Ohio Department of Education

OHIO GRADUATION TESTS

Mathematics

Scoring Guidelines and Samples of Scored Student Responses

Spring 2005

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Mathematics

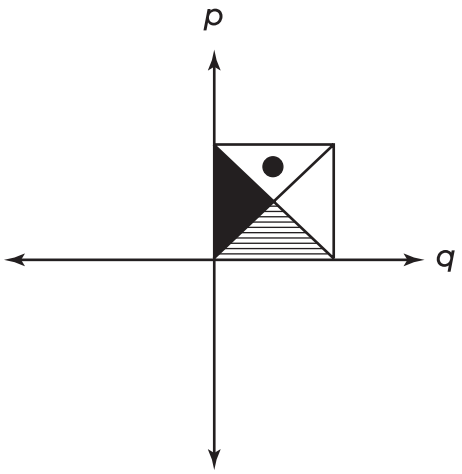
Item 6

Spring 2005

Item and Scoring Guidelines

Item

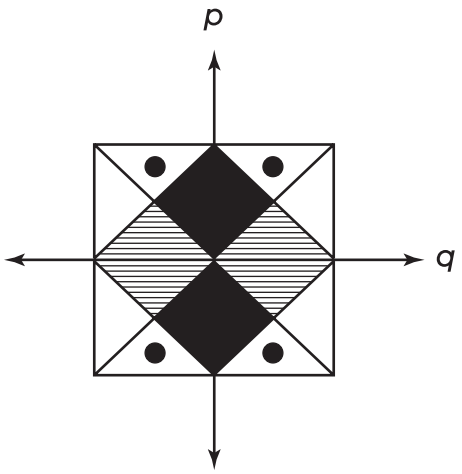
6. By the end of summer vacation, Callie had completed one-fourth of a quilt, as shown in the diagram below.



To finish the quilt, Callie plans to reflect the design of the completed portion over lines *p* and *q* until all 4 portions are complete.

In your **Answer Document**, copy the diagram above. Add the reflections of the completed portion of the quilt on the same sketch to show how the entire quilt will look when it is finished.

Sample Response for Item 6 (Short Answer):



Note: If students fill in the entire grid, focus on the center. Accurate reflections indicate the reflections are to scale.

Scoring Guidelines for Item 6:

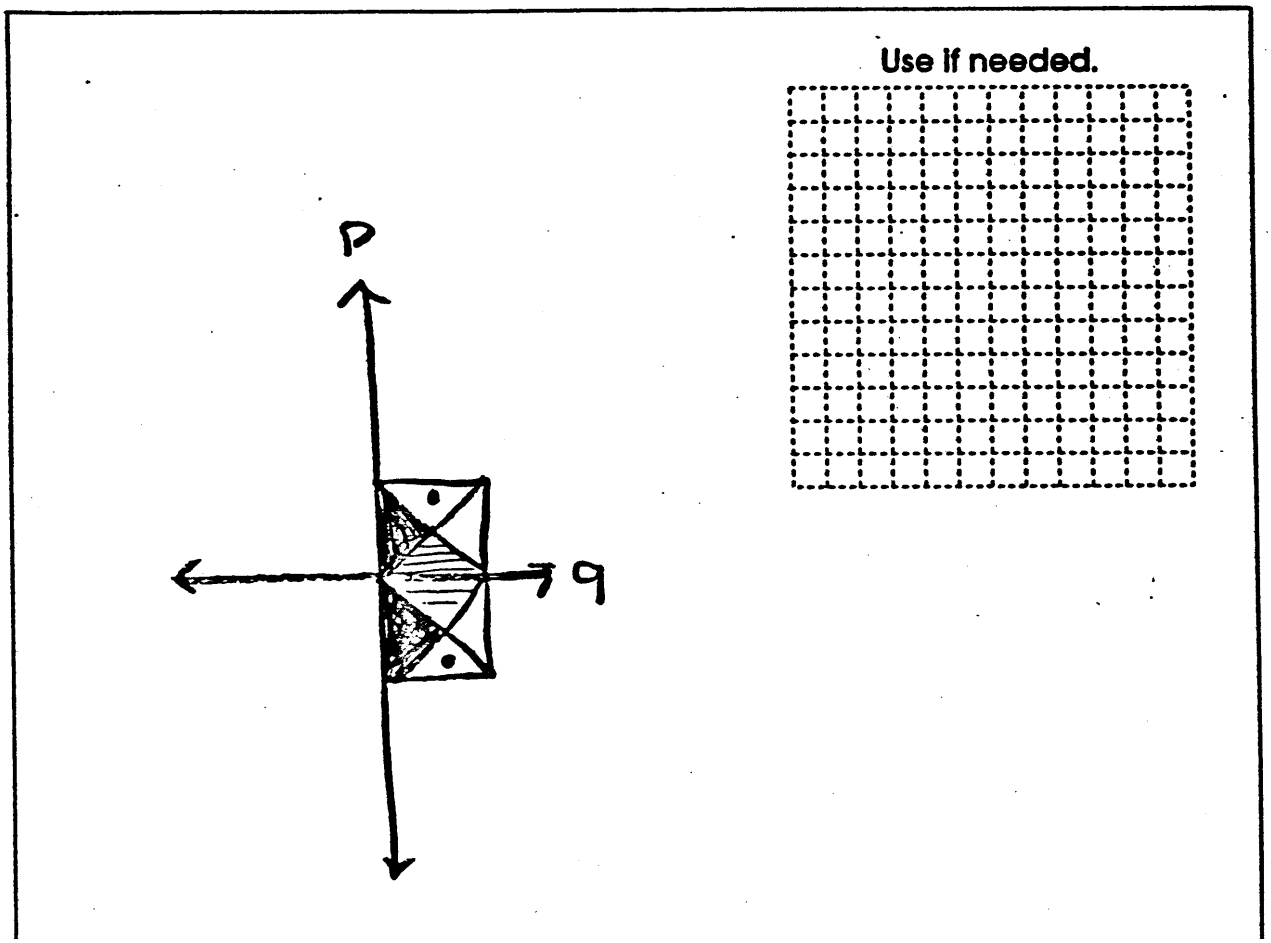
Score point	Description
2 points	The focus of the task is to draw reflections of the original design in each of the quadrants. The response shows the accurate reflections in each of the remaining three quadrants.
1 point	The response shows a partial understanding of the solution process or key elements of the task. The response may contain gaps or flaws in determining the solution. For example, the response may: Show only two accurate reflections of the design in two of the three remaining quadrants OR Show two accurate reflections based upon an initial incorrect reflection OR Show three accurate reflections based on a slightly flawed original design OR Show accurate translation or rotation of the original design in each of the quadrants.
0 points	The response fails to demonstrate minimal understanding of the task. For example, the response may: Only copy the original diagram OR Contain only translation or rotations in fewer than the three remaining quadrants. OR Be blank or the student writes, “I do not know” or unrelated statements.

Mathematics

Item 6

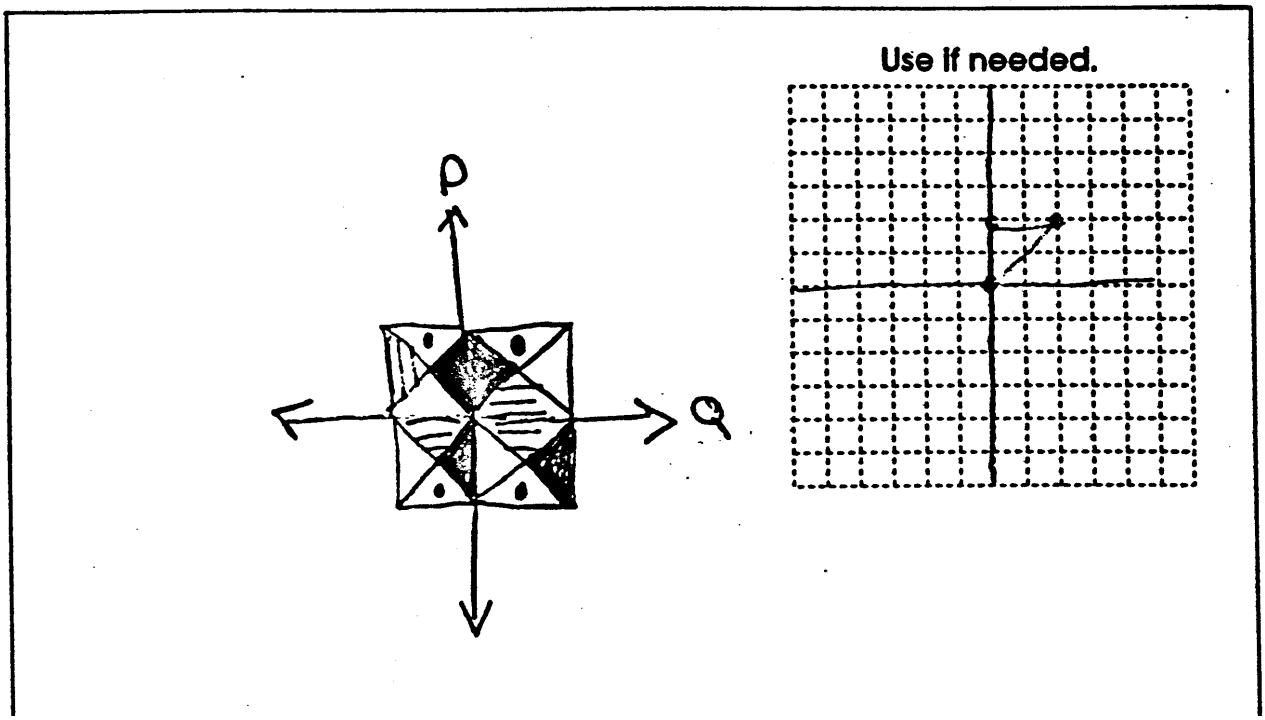
Spring 2005

Samples of Scored Student Responses



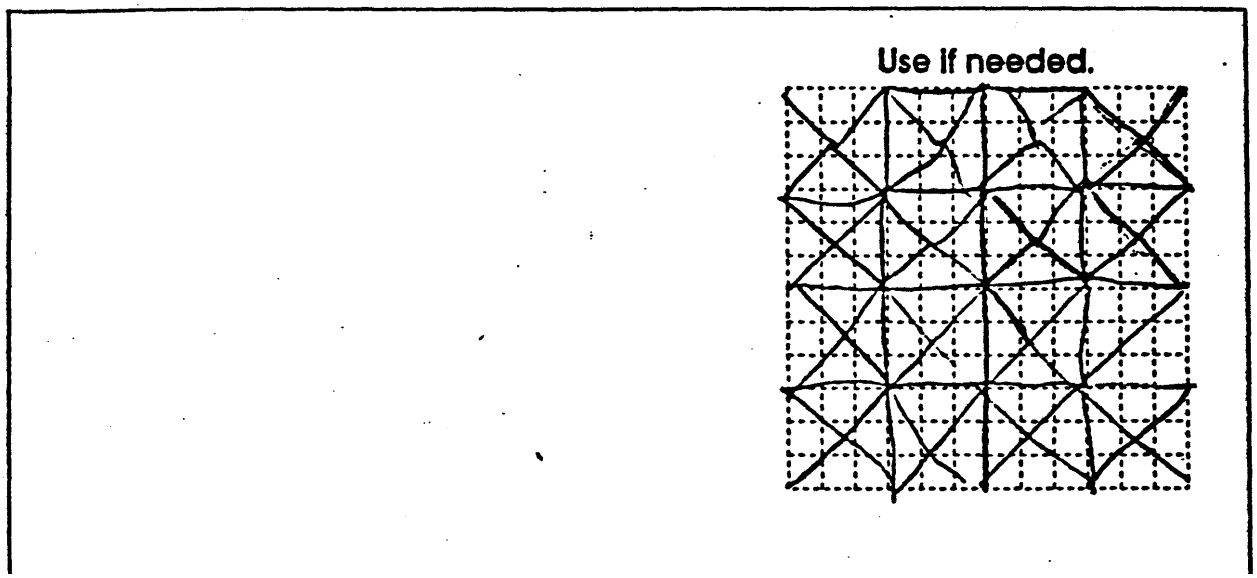
Score Point: 0

This response shows only one reflection of the original design.



