Standards for Mathematical Practice Progressions

Mathematical Practice 1: Make sense of problems and persevere in solving them.		
Kindergarten	**Grade 1**	**Grade 2**
In Kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, —Does this make sense? or they may try another strategy.	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They are willing to try other approaches.	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach. An example for this might be giving a student an equation and having him/her write a story to match.

Mathematical Practice 1: Make sense of problems and persevere in solving them.		
Grade 3	**Grade 4**	**Grade 5**
In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students may use concrete objects, pictures, or drawings to help them conceptualize and solve problems, such as “Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?” or “Describe another situation where there would be 5 groups of 3 or 5 × 3.” Students may check their thinking by asking themselves, “Does this make sense?” Students listen to other students’ strategies and can make connections between various methods for a given problem.	In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers. Students might use an equation strategy to solve the word problem. For example, students could solve the problem “Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?” with the equation 3 × $12 + $15 = a. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.	Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. For example, Sonia had $2\frac{1}{3}$ candy bars. She promised her brother that she would give him $\frac{1}{2}$ of a candy bar. How much will she have left after she gives her brother the amount she promised? They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”.

Mathematical Practice 1: Make sense of problems and persevere in solving them.		
Grade 6	**Grade 7**	**Grade 8**
In grade 6, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”. Students can explain the relationships between equations, verbal descriptions, and tables and graphs. Mathematically proficient students check their answers to problems using a different method.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”. When students compare arithmetic and algebraic solutions to the same problem, they identify correspondences between different approaches.	In grade 8, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”

Algebra 1	Math 1	Geometry	Math 2	Algebra 2/Math 3
Students learn that patience is often required to fully understand what a problem is asking. They discern between useful and extraneous information. They expand their repertoire of expressions and functions that can be used to solve problems.	Students persevere when attempting to understand the differences between linear and exponential functions. They make diagrams of geometric problems to help make sense of the problems.	Students construct accurate diagrams of geometry problems to help make sense of them. They organize their work so that others can follow their reasoning, e.g., in proofs.	Students persevere when attempting to understand the differences between quadratic functions and the linear and exponential functions they studied previously. They create diagrams of geometric problems to help make sense of the problems.	Students apply their understanding of various functions to real-world problems. They approach complex mathematics problems and break them down into smaller problems, synthesizing the results when presenting solutions.

(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)

Mathematical Practice 2: Reason abstractly and quantitatively.		
Kindergarten	**Grade 1**	**Grade 2**
Younger students begin to recognize that a number represents a specific quantity. Then, they connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. For example, a student may write the numeral 11 to represent an amount of objects counted, select the correct number card 17 to follow 16 on a calendar, or build two piles of counters to compare the numbers 5 and 8. In addition, kindergarten students begin to draw pictures, manipulate objects, or use diagrams or charts to express quantitative ideas. Students need to be encouraged to answer questions such as “How do you know?”, which reinforces their reasoning and understanding and helps student develop mathematical language.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. In first grade students make sense of quantities and relationships while solving tasks. They represent situations by decontextualizing tasks into numbers and symbols. For example, “There are 60 children on the playground and some children go line up. If there are 20 children still playing, how many children lined up?” Students translate the situation into the equation: 60 – 20 = ☐ and then solve the task. Students also contextualize situations during the problem-solving process. For example, students refer to the context of the task to determine they need to subtract 20 from 60 because the total number of children on the playground is the total number less the 20 that are still playing. Students might also reason about ways to partition two-dimensional geometric figures into halves and fourths.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length. In second grade students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, “There are 25 children in the cafeteria, and they are joined by 17 more children. How many students are in the cafeteria?” Students translate the situation into an equation, such as: 25 + 17 = ☐ and then solve the problem. Students also contextualize situations during the problem-solving process. For example, while solving the task above, students might refer to the context of the task to determine that they need to subtract 19 if 19 children leave.

