Mathematical Expectations for College and Career Readiness
2011

A large percentage of Ohio college freshmen find that they are not "ready" for entry level college coursework. With respect to mathematics, 32% of Ohio's recent graduates enrolling at Ohio public colleges or universities in fall 2008 took remedial mathematics during their first year of college.[1] Reducing the number and percentage of students taking remedial mathematics courses is challenging. Nonetheless, an important first step is to define clearly the mathematical expectations for students who enter two- and four-year colleges. These expectations should help guide college and high school faculty, and, most importantly, assist students and their parents in planning secondary and post secondary coursework.

Defining the mathematics and statistics students need to know as they enter college may sound simple. In fact, it is a challenging endeavor, requiring those who undertake it to examine the nature of mathematics and pedagogy and think seriously about the mathematics that will be needed in the future by *all* students. Certainly, students who will take calculus as entering freshmen require a different level of preparation and competence than students who take introductory credit-bearing courses. The first part of this document describes the standards for the latter group. Also included are the additional expectations for incoming college students who wish to enroll directly into college calculus courses. These expectations are drawn from the Precalculus outcomes in the Mathematics Transfer Assurance Guide.

This document is a revision of a set of expectations first articulated in 2006. The work, done under the auspices of the Ohio Board of Regents and the Ohio Department of Education, involved both higher education faculty and high school mathematics teachers. These revised expectations are consistent with the Common Core State Standards for Mathematics that have been adopted by Ohio, and in no way should be viewed as replacing those standards.

The panel creating this document made a conscious effort to limit the content to the essential mathematics needed for successful completion of entry-level college mathematics courses. This document represents minimal expectations. Thus, this document omits some traditional and appealing concepts that would provide an enriched mathematics background. It is understood and hoped that many secondary students will learn more mathematics than is reflected here. Students intending to pursue mathematically intensive programs and careers after high school would especially benefit from a more comprehensive and rigorous study of mathematics. The panel believes, however, that the core expectations outlined in this document will provide students with a sufficient knowledge base for success. And it is essential that students have more than a passing acquaintance with these ideas. Students must retain this foundation well beyond the confines of any one course and be able to apply these concepts and skills to both routine and nonroutine problems, drawn from a variety of contexts.

[1] See http://regents.ohio.gov/perfrpt/hs_2008/hs_trans_HS_rpt_AU08.pdf.

Expectation 1: Mathematical Processes

Mathematical processes are intertwined with content. In the best of all worlds they would intertwined to the point that it is unnecessary to mention them. However, it is important to make explicit that in addition to essential numeric, algebraic, geometry and data skills, students need to possess mathematical process skills in order to be successful in post-secondary education. These skills include communication and reasoning and using technology—but perhaps most important are problem solving skills. Students should have had plentiful and frequent experience with rich mathematical problems that engage them in problem solving, a process deeper than that of practicing a new technique on a classified category of word problems. Problem solving should contribute to the development of mathematical habits of mind (e.g., perseverance, questioning, independence, reflection, connecting) and develop an appreciation for and a disposition toward problem solving as the paramount aim of learning mathematics.

Students are expected to:

A. Solve Problems

1. Use a variety of problem solving strategies
2. Reflect on and analyze their own problem solutions and the solutions of others
3. Connect ideas in a variety of contexts
4. Solve complex, nonroutine and multi-step problems that may require student formulation of problems and/or sustained thought and effort

B. Communicate with Mathematical Ideas

1. Use correct mathematical terminology and notation
2. Show a logical progression of thought, clearly and coherently, orally and in writing
3. Read mathematical material with understanding and independence
4. Use appropriate degrees of precision based upon problem context; use exact answers (e.g., $\sqrt{2}$ or π) when appropriate

C. Reason Mathematically

1. Understand the need for proof in mathematics; recognize when a proof is required
2. Understand the difference between a statement verified by proof and one illustrated by using examples of data
3. Understand the meaning of logical terms (e.g., and, or, but, not, if … then)
4. Understand the significance of and roles played by definitions, assumptions, theorems/propositions, examples, and counterexamples in mathematics

D. Connect Mathematical Concepts

1. Connect mathematics with a variety of disciplines and workplace and everyday settings
2. Use connections among and within branches of mathematics (e.g., algebraic properties of a function and geometric properties of its graph)

E. Use Technology and other Tools

1. Use a variety of tools to solve mathematical problems—ranging from common tools (e.g., rulers, protractors) to technology-enhanced tools (e.g., calculators, computers, spreadsheets)
2. Use technology to collect organize and analyze information with the goal of interpretation, presentation and argumentation and as motivation for proof
3. Use appropriate technology to enhance and support student learning

Expectation 2: Number and Operations

Concepts from number and operations form the basis for understanding of algebra and work with symbols. Students should be proficient with arithmetic operations and their properties on integers, rational numbers and real numbers. They should demonstrate number sense and compute fluently, including mental methods, and make reasonable estimates. Students should possess a basic understanding of the real number system and the way the natural numbers, whole numbers, integers, rational numbers, and irrational numbers relate to one another.