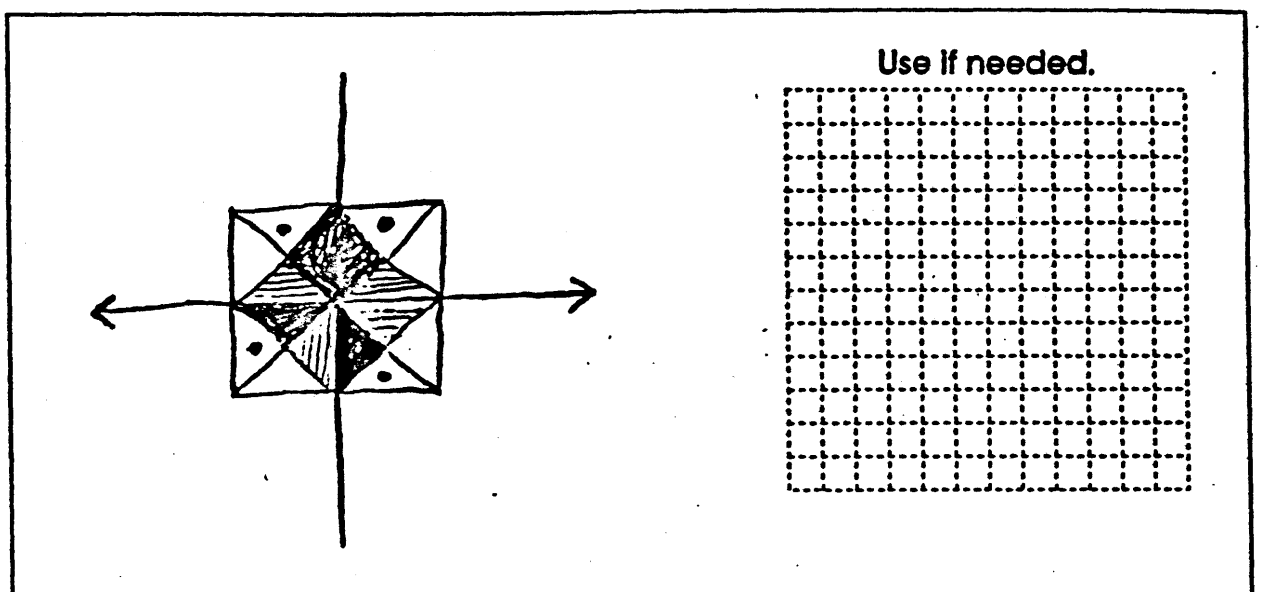
Score Point: 0

This response does not demonstrate the partial understanding required to achieve a "1." The response shows no correct reflections or rotations, and only one attempted translation (lower left quadrant, from lower right quadrant) from a flawed design.



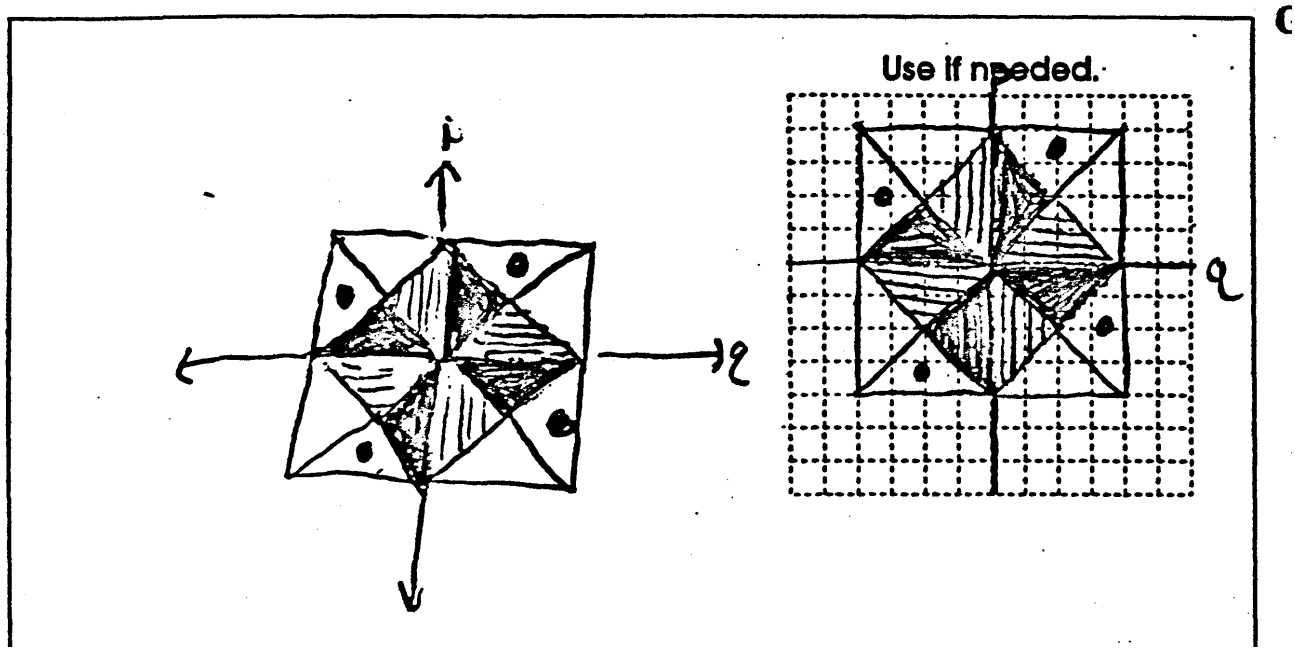
Score Point: 0

This response fails to demonstrate a minimal understanding of the task. The initial design in the response is flawed and contains insufficient differentiation to determine whether reflection, rotation, or translation has taken place.



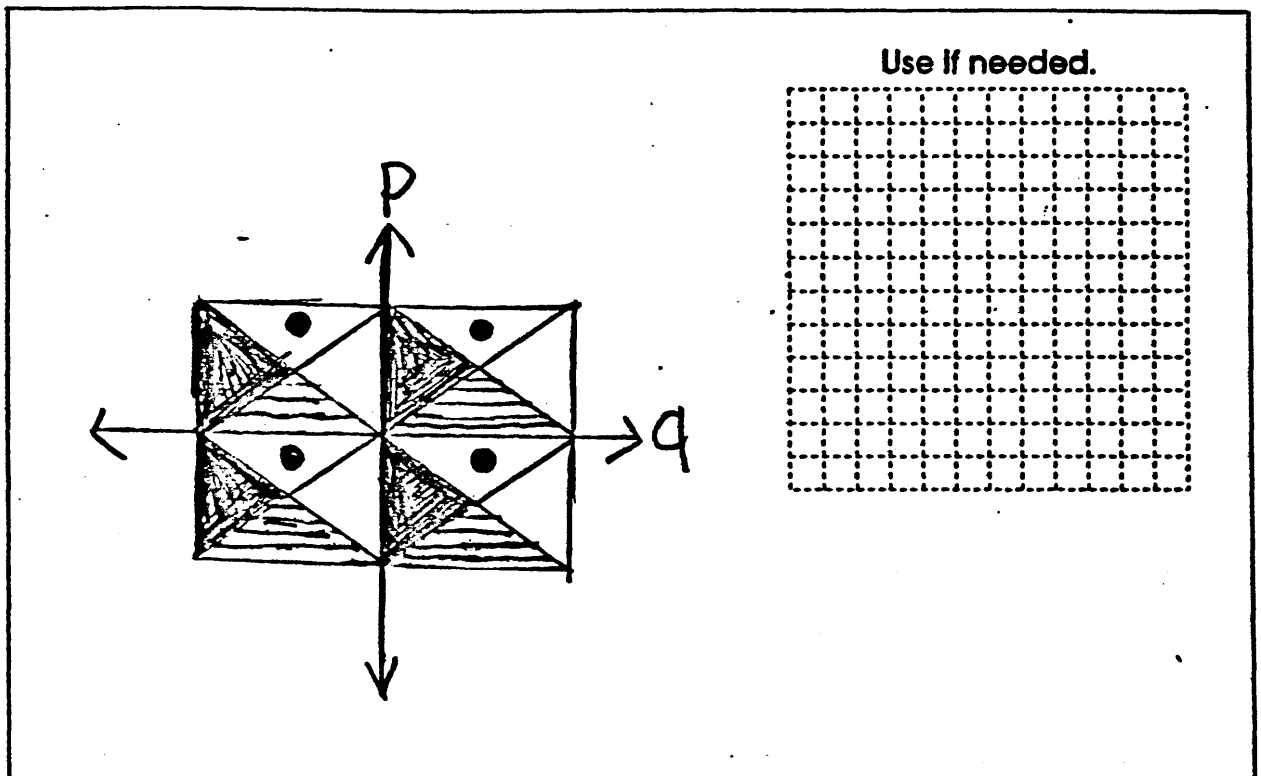
Score Point: 1

This response shows partial understanding of the solution process. The response contains two correct reflections of the initial drawing, one each across the *x*- and *y*-axes. The remaining quadrant (lower left) contains a rotation from the quadrant above.



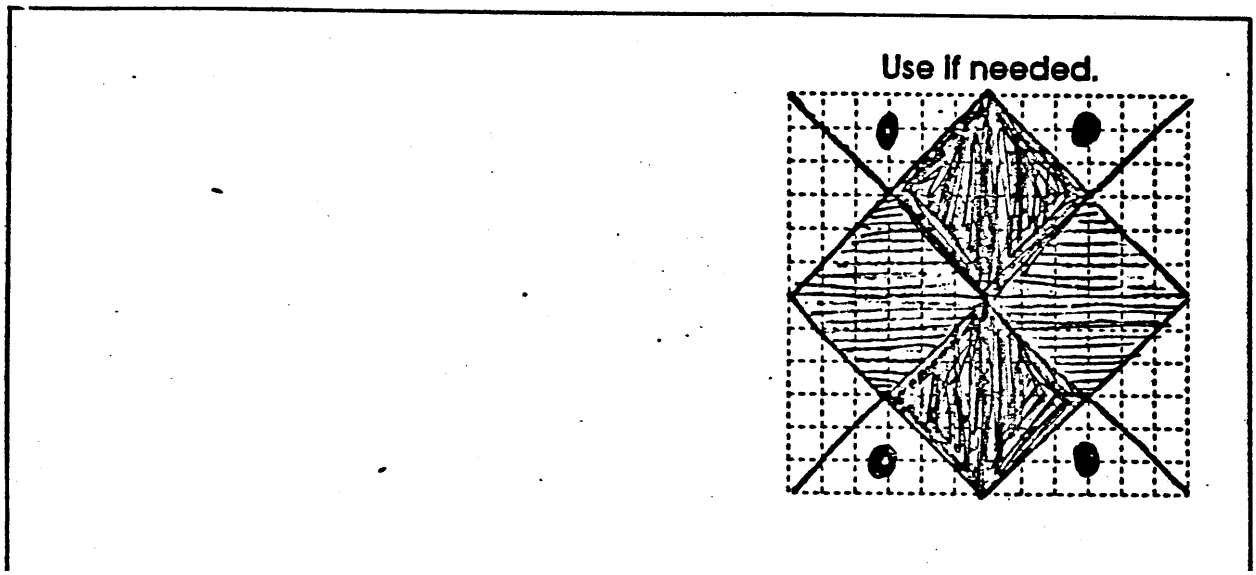
Score Point: 1

This response shows partial understanding of the key elements of the task. The response shows accurate rotations of the original design in each of the three remaining quadrants.



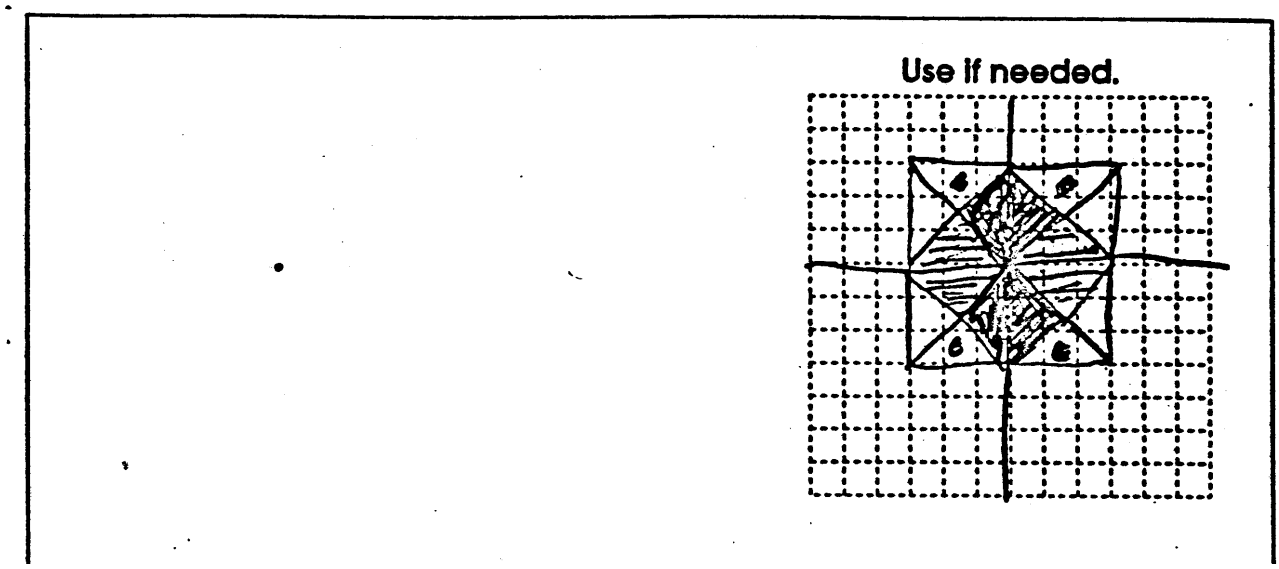
Score Point: 1

This response shows partial understanding of the key elements of the task. The response shows accurate translations of the original design in each of the three remaining quadrants.