<table>
<tr><th colspan="3">Mathematical Practice 2: Reason abstractly and quantitatively.</th></tr>
<tr><th>Grade 3</th><th>Grade 4</th><th>Grade 5</th></tr>
<tr>
<td>Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. For example: students apply their understanding of the meaning of the equal sign as “the same as” to interpret an equation with an unknown. When given 4 × ☐ = 40, they might think:
• 4 groups of some number is the same as 40
• 4 times some number is the same as 40
• I know that 4 groups of 10 is 40 so the unknown number is 10
• The missing factor is 10 because 4 times 10 equals 40.

Teachers might ask, “How do you know” or “What is the relationship between the quantities?” to reinforce students’ reasoning and understanding.</td>
<td>Fourth graders should recognize that a number represents a specific quantity. They connect the quality to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. Students might use base 10 blocks or drawings to demonstrate 154 × 6, as 154 added six times, and develop an understanding of the distributive property. For example:
154 × 6
= (100 + 50 + 4) × 6
= (100 × 6) + (50 × 6) + (4 × 6)
= 600 + 300 + 24 = 924</td>
<td>Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts. For example, students use abstract and quantitative thinking to recognize that 0.5 × (300 ÷ 15) is $\frac{1}{2}$ of (300 ÷ 15) without calculating the quotient.</td>
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<table>
<tr><th colspan="4">Mathematical Practice 2: Reason abstractly and quantitatively.</th></tr>
<tr><th colspan="2">Grade 6</th><th>Grade 7</th><th>Grade 8</th></tr>
<tr><td colspan="2">In grade 6, students represent a wide variety of real-world contexts using real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations or other meaningful moves. To reinforce students' reasoning and understanding, teachers might ask, "How do you know?" or "What is the relationship of the quantities?".</td><td>In grade 7, students represent a wide variety of real-world contexts using real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.</td><td>In grade 8, students represent a wide variety of real-world contexts using real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number(s) or variable(s) as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.</td></tr>
<tr><th>Algebra 1 & Math 1</th><th>Geometry</th><th>Math 2</th><th>Algebra 2/Math 3</th></tr>
<tr><td>Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</td><td>Students understand that the coordinate plane can be used to represent geometric shapes and transformations, and therefore they connect their understanding of number and algebra to geometry.</td><td>Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</td><td>Students deepen their understanding of transformations of graphs by changing the form of rational function y(x) = $\frac{a(x)}{b\ (x)}$, where a(x) and b(x) represent polynomials and b(x) is not 0, to reveal and interpret the key features of the function.</td></tr>
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Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.		
Kindergarten	**Grade 1**	**Grade 2**
Younger students construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking. They begin to develop the ability to reason and analyze situations as they consider questions such as “Are you sure that ___?”, “Do you think that would happen all the time?”, and “I wonder why ___?”	First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They also practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?”, “Explain your thinking.”, and “Why is that true?” They not only explain their own thinking but listen to others’ explanations. They decide if the explanations make sense and ask questions. For example, “There are 15 books on the shelf. If you take some books off the shelf and there are now 7 left, how many books did you take off the shelf?” Students might use a variety of strategies to solve the task and then share and discuss their problem-solving strategies with their classmates.	Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?”, “Explain your thinking.”, and “Why is that true?” They not only explain their own thinking but listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions. Students critique the strategies and reasoning of their classmates. For example, to solve 74 − 18, students may use a variety of strategies, and after working on the task, they might discuss and critique each other’s’ reasoning and strategies, citing similarities and differences between various problem-solving approaches.