Students are expected to:

A. Structure of the Number System

1. Understand and convert between different representations of numbers (decimal, percent, fraction, scientific notation, radicals…)
2. Explain the effects of operations on the magnitudes of quantities and signs of numbers

B. Operations

1. Perform arithmetic operations on various forms of real numbers
2. Compute and explain the solutions to problems involving ratio, proportion, percent, scientific notation, square roots and numbers with integer and rational exponents
3. Apply and generalize properties of operations (including order of operations) as a foundation for algebra

C. Estimation

1. Estimate the solutions to problems involving ratio, proportion, percent, scientific notation, square roots and numbers with integer and rational exponents

Expectation 3: Algebra

Algebra continues to be the most fundamental prerequisite for success in college mathematics. Algebra provides a language and structure that allows students to create representations, model and generalize mathematical ideas. It is concerned with change and patterns and dealing with concepts at a more abstract level than in arithmetic. In addition to competence with manipulation of algebraic objects students should be able to model and solve problems using a variety of algebraic methods.

Students are expected to:

A. Equations and Inequalities

1. Algebraically solve linear equations in one variable, including examples with no solution, one solution, and infinitely many solutions
2. Solve systems of linear equations with two unknowns by graphing, substitution, and addition/elimination; include examples with no solution, one solution, and infinitely many solutions
3. Solve quadratic equations by graphing, factoring, completing the square, and the quadratic formula including equations that have complex solutions
4. Algebraically solve linear inequalities and represent solutions in multiple ways such as graphically, inequality notation, and interval notation.
5. Algebraically solve absolute value equations in the form $|Ax + B| = C$ and related absolute value inequalities and represent solutions in multiple ways
6. Algebraically solve equations that include rational expressions or radicals including examples that generate extraneous solutions
7. Solve for specified variables in literal equations
8. Solve exponential equations in one variable using logarithms

B. Operations with Algebraic Objects

1. Perform operations with exponents and radicals, including laws of exponents, with both numerical and algebraic expressions
2. Add, subtract, multiply and divide rational expressions by hand and identify values where they are undefined. (Limit numerators and denominators to monomial, linear and quadratic expressions)
3. Evaluate and simplify algebraic expressions
4. Add, subtract, multiply and divide polynomial expressions (limit divisors to monomial and linear expressions)

C. Graphing

1. Graph linear equations and inequalities and quadratic equations in two variables, with and without technology. (Limit quadratic equations to circles and vertical and horizontal parabolas)
2. Graph common functions (e.g., absolute value, square root, linear, quadratic, rational, exponential, piecewise) with and without technology
3. Read a graph to interpret solutions to an equation and identify and interpret characteristics such as intercepts, extrema, and rates of change.
4. Graph transformations of functions (limit transformations to vertical and horizontal shifts, reflections, and stretches)
5. Interpret transformations of functions from both a graphical and algebraic perspective

D. Functions and Applications

1. Define functions; determine whether a relationship between two variables (represented in a variety of ways) represents a function; identify, as appropriate for the context, both the domain and range of a function; and use function notation
2. Describe how a change in one variable affects the value of a related variable, for example, problems involving direct and inverse variation
3. Interpret sequences as functions whose domain is a subset of the whole numbers. Solve problems with arithmetic and geometric sequences.
4. Adjust the parameters of function families to model relationships between variables. Function families include linear, quadratic, piecewise, absolute value, square root, power and exponential.
5. Formulate equations or functions that model problems in a variety of contexts.

Expectation 4: Geometry

Geometry is the place where students learn about shapes and space. It is also a natural place for students to use careful deductive reasoning. Students analyze mathematical situations and solve problems using geometric objects and ideas.

Students are expected to:

A. Structure

1. Describe and explain the different roles of assumptions, definitions, theorems and proofs in the logical structure of geometry
2. Use theorems about parallel and perpendicular lines, angles, congruent figures, similar figures, right triangles (e.g., Pythagorean Theorem), polygons, circles, polyhedrons, spheres, cylinders, and cones to solve problems
3. Prove theorems about lines, angles, triangles, and parallelograms
4. Use similarity to solve problems and to model proportional relationships
5. Use right triangle trigonometry to solve problems

B. Geometric Representations

1. Represent geometric objects algebraically using coordinates (analytic geometry)
2. Use algebra to solve geometric problems
3. Draw and define reflections, rotations, translations, and dilations of geometric objects and understand compositions of these transformations
4. Define, describe, and identify reflectional and rotational symmetry
5. Express transformations algebraically (i.e., using coordinates)

C. Measurement

1. Explain that the geometric measures (length, perimeter, area, volume) depend on the choice of unit, and that measurements are approximations
2. Explain the effect of a scale factor on length, perimeter, area, and volume
3. Calculate the perimeter and area of common plane figures and the surface area and volume of solids
4. Distinguish between exact and approximate values. Explain differences among accuracy, precision, and error, and describe how errors affect later calculations
5. Solve problems involving measurement, including problems requiring a choice of scale and unit
6. Convert fluently from one measurement unit to another, within and across systems

Expectation 5: Probability and Statistics

Statistics and probability form the basis for understanding situations involving variability. Beginning with questions, data are gathered, displayed, summarized, and interpreted in order to identify patterns and deviations from patterns and to make predictions. In a world increasingly inundated with data, it is essential that all students become familiar with ways data is used and misused.

