# MP.1 Make sense of problems and persevere in solving them.

Students learn that patience is often required to fully understand what a problem is asking. They discern between useful and extraneous information. They expand their repertoire of expressions and functions that can be used to solve problems.

# MP.2 Reason abstractly and quantitatively.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

# MP.3 Construct viable arguments and critique the reasoning of others.

Students reason through the solving of equations, recognizing that solving an equation involves more than simply following rote rules and steps. They use language such as "If \_\_\_\_\_, then \_\_\_\_" when explaining their solution methods and provide justification for their reasoning.

# MP.4 Model with mathematics.

Students also discover mathematics through experimentation and by examining data patterns from real-world contexts. They apply their new mathematical understanding of exponential, linear, and quadratic functions to real-world problems.

# MP.5 Use appropriate tools strategically.

Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret results. They construct diagrams to solve problems.

## MP.6 Attend to precision.

Students use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They make use of the definition of function when deciding if an equation can describe a function by asking, "Does every input value have exactly one output value?"

## MP.7 Look for and make use of structure.

Students develop formulas such as  $(a \pm b)^2 = a^2 \pm 2ab + b^2$  by applying the distributive property. Students see that the expression  $5 + (n - 2)^2$  takes the form of 5 plus "something squared," and because "something squared" must be positive or zero, the expression can be no smaller than 5.

# MP.8 Look for and express regularity in repeated reasoning.

Students see that the key feature of a line in the plane is an equal difference in outputs over equal intervals of inputs, and that the result of evaluating the expression  $\frac{y_2 - y_1}{x_2 - x_1}$  for points on the line is always equal to a certain number *m*. Therefore, if (x, y) is a generic point on this line, the equation  $m = \frac{y - y_1}{x - x_1}$  will give a general equation of that line.

(Adapted from Arizona Department of Education, California Mathematics Framework, and North Carolina Department of Public Instruction)

#### MP.1 Make sense of problems and persevere in solving them.

Students persevere when attempting to understand the differences between linear and exponential functions. They make diagrams of geometric problems to help make sense of the problems.

#### MP.2 Reason abstractly and quantitatively.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### MP.3 Construct viable arguments and critique the reasoning of others.

Students use formal and informal proofs to verify, prove, and justify geometric theorems with respect to congruence. These proofs can included paragraph proofs, flow charts, coordinate proofs, two-column proofs, diagrams without words, or the use of dynamic software.

#### MP.4 Model with mathematics.

Students apply their mathematical understanding of linear and exponential functions to many realworld problems, such as linear and exponential growth. Students also discover mathematics through experimentation and by examining patterns in data from real-world contexts.

### MP.5 Use appropriate tools strategically.

Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the results.

#### MP.6 Attend to precision.

Students use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem.

#### MP.7 Look for and make use of structure.

Students recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

#### MP.8 Look for and express regularity in repeated reasoning.

Students see that the key feature of a line in the plane is an equal difference in outputs over equal intervals of inputs, and that the result of evaluating the expression  $\frac{y_2 - y_1}{x_2 - x_1}$  for points on the line is always equal to a certain number *m*. Therefore, if (x, y) is a generic point on this line, the equation  $m = \frac{y - y_1}{x - x_1}$  will give a general equation of that line.

#### MP.1 Make sense of problems and persevere in solving them.

Students construct accurate diagrams of geometry problems to help make sense of them. They organize their work so that others can follow their reasoning, e.g., in proofs.

#### MP.2 Reason abstractly and guantitatively.

Students understand that the coordinate plane can be used to represent geometric shapes and transformations, and therefore they connect their understanding of number and algebra to geometry.

#### MP.3 Construct viable arguments and critique the reasoning of others.

Students use formal and informal proofs to verify, prove, and justify geometric theorems with respect to congruence and similarity. These proofs can included paragraph proofs, flow charts, coordinate proofs, two-column proofs, diagrams without words, or the use of dynamic software.

### MP.4 Model with mathematics.

Students apply their new mathematical understanding to real-world problems. They learn how transformational geometry and basic trigonometric functions can be used to model the physical world.