(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)

Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.		
Grade 3	**Grade 4**	**Grade 5**
Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as “How did you get that?” and “Why is that true?” Students explain their thinking to others and respond to others’ thinking. For example, after investigating patterns on the 100s chart, students might explain why the pattern makes sense.	In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?”, “Explain your thinking,” and “Why is that true?” They not only explain their own thinking but listen to others’ explanations. Students explain and defend their answers and solution strategies as they answer question that require an explanation. For example, “Vincent cuts 2 meters of string into 4-centimeter pieces for a craft. How many pieces of string does Vincent have? Explain your reasoning.” Students ask appropriate questions, and they decide if explanations make sense.	In fifth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking. Students use various strategies to solve problems and they defend and justify their work with others. For example, two afterschool clubs are having pizza parties. The teacher will order 3 pizzas for every 5 students in the math club; and 5 pizzas for every 8 students in the student council. If a student is in both groups, decide which party he/she should attend. How much pizza will each student get at each party? If a student wants to have the most pizza, which party should he/she attend?

Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.		
Grade 6	**Grade 7**	**Grade 8**
In grade 6, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. For example, as students notice when geometric conditions determine a unique triangle, more than one triangle, or no triangle, they have an opportunity to construct viable arguments and critique the reasoning of others. Students should be encouraged to answer questions such as these: “How did you get that?” “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.	In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.

<table>
<tr><th>Algebra 1</th><th>Math 1</th><th>Geometry</th><th>Math 2</th><th>Algebra 2/Math 3</th></tr>
<tr><td>Students reason through the solving of equations, recognizing that solving an equation involves more than simply following rote rules and steps. They use language such as “If ___, then ___” when explaining their solution methods and provide justification for their reasoning.</td><td colspan="2">Students use formal and informal proofs to verify, prove, and justify geometric theorems with respect to congruence. These proofs can include paragraph proofs, flow charts, coordinate proofs, two-column proofs, diagrams without words, or the use of dynamic software.</td><td>Students construct proofs of geometric theorems-based relationships between sine and cosine of complementary angles.</td><td>Students continue to reason through the solution of an equation and justify their reasoning to their peers. Students defend their choice of a function when modeling a real-world situation.</td></tr>
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(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)

Mathematical Practice 4: Model with mathematics.		
Kindergarten	**Grade 1**	**Grade 2**
In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all these representations as needed. For example, a student may use cubes or tiles to show the different number pairs for 5, or place three objects on a 10-frame and then determine how many more are needed to "make a ten." Students rely on manipulatives (or other visual and concrete representations) while solving tasks and record an answer with a drawing or equation.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all these representations as needed. First grade students model real-life mathematical situations with a number sentence or an equation and check to make sure equations accurately match the problem context. Students use concrete models and pictorial representations while solving tasks and write an equation to model problem situations. For example, to solve the problem, "There are 11 bananas on the counter. If you eat 4 bananas, how many are left?" students could write the equation $11 - 4 = 7$. Students also create a story context for an equation such as $13 - 7 = 6$.	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all these representations as needed. In grade two students model real-life mathematical situations with a number sentence or an equation and check to make sure that their equation accurately matches the problem context. They use concrete manipulatives and pictorial representations to explain the equation. They create an appropriate problem situation from an equation. For example, students create a story problem for the equation $43 + 17 = \chi$ such as "There were 43 gumballs in the machine. Tom poured in 17 more gumballs. How many gumballs are now in the machine?"

Mathematical Practice 4: Model with mathematics.		
Grade 3	**Grade 4**	**Grade 5**
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense. For example, students use various contexts and a variety of models (e.g., circles, squares, rectangles, fraction bars, and number lines) to represent and develop understanding of fractions. Students use models to represent both equations and story problems and can explain their thinking. They evaluate their results in the context of the situation and reflect on whether the results make sense. Students should be encouraged to answer questions, such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?”	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense. For example, students may use money (i.e., dollars and coins) or base10 blocks to solve the following problem: Elsie buys a drink for $1.39 and a granola bar for $0.89. How much change will she receive if she pays with a $5 bill?	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

