The sample space of a probability experiment can be modeled with a Venn diagram.
- The union of an event and its complement represent the entire sample space.
- The intersection of an event and its complement represent the empty set.
- Conditional probability is the probability of event A occurring given that event B has occurred. It is denoted by $A|B$ and is read "A given B."
- Two events occurring in succession are said to be independent if the outcome of one event has no effect on the outcome of the other, e.g., a coin tossed twice. Otherwise, the events are dependent, e.g., two cards are drawn in succession from a standard deck of cards.
- The intersection of two sets A and B is the set of elements that are common to both set A and set B. It is denoted by $A \cap B$ and is read "A intersection B" as well as "A and B."
- The union of two sets A and B is the set of elements, which are in A or in B or in both. It is denoted by $A \cup B$ and is read "A union B" as well as "A or B."
- If A and B are events that have no outcomes in common $(A \cap B \neq 0)$, they are said to be mutually exclusive. Mutually exclusive events cannot occur together.

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S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

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Expectations for Learning, continued

MATHEMATICAL THINKING

- Use appropriate vocabulary.
- Attend to precision.

INSTRUCTIONAL FOCUS

- Recognize and explain for two successive events, whether the outcome of the first event affects the outcome of the second event.
- Recognize and justify conceptually whether two events are independent.
- Make connections between conditional probability and independence. Recognize sample space subsets in everyday contexts.
- Identify an event and its complement.
- Identify which components of the sample space represent the union and intersection of two events.
- Explain what a conditional probability means within a context.
- Distinguish between a conditional probability (A given B) and the probability of an intersection (A and B).
- Use a two way frequency table to determine the following:
    - conditional probabilities;
    - probabilities of the sample space subsets;
    - event independence by comparing joint probabilities (P(A and B)) and the product of the separate probabilities (P(A) × P(B)); and
    - event independence by comparing the conditional probability (P(A given B)) and the probability P(A).

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 1, page 3

CONNECTIONS ACROSS STANDARDS

- This will lead into the cluster (S.CP.6-9) which includes the calculations of conditional probabilities, and the use of probability formulas and set notation with probability.

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Statistics and Probability **CONDITIONAL PROBABILITY AND RULES OF PROBABILITY** **Use the rules of probability to compute probabilities of compound events in a uniform probability model.** **S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ **S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★ (+) **S.CP.8** Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★ (G, M2) (+) **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.★ (G, M2)	**Expectations for Learning** In middle school, students develop basic probability skills including probability as relative frequencies; probabilities of compound events; development of a uniform/non-uniform probability model; and the use of tree diagrams. Also they are introduced to two-way frequency tables in middle school. Now in Geometry/Math 2 this cluster formalizes the concepts of conditional probability and independence in S.CP.1-5. The focus of this cluster is developing the Addition Rule and the (+) Multiplication Rule in everyday contexts. Although permutations and combinations are part of Geometry/Math 2 for students who pursue advanced mathematics, these concepts would also be appropriate in a fourth year course. Exploration of the Fundamental Counting Principle and factorials (!) may also be addressed in a fourth year course. **ESSENTIAL UNDERSTANDINGS** • Compound probabilities model real-world scenarios and must be interpreted within a context. • The conditional probability of A *given* B is the fraction of B's outcomes that also belong to A. This can be expressed by $P(A\mid B) = P(A \cap B)/P(B)$. • The addition rule is $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ and can also be expressed as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. • (+) The Multiplication Rule is $P(A\ and\ B) = P(A) * P(B\mid A) = P(B) * P(A\mid B)$. • (+) Permutations and combinations are strategies for counting the outcomes of a sample space. **MATHEMATICAL THINKING** • Use precise mathematical language. • Look for and make use of structure. • Compute accurately and efficiently. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Recognize and justify mathematically whether two events are independent. • Generalize probability rules using patterns. • Compute probabilities accurately and efficiently. • (+) Recognize and apply counting methods to compute probabilities. • Compute the conditional probability of A given B. • Interpret and explain conditional probability within a context. • Apply the Addition Rule. • Interpret and explain the Addition Rule within a context. • (+) Apply the Multiplication Rule. • (+) Interpret and explain the Multiplication Rule within a context. • (+) Know and explain the difference between a permutation and a combination. • (+) Calculate probabilities using permutations and combinations. • (+) Interpret and explain probabilities using permutations and combinations within a context. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Geometry, Number 1, page 3 **CONNECTIONS ACROSS STANDARDS** • Understand independence and conditional probability (S.CP.1-5).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

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Acknowledgments

Meghan Arnold
Teacher,
Miami East Local Schools, SW

Alex Blohm
Teacher,
South-Western City Schools, C

Kathleen Cooey
Curriculum Specialist/Coordinator,
West Geauga Local Schools, NE

Brett Doudican
Teacher, Dayton Early College, SW

Diane Farmer
Teacher,
Cleveland Catholic Dioceses, NE

Shari Fergusen
Teacher,
Tri-County North Local Schools, SW

Brad Findell (AC)
Higher Education,
Ohio State University, C

Ali Fleming (WG)
Teacher, Bexley City Schools, C

Christian Hamman
Curriculum Specialist/Coordinator,
Medina City Schools, NE

Aryn Hinkle
Teacher,
Perrysburg Exempted Village, NW

Margaret (Peggy) Kasten (AC)
Retired Educator, C

Endora Kight Neal (WG)
Curriculum Specialist/Coordinator,
Cleveland Metropolitan School District, NE

John Kramer
Teacher,
Flex High Charter School, C

Robert Krauss
Teacher, Manchester Local School District, SW

Emily Meister
Teacher,
Grandview Heights Schools, C

Stephen Miller
Curriculum Specialist/Coordinator,
Summit County ESC, NE

Stephanie Minor
Teacher,
Worthington School District, C

Tara Nicely
Teacher, Chillicothe City Schools, C

Marla Norman
Teacher, Lebanon City Schools, SW

Steve Phelps
Teacher, Madeira City Schools, SW

Stephanie Putzier
Curriculum Specialist/Coordinator,
Mahoning County ESC, NE

Tess Rivero (WG)
Teacher, Bellbrook-Sugarcreek Local Schools, SW

Parameswaran Sivagurunathan
Teacher, Cleveland Metropolitan School District, NE

Patricia Talarczyk
Teacher,
Mentor Exempted Village, NE

Heather VerStreate
Teacher, Mason City Schools, SW

Jennifer Walls (WG)
Teacher, Akron Public Schools, NE

Verdie Williams
Teacher, Pickaway Ross Joint Vocational School District, C

*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.