Discrete Mathematics/Computer Science
Critical Areas of Focus-DRAFT

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.

- What are the concepts?
- What are the procedures and skills?
- What are the key mathematical practices?
- What are the relationships students are to make?
- What further information is needed? For example, what does prove mean?
- What are the appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards *within* each course's Critical Areas of Focus.

- What are the relationships among the conceptual categories, domains, clusters and standards?
- Why is each relationship important?
- What are the differences?
- How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections *across* the Critical Areas of Focus within a course. This information will help create a sequence of units for the course.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.

- What understandings does this learning build upon?
- What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.

- What is taught now but not in Ohio's Learning Standards?
- What content is essentially the same? Identify the differences.
- What will be the new content for this grade?

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Critical Area of Focus #1

Communication and Analysis

Within this critical area, students develop conclusions based on quantitative information and critical thinking. They recognize, make and evaluate underlying assumptions in estimation, modeling and data analysis. Students then organize and present thoughts and processes using mathematical and statistical evidence. They communicate clear and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation. Students demonstrate numerical reasoning orally and in writing coherent statements and paragraphs.

In the context of real-world applications, students make and investigate mathematical conjectures. They can defend their conjectures and respectfully question conjectures made by their classmates. This leads to the development of mathematical arguments and informal proofs, which are ways of expressing certain kinds of reasoning and justification. Explanations (oral and written) include mathematical arguments and rationales, not just procedural descriptions or summaries. Listening to others' explanations gives students opportunities to develop their own understandings. Through communication, ideas become objects of reflection, refinement, discussion and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing and precise in their use of mathematical language. Additionally, conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. **This critical area of focus crosscuts all the other critical areas of focus.**

Standards for Mathematical Practices

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

This critical area crosscuts all the rest of the standards, so all standards in this document also fall under this crucial area!

CRITICAL AREA OF FOCUS #2

Combinatorial Games

This unit introduces and explores functions, algorithmic thinking, inference, modeling and computer programming concepts through a variety of combinatorial games. In a combinatorial game:

- Two or more players alternate moves.
- There must be no chance involved in the game, and both players must have complete information about all aspects of the game at all times.
- On each move, the player whose turn it is must have a finite number of possible actions.
- After a finite number of turns, each game must have a designated "end" where one player is the "winner."

Student discourse around a "winning strategy" is a central component of this unit as they analyze each game. Constructing viable arguments, demonstrating repeated reasoning, modeling with mathematics and critiquing the reasoning of others are critical components of the unit as students develop an understanding of *why* a strategy works and determine whether the strategy will always work.

MATHEMATICS

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.★

c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

Functions – Interpreting Functions

Understand the concept of a function and use function notation.

F.IF.2 Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of context.

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations.
For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3)

(+) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Geometry – Congruence

Experiment with transformations in the plane.

G.CO.2 Represent transformations in the plane using, *e.g.*, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, *e.g.*, translation versus horizontal stretch.

COMPUTER SCIENCE

Computer Science 9-12 Foundational Level – Data and Analysis

Inference and Modeling

DA.IM.9-12.F.a Evaluate a model by creating a hypothesis, testing it and refining it to discover connections and trends in the data.

Computer Science 9-12 Foundational Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.F.a Define and use appropriate problem-solving strategies and visual artifacts to create and refine a solution to a real-world problem.

ATP.A.9-12.F.b Define and implement an algorithm by decomposing problem requirements from a problem statement to solve a problem.

Variables and Data Representation

ATP.VDR.9-12.F.a Identify types of variables and data and utilize them to create a computer program that stores data in appropriate ways.

Control Structures

ATP.CS.9-12.F.a Define control structures and Boolean logic and use them to solve real-world scenarios.

ATP.CS.9-12.F.b Use appropriate syntax to create and use a method.

Modularity

ATP.M.9-12.F.a Break down a solution into procedures using systematic analysis and design.

Computer Science 9-12 Advanced Level – Algorithmic Thinking and Programming

Control Structures

ATP.CS.9-12.A.a Write programs that use library methods and control structures and methods to solve a problem.

ATP.CS.9-12.A.b Refactor a program to be smaller and more efficient.