Mathematical Practice 4: Model with mathematics.		
Grade 6	**Grade 7**	**Grade 8**
In grade 6, students model problem situations symbolically, graphically, in tables, contextually and with drawings of quantities as needed. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students begin to represent two quantities simultaneously. Students use number lines to compare numbers and represent inequalities. They use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all these representations as appropriate and apply them to a problem context. Students should be encouraged to answer questions such as “What are some ways to represent the quantities?” or “What formula might apply in this situation?”	In grade 7, students model problem situations symbolically, graphically, in tables, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use experiments or simulations to generate data sets and create probability models. Proportional relationships present opportunities for modeling. For example, for modeling purposes, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building. Students should be encouraged to answer questions such as “What are some ways to represent the quantities?” or “How might it help to create a table, chart, or graph?”	In grade 8, students model problem situations symbolically, graphically, in tables, and contextually. Working with the new concept of a function, students learn that relationships between variable quantities in the real-world often satisfy a dependent relationship, in that one quantity determines the value of another. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all these representations as appropriate to a problem context. Students should be encouraged to answer questions such as “What are some ways to represent the quantities?” or “How might it help to create a table, chart, graph or ___?”

Mathematical Practice 4: Model with mathematics.				
Algebra 1	**Math 1**	**Geometry**	**Math 2**	**Algebra 2/Math 3**
Students also discover mathematics through experimentation and by examining data patterns from real-world contexts. They apply their new mathematical understanding of exponential, linear, and quadratic functions to real-world problems.	Students apply their mathematical understanding of linear and exponential functions to many real-world problems, such as linear and exponential growth. Students also discover mathematics through experimentation and by examining patterns in data from real-world contexts.	Students apply their new mathematical understanding to real-world problems. They learn how transformational geometry and basic trigonometric functions can be used to model the physical world.	Students apply their mathematical understanding of quadratic functions to real-world problems. They also discover mathematics through experimentation and by examining patterns in data from real-world contexts.	Students apply their new mathematical understanding to real-world problems, making use of their expanding repertoire of functions in modeling. Students also discover mathematics through experimentation and by examining patterns in data from real-world contexts.

(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)

Mathematical Practice 5: Use appropriate tools strategically.		
Kindergarten	**Grade 1**	**Grade 2**
Younger students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide that it might be advantageous to use linking cubes to represent two quantities and then compare the two representations side-by-side or later, make math drawings of the quantities. Students decide which tools may be helpful to use depending on the problem or task and explain why they use particular mathematical tools.	In first grade, students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem. In first grade students use tools such as counters, place value (base ten) blocks, hundreds number boards, number lines, concrete geometric shapes (e.g., pattern blocks, 3-dimensional solids), and virtual representations to support conceptual understanding and mathematical thinking. Students determine which tools are the most appropriate to use. For example, when solving 12 + 8 = ☐ , students explain why place value blocks are more appropriate than counters.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation. Students may use tools such as snap cubes, place value (base ten) blocks, hundreds number boards, number lines, rulers, virtual manipulatives, and concrete geometric shapes (e.g., pattern blocks, three-dimensional solids). Students understand which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a ruler.

Mathematical Practice 5: Use appropriate tools strategically.		
Grade 3	**Grade 4**	**Grade 5**
Third graders consider the available tools (including drawings and estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table and determine whether they have all the possible rectangles. Students should be encouraged to answer questions such as, "Why was it helpful to use ___?"	Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper, a number line, or base 10 blocks to represent, compare, add, and subtract decimals to the hundredths. Students in fourth grade use protractors to measure angles. They use other measurement tools to understand the relative size of units within a given system and express measurements given in larger units in terms of smaller units.	Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real-world data.
Grade 6	**Grade 7**	**Grade 8**
Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent figures on the coordinate plane to calculate area. Number lines are used to create dot plots, histograms, and box plots to visually compare the center and variability of the data. Visual fraction models can be used to represent situations involving division of fractions. Additionally, students might use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures. Students should be encouraged to answer questions such as "What approach did you try first?" or "Why was it helpful to use?"	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms. Teachers might ask, "What approach are you considering?" or "Why was it helpful to use ___?"	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the between the angles created by a transversal that intersects parallel lines. Teachers might ask, "What approach are you considering?" or "Why was it helpful to use ___?"

