Ohio's Model Curriculum | Mathematics

with Instructional Supports

Grade 5

Ohio | Department of Education

Mathematics Model Curriculum
with Instructional Supports
Grade 5

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. In addition there are ideas for adapting instructions to meet the needs of all students. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision.

Standards for Mathematical Practice—Grade 5

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.
Students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. For example, Sonia had $2\frac{1}{3}$ candy bars. She promised her brother that she would give him $\frac{1}{2}$ of a candy bar. How much will she have left after she gives her brother the amount she promised? They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?".

MP.2 Reason abstractly and quantitatively.
Fifth graders should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts. For example, students use abstract and quantitative thinking to recognize that $0.5 \times (300 \div 15)$ is $\frac{1}{2}$ of $(300 \div 15)$ without calculating the quotient.

MP.3 Construct viable arguments and critique the reasoning of others.
In Grade 5, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

Students use various strategies to solve problems, and they defend and justify their work with others. For example, two afterschool clubs are having pizza parties. The teacher will order 3 pizzas for every 5 students in the math club and 5 pizzas for every 8 students in the student council. If a student is in both groups, decide which party he/she should to attend. How much pizza will each student get at each party? If a student wants to have the most pizza, which party should he/she attend?
Continued on next page

Standards of Mathematical Practice, continued

MP.4 Model with mathematics.
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

MP.5 Use appropriate tools strategically.
Fifth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real-world data.

MP.6 Attend to precision.
Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism, they record their answers in cubic units.

MP.7 Look for and make use of structure.
In Grade 5, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

MP.8 Look for and express regularity in repeated reasoning.
Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers. The also perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.

Mathematics Model Curriculum
with Instructional Supports
Grade 5

STANDARDS	MODEL CURRICULUM
OPERATIONS AND ALGEBRAIC THINKING **Write and interpret numerical expressions.** **5.OA.1** Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary. **5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.*	**Expectations for Learning** In previous grade levels, students used parentheses for clarification but were not expected to formally use the algebraic order of operations. In Grade 5, students explore the need for parentheses in numerical expressions. However, formal use of algebraic order of operations is still not necessary when evaluating expressions. The reason is to discourage teachers from teaching a mneumonic. Instead, students should be deepening their understanding of numbers and operations appropriate for the grade level. Therefore, in Grade 5, work with 5.OA.1 should be viewed as exploratory rather than for attaining mastery; students may use parenthesis, brackets, or braces, but they should not be using nested expressions. Also problems should be no more complex than the expressions one finds in an application of the associative or distributive property, e.g., (8 + 27) + 2 or (6 × 30) + (6 × 7). Also, in Grade 5 students learn to write simple expressions from a contextual situation. In addition, they create contextual situations from given numerical expressions without evaluating them. *Note: the numbers in expressions need not always be whole numbers.* Students in Grade 6 will use the conventions for order of operations to interpret as well as evaluate expressions. **ESSENTIAL UNDERSTANDINGS** • Calculations with parentheses are evaluated first within an expression. • Expressions can be written using words or symbols. • It is acceptable to change the order of an expression. For example, "add seven and six, then multiply by two" mathematically would get the same answer as (6 + 7) × 2 or 2 × (6 + 7).

Expectations for Learning, continued

MATHEMATICAL THINKING

- Make and analyze mathematical conjectures related to expressions.
- Attend to precision when recording mathematical expressions.
- Reflect on whether the results are reasonable.
- Use grade-level appropriate mathematical language and notation to explain reasoning.

INSTRUCTIONAL FOCUS

- Evaluate and interpret numerical expressions, including whole numbers, fractions, and decimals.
- Use conceptual understanding to interpret multiplicative comparisons without evaluating them.
- Explain the relationship between two number expressions without calculating the answers.
- Translate a numerical expression into words.
- Translate an expression written in words symbolically. For example, twice the sum of seven and six.
- Explore the use of parentheses to indicate what operation(s) would be performed first when multiple operations exist in an expression.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 5, Number 2, pages 30-31
- Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10

CONNECTIONS ACROSS STANDARDS

- Apply and extend previous understandings of multiplication and division to multiply and divide fractions (5.NF.5).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
OPERATIONS AND ALGEBRAIC THINKING **Analyze patterns and relationships.** **5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**Expectations for Learning** In Grade 4, students reasoned about number or shape patterns, and they generated terms that followed a given rule. In Grade 5, students work with two numerical patterns that are related. They examine these relationships using ordered pairs in the first quadrant of the coordinate plane. In later grades, this work prepares students for studying proportional relationships, functions, and graphing in all four quadrants. **ESSENTIAL UNDERSTANDINGS** • A relationship can exist between two numerical patterns generated from two given rules. • Ordered pairs generated from given rules can be graphed on a coordinate plane. **MATHEMATICAL THINKING** • Explore and generalize relationships based on patterns and structures. • Make and analyze mathematical conjectures related to patterns. • Use grade-level appropriate mathematical language and notation to explain reasoning. • Justify mathematical models used. • Reflect on whether the results are reasonable. **INSTRUCTIONAL FOCUS** • Generate two numerical patterns from two given rules. • Align the two number sequences generated from the given rules to form corresponding terms. • Generate ordered pairs using the corresponding terms of two given rules. o Graph the ordered pairs in the first quadrant of the coordinate plane. • Apply the orientation of the *x*- and *y*-axis in relation to the ordered pairs. • Informally compare the relationship of the *x*- and *y*-coordinates of two different rules when graphed on a coordinate plane. • Discuss and apply the relationship between the two results, when two rules are given. *Continued on next page*