Modularity

ATP.M.9-12.A.a Construct solutions to problems using student-created components (e.g., procedures, modules, objects).

CRITICAL AREA OF FOCUS #3

Counting and Combinatorics

Combinatorics is a branch of mathematics dealing with combinations of objects belonging to a finite set with certain constraints. In this unit, *combinatorics* are used to explore new real-world applications and extend understanding of previous work, such as recursive formulas, exponential functions and sets. Students develop an understanding of the importance of labeling objects in a set with unique names. Models such as trees, lists, diagrams, tables and formulas are used to explore the concept of overcounting and its applications. Students refine their methods to be more efficient as they keep track of the number of objects in a set. Students construct meaningful mathematical justifications for a variety of counting formulas.

MATHEMATICS

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
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4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.★

c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

Algebra – Seeing Structure in Expressions

Interpret the structure of expressions.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 23^t.*

Statistics and Probability – Conditional Probability and The Rules Of Probability

Understand independence and conditional probability and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections or complements of other events (“or,” “and,” “not”).★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★

(+) S.CP.8 Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.★ (G, M2)

(+) S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.★ (G, M2)

COMPUTER SCIENCE

Computer Science 9-12 Foundational Level – Networks and The Internet

Cybersecurity

NI.C.9-12.F.a Examine and employ principles of cybersecurity.

NI.C.9-12.F.b Identify physical, social and digital security risks to address possible attacks from both existing and emergent technologies, including cluster computing and quantum key distribution.

Computer Science 9-12 Foundational Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.F.a Define and use appropriate problem-solving strategies and visual artifacts to create and refine a solution to a real-world problem.

ATP.A.9-12.F.b Define and implement an algorithm by decomposing problem requirements from a problem statement to solve a problem.

Control Structures

ATP.CS.9-12.F.a Define control structures and Boolean logic and use them to solve real-world scenarios.

ATP.CS.9-12.F.b Use appropriate syntax to create and use a method.

Modularity

ATP.M.9-12.F.a Break down a solution into procedures using systematic analysis and design.

ATP.M.9-12.F.b Create computational artifacts by systematically organizing, manipulating and/or processing data.

Computer Science 9-12 Foundational Level – Impacts of Computing

Safety, Law and Ethics

IC.SLE.9-12.F.b Analyze the concepts of usability and security to explain typical tradeoffs between them.

Computer Science 9-12 Advanced Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.A.b Use iteration to effectively solve problems.

Modularity

ATP.M.9-12.A.a Construct solutions to problems using student-created components (e.g., procedures, modules, objects).

Computer Science 9-12 Advanced Level – Impacts of Computing

Safety, Law and Ethics

IC.SLE.9-12.A.a Create a scenario to demonstrate typical tradeoffs between usability and security and recommend security measures based on these or other tradeoffs.

CRITICAL AREA OF FOCUS #4

Probability

Content associated with Statistics and Probability has a wide range of applications in many degree programs and careers. To prepare students for these careers, this unit is designed to highlight discrete mathematics and computer science connections to probability.

The probability tasks in the unit develop students' reasoning associated with the probability of a finite number of events. Students use JavaScript® to apply discrete math and computer science concepts to predict and verify prescribed outcomes through increasing amounts of simulated trials. Students construct mathematical arguments while looking for patterns in different mathematical models and simulations to determine whether the model or simulated results are consistent with the outcome(s) of the event.

MATHEMATICS

Standards for Mathematical Practices

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5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
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Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.★
c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★
b. Combine standard function types using arithmetic operations.
For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3)
(+) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Geometry – Modeling with Geometry

Apply geometric concepts in modeling situations.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★

G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

Statistics and Probability – Conditional Probability and The Rules Of Probability

Understand independence and conditional probability and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★

Statistics and Probability – Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?★

Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★

Statistics and Probability – Using Probability to Make Decisions

Calculate expected values, and use them to solve problems.

(+) S.MD.1 Define a random variable[G] for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution[G] using the same graphical displays as for data distributions.★

(+) S.MD.2 Calculate the expected value[G] of a random variable; interpret it as the mean of the probability distribution.★

Use probability to evaluate outcomes of decisions.