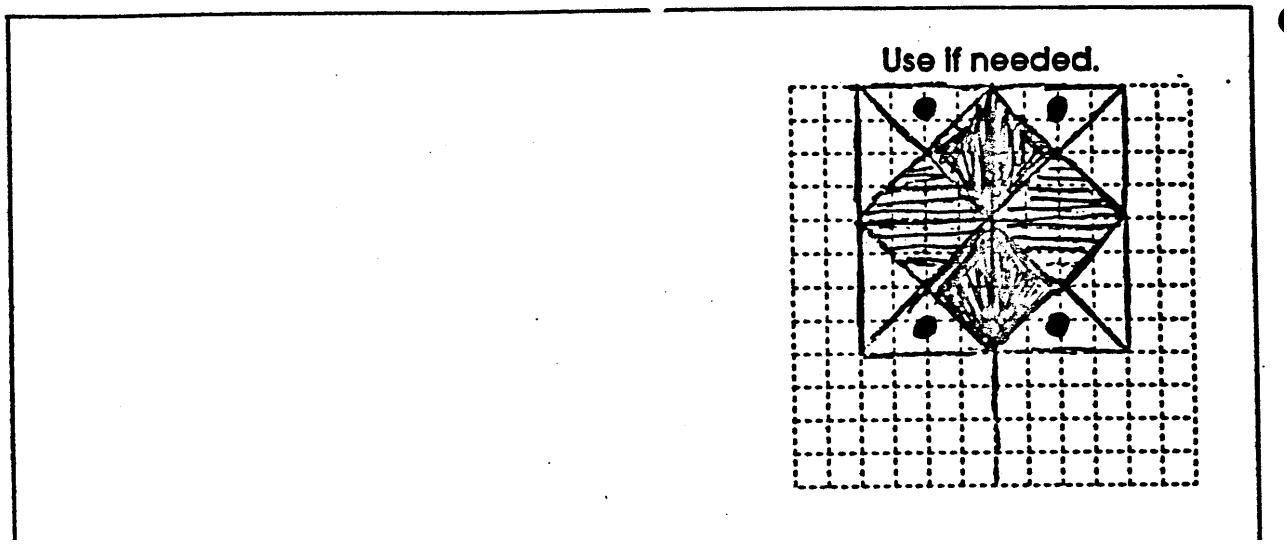
Score Point: 2

This response shows the original design and accurate reflections of the original design in each of the three remaining quadrants.



Score Point: 2

This response shows the original design and accurate reflections of the original design in each of the three remaining quadrants.



Score Point: 2

This response shows the original design and accurate reflections of the original design in each of the three remaining quadrants.

Mathematics

Item 12

Spring 2005

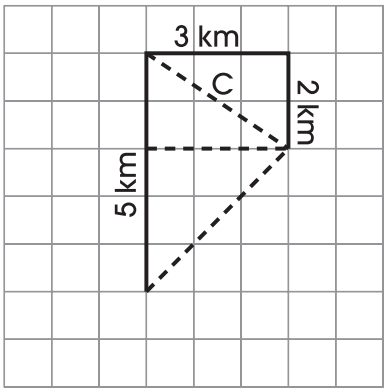
Item and Scoring Guidelines

Item

12. Brian and Caleb walked 5 kilometers north from their car to set up their tent. They hiked 3 kilometers east from their campsite to look for firewood. Then they walked 2 kilometers south. Caleb said that after they had walked the 2 kilometers south, they were the same distance from their car as they were from their tent.

    In your **Answer Document**, determine whether Caleb was correct by drawing a sketch of their hike and comparing the distances. Show your work.

Sample Response for Item 12 (Short Answer):



AND

They are 3 kilometers east and 2 kilometers south from their tent.
$c^2 = 3^2 + 2^2$
$c = 3.6$
They are 3.6 kilometers from their tent.

They are 3 kilometers north and 3 kilometers east of the car.
$c^2 = 3^2 + 3^2$
$c = 4.2$
Caleb is wrong. The distance from their car would be 4.2 kilometers, not 3.6.

OR

Caleb is wrong. For the distance from the car to their location to be the same, the two triangles formed would have to be congruent since they would have a hypotenuse and leg of the same length. However, they are not: one is two kilometers long and the other three kilometers long.

OR

Caleb is wrong. At the point where Caleb makes his statement about the distances to the car and the tent, two triangles are formed. The statement is true if the hypotenuse of these other two triangles are equal. Since the triangles share a common side (leg), the two triangles would be congruent by having the hypotenuse and the legs equal. However, the distances of the remaining legs are not equal ($2km \neq 3km$), thus the triangles are not congruent. This means the distances to the tent and car from that point are not equal.

Scoring Guidelines for Item 12:

Score point	Description
2 points	The focus of this item is to determine the validity of Caleb's statement with adequate mathematical support through work or reasoning. The response contains a scaled or labeled sketch of the hike and a mathematically correct explanation for why Caleb is wrong.
1 point	The response provides evidence of a partially correct answer or solution process. The response shows understanding of most of the key elements of the task, but contains gaps or flaws. For example, the response may: Contain a sketch to represent the hike drawn to scale or the distances labeled with numbers, with no further work or explanation. OR Contain a sketch that is slightly flawed or incomplete and compares the two distances based on the sketch as drawn with supporting work or valid explanation. OR Contain an accurate comparison of the distance with an an incorrect or missing sketch.
0 Points	The response fails to demonstrate a minimal understanding of the task. For example, the response may: State that Caleb is wrong with no additional work that supports this conclusion.

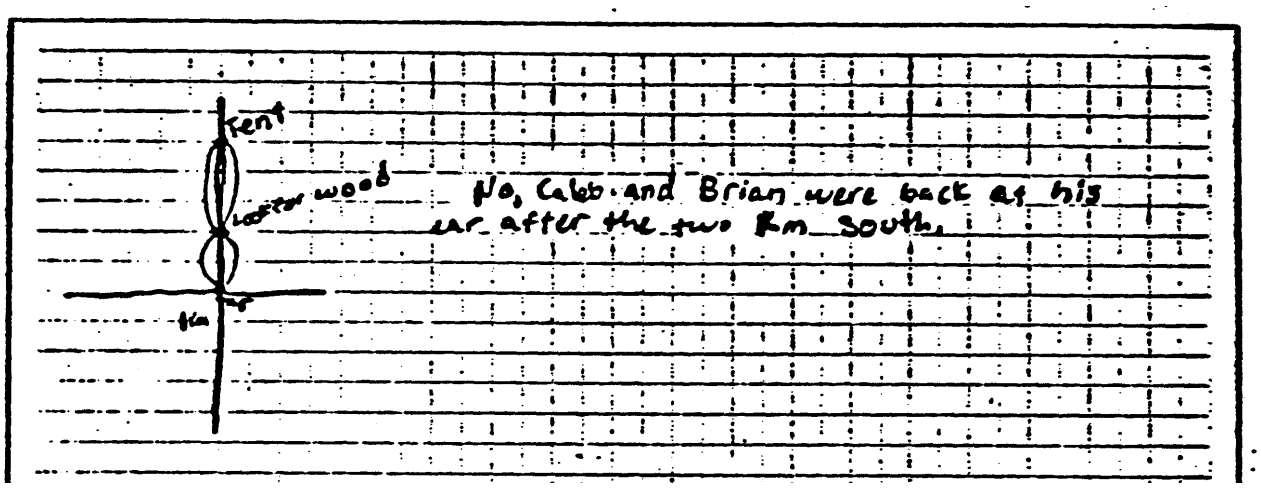
OR
Recopy information provided in the question with no work toward a solution.
OR
Be blank or state “I do not know” or include unrelated statements or work.

Mathematics

Item 12

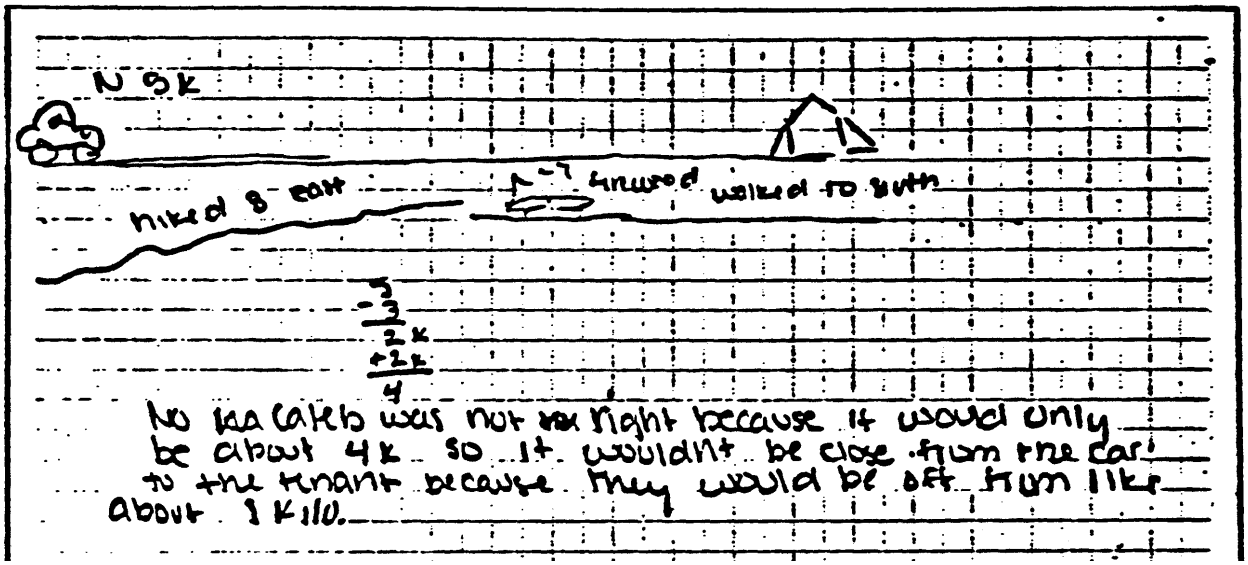
Spring 2005

Samples of Scored Student Responses



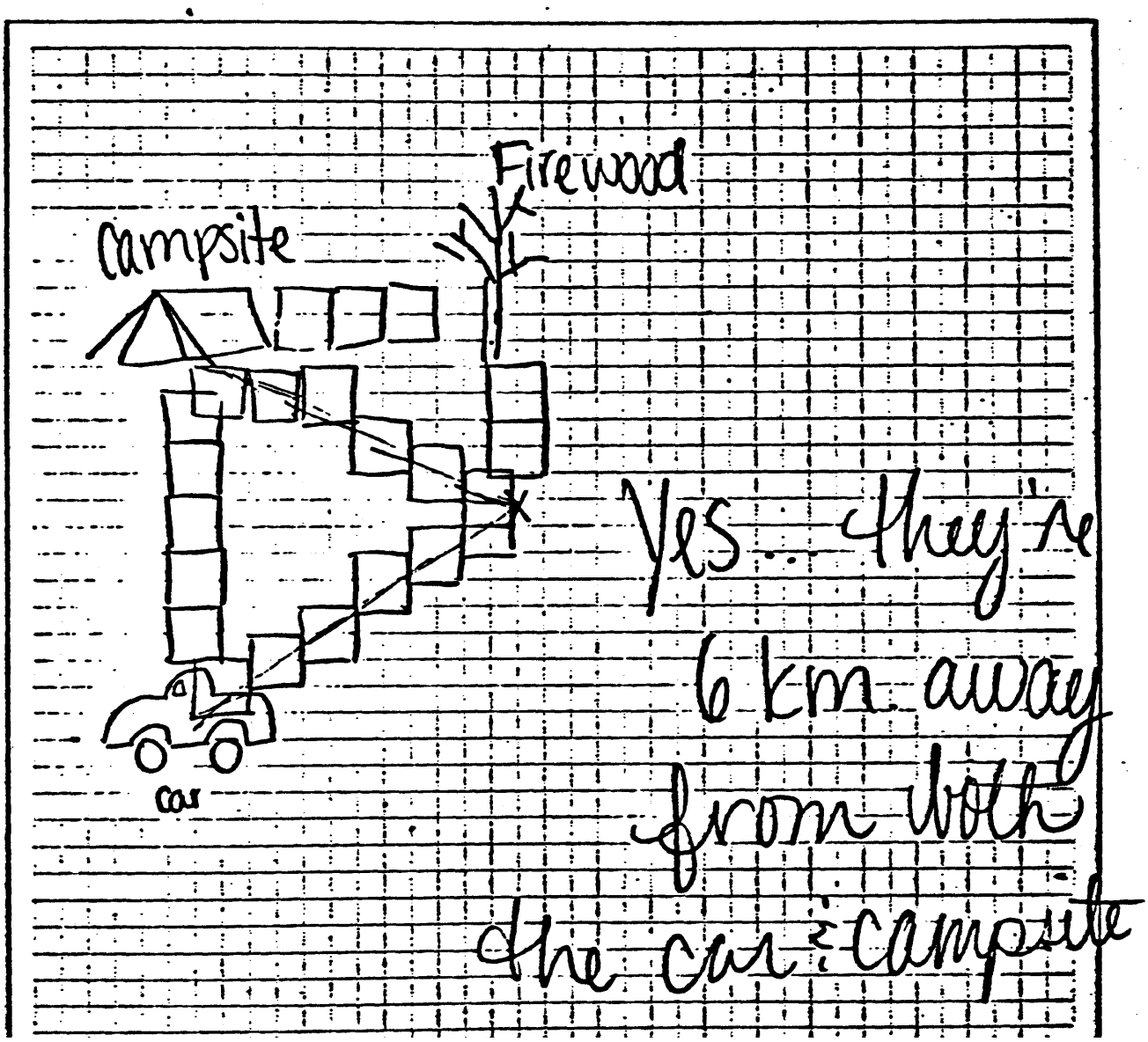
Score Point: 0

This response fails to demonstrate a minimal understanding of the task. The sketch and explanation are inaccurate, showing only north/south travel.