	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 5, Number 4, page 34 • Ohio's K-8 Learning Progressions, Operations and Algebraic Thinking, pages 8-10 **CONNECTIONS ACROSS STANDARDS** • Graph points on the coordinate plane to solve real-world and mathematical problems (5.G.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
NUMBERS AND OPERATIONS IN BASE TEN **Understand the place value system.** **5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $^1/_{10}$ of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10. **5.NBT.3** Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form[G], e.g., 347.392 = $3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (^1/_{10}) + 9 \times (^1/_{100}) + 2 \times (^1/_{1000})$. **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NBT.4** Use place value understanding to round decimals to any place, millions through hundredths.	**Expectations for Learning** In Grade 4, students explored the concept that a digit in one place represents ten times what it represents in the place to its right. Also, they compared decimals to the hundredths and rounded whole numbers to a given place. In Grade 5, students extend their conceptual understanding of the base-ten system to the relationship that a digit in one place represents $\frac{1}{10}$ of what it represents in the place to its left. Decimals move from the domain Number and Operations—Fractions to the domain Number and Operations in Base Ten. Students extend base-ten relationships as they explain patterns in the number of zeros when multiplying by powers of 10 and in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. They also read and write decimals to thousandths using base-ten numerals, number names, and expanded form. In addition students compare two decimals to the thousandths place using the symbols >, =, and <. In addition, students round decimals to any given place value, millions through hundredths. In future grades, they will extend the base-ten system to include negative numbers and scientific notation. **ESSENTIAL UNDERSTANDINGS** • In the base-ten system, the value of each place is 10 times the value of the place to the immediate right and $\frac{1}{10}$ of the value to its immediate left. • There are patterns in the number of zeros when multiplying a number by a power of ten. • Each period of three digits separated by commas is read as hundreds, tens, and ones, followed (when appropriate) by the name of the period, e.g., 123,456 is read as one hundred twenty-three thousand, four hundred fifty-six. • In a decimal number, digits to the right of the decimal point are named by the appropriate unit: tenths, hundredths, thousandths. • In a decimal number, the digits to the right of the decimal point are read followed by the name of the appropriate unit. • When reading a decimal number, the decimal point is read as *and*. • Decimals to thousandths can be expressed in standard form, word form, and expanded form. • Two decimals to thousandths can be compared using the symbols >, =, and <. • Rounding helps solve problems mentally and assess the reasonableness of an answer.

Expectations for Learning, continued

MATHEMATICAL THINKING

- Pay attention to and make sense of quantities.
- Use grade-level appropriate mathematical language, models, and notation to explain and justify reasoning.
- Determine reasonableness of results.
- Explore and generalize concepts based on patterns and structures.

INSTRUCTIONAL FOCUS

Understanding Place Value

- Relate multiplication and division to place value.
- Explore using place value, multiplication, or division with whole numbers and/or decimal numbers:
  - A digit in the tens place represents a number that is ten times more than the number resulting from the same digit in the ones place.
  - A digit in the hundreds place represents a number that is ten times more than the number resulting from the same digit in the tens place.
  - A digit in the thousands place represents a number that is ten times more than the number resulting from the same digit in the hundreds place.
  - A digit in the tenths place is $^1/_{10}$ of the digit in the ones place.
  - A digit in the hundredths place is $^1/_{10}$ of the digit in the tenths place.
  - A digit in the thousandths place is $^1/_{10}$ of the digit in the hundredths place.
- Explore and explain why multiplying by a power of 10 changes the value of the number.
- Use whole number exponents to denote powers of 10.
- Represent, read, and write decimals to the thousandths in various forms (standard, word, expanded).
- Use patterns in the place value system to read and write numbers.
- Create numbers given specific criteria, e.g., Create a number that has 3 in the thousandths place, 5 in the hundredths place, 7 in the ones place, etc.

Continued on next page

Expectations for Learning, continued

INSTRUCTIONAL FOCUS, CONTINUED

Comparing Numbers

- Compare numbers based on place-value understanding—
  - with the same number of digits;
  - with the same leading digits;
  - with different leading digits and different number of digits; and
  - with the same whole number value and different decimal values.
- Connect the mathematical language to the use of symbols >, =, and < when describing the relationship between the numbers.
- Write two true inequality statements using symbols and words for a pair of decimals, e.g., 3.012 < 3.102 and 3.102 > 3.012.
- Compare the value of a numeral in a number to the same numeral in a different place in a different number, e.g. Given 3.42 and 4.32, compare the value of 3.

Rounding

- Round numbers based on place-value understanding.
- Explore rounding by using the location of a given number on a model, e.g., number line, number chart, etc.
- Round numbers based on place-value understanding.
- Explain reasoning when rounding.
- Develop and generalize rounding rules for decimals.
- Identify or create numbers that will round to a chosen number, e.g., Create a number that will round to 1.05.
- Explore the purposes of rounding.

Continued on next page

	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 5, Number 2, pages 30-32 • Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5 • Ohio's K-8 Learning Progressions, The Number System, pages 16-17 • Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19 **CONNECTIONS ACROSS STANDARDS** • Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings (5.NBT.7).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
NUMBER AND OPERATIONS IN BASE TEN **Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.5** Fluently[G] multiply multi-digit whole numbers using a standard algorithm[G]. **5.NBT.6** Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.7** Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. **a.** Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place). *Continued on next page*	**Expectations for Learning** In Grade 4, students explored the concepts of division with one-digit divisors; division with remainders; and multiplication of whole numbers. Previously, they also understood decimals in relation to fractions with denominators of 10 and 100. In Grade 5, students fluently multiply multi-digit whole numbers using a standard algorithm. They explore the conceptual understanding of division with remainders to include up to two-digit divisors and up to four digit dividends by applying equations, area models, and arrays to illustrate and explain strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Decimals move from the domain Number and Operations—Fractions to the domain Number and Operations in Base Ten. Now students extend addition and subtraction (whole numbers to the hundreds place and decimals to the hundredths place). Also, they multiply whole numbers by decimals (whole numbers to the hundreds place and decimals to the hundredths place). Students divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place). In Grade 6, students will demonstrate fluency with division of multi-digit numbers and fluency in all four operations with multi-digit decimals. *(Fluency is the ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests)*. **ESSENTIAL UNDERSTANDINGS** **Multiplication and Division** • There are different algorithms that can be used to multiply. • Fluency is being efficient, accurate, and flexible with strategies. • There is a relationship between multiplication and division. • Equations, rectangular arrays, and/or area models can be used to illustrate and explain division. • Remainders can be interpreted symbolically and in context. • Real-world mathematical situations can be represented using concrete models or drawings. • Patterns and structures can be generalized when multiplying and dividing whole numbers.

b. Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). **c.** Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). *For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.*	**Expectations for Learning, continued** **ESSENTIAL UNDERSTANDINGS, CONTINUED** **Operations with Decimals to Hundredths** • Patterns and structures can be generalized when multiplying and dividing decimals. • There is a relationship between addition and subtraction. • Real-world mathematical situations can be represented using concrete models or drawings when adding and subtracting decimals (including decimals with whole numbers through hundreds place and decimals through hundredths place). • Real-world mathematical situations can be represented using concrete models or drawings when multiplying whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). • Real-world mathematical situations can be represented using concrete models or drawings when dividing whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). **MATHEMATICAL THINKING** • Justify mathematical models used. • Pay attention to and make sense of quantities. • Reflect on whether the results are reasonable. • Use grade-level appropriate mathematical language to illustrate and explain reasoning. • Compute accurately, efficiently, and flexibly with grade-level numbers. • Explore and generalize concepts based on patterns and structures. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Note: Conversions within the metric system are addressed in 5.NBT.7 when students solve real-world problems using decimals. This is an application and extension of 4.MD.1.

Multiplication

- Estimate the solution of a multiplication situation.
- Connect a standard algorithm to an efficient strategy.
- Explain and justify the reasoning used in a standard algorithm.
- Analyze other students' use of a standard algorithm, and explain any errors.
- Use an efficient standard algorithm accurately and flexibly.

Division

- Explore number relationships and look for patterns.
- Divide finding whole number quotients with up to four-digit dividends and two-digit divisors.
- Explore division problems that result in remainders.
- Determine whether the remainder is left alone, is discarded, or forces the quotient to increase.
- Explore and explain how zeroes affect division: in the dividends, within the process of dividing, and in the quotient.
- Illustrate and explain the relationship between multiplication and division.
- Solve division problems using strategies that may include the following: decomposing factors; using the relationship between multiplication and division; creating equivalent but easier or known products; and properties of operations, etc.
- Apply the conceptual understanding of properties to division.
- Estimate the solution of a division problem.
- Use visual representations such as area models and arrays to draw connections to equations.
- Solve real-world problem types: equal groups, arrays/area, and compare. See Table 2, page 96.
- Determine reasonableness of a solution and compare to initial estimation with multiplication and division.

Expectations for Learning, continued

INSTRUCTION FOCUS, CONTINUED

Operations with Decimals to Hundredths

- Estimate solutions when solving problems with decimals before computing.
- Explore and explain mathematical operations in context of real-world problems.
- Illustrate and explain calculations with decimals through the use of concrete models, drawings, or strategies based on place value.
- Pay attention to and make sense of quantities when using decimals in real-world problems.
- Solve problems using the different problem types. See Table 1, page 95 and Table 2, page 96.
- Use concrete models or drawings to relate strategies to a written method:
  - Add and subtract decimals.
  - Connect the addition and subtraction of decimals to fractions.
  - Multiply whole numbers by decimals.
  - Compare a decimal product problem to the same problem without decimals, e.g., 24.8 × 3.5 to 248 × 35.
  - Compare a decimal product problem to the same problems with the decimal located in different positions, e.g., 24.8 × 3.5 to 2.48 × 0.35.
  - Divide whole numbers by decimals and decimals by whole numbers using numbers whose division can be readily modeled, e.g., 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.
- Determine reasonableness of a solution and compare to initial estimation with decimals in all four operations.

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	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 5, Number 2, pages 30-31 • Ohio's K-8 Learning Progressions, Number and Operations in Base Ten, pages 4-5 • Ohio's K-8 Learning Progressions, The Number System, pages 16-17 **CONNECTIONS ACROSS STANDARDS** • Understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left (5.NBT.2). • Apply and extend previous understandings of multiplication and division to multiply and divide fractions (5.NF.1-7).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
NUMBER AND OPERATIONS—FRACTIONS **Use equivalent fractions as a strategy to add and subtract fractions. (Fractions need not be simplified).** **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show* $^{2}/_{3} + {}^{5}/_{4} = {}^{8}/_{12} + {}^{15}/_{12} = {}^{23}/_{12}$. *In general,* $^{a}/_{b} + {}^{c}/_{d} = ({}^{a}/_{b} \times {}^{d}/_{d}) + ({}^{c}/_{d} \times {}^{b}/_{b}) = {}^{(ad + bc)}/_{bd}$. **5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models[G] or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $^{2}/_{5} + {}^{1}/_{2} = {}^{3}/_{7}$, *by observing that* $^{3}/_{7} < {}^{1}/_{2}$.	**Expectations for Learning** In Grade 3, students explored the meaning and relationships in fractions; the significance of the whole; the unit fraction; and the initial understanding of equivalence of fractions using models. These explorations used denominators of 2, 3, 4, 6, and 8. The models used were primarily rectangular area models and arrays and length models including number lines and fraction strips. In Grade 4, students used models to compare fractions with different numerators and different denominators; recorded the results using > , =, or <; and used the additional denominators of 5, 10, 12, and 100. The models used were primarily rectangular area models and length models including number lines. Students utilized the renaming of fractions to solve real-world word problems involving addition and subtraction of like denominators. Fractional understanding and operations were extended to tenths and hundredths using both fraction and decimal symbols. Students have developed an understanding of the relationship and equivalence of fractions and decimals expressed as tenths and hundredths. When working with money and in the metric measurement system, the foundation and application of the place value system for decimals is developed. In Grade 5, students add and subtract fractions with unlike denominators. Although there are no limitations on denominators, the focus should be on using denominators with which students can relate, visualize, and model in order to develop a conceptual understanding of adding and subtracting fractions with unlike denominators. Solutions are permitted to be expressed un-simplified. It is important to reinforce using models to find solutions while pairing the representations with equations. Students can use benchmark fractions and/or number sense to estimate a fraction problem. Also, they assess the reasonableness of their solutions. In future grades, students will continue to explore the importance of the unit and to solidify fractional understanding needed in solving algebraic equations. *Continued on next page*