Mathematical Practice 5: Use appropriate tools strategically.				
Algebra 1	**Math 1**	**Geometry**	**Math 2**	**Algebra 2/Math 3**
Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret results. They construct diagrams to solve problems.	Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the results.	Students make use of visual tools for representing geometry, such as simple patty paper, transparencies, or dynamic geometry software.	Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the result.	Students continue to use graphing technology to deepen their understanding of the behavior of polynomial, rational, square root, and trigonometric functions.

(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)

Mathematical Practice 6: Attend to precision.		
Kindergarten	**Grade 1**	**Grade 2**
Kindergarten students begin to develop precise communication skills, calculations, and measurements. Students describe their own actions, strategies, and reasoning using grade level appropriate vocabulary. Opportunities to work with pictorial representations and concrete objects can help students develop understanding and descriptive vocabulary. For example, students describe and compare two- and three-dimensional shapes and sort objects based on appearance. While measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions. Students should be encouraged to answer questions such as, “How do you know your answer is reasonable?”	As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning. In grade one, students use precise communication, calculation, and measurement skills. Students can describe their solutions strategies to mathematical tasks using grade-level appropriate vocabulary, precise explanations, and mathematical reasoning. When students measure objects iteratively (repetitively), they check to make sure there are no gaps or overlaps. Students regularly check their work to ensure the accuracy and reasonableness of solutions.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning. Second grade students communicate clearly, using grade-level appropriate vocabulary accurately and precise explanations and reasoning to explain their process and solutions. For example, while measuring an object, students carefully line up the tool correctly to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.

Mathematical Practice 6: Attend to precision.		
Grade 3	**Grade 4**	**Grade 5**
As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. For instance, they may use graph paper or a number line to represent, compare, add, and subtract decimals to the hundredths. Students in fourth grade use protractors to measure angles. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
Grade 6	**Grade 7**	**Grade 8**
In grade 6, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities. When using ratio reasoning in solving problems, students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. Students also learn to express numerical answers with an appropriate degree of precision when working with rational numbers in a situational problem. Teachers might ask, “What mathematical language, definitions, or properties can you use to explain ___?”	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities. Teachers might ask, “What mathematical language, definitions, or properties can you use to explain ___?	In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Teachers might ask, “What mathematical language, definitions, or properties can you use to explain ____?”

Mathematical Practice 6: Attend to precision.				
Algebra 1	**Math 1**	**Geometry**	**Math 2**	**Algebra 2/Math 3**
Students use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They make use of the definition of function when deciding if an equation can describe a function by asking, “Does every input value have exactly one output value?”	Students use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem.	Students develop and use precise definitions of geometric terms. They verify that a shape has specific properties and justify the categorization of the shape, e.g., a rhombus versus a quadrilateral.	To avoid the extraneous solutions, students make a use of the definition of the solution of the equation by asking, “Does this value make the equation a correct statement?”	Students make note of the precise definition of complex number, understanding that real numbers are a subset of complex numbers. They pay attention to units in real-world problems and use unit analysis as a method for verifying their answers.

(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)