(+) S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★

a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.

b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

(+) S.MD.6 Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★

(+) S.MD.7 Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

COMPUTER SCIENCE

Computer Science 9-12 Foundational Level – Data and Analysis

Data Collection and Storage

DA.DCS.9-12.F.a Analyze patterns in a real-world data store through hypothesis, testing and use of data tools to gain insight and knowledge.

Visualization and Communication

DA.VC.9-12.F.a Analyze the benefits and limitations of data visualization or multisensory artifacts and tools to communicate which is most appropriate to solve a real-world problem.

Inference and Modeling

DA.IM.9-12.F.a Evaluate a model by creating a hypothesis, testing it and refining it to discover connections and trends in the data.

Computer Science 9-12 Foundational Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.F.a Define and use appropriate problem-solving strategies and visual artifacts to create and refine a solution to a real-world problem.

ATP.A.9-12.F.b Define and implement an algorithm by decomposing problem requirements from a problem statement to solve a problem.

Variables and Data Representation
ATP.VDR.9-12.F.a Identify types of variables and data and utilize them to create a computer program that stores data in appropriate ways.

Modularity
ATP.M.9-12.F.b Create computational artifacts by systematically organizing, manipulating and/or processing data.

Computer Science 9-12 Foundational Level – Artificial Intelligence
Representation & Reasoning
AI.RR.9-12.F.a Categorize real-world problems as classification, prediction, sequential decision problems, combination search, heuristic search, adversarial search, logical deduction or statistical inference.

Computer Science 9-12 Advanced Level – Data and Analysis
Visualization and Communication
DA.VC.9-12.A.a Create visualization or multisensory artifacts to communicate insights and knowledge gained from complex data analysis that answers real-world questions.

Computer Science 9-12 Advanced Level – Algorithmic Thinking and Programming
Algorithms
ATP.A.9-12.A.b Use iteration to effectively solve problems.

Variables and Data Representation
ATP.VDR.9-12.A.a Utilize different data storage structures to store larger and more complex data than variables can contain.
ATP.VDR.9-12.A.b Identify the appropriate data structures or variables to use to design a solution to a complex problem.

CRITICAL AREAS OF FOCUS #5

Connectivity and Graph Theory

A graph is a connected graph if, for each pair of vertices, there exists at least one single path which joins them.

Vertex-edge graphs and their properties are introduced and explored through a variety of real-world applications. These types of graphs can be represented by a table, drawing or list where vertices (nodes or “dots”) may represent the objects, people or places while relationships among them are represented by edges (connecting lines or curves). Applications of contexts allow students to explore foundational concepts related to both fully and minimally connected graphs along with their properties. Students extend their understanding of the ways (and how) types of graphs can be useful tools for tracking relationships and specific connections among objects or people. Mathematical structures such as weighted planar, directed and undirected graphs for modeling pairwise relations are introduced as part of this exploration and application.

MATHEMATICS

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
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6. Attend to precision.

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.★
c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★
b. Combine standard function types using arithmetic operations.
For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3)
(+) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★
a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of context.★

Geometry – Modeling with Geometry

Apply geometric concepts in modeling situations.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★

G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

COMPUTER SCIENCE

Computer Science 9-12 Foundational Level – Networks and The Internet

Networking

NI.N.9-12.F.c Understand scalability and reliability of networks to describe the relationships.

Computer Science 9-12 Foundational Level – Data and Analysis

Data Collection and Storage

DA.DCS.9-12.F.a Analyze patterns in a real-world data store through hypothesis, testing and use of data tools to gain insight and knowledge.

Visualization and Communication

DA.VC.9-12.F.a Analyze the benefits and limitations of data visualization or multisensory artifacts and tools to communicate which is most appropriate to solve a real-world problem.

Computer Science 9-12 Foundational Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.F.a Define and use appropriate problem-solving strategies and visual artifacts to create and refine a solution to a real-world problem.

Variables and Data Representation

ATP.VDR.9-12.F.a Identify types of variables and data and utilize them to create a computer program that stores data in appropriate ways.

Control Structures

ATP.CS.9-12.F.a Define control structures and Boolean logic and use them to solve real-world scenarios.

Computer Science 9-12 Foundational Level – Impacts of Computing

Social Interactions

IC.SI.9-12.F.a Evaluate tools to increase connectivity of people in different cultures and career fields.