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS

- Fractions can be added and subtracted when the wholes are the same size and the fractional parts (denominators) are the same.
- Fractions with different denominators are called *unlike fractions*.
- Fractions with different denominators can be added and subtracted by replacing each fraction with an equivalent fraction expressed with a like denominator.
- An equation can be used to describe a mathematical situation involving fractions.
- There is usually more than one way to describe and solve a mathematical situation involving fractions.
- A fraction with a numerator larger than the denominator can be expressed as a mixed number or a fraction greater than one; both are correct representations.
- Expressing a mixed number as a fraction, e.g., $2\frac{3}{5} = \frac{13}{5}$, may be useful when solving a fraction problem.
- Benchmark fractions may be used to estimate and to check whether answers are reasonable.
- Common denominators are needed to add and subtract fractions with unlike denominators.
- Multiples may be used to find common denominators.

MATHEMATICAL THINKING

- Use mathematical models to solve problems.
- Explore and generalize concepts based on patterns and structures.
- Use grade-level appropriate mathematical language and notation to illustrate and explain reasoning.
- Compute accurately, efficiently, and flexibly with grade-level numbers.
- Reflect on whether results are reasonable.

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Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Add and subtract fractions and mixed numbers with like and unlike denominators using models.
- Discuss and explore the use of models (e.g., rectangular area models, fraction strips, number lines, clock models, etc.) to find an appropriate model to represent both fractions when the denominators are difficult to represent.
- Represent two fractions with unlike denominators using the same model.
- Express two fractions with unlike denominators as fractions with like denominators using visual models and renaming strategies; then write an equation using each fraction.
- Decompose each of two fractions into a sum of fractions with the same denominator, e.g., To solve $\frac{1}{2}+\frac{3}{4}$ a student may think $\frac{1}{2}=\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$, so $\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$.
- Add and subtract combinations of fractions whose denominators are multiples: (2, 4, 6, 8, 10, 12 or 3, 6, 12 or 5, 10, and 100) by using models and applying renaming fractions strategies.
- Explain and justify thinking when adding and subtracting combinations of fractions with—
  - unlike fractions where only *one* of the fractions needs to be changed, e.g., $\frac{7}{8}-\frac{1}{2}$; $\frac{3}{4}+\frac{1}{8}$; $1\frac{1}{2}-\frac{3}{4}$. Rewrite as an equivalent mathematical problem in order to add or subtract equal sized parts, and
  - unlike fractions where *both* fractions need to be changed, e.g., $\frac{2}{3}-\frac{1}{2}$, $1\frac{2}{3}-\frac{3}{4}$. Find the best model; rewrite as an equivalent mathematical problem; then add or subtract the equal sized parts.
- Using models and equations, add and subtract to solve word problems with two or more fractions with like and unlike denominators For example, $\frac{45}{100}+\frac{6}{10}$; $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$; $2-\frac{1}{10}$; $\frac{3}{4}+1\frac{1}{2}+\frac{3}{8}$ etc.

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Expectations for Learning, continued

INSTRUCTIONAL FOCUS, CONTINUED

- Solve multiple groups problems involving groups of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{12}$, which include mixed numbers, e.g., Three children are having breakfast. Each child is to get $1\frac{1}{2}$ waffles. How many waffles are needed? $1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = 3 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4 + \frac{1}{2} = 4\frac{1}{2}$ waffles.
- Represent real-world problems with visual models and with equations; justify the solutions using the relationship between addition and subtraction and properties of operations.
- Explore and explain estimates of fraction problems using number sense or benchmark fractions.
- Assess solutions to determine if the solutions are reasonable.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 5, Number 1, pages 28-29
- Ohio's K-8 Learning Progressions, Number and Operations—Fractions, pages 6-7
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17

CONNECTIONS ACROSS STANDARDS

- Write and interpret numerical expressions (5.OA.1-2).
- Generate a pattern given a rule (5.OA.3).
- Add and subtract decimals (5.NBT.7).
- Display and interpret data in graphs (5.MD.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
NUMBER AND OPERATIONS—FRACTIONS **Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified).** **5.NF.3** Interpret a fraction as division of the numerator by the denominator ($^a/_b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $^3/_4$ as the result of dividing 3 by 4, noting that $^3/_4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $^3/_4$. If 9 people want to share a 50 pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* *Continued on next page*	**Expectations for Learning** In previous grades, students developed the meaning of fractions. They developed the understanding of equivalent fractions and used this learning to rewrite expressions to solve addition and subtraction problems of fractions with like denominators. They compared fractions by first comparing fractions with like numerators or like denominators. Then in Grade 4, they compared fractions with different numerators and different denominators by creating equivalent fractions or by using benchmark fractions. Students developed an understanding of the need for equal sized parts and used various strategies for finding them without memorizing common denominator methods. Fraction work began with representing unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{100}$) on rectangular area models and length models. Then the work expanded to show iteration of unit fractions (repeated addition equivalent to the multiplication of a unit fraction by a whole number, e.g., $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 3 \times \frac{1}{8}$). Students learned to use fraction symbols and compared fractions using >, =, or <. In Grade 5, students expand their interpretation of fractions to that of representing division. Students apply the understanding of the relationship of multiplication and division, and they also apply the effect of multiplication to fractions. In addition, they multiply whole numbers by fractions and multiply fractions by fractions. Also, they divide whole numbers by fractions and divide fractions by whole numbers. Although students reason about the solution of multiplication and division of whole numbers and fractions, there is no expectation that students divide fractions by fractions at this grade. Students continue to use models paired with expressions and equations to represent problem situations. They find the area of a rectangle with fractional side lengths both by tiling and by multiplying side lengths. A major focus of this cluster is developing the understanding of how real-world situations are represented and how solutions are found when fractions are involved. This learning sets the foundation for future work with ratios and proportions (in the Ratios and Proportional Relationships domain) and operations with rational numbers (in the Number System domain). *Continued on next page*