Mathematical Practice 7: Look for and make use of structure.		
Kindergarten	**Grade 1**	**Grade 2**
Younger students begin to discern a pattern or structure in the number system. For instance, students recognize that 3 + 2 = 5 and 2 + 3 = 5. Students use counting strategies, such as counting on, counting all, or taking away, to build fluency with facts to 5. Students notice the written pattern in the "teen" numbers—that the numbers start with 1 (representing 1 ten) and end with the number of additional ones. Teachers might ask, "What do you notice when ___?"	First graders begin to discern a pattern or structure. For instance, if students recognize 12 + 3 = 15, then they also know 3 + 12 = 15. (Commutative property of addition.) To add 4 + 6 + 4, the first two numbers can be added to make a ten, so 4 + 6 + 4 = 10 + 4 = 14. While solving addition problems, students begin to recognize the commutative property, for example 7 + 4 = 11, and 4 + 7 = 11. While decomposing two-digit numbers, students realize that any two-digit number can be broken up into tens and ones, e.g. 35 = 30 + 5, 76 = 70 + 6. Grade one students make use of structure when they work with subtraction as a missing addend problem, such as 13 − 7 = ☐ can be written as 7 + ☐ = 13 and can be thought of as how much more do I need to add to 7 to get to 13?	Second grade students look for patterns and structures in the number system. For example, students notice number patterns within the tens place as they connect skip counting by 10s to corresponding numbers on a 100s chart. Students see structure in the base-ten number system as they understand that 10 ones equal a ten, and 10 tens equal a hundred. Students adopt mental math strategies based on patterns (making ten, fact families, doubles). They use structure to understand subtraction as a missing addend problems (e.g., 50 − 33 = ☐ can be written as 33 + ☐ = 50 and can be thought of as "How much more do I need to add to 33 to get to 50?")
Grade 3	**Grade 4**	**Grade 5**
Students look closely to discover a pattern or structure. For instance, students use properties of operations (e.g., commutative and distributive properties) as strategies to multiply and divide. Teachers might ask, "What do you notice when ___?" or "How do you know if something is a pattern?"	In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as arrays and area models to the multiplication principal of counting. They generate number or shape patterns that follow a given rule using two-column tables.	In fifth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

Mathematical Practice 7: Look for and make use of structure.		
Grade 6	**Grade 7**	**Grade 8**
Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. Students apply properties to generate equivalent expressions (i.e., 6 + 2n = 2 (3 + n) by distributive property) and solve equations (i.e., 2c + 3 = 15, 2c = 12 by subtraction property of equality; c = 6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume. Teachers might ask, "What do you notice when ___?" or "What parts of the problem might you eliminate, simplify, or ___?"	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e., 6 + 2n = 2 (3 + n) by distributive property) and solve equations (i.e., 2c + 3 = 15, 2c = 12 by subtraction property of equality; c = 6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities. Solving an equation such as 8 = 4 $(n - \frac{1}{2})$ is easier if students can see and make use of structure, temporarily viewing $(n - \frac{1}{2})$ as a single entity.	Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

Mathematical Practice 7: Look for and make use of structure.				
Algebra 1	**Math 1**	**Geometry**	**Math 2**	**Algebra 2/Math 3**
Students develop formulas such as $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. Students see that the expression $5 + (n - 2)^2$ takes the form of 5 plus "something squared," and because "something squared" must be positive or zero, the expression can be no smaller than 5.	Students recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.	Students construct triangles in quadrilaterals or other shapes and use congruence criteria of triangles to justify results about those shapes.	Students develop formulas such as $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. Students see that the expression $5 + (n - 2)^2$ takes the form of 5 plus "something squared," and because "something squared" must be positive or zero, the expression can be no smaller than 5.	Students see the operations of complex numbers as extensions of the operations for real numbers. They understand the periodicity of sine and cosine and use these functions to model periodic phenomena.

(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)

Mathematical Practice 8: Look for and express regularity in repeated reasoning.		
Kindergarten	**Grade 1**	**Grade 2**
In the early grades, students notice repetitive actions in counting, computations, and mathematical tasks. For example, the next number in a counting sequence is 1 more when counting by ones and 10 more when counting by tens (or 1 more group of 10). Students should be encouraged to answer questions such as, "What would happen if ___?" and "There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?" Kindergarten students realize 8 crayons could include 4 of each color (8 = 4 + 4), 5 of one color and 3 of another (8 = 5 + 3), and so on. For each solution, students repeatedly engage in the process of finding two numbers to join together to equal 8.	Grade one students begin to look for regularity in problem structures when solving mathematical tasks. For example, students add three one-digit numbers by using strategies such as "make a ten" or doubles. Students recognize when and how to use strategies to solve similar problems. For example, when evaluating 8 + 7 + 2, a student may say, "I know that 8 and 2 equals 10, then I add 7 to get to 17. It helps if I can make a 10 out of two numbers when I start." Students use repeated reasoning while solving a task with multiple correct answers. For example, solve the problem, "There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?" Students use repeated reasoning to find pairs of numbers that add up to 12 (e.g., the 12 crayons could include 6 of each color (6 + 6 = 12), 7 of one color and 5 of another (7 + 5 = 12), etc.)	Second grade students notice repetitive actions in counting and computation (e.g., number patterns to skip count). When children have multiple opportunities to add and subtract, they look for shortcuts, such as using estimation strategies and then adjust the answer to compensate. Students continually check for the reasonableness of their solutions during and after completing a task by asking themselves, "Does this make sense?"