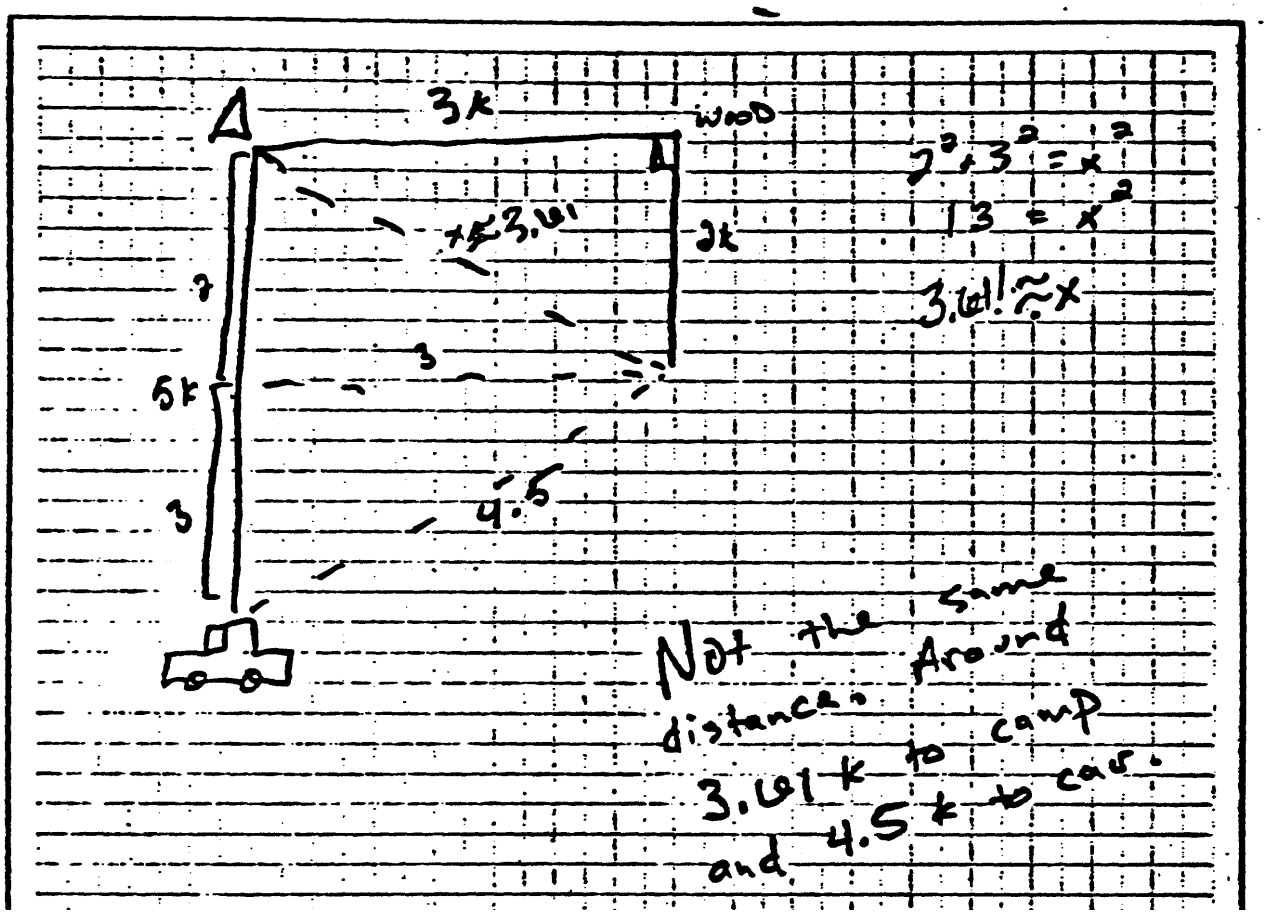
Score Point: 0

This response does state that Caleb was wrong, but neither the sketch nor the explanation supports this conclusion.



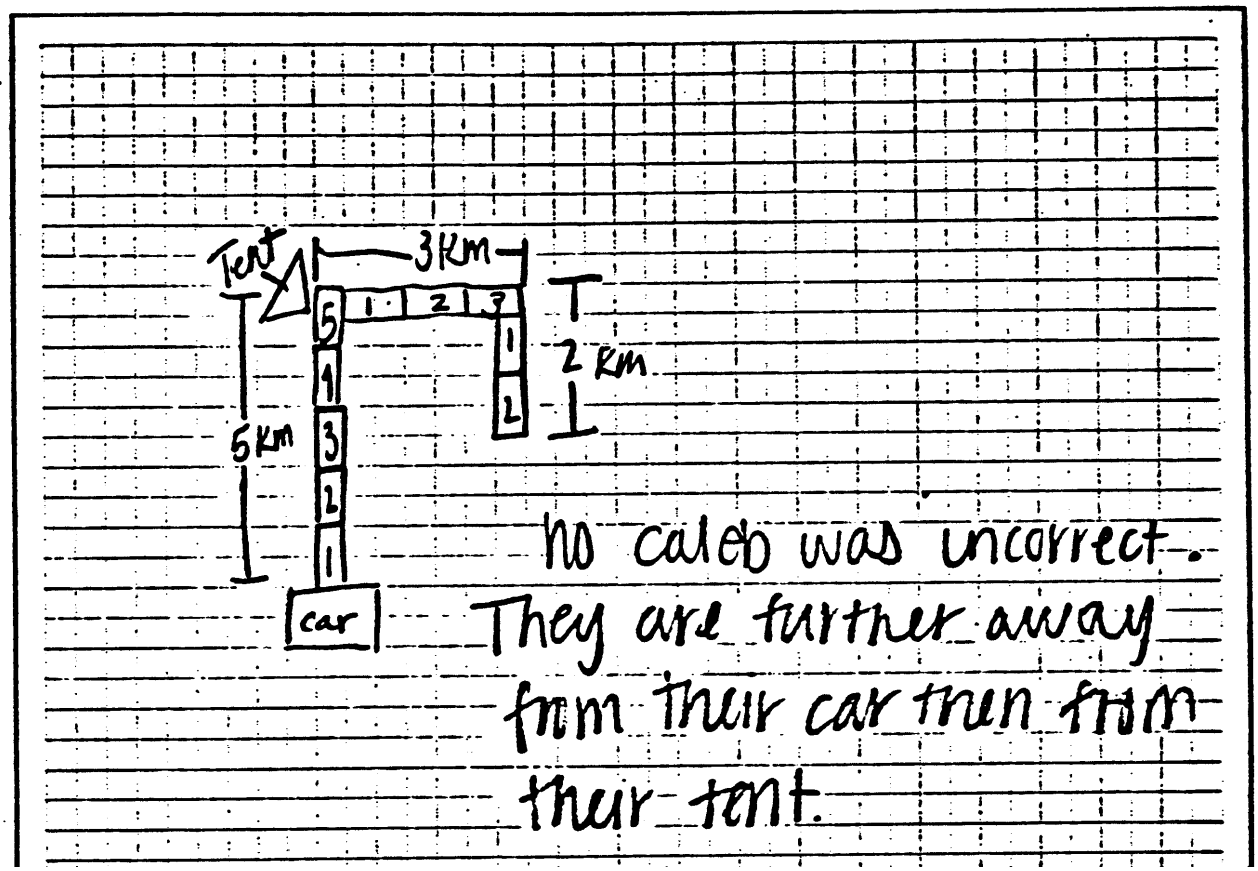
Score Point: 0

This response fails to demonstrate a minimal understanding of the task. The conclusion is incorrect, and the sketch is not drawn to scale nor properly labeled.



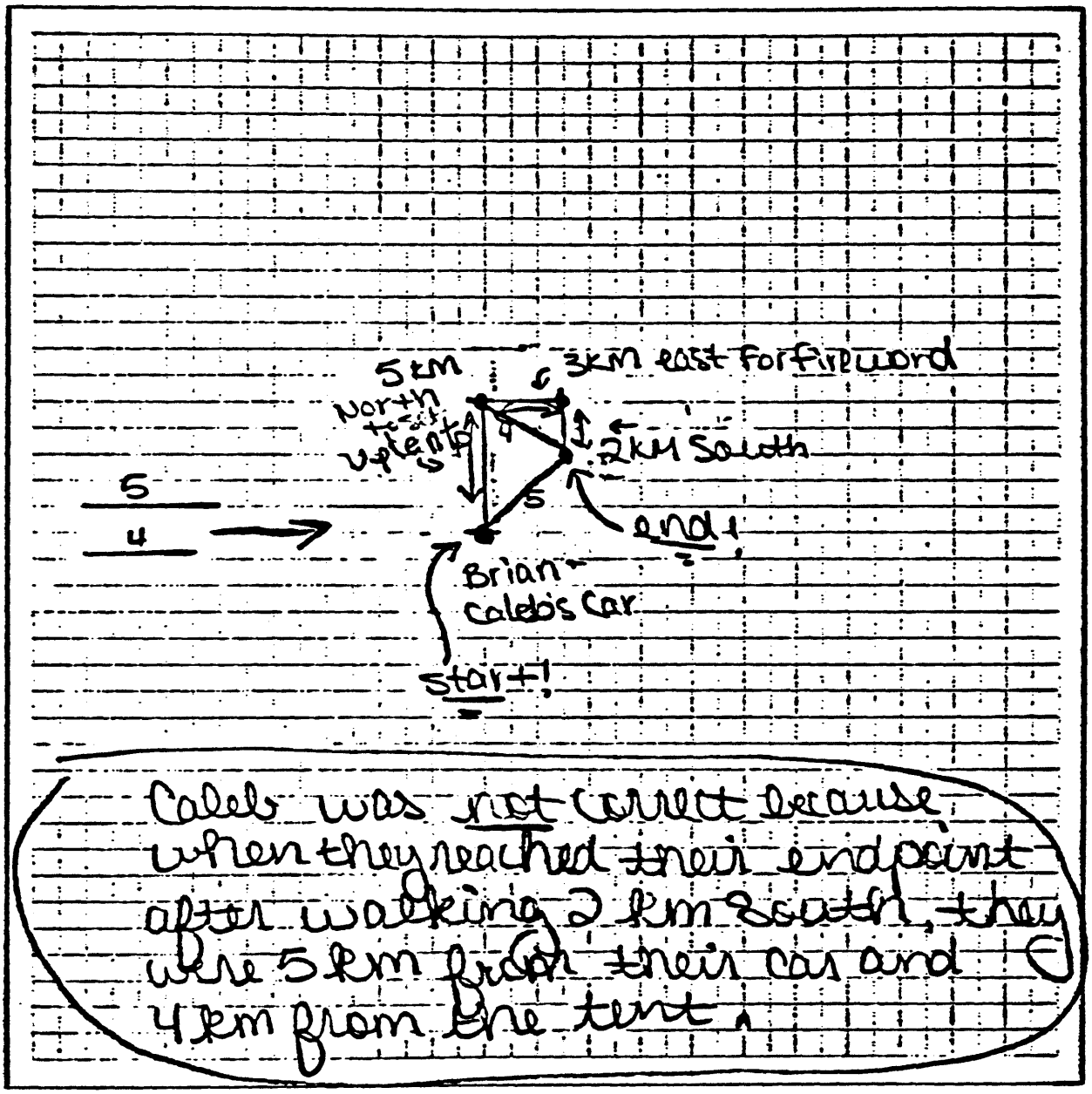
Score Point: 1

This response is partially correct but contains a calculation error. The response contains a sketch to represent the hike that has the distances labeled with numbers. It shows work for one distance (3.61). The other distance is incorrect (4.5 rather than 4.24), and no work is shown to support it.