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(^a/_b) \times q$ as a parts of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $(^2/_3) \times 4 = {}^8/_3$*, and create a story context for this equation. Do the same with* $(^2/_3) \times (^4/_5) = {}^8/_{15}$*. (In general,* $(^a/_b) \times (^c/_d) = {}^{ac}/_{bd}$*.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5 Interpret multiplication as scaling (resizing).

a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Continued on next page

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS

Interpreting Fractions

- The denominator describes what number of equal parts a whole has been divided into.
- The numerator describes how many of the parts are considered.
- The numerator is a multiplier, e.g., $\frac{4}{5} = 4 \times \frac{1}{5}$.
- A fraction represents division, so $a \div b = \frac{a}{b}$, e.g., $3 \div 4 = \frac{3}{4}$.
  - The denominator is the *divisor*.
  - The numerator is the *dividend*.
- Equal shares means each sharer gets the same sized part and no parts are discarded.
- The solution to an equal sharing problem can be shown with a fraction representing the relationship of the sharers and the amount.
- When adding or subtracting unlike fractions, all fractions must be represented with equal sized parts of the same whole.

Continued on next page

b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $^{a}/_{b} = {}^{(n\times a)}/_{(n\times b)}$ to the effect of multiplying $^{a}/_{b}$ by 1.

5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade.

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Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Multiplication of Fractions

- The idea of the numerator as a multiplier can be used when a fraction is being multiplied by a whole number, e.g., Just as $\frac{5}{8} = 5\times\frac{1}{8}$, 5 groups of $\frac{3}{8}$ equals $5\times\frac{3}{8} = (5\times3)\times\frac{1}{8}$ which equals $\frac{15}{8}$.
- Arrays, number lines, fraction strips, or sets can be used to find the solution to multiplying a whole number by a fraction.
- The product of a fraction $\left(\frac{a}{b}\right)$ and a whole number (q) shown as $\left(\frac{a}{b}\times q\right)$ can be found by partitioning the whole number (q) into equal sized parts (b) with the result being $a\times q$ parts of size $\frac{1}{b}$, i.e., $\frac{(a\times q)}{b}$.
- The product of two fractions $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is found by multiplying the numerators (a and c) and then multiplying the denominators (b and d) which is then shown as $\frac{(a\times c)}{(b\times d)}$.
- Multiplying any number by a value of one maintains the original relationship.
- The relationship between multiplication and division is applied to fractions just as it is applied to whole numbers.
- The area of a rectangle with fractional side lengths can be computed.
- Multiplication can be used to solve division problems involving fractions.
- When a number is multiplied by a number greater than one, the product will be greater than the original number, e.g., $3\times\frac{5}{4}$ will be greater than 3.
- When a number is multiplied by a fraction less than one the product is smaller than the original number, e.g., $5\times\frac{3}{4}$ will be less than 5).
- When two fractions less than one are multiplied, the product is smaller than both of the original fractions.

Division of Fractions

- A whole number can be divided by a non-zero fraction.
- A fraction can be divided by a non-zero whole number.

Continued on next page

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = (1/12)$ because $(1/12) \times 4 = (1/3)$.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.*

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $1/2$ pound of chocolate equally? How many $1/3$ cup servings are in 2 cups of raisins?*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use mathematical models to solve problems.
- Explore and generalize concepts based on patterns and structures.
- Use grade-level appropriate mathematical language and notation to illustrate and explain reasoning.
- Compute accurately, efficiently, and flexibly with grade-level numbers.
- Reflect on whether results are reasonable.

INSTRUCTIONAL FOCUS

Division Problems Represented as Fractions

- Represent fractions as division problems and vice versa.
- Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.
    - Create, explain, and solve real-world word problems involving equal shares or multiple groups using models and equations.
        - When the answer is a mixed number, explain what two whole numbers the answer lies between.
    - Solve equal sharing problems where the amount shared is less than the number of sharers by writing the fraction, e.g., When three pizzas are shared with 8 students, each student gets $\frac{3}{8}$ of a pizza.
    - Solve equal sharing problems involving comparisons where the amount shared is less than the number of sharers, e.g., Who gets more? A student in a group of 6 sharing 4 brownies or a student in a group of 5 sharing 3 brownies?

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Expectations for Learning, continued

INSTRUCTIONAL FOCUS, CONTINUED

Multiplication of Fractions

- Explore and explain that the product of a fraction $\left(\frac{a}{b}\right)$ and a whole number (q) shown as $\left(\frac{a}{b} \times q\right)$ can be found by partitioning the whole number (q) into equal sized parts (b) with the result being $a \times q$ parts of size $\frac{1}{b}$, i.e., $\frac{(a \times q)}{b}$.
- Explore and explain that the product of two fractions $\left(\frac{a}{b} \times \frac{c}{d}\right)$ is found by multiplying the numerators (a and c) and then multiplying the denominators (b and d) which is then shown as $\frac{(a \times c)}{(b \times d)}$.
- Model and find the area of a rectangular region with sides of fractional lengths by tiling.
- Scaffold area of a rectangular region with sides of fractional lengths from concrete (tiling) to symbolic representation (equation).
- Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- Explore and explain the value of the solutions when multiplying the following:
  - a given number by a fraction greater than one; and
  - a given number by a fraction less than one.
- Relate the principle of fraction equivalence $\frac{a}{b} = \frac{(n \times a)}{(n \times b)}$ to the effect of multiplying $\frac{a}{b}$ by 1.
- Represent and create real-world problems with visual models and a corresponding equation, justifying the solution:
  - fractions by whole numbers;
  - fractions by unit fractions;
  - two fractions; and
  - fractions and mixed numbers.