#### MP.5 Use appropriate tools strategically.

Students make use of visual tools for representing geometry, such as simple patty paper, transparencies, or dynamic geometry software.

#### MP.6 Attend to precision.

Students develop and use precise definitions of geometric terms. They verify that a particular shape has specific properties and justify the categorization of the shape, e.g., a rhombus versus a quadrilateral.

#### MP.7 Look for and make use of structure.

Students construct triangles in quadrilaterals or other shapes and use congruence criteria of triangles to justify results about those shapes.

#### MP.8 Look for and express regularity in repeated reasoning.

Students explore rotations, reflections, and translations, noticing that some attributes of shapes (e.g., parallelism, congruency, orientation) remain the same. They develop properties of transformations by generalizing these observations.

#### MP.1 Make sense of problems and persevere in solving them.

Students persevere when attempting to understand the differences between quadratic functions and the linear and exponential functions they studied previously. They create diagrams of geometric problems to help make sense of the problems.

## MP.2 Reason abstractly and quantitatively.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### MP.3 Construct viable arguments and critique the reasoning of others.

Students construct proofs of geometric theorems based relationships between sine and cosine of complementary angles.

### MP.4 Model with mathematics.

Students apply their mathematical understanding of quadratic functions to real-world problems. They also discover mathematics through experimentation and by examining patterns in data from real-world contexts.

## MP.5 Use appropriate tools strategically.

Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the result.

## MP.6 Attend to precision.

To avoid the extraneous solutions, students make a use of the definition of the solution of the equation by asking, "Does this value make the equation a correct statement?"

#### MP.7 Look for and make use of structure.

Students develop formulas such as  $(a \pm b)^2 = a^2 \pm 2ab + b^2$  by applying the distributive property. Students see that the expression  $5 + (n - 2)^2$  takes the form of 5 plus "something squared," and because "something squared" must be positive or zero, the expression can be no smaller than 5.

#### MP.8 Look for and express regularity in repeated reasoning.

Students understand that when figures are scaled by a factor of k, the effect on their lengths, areas and volumes remain the same such that they are multiples of k,  $k^2$ , and  $k^3$ .

#### MP.1 Make sense of problems and persevere in solving them.

Students apply their understanding of various functions to real-world problems. They approach complex mathematics problems and break them down into smaller problems, synthesizing the results when presenting solutions.

### MP.2 Reason abstractly and quantitatively.

Students deepen their understanding of transformations of graphs by changing the form of rational function y(x) = a(x)/b(x), where a(x) and b(x) represent polynomials and b(x) is not 0, to reveal and interpret the key features of the function.

#### MP.3 Construct viable arguments and critique the reasoning of others.

Students continue to reason through the solution of an equation and justify their reasoning to their peers. Students defend their choice of a function when modeling a real-world situation.

### MP.4 Model with mathematics.

Students apply their new mathematical understanding to real-world problems, making use of their expanding repertoire of functions in modeling. Students also discover mathematics through experimentation and by examining patterns in data from real-world contexts.

## MP.5 Use appropriate tools strategically.

Students continue to use graphing technology to deepen their understanding of the behavior of polynomial, rational, square root, and trigonometric functions.

## MP.6 Attend to precision.

Students make note of the precise definition of complex number, understanding that real numbers are a subset of complex numbers. They pay attention to units in real-world problems and use unit analysis as a method for verifying their answers.

#### MP.7 Look for and make use of structure.

Students see the operations of complex numbers as extensions of the operations for real numbers. They understand the periodicity of sine and cosine and use these functions to model periodic phenomena.

## MP.8 Look for and express regularity in repeated reasoning.

Students observe a pattern that powers of the imaginary number *i* cycles through the same four outcomes, *i*, -1, -i and 1, since  $i^4 = 1$  and any power of *i* with an integer exponent that is a multiple of 4 has a value 1.

$$\begin{array}{ll} i = i & i^5 = i \\ i^2 = -1 & i^6 = -1 \\ i^3 = -i & i^7 = -i \\ i^4 = 1 & i^8 = 1 \end{array}$$

Students use this observation to make a conjecture about any power of *i*.

(Adapted from Arizona Department of Education, California Mathematics Framework, and North Carolina Department of Public Instruction)