Score Point: 1

This response provides evidence of a partially correct answer. The response contains a sketch with the distances labeled to represent the hike but shows no work or explanation.



Caleb was not correct because when they reached their endpoint after walking 2 km South, they were 5 km from their car and 4 km from the tent.

Score Point: 1

This response contains a sketch to represent the hike that is to scale and has the distances labeled with numbers. The response then concludes that Caleb was not correct but contains no correct work or explanation to support the conclusion.

distance $\sqrt{(0-3)^2+(5-3)^2}$

$\sqrt{9+4}=\sqrt{13}$

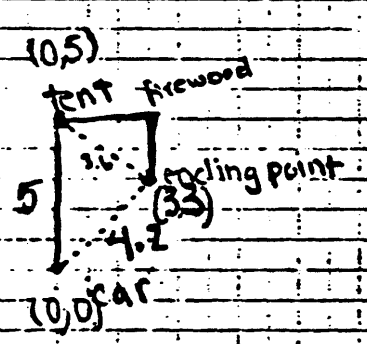
$\sqrt{13}\approx 3.6$

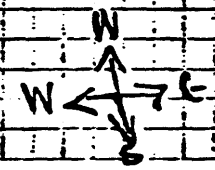
distance $\sqrt{(0-3)^2+(0-3)^2}=\sqrt{9+9}=\sqrt{18}$

$\sqrt{18}\approx 4.2$

they are closer to their tent

Caleb was wrong

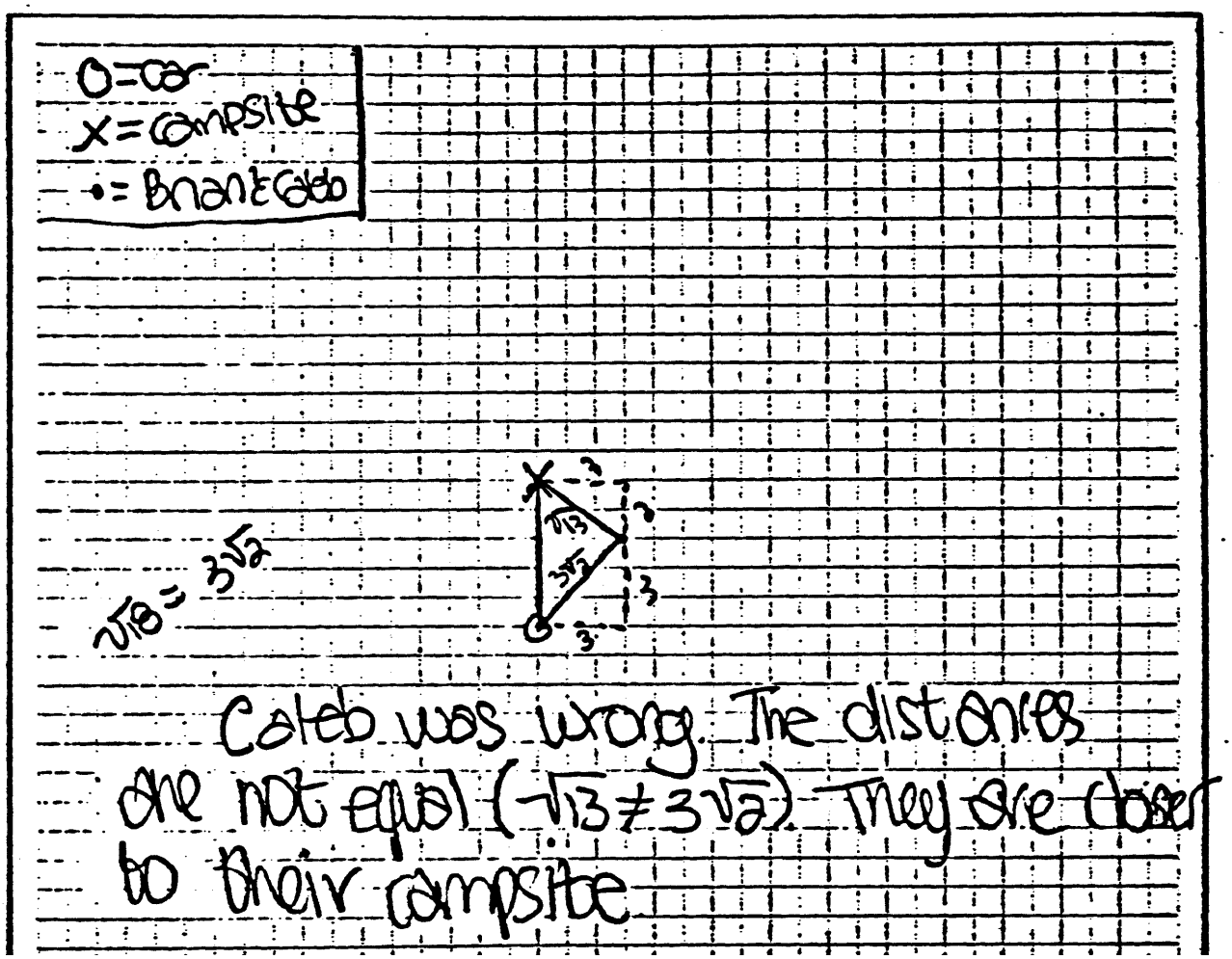




1 unit = 1 kilometer

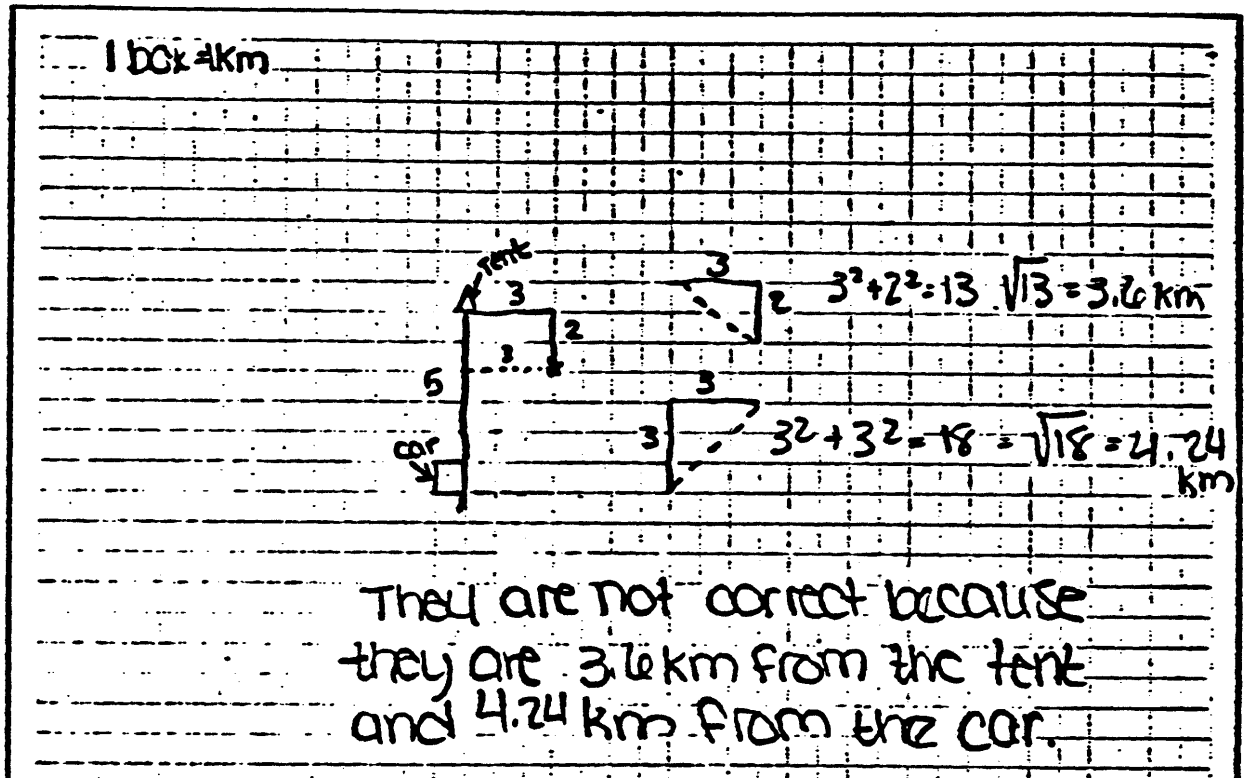
Score Point: 2

This response correctly determines the validity of Caleb's statement with adequate mathematical support. The response contains a scaled sketch of the hike and, by use of the distance formula, shows that the hypotenuse of the triangles formed (by drawing lines from the position of Brian and Caleb to the positions of the tent and the car) are not equal.



Score Point: 2

This response correctly determines the validity of Caleb's statement with adequate mathematical support. The response contains a scaled sketch of the hike. It states that Caleb was wrong and contains an explanation showing that the distances are not equal. The triangles and their hypotenuse are included in the sketch and the correct length of each hypotenuse is labeled.



Score Point: 2

This response correctly determines the validity of Caleb's statement with adequate mathematical support. The response contains a labeled sketch of the hike and an explanation using the Pythagorean theorem to show that the hypotenuse of the triangles formed (by drawing lines from the position of Brian and Caleb to the positions of the tent and the car) are not equal.

Mathematics

Item 18

Spring 2005

Item and Scoring Guidelines

Item

18. An organization distributes money to various groups in the Toledo area. This organization expects to receive a gift of 15% of a $5.8 million grant. Sixty-seven percent of this gift will be distributed to the Toledo Arts Festival.

    In your **Answer Document**, compute the amount that the Toledo Arts Festival will receive. Show or explain all steps used in finding the answer.

Sample Response for Item 18 (Short Answer):

The following are possible methods that could be used to determine the amount of money the Toledo Arts Festival will receive from the gift given. This list of examples is not exhaustive.

$0.15 \times \$5{,}800{,}000 = \$870{,}000$
$0.67 \times \$870{,}000 = \$582{,}900$

OR

$0.15 \times .67 = .1005$
$0.1005 \times \$5{,}800{,}000 = \$582{,}900$

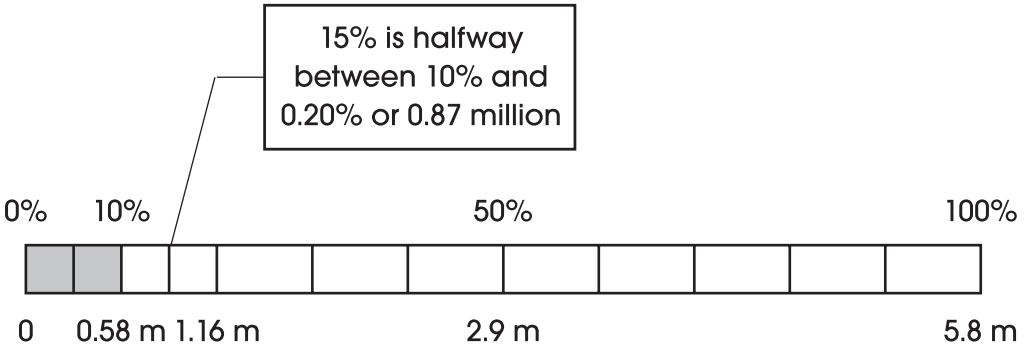
OR

$67\% \approx 2/3.$
$2/3 \text{ x } 15 = 10\%.$
10% of 5.8 million = 580,000

OR

$$\frac{n}{\$5{,}800{,}000} = \frac{15}{100} \qquad n = \$870{,}000$$
$$\frac{n}{\$870{,}000} = \frac{67}{100} \qquad n = \$582{,}900$$

OR



$\frac{2}{3}$ of 15% is the same as 10% or 0.58 million.

Note: A value of 0.58 million will be accepted as a correct amount of money given to the Toledo Arts Festival.

Scoring Guidelines for Item 18:

Score point	Description
2 points	The focus of this item is to find the final amount of money that the Toledo Arts Festival will receive by finding 67% of the 15% that was donated. The response contains a correct value ($580,000 or $582,900 or $583,000 or $0.58 million) for the contribution to the Toledo Arts Festival. Supporting work or detailed explanation is provided.
1 point	The response provides evidence of a partially correct answer or solution process. The response shows understanding of most of the key elements of the task but contains gaps or flaws. For example, the response may: Provide a correct solution process but give an incorrect amount for the Arts Festival due to errors in computation. OR Provide evidence of a correct first step in the process; e.g., method for finding 15% of $5.8 million.

OR
Contain a correct amount—$580,000 or $582,900 or $583,000 or $0.58 million—with no work or explanation.

0 points

The response fails to demonstrate a minimal understanding of the task.

For example, the response may:

Include an incorrect amount for the Toledo Arts Festival with no work or explanation to support the solution.
OR
Show no apparent understanding of the task or possible solution process; e.g., simply adds the three values given in the item.
OR
Recopy information provided within the item.
OR
Be blank or state "I do not know," or include unrelated statements or work.