Continued on next page

Expectations for Learning, continued

INSTRUCTIONAL FOCUS, CONTINUED

Division of Fractions

- Use the understanding of the relationship between whole number multiplication and division to reason about solving problems involving the division of a whole number by a unit fraction.
- Interpret division of a whole number by a unit fraction to solve real-world problems. Model, explain, and justify results, e.g., A cookie recipe needs $\frac{1}{2}$ cup of sugar. How many recipes (batches) can be made with 4 cups of sugar?
- Interpret the division of a unit fraction by a whole number to solve real-world problems. Using visual models, explain, and justify results, e.g., A $\frac{1}{2}$ of a sheet of pizza is left over, and 8 students want to share it for lunch the next day. How much of the pizza will each student get?

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 5, Number 1, pages 28-29
- Ohio's K-8 Critical Areas of Focus, Grade 5, Number 3, pages 32-33
- Ohio's K-8 Learning Progressions, Number and Operations—Fractions, pages 6-7
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17

CONNECTIONS ACROSS STANDARDS

- Write and interpret numerical expressions (5.OA.1-2).
- Generate a pattern given a rule (5.OA.3).
- Multiply and divide decimals (5.NBT.7).
- Represent and interpret data (5.MD.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
MEASUREMENT AND DATA **Convert like measurement units within a given measurement system.** **5.MD.1** Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.	**Expectations for Learning** In grades 2 and 3, students measured and estimated in both the metric system and the U.S. customary system. In Grade 4, students used metric units and expressed larger measurement units in terms of smaller units. In Grade 5, students use their understanding of relative sizes of units (pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds) to convert between U.S. customary system (pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds). They solve multi-step, real-world problems using U.S. customary measures. (*Note: Conversions within the metric system are addressed in 5.NBT.7 when students solve real-world problems using decimals. This is an application and extension of 4.MD.1.*) In Grade 6, students will use proportional reasoning and applications of measurement in other contexts. **ESSENTIAL UNDERSTANDINGS** • Two measurement systems (U.S. customary and metric) are currently used in the United States. • Relationships between units vary depending on the measurement system. • Conversions in the U.S. customary system vary depending upon what is being measured. • Conversions in the metric system are based on powers of ten. • When converting from a larger unit to a smaller unit, there will be more iterations of the smaller unit. For example, when converting from yards to feet, there will always be a greater number of feet than yards. • When converting from a smaller unit to a larger unit, there will be less iterations of the larger unit. For example, when converting from cups to gallons, there will always be fewer gallons than cups • Measurements can be converted to solve multi-step real-world problems. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Recognize a pattern or structure when using different types of measurement.
- Measure using appropriate tools and units.
- Create models, tables, and drawings to represent measurements.
- Use grade-level appropriate mathematical language and notation to explain reasoning.
- Make and test conjectures about conversions; then justify reasoning.
- Solve real-world problems accurately, and consider the reasonableness of the solution(s).

INSTRUCTIONAL FOCUS

- Explore the U.S. customary system using appropriate tools (rulers, yardsticks, scales, measuring containers, clocks, etc.)
- Explain relative sizes of these U.S. customary units:
  - weight—pounds, ounces;
  - length—miles, yards, feet, inches;
  - capacity—gallons, quarts, pints, cups, fluid ounces; and
  - time—hours, minutes, seconds.
- Explore, record, and look for a pattern when doing conversions in a two-column table.
- Convert between units using these conversions:
  - 1 pound = 16 ounces,
  - 1 mile = 5,280 feet,
  - 1 yard = 3 feet; 1 foot = 12 inches; 1 yard = 36 inches,
  - 1 gallon = 4 quarts or 8 pints or 16 cups or 128 fluid ounces,
  - 1 quart = 2 pints or 4 cups or 32 fluid ounces,
  - 1 pint = 2 cups or 16 fluid ounces,
  - 1 cup = 8 fluid ounces, and
  - 1 hour = 60 minutes; 1 minute = 60 seconds; 1 hour = 3,600 seconds.
- Solve multi-step, real-world problems involving conversions using all four operations. *Note: See the Ohio State Test Grade 5 Reference Sheet for conversions that will be given.*

Continued on next page

	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 5, Number 1, pages 28 – 29 • Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14 **CONNECTIONS ACROSS STANDARDS** • Add, subtract, multiply, and divide decimals to hundredths (5.NBT.7). • Perform operations with fractions (5.NF.1-7). • Generate numerical patterns given rules (5.OA.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
MEASUREMENT AND DATA **Represent and interpret data.** **5.MD.2** Display and interpret data in graphs (picture graphs, bar graphs, and line plots[G]) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions $^1/_2$, $^1/_4$, $^1/_8$, or decimals.	**Expectations for Learning** In Grade 4, students displayed and interpreted data in picture graphs, bar graphs, and line plots and solved grade-level appropriate problems. In Grade 5, students display and interpret data in graphs and solve problems using numbers and operations for this grade, which include the use of fractions $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right)$ and decimals. *Note: Students may use their knowledge of metric conversions from fourth grade.* In Grade 6, students will use proportional reasoning and applications of measurement within other areas, such as measurement within Statistics and Probability. **ESSENTIAL UNDERSTANDINGS** • Picture graphs, bar graphs, and line plots are used to display data. • The key of a picture graph tells how many items each picture or symbol represents. • The scale of a bar graph varies depending on the data set. • The scale of a line plot can be whole numbers, halves, quarters, eighths, sixteenths, tenths, or hundredths. • Symbols used in picture graphs and line plots should be consistently spaced and sized. • Information presented in a graph can be used to solve problems using metric or U.S. customary measurements. **MATHEMATICAL THINKING** • Interpret word problems to determine the operation(s) to be used. • Use a graph to organize, represent, and solve real-world mathematical situations accurately. • Measure using appropriate tools and units; justify mathematical models used. • Attend to precision when graphing fractional quantities. • Reflect on whether the results are reasonable. • Use grade-level appropriate mathematical language and notation to explain reasoning. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Picture Graph

- Display and interpret data using real-world problems with grade-level appropriate units for data sets:
  - Use units of halves and quarters in situations where these fractions are appropriate.
  - The amount of items represented by a fractional picture or symbol is determined by the key and the fraction ($\frac{1}{2}$ or $\frac{1}{4}$) of the picture or symbol that is present.