Students are expected to:

A. Data Displays and Interpretation

1. Create and/or interpret graphical displays to describe sets of data (e.g., box-and-whisker, scatterplot, frequency distribution, normal distribution)
2. Find and interpret measures of central tendency and variability for sets of data

B. Representations and Use of Data

1. Use the context to determine appropriate way(s) to represent data, and understand the advantages and disadvantages of various representations
2. Identify misuses of data
3. Distinguish between correlation and causation
4. Understand the characteristics of well-designed studies (e.g., lack of bias, sampling methods, randomness) in order to interpret results

C. Probability Concepts

1. Use the fundamental counting principle to determine the number of possible outcomes
2. Compute probability of compound events, independent events, and simple dependent events
3. Compare experimental and theoretical results for simple experiments

Additional Expectations for Calculus

The expectations outlined above will help assure that students are ready for college. If a student plans to enroll in a calculus course upon entering college that student should also have facility with the following which are drawn from the Ohio Board of Regents description of a Precalculus Course (TMM002). The codes in parentheses indicate Precalculus outcomes that are already included or partially included among the expectations for all students.

1. Functions

1.1 Represent functions verbally, numerically, graphically and algebraically, including linear, quadratic, polynomial, rational, root/radical/power, piecewise-defined, exponential, logarithmic, trigonometric and inverse trigonometric functions (3.C.2, 3.C.3, except logarithmic and trigonometric functions)

1.2 Determine whether an algebraic relation or given graph represents a function (3.D.1)

1.3 Perform transformations of functions—translations, reflections and stretching and shrinking (3.C.4, 3.C.5)

1.4 Perform operations with functions—addition, subtraction, multiplication, division and composition

1.5 Analyze algebraic structure and graph of a function, including those listed in 1.1 to determine the intercepts, domain, range, intervals on which the function is increasing, decreasing or constant, the vertex of a quadratic function, asymptotes, whether the function is one-to-one, whether the graph as symmetry (even/odd), etc. and given the graph of the function to determine possible algebraic definitions. (3.C.3; 3.D.1,2,4)

1.6 Find inverse of functions listed in 1.1 and understand the relationship of the graph of a function to that of its inverse

1.7 Use the Remainder and Factor Theorems for polynomial functions

1.8 Use functions, including those listed in 1.1 to model a variety of real-world problem solving applications. (3.D.4)

2. Equations/Systems

2.1 Understand the difference between an algebraic equation of one, two or more variables and a function, and the relationship among the solutions of an equation in one variable, the zeros of the corresponding function, and the coordinates of the x-intercepts of the graph of that function.

2.2 Determine algebraically and graphically whether the graph of an equation exhibits symmetry.

2.3 Solve a variety of equations, including polynomial, rational, exponential, and logarithmic, trigonometric and inverse trigonometric, including equations arising in application problems. (3.A.1,3,5,6, except logarithmic and trigonometric equations)

2.4 Solve a system of linear equations graphically and algebraically by substitution and elimination and solve application problems that involve systems of linear equations. (3.A.2)

2.5 Identify and express the conics (quadratics in two variables) in standard rectangular form, graph the conics, and solve applied problems involving conics.

2.6 Solve polynomial and rational inequalities graphically and algebraically

3. Sequences/Series

3.1 Represent sequences verbally, numerically, graphically and algebraically, including both the general form and recursively. (3.D.3)
3.2 Write series in summation notation, and represent sequences of partial sums verbally, numerically and graphically.
3.3 Identify and express the general term of arithmetic and geometric sequences, and find the sum of arithmetic and geometric series. (3.D.3)

4. More Trigonometry
4.1 Express angles in both degree and radian measure.
4.2 Define the six trigonometric functions in terms of right triangles and the unit circle.
4.3 Solve right and oblique triangles in degrees and radians for both special and non-special angles, solve application problems that involve right and oblique triangles. (4.A.5)
4.4 Verify trigonometric identities by algebraically manipulating trigonometric expressions using fundamental trigonometric identities, including the Pythagorean, sum and difference of angle, double-angle and half-angle identities.
4.5 Solve a variety of trigonometric and inverse trigonometric equations, including those requiring the use of the fundamental trigonometric identities listed in 4.4, in degrees and radians for both special and non-special angles. Solve application problems that involve such equations.

5. Vectors
5.1 Represent vectors graphically in both rectangular and polar coordinates and understand the conceptual and notational difference between a vector and a point in the plane.
5.2 Perform basic vector operations both graphically and algebraically—addition, subtraction and scalar multiplication.
5.3 Solve application problems using vectors.