Mathematics

Item 18

Spring 2005

Samples of Scored Student Responses

$870,000

Score Point: 0

This response includes an incorrect amount for the Toledo Arts festival ($870,000) with no work or explanation to support the solution.

.15 / 5.8 mill = .026 · 5.8 = 1.508

5.8000000

Score Point: 0

This response fails to demonstrate a minimal understanding of the task. The response divides 0.15 by 5.8 million (rather than multiplying 5.8 million by 0.15) and then multiplies that result by 5.8, showing no apparent understanding of the task or possible solution process.

5,800,000 ÷ .67 = 5,130,000 → Toledo Arts Festival

Score Point: 0

This response fails to demonstrate a minimal understanding of the task. The work provided indicates an incorrect solution process (dividing by 0.67 rather than multiplying). The listed operation is not performed. Instead, 670,000 is subtracted from 5,800,000, which results in the incorrect amount of 5,130,000.

$5.8 million x .15 (15%)
=
870,000

870,000 x .6 (60%)
=
522,000

Toledo Arts Festival recieves
$522,000

Score Point: 1

This response provides a correct solution process but gives an incorrect amount for the Arts Festival. This is due to the calculation error of using .6 (60%) instead of 67% for the final step, which results in an incorrect final amount of $522,000.

5,800,000 (5.8 million)
x 000015 (15%)
$870,000.00
gift

Score Point: 1

This response provides evidence of a partially correct solution process. A correct first step in the solution process, a calculation of 15% of 5.8 million with a result of $870,000, is shown, but the response does not take the remaining step of calculating 67% of $870,000.

5.8
x .67
3.886 million

Toledo Arts Festival will receive 3.886 million dollars.

Score Point: 1

This response provides evidence of a partially correct solution process. A correct first step in the solution process, a calculation of 67% (.67) of 5.8 million with a result of $3.886 million, is shown, but the step of calculating 15% is omitted.

$5,800,000 × .15 = $870,000

Gift: $870,000 × .67 = $582,900

The Toledo Arts Festival will receive: $582,900.

This was calculated by taking 15% of the grant and then 67% of that 15%.

Score Point: 2

This response contains a correct value ($582,900) for the contribution with supporting work and an explanation.

$$\frac{15}{100} = \frac{x}{5.8}$$

$$\frac{87}{100} = 100x$$

$$.87 = x$$

$$x = \$870000$$

$$\frac{67}{100} = \frac{x}{870000}$$

$$\frac{100x}{100} = \frac{58290000}{100}$$

$$x = \$582,900$$

Score Point: 2

This response contains a correct value ($582,900) for the contribution with supporting work.

5.8 · .15 = .87

67% of that

.67 · .87 = .583

The Toledo Arts Festival will receive $583,000

Score Point: 2

This response contains a correct value ($583,000) for the contribution with supporting work. The value was determined by rounding to .583 in the decimal calculation. $583,000 is accepted as a correct value.

Mathematics

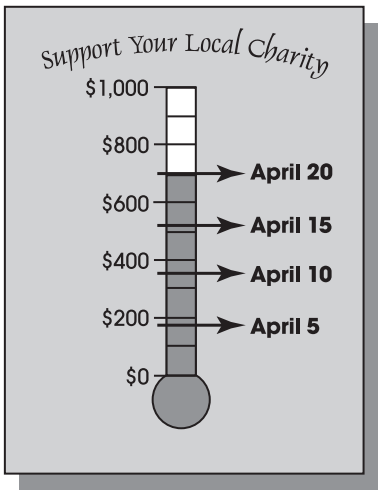
Item 24

Spring 2005

Item and Scoring Guidelines

Item

24. Last spring, the employees of an Akron tire company contributed to a local charity. The graphic represents the total dollar contributions as of several dates in April.



In your **Answer Document**, determine on what date the charity could have expected to reach its goal of $1,000, if the pattern of donations remains about the same. Show your work or provide an explanation for your answer.

Sample Response for Item 24 (Short Answer):

Find the average amount received every 5 days.
700 ÷ 4 = $175/5 days
700 + 175 = 875 on April 25.
875 + 175 = 1050 on April 30
The tire company should reach their goal on the 28th or 29th of April because, by the 30th, they will have exceeded the goal.

OR

Find the average amount received every day.
700 ÷ 20 = $35/day
$35 x 28 = $980 on April 28.
$35 x 29 = $1015 on April 29.
OR
$1,000 ÷ $35 = 228.57 days
The tire company should reach its goal during the day on the 29th.

OR

This response contains a description of how the diagram was used to estimate when the tire company would reach their goal on the (range of 28th to 30th) of April.

Scoring Guidelines for Item 24:

Score point	Description
2 Points	The focus of this item is to determine the approximate date the company will meet the $1,000 goal based on the current rate of giving. The response indicates a date from the 28th to the 30th of April with supporting work or detailed explanation based on the information provided in the item.
1 Point	The response shows a partial understanding of the solution process or key elements of the task. The response contains gaps or flaws in determining the solution. For example the response may: Indicate the correct date but show no work or the reasoning may be flawed. OR Indicates a date outside the acceptable range resulting from an error in computation. An appropriate solution process is shown or explained. OR Contain a correct method for determining the date, but the date is incorrect or missing.

0 Points	The response fails to provide evidence of minimal understanding of the task. For example the response may: Recopy information provided in the question with no work toward a solution. OR State a date outside the acceptable range with no supporting work or explanation. OR Be blank or the student writes “I do not know” or includes unrelated statements or work.

Mathematics

Item 24

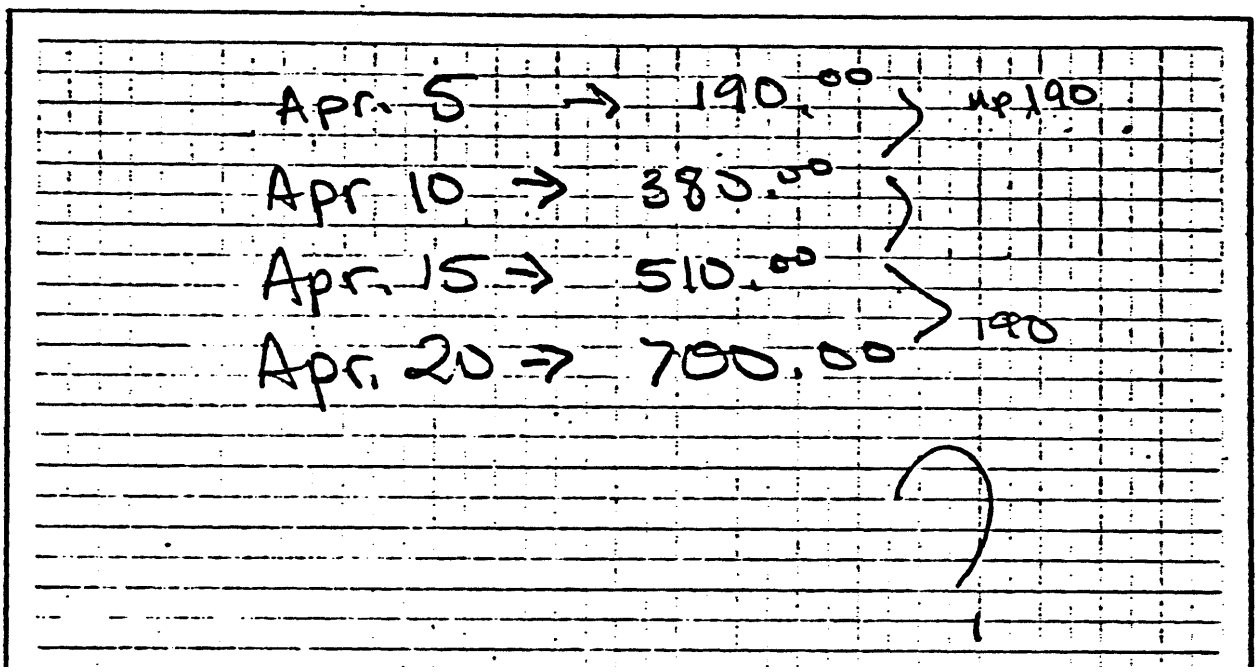
Spring 2005

Samples of Scored Student Responses

May 5th because the numbers keep going up by five.

Score Point: 0

This response fails to provide evidence of minimal understanding of the task. The response indicates the date, May 5th, that is not in the acceptable range of April 28–30. The remainder of the response (because the numbers keep going up by five) appears only to refer to the interval between dates shown on the graph. The response provides no further explanation or supporting work.



Score Point: 0

This response fails to provide evidence of minimal understanding of the task. The information from the graph is copied, and two intervals of $190 are noted. No further work toward a solution is attempted, and no date for reaching the goal is indicated.

May 4th

add 5 days 3 times = over 1,000

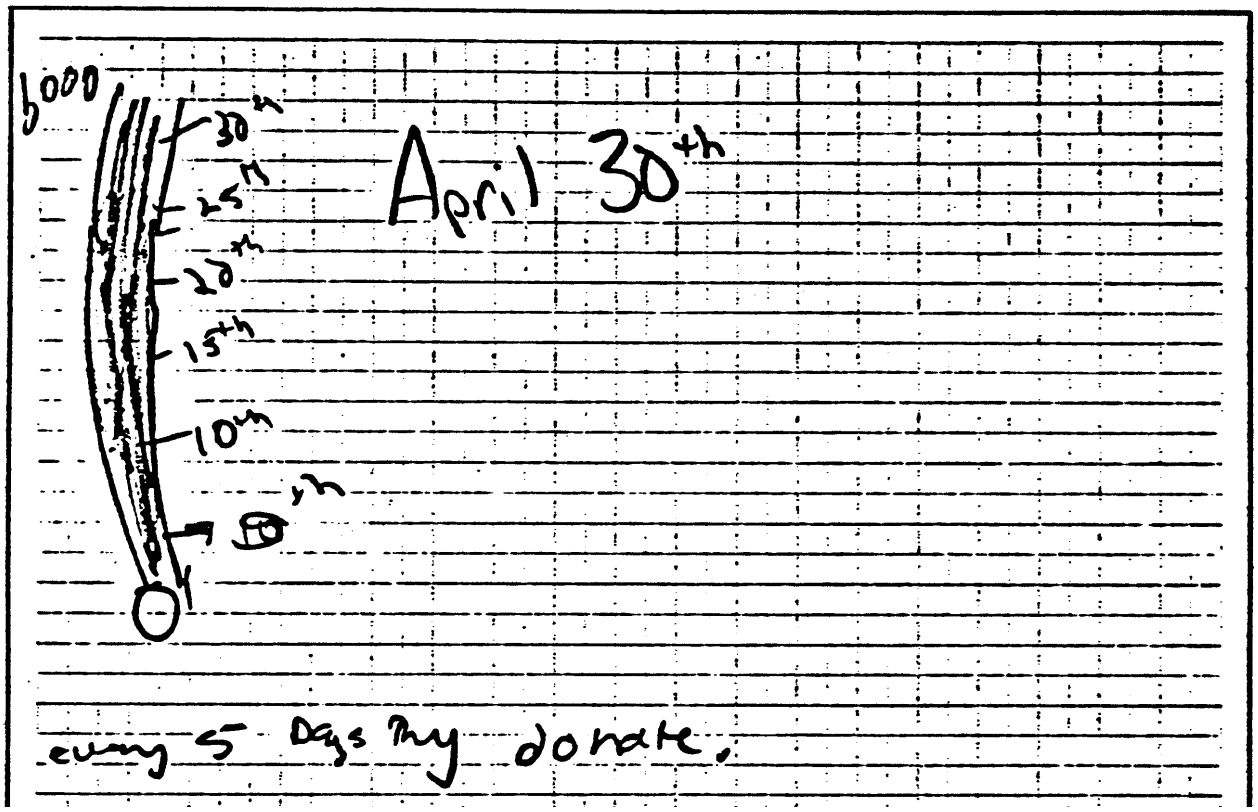
Score Point: 0

This response fails to provide evidence of minimal understanding of the task. The date indicated, May 4, is incorrect (not in the acceptable range of April 28–30). The explanation does not indicate the amount of the donation in each of the five-day intervals mentioned, and no work is shown.

Every five days the charity goes up about $180. Therefore the next five days from April 20 the charity will be $880.00. I'm guessing it will take two days to reach $1,000.00. They will reach their goal on April 27.

Score Point: 1

This response shows a partial understanding of the solution process but contains flaws in determining the date. The response estimates from the graph a rate of donation (every 5 days ... about $180) that is slightly larger than the actual average amount of $175. This results in the calculation of a date of April 27, which is not in the acceptable range of April 28–30.



Score Point: 1

This response indicates a correct approximate date for meeting the $1,000 goal, but no supporting work is shown. The graph shown lists only dates, not amounts. The only explanation (every 5 days they donate) again refers only to the time involved and not to the amount of money donated. There is no way to determine what rate of donation was used in the response to arrive at the date of April 30.