Bar Graph

- Display and interpret data using real-world problems with grade-level appropriate units for data sets:
  - Use whole number units for a large variety of data sets.
  - Use units of halves, quarters, and eighths in situations where these fractions are appropriate.
  - Use units of currency in situations when appropriate.
  - Use decimal units in situations involving metric measurements.

Line Plots

- Display and interpret data using real-world problems with grade-level appropriate units for data sets:
  - Use whole number units for a large variety of data sets.
  - Use units of halves, quarters, eighths, and sixteenths in situations where these fractions are appropriate.
  - Use units of currency in situations when appropriate.
  - Use decimal units in situations involving metric measurements.

Circle Graphs

- Explore circle graphs in connection with social studies standards.

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	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 5, Number 1, pages 28 – 29 • Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14 • Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23 **CONNECTIONS ACROSS STANDARDS** • Apply fraction operations and ordering (5.NF.1-7). • Solve real-world problems with decimal operations (5.NBT.7).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
MEASUREMENT AND DATA **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole number products as volumes, e.g., to represent the Associative Property of Multiplication. *Continued on next page*	**Expectations for Learning** In third and fourth grades, students developed the concepts of area as the idea of covering and liquid volume as the idea of filling. In Grade 5, students extend the concept of area as covering to include covering the base of a three-dimensional object; they find volume by using that information along with the height (how many layers of the base) of the object. They count unit cubes, using cubic cm, cubic in, cubic ft, and improvised units to find volume. Also, they relate volume to the operations of multiplication (base layer $B = \ell \times w$) and addition (how many layers high) to solve real-world and mathematical problems involving volume. Students apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ to rectangular prisms using real-world and mathematical problems involving whole numbers. Then students find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems. In Grade 6, students will extend this whole number understanding of volume to find the volume of rectangular prisms with fractional side lengths. **ESSENTIAL UNDERSTANDINGS** • Volume is an attribute of a three-dimensional solid figure that is measured in cubic units. • Volume can be measured (or determined) by finding the total number of cubic units required to fill the space without gaps or overlaps. • The process of finding volume shifts from building with cubes and counting to the multiplication of side lengths. • The area of a *Base* of a rectangular prism is found by multiplying the length by width ($B = \ell \times w$). • In a right rectangular prism, any two parallel faces can be the Bases. • The volume of a rectangular prism can be found by multiplying the length by width by height ($\ell \times w \times h$) or by multiplying the area of the Base by height ($B \times h$). • A figure composed of rectangular prisms may be decomposed into two non-overlapping rectangular prisms whose volumes may be added to find the volume of the figure. *Continued on next page*

b. Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Expectations for Learning, continued

MATHEMATICAL THINKING

- Select appropriate units to estimate and measure volume.
- Use spatial reasoning.
- Measure using appropriate tools and units; justify mathematical models used.
- Create models to represent volume.
- Recognize and use structure.
- Make and test conjectures about volume; then justify reasoning.
- Use grade-level appropriate mathematical language and notation to explain reasoning.
- Solve real-world problems accurately, and consider the reasonableness of the solution(s).

INSTRUCTIONAL FOCUS

- Explore and develop the conceptual understanding of "a unit cube" with volume as "one cubic unit."
- Recognize volume as an attribute of a three-dimensional object.
- Use packing of unit cubes (without gaps or overlaps) to find the volume of a rectangular prism by counting the unit cubes.
- Use appropriate units (cubic cm, cubic in, cubic ft, and improvised units).
- Explore and explain finding the volume of a rectangular prism with whole number side lengths by packing with unit cubes to find that the volume is the same as would be by multiplying the side lengths.
- Decompose a prism built from cubes into layers.
- Develop a connection between building layers from the base to applying formulas for finding volume.
- Explore and explain the volume of a figure composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts.

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	Content Elaborations • Ohio's K-8 Critical Areas of Focus, Grade 5, Number 3, pages 32 – 33 • Ohio's K-8 Learning Progressions, Measurement and Data, pages 12-14 • Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21 **CONNECTIONS ACROSS STANDARDS** *There are no direct connections to these standards within Grade 5.The ideas developed in these standards will be used in later grades.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
GEOMETRY **Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate. **5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**Expectations for Learning** In Grade 4, students drew and identified perpendicular lines. In Grade 5, students develop understanding of the coordinate plane (limited to the first quadrant) as a tool to model numerical relationships. Students apply their understanding of distance and direction to an ordered pair's horizontal and vertical position on the coordinate plane. Students name these *axes* and *coordinates* as the *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate. Students represent real-world and mathematical situations by graphing points and interpreting coordinate values of points in context. In Grade 6, students will work with negative numbers, ratios, and proportional relationships. The student understanding of this cluster aligns with van Hiele Level 0 (Visualization). **ESSENTIAL UNDERSTANDINGS** • Coordinate graphs show relationships between numbers on a coordinate grid. • The coordinate system is created from a horizontal number line (*x*-axis) and a vertical number line (*y*-axis) with the intersection of the lines at zero (the *origin*). • A given point can be located in the plane by using an ordered pair of numbers (*x*, *y*). • The *origin* of the coordinate plane is represented by the ordered pair (0, 0). • The first number in an ordered pair, the *x-coordinate* or *x*, indicates how far to travel from the origin in the horizontal direction. • The second number in an ordered pair, the *y-coordinate* or *y*, indicates how far to travel in the vertical direction. • Distance is found by counting intervals rather than counting the grid marks. • Real-world situations can be represented by graphing points in the coordinate plane. • Coordinate values can be interpreted in the context of real-world situations. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Pay attention to and make sense of quantities.
- Use spatial reasoning.
- Create visual representations of ordered pairs on a coordinate system with precision.
- Recognize and use structure of a coordinate system.
- Represent and interpret real-world and mathematical situations.
- Use grade-level appropriate mathematical language to explain reasoning.