Mathematical Practice 8: Look for and express regularity in repeated reasoning.		
Grade 3	**Grade 4**	**Grade 5**
Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 × 8, they might decompose 7 into 5 and 2 and then multiply 5 × 8 and 2 × 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense? Students should be encouraged to answer questions, such as, "What is happening in this situation?" or "What predictions or generalizations can this pattern support?"	Students in fourth grade should notice repetitive actions in computation to generalize. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.	Fifth graders use repeated reasoning to understand algorithms and generalize about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Mathematical Practice 8: Look for and express regularity in repeated reasoning.		
Grade 6	**Grade 7**	**Grade 8**
In grade 6, students use repeated reasoning to understand algorithms and generalize about patterns. During multiple opportunities to solve and model problems, they may notice that $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ and construct other examples and models that confirm their generalization. Students connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. Students informally begin to make connections between rates and representations showing the relationships between quantities. Students should be encouraged to answer questions such as, "How would we prove that ___?" or "How is this situation like and different from other situations?"	In grade 7, students use repeated reasoning to understand algorithms and generalize about patterns. During multiple opportunities to solve and model problems, they may notice that $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$ and construct other examples and models that confirm their generalization. Students should be encouraged to answer questions such as "How would we prove that ___?" or "How is this situation both like and different from other situations using these operations?"	In grade eight, students use repeated reasoning to understand the slope formula and to make sense of rational and irrational numbers. Through multiple opportunities to model linear relationships, they notice that the slope of the graph of the linear relationship and the rate of change of the associated function are the same. For example, as students repeatedly check whether points are on the line with a slope of 3 that goes through the point (1, 2), they might abstract the equation of the line in the form $\frac{y-2}{x-1} = 3$. Students should be encouraged to answer questions such as "How would we prove that ___?" or "How is this situation like and different from other situations using these operations?"

Mathematical Practice 8: Look for and express regularity in repeated reasoning.				
Algebra 1	**Math 1**	**Geometry**	**Math 2**	**Algebra 2/Math 3**
Students see that the key feature of a line in the plane is an equal difference in outputs over equal intervals of inputs, and that the result of evaluating the expression $\frac{y_2 - y_1}{x_2 - x_1}$ for points on the line is always equal to a certain number m. Therefore, if (x, y) is a generic point on this line, the equation $m = \frac{y - y_1}{x - x_1}$ will give a general equation of that line.		Students explore rotations, reflections, and translations, noticing that some attributes of shapes (e.g., parallelism, congruency, orientation) remain the same. They develop properties of transformations by generalizing these observations.	Students understand that when figures are scaled by a factor of k, the effect on their lengths, areas and volumes remain the same such that they are multiples of k, k^2, and k^3.	Students observe a pattern that powers of the imaginary number ii cycles through the same four outcomes, i, −1, $-i$ and 1, since $i^4 = 1$ and any power of ii with an integer exponent that is a multiple of 4 has a value 1. $i = i$ $i^5 = i$ $i^2 = -1$ $i^6 = -1$ $i^3 = -i$ $i^7 = -i$ $i^4 = 1$ $i^8 = 1$ Students use this observation to make a conjecture about any power of i.

(Adapted from Arizona Department of Education and North Carolina Department of Public Instruction)