Geometry Critical Areas of Focus

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. These descriptions have been used to develop critical areas for each of the courses in both the Traditional and Integrated pathways. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.
- What are the concepts?
- What are the procedures and skills?
- What are the key mathematical practices?
- What are the relationships students are to make?
- What further information is needed? For example, what does prove mean?
- What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards within each course's Critical Areas of Focus.
- What are the relationships among the conceptual categories, domains, clusters and standards?
- Why is each relationship important?
- What are the differences?
- How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections across the Critical Areas of Focus within a course. This information will help create a sequence of units for the course. For example, Critical Area of Focus #3 needs to be addressed prior to Critical Area of Focus #5.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.
- What understandings does this learning build upon?
- What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.
- What is taught now but not in Ohio's Learning Standards?
- What content is essentially the same? Identify the differences.
- What will be new content for this grade?

Geometry Table of Contents

Geometry Critical Areas of Focus

Critical Area of Focus #1

Applications of Probability

Building on probability concepts that began in grade 7, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions related to real-world situations.

Statistics and Probability – Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). ★

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★

S.CP.3 Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ★

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. ★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. ★

S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model. ★

(+) S.CP.8 Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model. ★

(+) S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems. ★

CRITICAL AREA OF FOCUS #2

Congruence, Proof and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent or to have symmetries of itself, rotational or reflected. Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal and informal proof. Students prove theorems—using a variety of formats—and apply them when solving problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work. Students will extend prior experience with geometric shapes toward the development of a hierarchy of two-dimensional figures based on formal properties.

Geometry - Congruence

Experiment with transformations in the plane.

G.CO.1 Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.

a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes.

b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

CONTINUED ON THE NEXT PAGE

Geometry Critical Areas of Focus

CRITICAL AREA OF FOCUS #2, CONTINUED

Congruence, Proof and Constructions

Geometry - Congruence

Prove geometric theorems both formally and informally using a variety of methods.

G.CO.9 Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

G.CO.10 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

G.CO.11 Prove and apply theorems about parallelograms. Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Classify and analyze geometric figures.

G.CO.14 Classify two-dimensional figures in a hierarchy based on properties.

Geometry Critical Areas of Focus

Critical Area of Focus #3

Similarity, Proof, and Trigonometry

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use it as a familiar foundation for the development of informal and formal proofs, problem solving and applications to similarity in right triangles. This will assist in the further development of right triangle trigonometry, with particular attention to special right triangles, right triangles with one side and one acute angle given and the Pythagorean Theorem. Students apply geometric concepts to solve real-world, design and modeling problems.

Geometry – Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations.

G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove and apply theorems both formally and informally involving similarity using a variety of methods.

G.SRT.4 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

Define trigonometric ratios and solve problems involving right triangles.

G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G.SRT.8 Solve problems involving right triangles. ★

- **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)

Apply geometric concepts in modeling situations.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. ★

G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot. ★

G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. ★

Geometry Critical Areas of Focus

CRITICAL AREA OF FOCUS #4

Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Geometry – Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2)

G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

Geometry Critical Areas of Focus

CRITICAL AREA OF FOCUS #5

Circles With and Without Coordinates

Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. Students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done with systems of equations in the first course to determine intersections between lines and circles. Students model and solve real-world problems applying these geometric concepts.

Geometry - Circles

Understand and apply theorems about circles.

G.C.1 Prove that all circles are similar using transformational arguments.

G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G.C.3 Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.

(+) G.C.4 Construct a tangent line from a point outside a given circle to the circle. Find the arc lengths and areas of sectors of circles.

Find arc lengths and areas of sectors of circles.

G.C.5 Find arc lengths and areas of sectors of circles.

a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. (G, M2)

b. Derive the formula for the area of a sector, and use it to solve problems (G, M2)

Geometry – Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2)

Geometry – Modeling with Geometry

Apply geometric concepts in modeling situations.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. ★

Geometry Critical Areas of Focus

CRITICAL AREA OF FOCUS #6

Extending to Three Dimensions

Students' experience with two-dimensional and three dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Students develop the understanding of how changes in dimensions result in similar and non-similar shapes and how scaling changes lengths, areas and volumes. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. They solve real-world problems applying these geometric concepts.

Geometry – Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

Visualize the relation between two-dimensional and three-dimensional objects.

G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Understand the relationships between lengths, area, and volumes.

G.GMD.5 Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.

G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.

Geometry – Modeling with Geometry

Apply geometric concepts in modeling situations.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. ★