INSTRUCTIONAL FOCUS

- Identify the horizontal number line as the *x*-axis.
- Identify the vertical number line as the *y*-axis.
- Identify the intersection of the number lines as the origin (0, 0).
- Identify *x*- and *y*-coordinates within an ordered pair (limited to whole numbers).
- Identify ordered pairs when given points in the first quadrant.
- Graph points in the first quadrant when given ordered pairs.
- Represent real-world and mathematical problems by graphing points in the first quadrant.
- Explore and explain paths (horizontally and vertically) between two sets of ordered pairs on a coordinate plane.
- Interpret coordinate values of points within the context of a situation.
- Represent geometric shapes on the coordinate grid, e.g., Given three points, plot the fourth point to create a rectangle).

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 5, Number 4, page 34
- Ohio's K-8 Learning Progressions, K-5 Geometry, page 11
- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21
- Ohio's K-8 Learning Progressions, The Number System, pages 16-17
- Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15

CONNECTIONS ACROSS STANDARDS

- Use patterns to create ordered pairs, and graph them in the first quadrant of a coordinate plane (5.OA.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

STANDARDS	MODEL CURRICULUM
GEOMETRY **Classify two-dimensional figures into categories based on their properties.** **5.G.3** Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles). **5.G.4** Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids[G], and rhombuses.	**Expectations for Learning** In Grade 4, students measured angles in whole-number degrees using a protractor and sketched angles of specified measure. They also used types of angle measures, side lengths, and parallel and perpendicular lines to identify, draw, and classify polygons. In Grade 5, students explore the commonalities and differences of triangles and quadrilaterals. They classify triangles by angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles). Also, students classify quadrilaterals (squares, rectangles, parallelograms, trapezoids, and rhombuses) by angle measures, side lengths, and the presence or absence of parallel and perpendicular lines. In high school, students will classify two-dimensional figures in a hierarchy based on properties. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moved toward Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Triangles can be named and classified by angle measures (equiangular, acute, right, and obtuse) and/or side lengths (scalene, isosceles, and equilateral). • Triangles can be compared. • Quadrilaterals can be named and classified by angle measures, side lengths, or the presence or absence of parallel and perpendicular lines. • Quadrilaterals can be compared. **MATHEMATICAL THINKING** • Use spatial reasoning. • Create models and drawings to represent figures. • Recognize and use a pattern or structure. • Make and test conjectures about the classification of triangles and quadrilaterals; then justify reasoning. • Use grade-level appropriate mathematical language to explain reasoning. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Note: Students are not required to measure angles with a protractor for this cluster, but should be comparing angles to greater than, less than, or equal to 90 degrees.

- Describe an equilateral triangle as having three equal side lengths.
- Describe a scalene triangle as having three different side lengths.
- Explore and describe an isosceles triangle as having at least two sides the same length.
- Explore and describe an equilateral triangle as a special type of an isosceles triangle.
- Identify and describe triangles by the following:
  - side lengths (isosceles, equilateral, scalene)
  - angle measures (obtuse, acute, right, equiangular)
- Sort and compare types of triangles.
- Explore and describe squares, rectangles, parallelograms, trapezoids, and rhombuses based on side lengths, angle measures, and the presence or absence of parallel and/or perpendicular sides.
- Identify and describe quadrilaterals by the following:
  - side lengths;
  - angle measures;
  - the presence or absence of parallel and/or perpendicular lines; and/or
  - the presence or absence of symmetry.
- Sort and compare types of quadrilaterals.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 5, Number 5, page 35
- Ohio's K-8 Learning Progressions, K-5 Geometry, page 11
- Ohio's K-8 Learning Progressions, 6-8 Geometry, page 21
- Glossary – trapezoid

CONNECTIONS ACROSS STANDARDS

There are no direct connections to these standards within Grade 5. The ideas developed in these standards will be used in later grades, including grade 7 and high school.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision.

Instructional Tools/Resources

This section is under revision.

Acknowledgments

Ben Ayers
Curriculum Specialist/Coordinator, Huber Heights City Schools, SW

Jodie Bailey
Teacher, Hilliard City Schools, C

Terra Baker
Curriculum Specialist/Coordinator, Columbus City Schools, C

Charesha Barrett
Consultant, NE

Bridgette Beeler (WG)
Teacher, Perrysburg Exempted Village, NW

Angie Binegar
Teacher, Switzerland of Ohio Local Schools, SE

Sheri Bobeck
Curriculum Specialist/Coordinator, Columbiana County ESC, NE

Corey Bubon
Curriculum Specialist/Coordinator, Mahoning County ESC, NE

Susan Gahler
Teacher, Genoa Area Local Schools, NW

Leigh Ann Hewitt
Teacher, Canal Winchester Local Schools, C

Susie Huston
Curriculum Specialist/Coordinator, Lancaster City Schools, C

Laura Inkrott
Curriculum Specialist/Coordinator, Northridge Local Schools, C

Shaundra Jones
Curriculum Specialist/Coordinator, Toledo Public Schools, NW

Amy LaMotte Davis
Curriculum Specialist/Coordinator, Shaker Heights City Schools, NE

Sharilyn Leonard (WG)
Teacher, Oak Hill Union Local Schools, SE

Karen LeScoezec
Curriculum Specialist/Coordinator, Riverside Local Schools, NE

Anita O'Mellan
Higher Education, Youngstown State University, NE

Sandra Peloquin
Teacher, Lorain City Schools, NE

Erika Poston
Curriculum Specialist/Coordinator, Stow-Monroe Falls City Schools, NE

Diane Reisdorff (WG)
Teacher, Westlake City Schools, NE

Benjamin Shaw (WG)
Curriculum Specialist/Coordinator, Mahoning County ESC, NE

Pam Steinkirchner
Teacher, Green Local Schools, NE

Keri Stoyle
Teacher, Aurora City Schools, NE

Alison Tobias
Curriculum Specialist/Coordinator, South-Western City Schools, C

Barbara Weidus (WG)
Curriculum Specialist/Coordinator, New Richmond Exempted Village, SW

Karen Zagorec
Administrator, Warren City Schools, NE

*(WG) refers to a member of the Working Group in the Standards Revision Process.