The company will reach its goal of $1000 on May 2
According to the graphic, every 5 days, the company's contributions go up $175. Now, it is April 20 and the company has $700. If the company goes for 10 more days, they will have $350.

700
+350
1050

April 5 - 175 175
10 - 350 175
15 - 525
20 - 700

Score Point: 1

This response indicates a correct method for determining the date, but the date is incorrect. The response calculates the rate of donation at $175 per five-day interval based on a $700 total on April 20. It notes that in 10 more days the goal will be exceeded. However, the response shows the completion date as May 2 (not in the acceptable range of April 28–30).

Approximately on April 29

It rises about 35$ a day.

On April 20 its exactly on 700. So if you take 5 days times four intervals, you get 20. 700 ÷ 20 = 35

A 19
A 20
A 21 = 735
A 22 = 770
A 23 = 805
A 24 = 840
A 25 = 875
A 26 = 910
A 27 = 945
A 28 = 980
A 29 = 1015

On April 29th, it will reach about 1000

Score Point: 2

This response indicates a correct approximate date for meeting the $1,000 goal and provides supporting work and a detailed explanation. The response uses the last benchmark of donations ($700 on April 20) on the graph provided in the item to compute a donation rate of $35/day. With that rate, the response calculates the length of time it will take for the donations to reach $1,000 (April 29).

April 5 $180

10 $360

15 $540

20 $720

25 $900

about 29 $1,000

about $36 a day

* On April 29 the charity could expect to reach it's goal of $1000.

Score Point: 2

This response indicates a correct approximate date for meeting the $1,000 goal and provides a detailed explanation. The response estimates a donation rate of $180/5 days or $36/day. The response calculates that the goal would be reached on April 29.

$\frac{700}{4} = \$175/\text{donation}$

$\frac{1{,}000}{175}$ 5.7 donations

on its sixth donation which would have been April 30th. I took the total amount paid after 4 donations which was $700. 700 ÷ 4 equals 175. The gave $175 every 5 days. Then I took 1,000 ÷ 175 and got 5.7 donations which means they would expect to break it on the 6th. which would be on April 30

Score Point: 2

This response indicates a correct approximate date for meeting the $1,000 goal and provides supporting work and a detailed explanation. The response calculates the average donation per five-day interval ($175) and determines the number of those donations it would take to meet the $1,000 goal (5.7). The response concludes that the goal will be reached with the sixth donation on April 30.

Mathematics

Item 36

Spring 2005

Item and Scoring Guidelines

Item

36. The average salary for all department store workers in a certain area is $255 a week. The weekly salaries of the 7 employees in the Acme Department Store are given in the table below.

Acme Employees' Salaries

Employee Number	Salary
Employee 1	$240
Employee 2	$245
Employee 3	$245
Employee 4	$250
Employee 5	$252
Employee 6	$260
Employee 7	$420

In your **Answer Document**, determine the measures of center (mean, median and mode) of the 7 salaries.

Specify which of these measures of center the management could use to represent the salaries in an argument against pay increases. Explain your answer.

Specify which of these measures of center the labor union could use to represent the salaries in an argument for pay increases. Explain your answer.

Sample Response for Item 36 (Extended Response):

The mean salary is $273.14. The median salary is $250. The mode salary is $245.

The management could use the mean salary to argue against pay increases as the mean salary is greater than the average salary in the area.

The labor unions could use the median or mode salary as the median and mode salaries are less than the average salary in the area.

Scoring Guidelines for Item 36:

Score point	Description
4 points	The focus of this item is to find the three measures of center (mean, median and mode), compare these three measures of center of the Acme employees to the average for the area ($255), and make an argument for management's position that no pay increase is needed and for the labor union's position that a pay increase is needed. The response contains the correct values for the measures of center that are labeled correctly. The response identifies a measure to support each of the two positions with an appropriate explanation.
3 points	The response clearly addresses the task; however, it includes errors in completing one or two of the parts. For example, the response may: Contain the correct values for the measures of center labeled correctly. The response contains an appropriate measure choice for one position (management or labor union) with a valid explanation for that choice. OR Contain two correct and one incorrect or missing measure of center. The response contains appropriate measure choices for the two positions, based on the calculated measures, with valid explanations for the choices. OR Contain three correct measures of center that are labeled correctly and two arguments that demonstrate some understanding of the two positions but are not clearly stated.
2 points	The response provides evidence of a partial understanding of the task. The response may adequately address some parts of the task but contains major gaps or flaws in other parts.

For example, the response may:

Contain two correct measures and one incorrect or missing measure. The response contains an appropriate measurement choice for one of the two positions with an appropriate explanation for that choice.
OR
Contain incorrect values for all measures. The response contains two appropriate measure choices for the two positions based on the values of the measures as calculated with an explanation for the choices.
OR
Contain correct values for all measures of center that are labeled accurately, but the arguments for the two positions do not support the correct choice made.
OR
Contain three correct values for the measures of center that are labeled correctly and one argument that demonstrates some understanding of the two positions but is not clearly stated.

1 point

The response has significant errors or omissions in the solution of the task. There is evidence of minimal understanding of the task or solution process; however, the response includes significant errors or omissions in most parts of the solution to the task.

For example, the response may:

Contain two or three correct values for the measures that are correctly labeled. The response contains invalid measure choices and explanations or no attempt to address the two positions of the task.
OR
Contain no correct values for the measures of center. However, the response makes a valid choice and explanation for one position based on the incorrect values for measures of center.
OR
Contain correct values for the measures of center that are missing labels or labeled incorrectly. NO attempt is made to address the two positions.

0 points	The response fails to demonstrate minimal understanding of the task. For example, the response may: Only have one value for a measure of center identified correctly with invalid or no attempt to address the two positions. OR Recopy information from the item with no attempt to answer. OR Be blank or state “I do not know” or include unrelated statements or work.

Mathematics

Item 36

Spring 2005

Samples of Scored Student Responses

G-

mean - 240
median - 250
mode - 420

Employee 7 could be used as an example against pay increases
Employee 1 could be used for an example that pay checks raises are needed

Score Point: 0

This response fails to demonstrate minimal understanding of the mathematical concepts involved in this task. Only one measure of center (median) has been computed and labeled correctly. An attempt was made to argue for and against pay raises, but individual employees were used to argue the case instead of the measures of center, which is not a valid argument. This response receives a score of "0."

G-14

273 is the adverage salery.
It all depends on what job they
have what they make.

Score Point: 0

This response shows minimal understanding of the mathematical concepts involved in this task. Average salary (mean) of $273 has been computed, but no further work or explanation is provided that is relevant to the question. One correct measure of center is insufficient for a score of "1," so this response receives a "0."

Mean = 273.14

median = 250

mode = 245

Mean becouse it equals out all the Salarys and makes them even.

Score Point: 1

This response only shows an understanding of how to compute and label correctly the three measures of center (mean, median and mode). A choice seems to have been made (mean), but it is not clear whom it favors. As a result, this response receives a score of "1" for having three correct values for the measure of center labeled correctly.

G-11

240
245 } mode
245
+ 250 — median
252
260
420

$\boxed{1912 = \text{mean}}$

240 because 250 was used the most.

Score Point: 1

This response shows only minimal understanding of the task with significant errors and omissions. Two measures of center (mode and median) have been indicated correctly. While an attempt was made to compute the mean by summing the salaries for all employees, an average was not found. The statement (240 because 250 was used the most) did not support either position. No other attempt was made to choose measures for management or labor union. This response receives a score of "1" for two correct measures of center.

G-12

by taking the mean
the average pay is $273.14.
which is over other averages
so no pay increase
is necessary

Score Point: 1

This response indicates some understanding of the task, but little work is shown. The average pay has been computed ($273) and compared to the average pay in the area (other averages) to show that no pay increase is needed. Since a valid choice with an explanation for one position is given, this response receives a score of "1."

Employee 1 - 240
Employee 2 - 245
Employee 3 - 245
Employee 4 - 250
Employee 5 - 252
Employee 6 - 260
Employee 7 - 420

mean - $\boxed{\$273.14}$

median - 420 - 240 - $\boxed{\$180}$

mode - $\boxed{\$245}$

$7\overline{)1912}$

$$\begin{array}{r} 273 \\ 7\overline{)1912} \\ -14 \\ \hline 51 \\ -49 \\ \hline 22 \\ -21 \\ \hline 40 \end{array}$$

The management could use mean. The average pay is around $273. The average salary for all department store workers is $255. That is $18 more for Acme workers than other department stores.

Workers could use median for an argument for higher pay because worker #1 is making $240 a week, while worker #7 is making $420/wk. Worker #7 is making over $100 more than the other workers.

Score Point: 2

This response shows partial understanding of the mathematical concepts and procedures involved in the task. Two measures of center (mean and mode) have been computed and labeled correctly; however, median is incorrectly calculated as a range. An appropriate measure choice (mean) has been made for management with adequate support, including the comparison of the average salary for all department store workers ($255) and calculation of the difference between the two means ($18). The choice of median for the workers would have been acceptable since it is the lowest, but the argument compares individual employees (highest and lowest paid) instead of using the measures of center. Therefore, the response receives a score of "2."

G-9

mean = 273
median = $250
mode = $245

The median shows where most of the jobs pay + and compared to the one with $420 a week, it isn't very much.

The labor union could use the mode because the average is $273 and the ones payed the same are the ones down at $245

Score Point: 2

This response shows partial understanding of the mathematical concepts involved in the task. The three measures of center (mean, median and mode) have been correctly computed and labeled. The two arguments seem to show support for the position of the labor union. The median is compared to the maximum salary of $420 and the mode of $245 is compared to the mean of $273 showing discrepancies in the salaries. With three correct measures of center and some understanding of one of the positions, this response receives a score of "2."

G-4

240
245
245
250
252
260
420

mean = $273.14
median = $250
mode = $245

The management could use the mean to represent the salaries in an argument against pay increases. They could say the average department store worker makes $255 a week and at their store the average is $273.14 a week.

Score Point: 3

This response shows understanding of the mathematical concepts involved in the task, but one element is missing. The three measures of center (mean, median and mode) have been computed and labeled correctly. The response contains an appropriate measure choice for management (mean) with appropriate support for the choice including a comparison with the average department store salary of $255. However, there is no indication of what the labor union should use to argue their position, thereby missing an important task. As a result, this response is incomplete and receives a score of "3."

Mean: $308
Median: $250
Mode: $245

The mean (average) could be used to argue against pay increases because the average pay is $308 (more than the dept. store average.)

The mode and median could be used to argue for pay increases because the middle pay is $250, less than average, and a common salary is $245 which is also less than average.

Score Point: 3

This response shows understanding of the task but includes an error in completing one of the parts. While the three measures of center (mean, median and mode) have been computed and labeled, a mistake was made in calculating the mean ($308 instead of $273.14). Appropriate measure choices were made against pay increases (mean) and for pay increases (mode and median) using the calculated values, with adequate support, including comparison with the department store average. The mistake in the calculation of the mean led this response to a score of "3."

Mean = $273.14
Median = $250
Mode = $245

The Mean could be used against pay increase Because the mean is fairly high at $273.14.

The Mode could be used in favor of pay increase. It is the smallest of the numbers at $245.

Score Point: 3

This response shows understanding of the mathematical concepts involved, and most tasks are performed appropriately. The three measures of center (mean, median and mode) have been computed and labeled correctly. Appropriate measure choices have been made against pay increases (mean) and in favor of pay increases (mode). However, while there is some understanding of the two positions, the support is weak (mean is fairly high at $273.14, smallest of the numbers at $245), and there is no comparison to the area average. As a result, this response receives a score of "3."

mean
$273.14

median
240, 245, 245, 250, 252, 260, 420
$250

mode
$245

management would use mean of $273.14. He/she could argue that the employees do not need a pay increase because the average salary is $255 and his/her workers are averaging $273.14, which is well over $255

The Labor union would use mode $245, because that is $10 under the average salary in the area of $255. He could argue that more than one of the workers was only making $245 and that could help for a pay increase.

Score Point: 4

This response is correct and complete, demonstrating a thorough understanding of the mathematical concepts and procedures embodied in the task. The three measures of center (mean, median and mode) for Acme employees have been computed and labeled correctly. Appropriate measures of center have been identified for both management and labor union (mean and mode respectively) and adequate support, including a comparison with the average salary in the area of $255, is provided to support their positions i.e.

management against pay increase and labor union for an increase. With all elements included appropriately, this response receives a score of “4.”

Mean: $273

Median: $250

Mode: $245

The center management could use the Mean to represent the salaries in an arguement against pay increases because it is the largest amount of money out of the three and it is higher than the average salary for all depart+ment workers. They could say these workers are getting more than the average worker, which makes no need to increase the pay.

The labor union could use the Mode to represent the salaries in an arguement for pay increases because it is the smallest amount of money out of the three and it is smaller than the average salarie of depar+ment store workers. They could say they aren't getting what the average worker gets and that it could be increased.