Computer Science 9-12 Advanced Level – Algorithmic Thinking and Programming

Variables and Data Representation

ATP.VDR.9-12.A.a Utilize different data storage structures to store larger and more complex data than variables can contain.

Computer Science 9-12 Advanced Level – Artificial Intelligence

Representation and Reasoning

AI.RR.9-12.A.b Illustrate breadth-first, depth-first and best-first search algorithms to grow a search tree.

CRITICAL AREAS OF FOCUS #6

Iteration and Recursion

Recursion is when an object is defined in terms of itself or objects of the same type. While recursive functions repeat by calling on themselves, iteration functions repeat using a set of directions. In this unit, students work flexibly with iterations and recursion as they generalize patterns to identify efficient processes in a variety of applications. Previous understanding of functions is extended to include recursive and explicit applications through real-world contexts. Factorial functions are introduced and explored through work with iteration and recursive functions.

MATHEMATICS

Standards for Mathematical Practices

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(+) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Functions – Interpreting Functions

Understand the concept of a function and use function notation.

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

COMPUTER SCIENCE

Computer Science 9-12 Foundational Level – Data and Analysis

Inference and Modeling

DA.IM.9-12.F.a Evaluate a model by creating a hypothesis, testing it and refining it to discover connections and trends in the data.

Computer Science 9-12 Foundational Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.F.a Define and use appropriate problem-solving strategies and visual artifacts to create and refine a solution to a real-world problem.
ATP.A.9-12.F.c Define and explain iterative algorithms to understand how and when to apply them.
ATP.A.9-12.F.d Define and explain recursive algorithms to understand how and when to apply them.

Computer Science 9-12 Advanced Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.A.b Use iteration to effectively solve problems.
ATP.A.9-12.A.c Use recursion to effectively solve problems.

CRITICAL AREAS OF FOCUS #7

Cryptography

Cryptography is the human endeavor of trying to maintain privacy in communication. Students use inverse functions, combinations, and permutations to encrypt and decrypt messages. The concept of *modulus* or *mods* is introduced as students explore remainders through a variety of computer science applications. Students extend their understanding of non-Base-10 numbers to discover their limitations and applications in cryptography.

MATHEMATICS

Standards for Mathematical Practices

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7. Look for and make use of structure.

Functions – Building Functions

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For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3)
(+) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Build new functions from existing functions.

F.BF.4 Find inverse functions.
(+) b. Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)
(+) c. Verify by composition that one function is the inverse of another. (A2, M3)
(+) d. Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)

Statistics and Probability – Conditional Probability and The Rules Of Probability

Understand independence and conditional probability and use them to interpret data.

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

(+) S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.★ (G, M2)

COMPUTER SCIENCE

Computer Science 9-12 Foundational Level – Networks and The Internet

Cybersecurity

NI.C.9-12.F.a Examine and employ principles of cybersecurity.

NI.C.9-12.F.b Identify physical, social and digital security risks to address possible attacks from both existing and emergent technologies, including cluster computing and quantum key distribution.

NI.C.9-12.F.d Explore and utilize examples of encryption methods, e.g., Vigenere, Bacon's cipher, and Enigma.

Computer Science 9-12 Foundational Level – Data and Analysis

Inference and Modeling

DA.IM.9-12.F.a Evaluate a model by creating a hypothesis, testing it and refining it to discover connections and trends in the data.

Computer Science 9-12 Foundational Level – Algorithmic Thinking and Programming

Algorithms

ATP.A.9-12.F.a Define and use appropriate problem-solving strategies and visual artifacts to create and refine a solution to a real-world problem.

Variables and Data Representation

ATP.VDR.9-12.A.b Identify the appropriate data structures or variables to use to design a solution to a complex problem.

ATP.VDR.9-12.F.a Identify types of variables and data and utilize them to create a computer program that stores data in appropriate ways.

Computer Science 9-12 Foundational Level – Algorithmic Thinking and Programming

Control Structures

ATP.CS.9-12.A.b Refactor a program to be smaller and more efficient.

Computer Science 9-12 Advanced Level – Networks and the Internet

Cybersecurity

NI.C.9-12.A.d Explore and utilize examples of encryption methods (e.g., Vigenére, Bacon's cipher and Enigma).