Score Point: 4

This response is clear and complete, demonstrating a thorough understanding of the underlying mathematical concepts and procedures. The three measures of center (mean, median and mode) have been computed and labeled correctly with the mean being rounded to 273 (instead of 273.14). Appropriate measures have been chosen for management (mean) and labor union (mode) to argue their respective positions. Adequate support is provided for these choices, including a comparison of the measures for Acme employees with the average salary for all department store workers.

G-3

$240
$245
$245
$250
$252
$260
$420

mean = $273.14
mode = 245
median = 250

Management could use the mean in an argument against pay increases. The mean is the average and Acme Department Stores mean is $18.14 more than the average.

Labor unions could use the mode and median to argue about pay raises. Both of them, are below the average salary.

Score Point: 4

This response is clear and complete, and all tasks have been accomplished appropriately. The three measures of center (mean, median and mode) have been computed and labeled correctly. Appropriate measures have been chosen for management (mean) and labor union (mode and median) to argue their respective positions. Adequate support is provided for each position including a comparison with the average salary for the area.

Mathematics

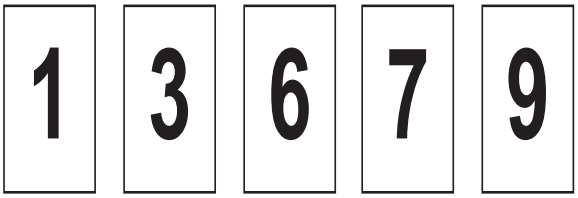
Item 42

Spring 2005

Item and Scoring Guidelines

Item

42. The following numbers are written on individual pieces of paper and placed in a bag.



Four numbers are randomly drawn from the bag and are placed in the order in which they were drawn to form a four-digit number.

In your **Answer Document**, determine the probability that a number created in this way has a value greater than 6,000. Show your work or provide an explanation to support your answer.

Sample Response for Item 42 (Short Answer):

$\frac{3}{5}$ or 60% The answer depends only on the first number drawn. If the first number drawn is 6, 7 or 9, then the four-digit number will be greater than 6,000. The probability of drawing any one of these numbers first is $\frac{3}{5}$ or 60%.

OR

The number of possible four-digit numbers from the five cards. $(5 \times 4 \times 3 \times 2 = 120)$
The number of possible four-digit numbers having a 6, 7 or 9 as its first digit.
$(3 \times 4 \times 3 \times 2 = 72)$
Probability of the number being over 6,000 is $\frac{72}{120}$ or $\frac{3}{5}$.

Scoring Guidelines for Item 42:

Score point	Description
2 points	The focus of this item is to find the probability of creating a four-digit number having a value greater than 6,000 when randomly drawing from the specified number cards. The response identifies the probability as $\frac{3}{5}$ or 0.6 or 60% or equal value and provides a clear explanation that the answer depends only on the first number drawn or accounts for all possibilities.
1 point	The response shows a partial understanding of the solution process or key elements of the task. The response may contain gaps or flaws in determining the solution. For example, the response may: Contain a correct fraction, decimal or percent for the probability but not state that only the first number matters or does not include an appropriate or complete explanation to support the answer. OR State that only the first number is important or provide appropriate explanation but miscalculate the probability or fail to include the probability. OR Determine the total possible outcomes (120) or the total possible numbers greater than 6,000 (72) with support; e.g., diagrams, lists, trees or mathematical processes.
0 points	The response fails to provide evidence of minimal understanding. For example the response may: Include an incorrect answer with flawed work and/or explanation. OR Recopy information provided in the question with no work toward a solution. OR Be blank or state “I do not know” or include unrelated statements or work.

Mathematics

Item 42

Spring 2005

Samples of Scored Student Responses

Probability 1 out of 5

Score Point: 0

This response fails to provide evidence of minimal understanding. The response includes only an incorrect answer with no further work or explanation.

1, 3, 6, 7, 9

$1 \times 3 \times 6 \times 7 \times 9 = 1134$

Even with the greatest amount of numbers, their product does not come out to be greater than 6,000. Therefore, there is a 0% probability that the value will be greater than 6,000.

Score Point: 0

This response fails to provide evidence of minimal understanding. The response multiplies the numbers on the cards, showing no apparent understanding of a possible solution process.

The probability that the number will be greatter than 6,000 is 100% because there are 5 numbers there and anyway you arrange them they will be greatter than 6000.

Score Point: 0

This response fails to provide evidence of minimal understanding. The response assumes that all five numbers are to be drawn, resulting in a five-digit number (greater than 6,000 is 100%) rather than the four-digit number specified in the item.

60% chance the number
will be greater than 6,000

Score Point: 1

This response shows a partial understanding of the solution process. The response contains a correct percent for the probability but does not include any explanation to support the answer.

If u pull out a 6, 7, 9 first then it will have a greater value than 6,000

Score Point: 1

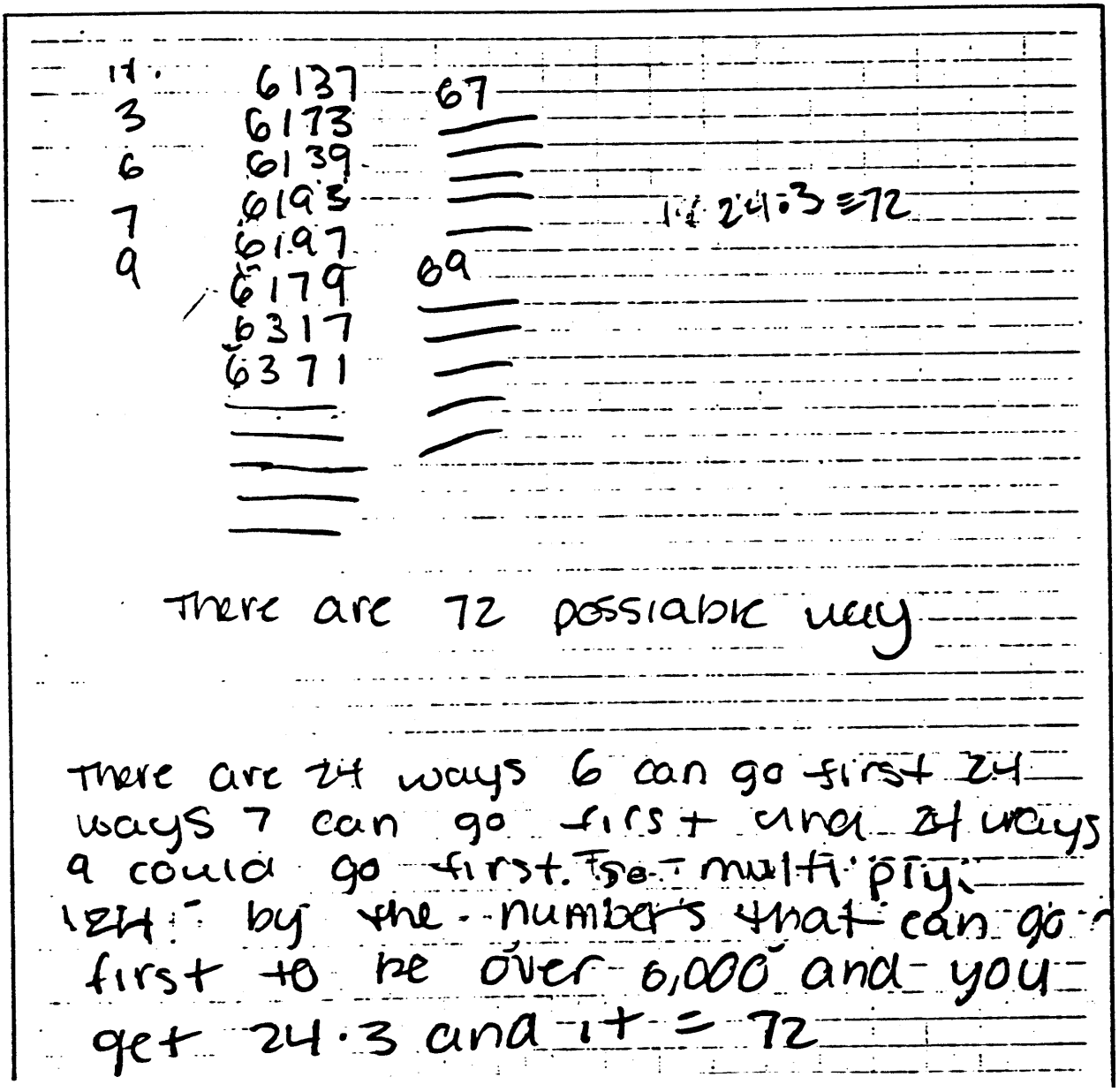
This response shows a partial understanding of the key elements of the task. The response states that the first number chosen determines whether the value will be over 6,000, but the probability is missing.

1	6137	67
3	6173	
6	6139	
7	6193	
9	6197	
	6179	69
	6317	
	6371	

$1 \cdot 24 \cdot 3 = 72$

There are 72 possiable way

There are 24 ways 6 can go first 24 ways 7 can go first and 24 ways 9 could go first. So multiply 124 by the numbers that can go first to be over 6,000 and you get 24·3 and it = 72



Score Point: 1

This response shows a partial understanding of the solution process. The response determines the total of possible numbers greater than 6,000. It notes 24 possibilities each for 6, 7 and 9 (the numbers that can go first to be over 6,000) and correctly indicates that the total is 72 possibilities. It does not complete the process of determining total possible outcomes (120) to find the probability of 0.6.

The probability that the number drawn will be over 6,000 is 60%.

If a '1' is drawn first, the # will be less than 6,000

If a '3' is drawn first, the # will be < 6,000

- however:

if a '6' is drawn, the # will be > 6,000

if a '7' is drawn, the # will be > 6,000

if a '9' is drawn, the # will be > 6,000

3/5 chance the # will be > 6,000

3 ÷ 5 = 60%

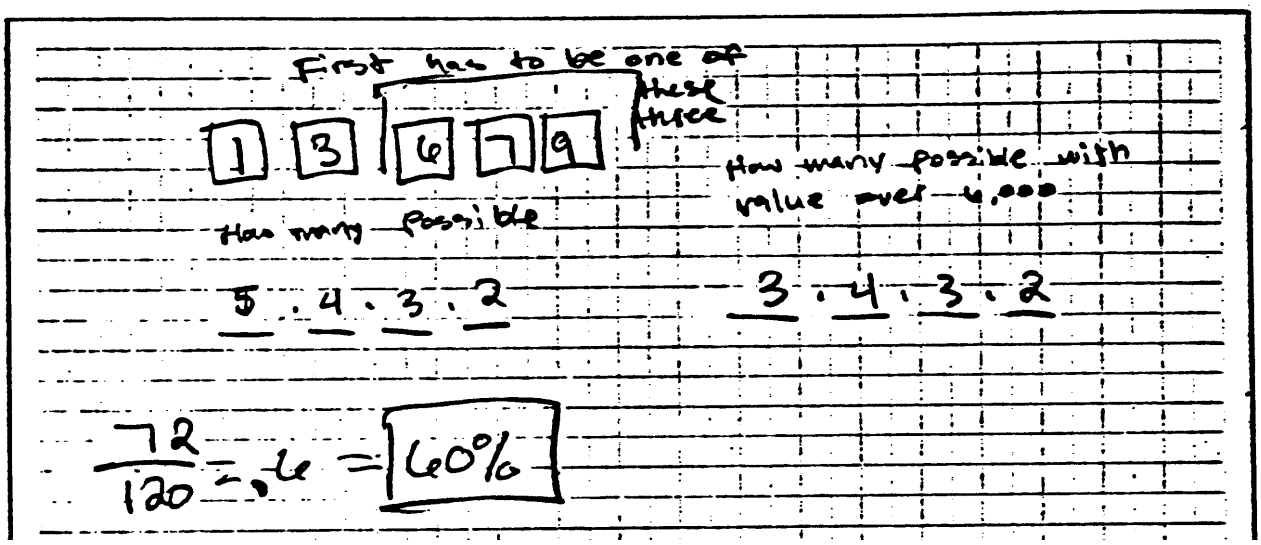
Score Point: 2

This response identifies the probability of drawing a number having a value greater than 6,000 as 60% and provides a clear explanation that the answer depends on the first number drawn.

The only way that the number can be greater than 6,000 is if 6, 7, or 9 is the first digit. If the first digit is 1 or 3, the number cannot be greater than 6,000. The chances of drawing a 6, 7, or 9 first is 3 in 5 therefore the chance of the first number being a 6, 7, or 9 in also 3 in 5. Therefore the chances of the four digit number being greater than 6,000 is 3 in 5.

Score Point: 2

This response provides a clear explanation that the answer depends on the first number drawn and identifies the probability of drawing a number having a value greater than 6,000 as 3 in 5.



Score Point: 2

This response indicates that in order to draw a number with a value over 6,000, the first number has to be 6, 7 or 9. The response then identifies the probability of drawing one of those numbers as 60% of the total possible combinations.