Ohio's Model Curriculum | Mathematics

with Instructional Supports

Grade 7

Ohio | Department of Education

Mathematics Model Curriculum
with Instructional Supports
Grade 7

Table of Contents

Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is a model curriculum that supports the standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards. The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, possible connections between topics, and some common misconceptions.

To be noted, the emphasis of the model curriculum is on <u>instruction</u> not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Also, examples presented in this document may need to be rewritten to accommodate the needs of each individual classroom.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

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Introduction, continued

COMPONENTS OF INSTRUCTIONAL SUPPORTS

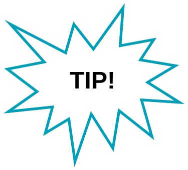
The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

There are several icons that help identify various tips in the instructional strategies section:

 = a common misconception

 = a technology tip

 = a career connection



= a general tip which may include diverse leaner or English learner tips.

Standards for Mathematical Practices—Grade 7

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.

In Grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" When students compare arithmetic and algebraic solutions to the same problem, they identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

In Grade 7, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

MP.3 Construct viable arguments and critique the reasoning of others.

In Grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. For example, as students notice when geometric conditions determine a unique triangle, more than one triangle, or no triangle, they have an opportunity to construct viable arguments and critique the reasoning of others. Students should be encouraged to answer questions such as these: "How did you get that?", "Why is that true?", or "Does that always work?" They explain their thinking to others and respond to others' thinking.

MP.4 Model with mathematics.

In Grade 7, students model problem situations symbolically, graphically, in tables, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use experiments or simulations to generate data sets and create probability models. Proportional relationships present opportunities for modeling. For example, for modeling purposes, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building. Students should be encouraged to answer questions such as "What are some ways to represent the quantities?" or "How might it help to create a table, chart, or graph?"

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Standards for Mathematical Practice, continued

MP.5 Use appropriate tools strategically.

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in Grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms. Teachers might ask, "What approach are you considering?" or "Why was it helpful to use ____ ?"

MP.6 Attend to precision.

In Grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations, or inequalities. Teachers might ask, "What mathematical language, definitions, or properties can you use to explain ____ ?"

MP.7 Look for and make use of structure.

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (e.g., $6 + 2n = 2(3 + n)$ by distributive property) and solve equations (e.g., $2c + 3 = 15, 2c = 12$ by subtraction property of equality; c = 6 by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities. Solving an equation such as $8 = 4(n - \frac{1}{2})$ is easier if students can see and make use of structure, temporarily viewing $(n - \frac{1}{2})$ as a single entity.

MP.8 Look for and express regularity in repeated reasoning.

In Grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$ and construct other examples and models that confirm their generalization. Students should be encouraged to answer questions such as "How would we prove that ____?" or "How is this situation both similar to and different from other situations using these operations?"

Mathematics Model Curriculum
with Instructional Supports
Grade 7

STANDARDS	MODEL CURRICULUM (7.RP.1-3)
RATIOS AND PROPORTIONAL RELATIONSHIPS **Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $^1/_2$ mile in each $^1/_4$ hour, compute the unit rate as the complex fraction* [G] $^{(1/2)}/_{(1/4)}$ *miles per hour, equivalently 2 miles per hour.* **7.RP.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. *Continued on next page*	**Expectations for Learning** In Grade 6, students begin reasoning about ratios, rates, and percents using models. In Grade 7, students extend this reasoning to proportions, direct variation equations, and more advanced percent problems. They identify unit rates in representations of proportional relationships. Students work with equations in two variables to represent and analyze proportional relationships. Also, they solve multi-step mathematical and real-world ratio and percent problems, such as problems involving percent increase and decrease. They also extend their learning of ratios to those specified by rational numbers. The study of proportional relationships is a foundation for the study of functions, which continues through high school and beyond. **ESSENTIAL UNDERSTANDINGS** **Percents** • A percent is a specific kind of ratio with a whole of 100. • All percent problems involve a part, a whole measured in some unit, and the same part and whole measured in hundredths. • Percents can be bigger than 100% and less than 1%. • Percent problems can be represented with a proportion or an equation. • Percent increase or percent decrease problems require careful attention to the referent whole by determining to what the whole (or 100% amount) a percentage refers. • Percent of change that involves an increase includes the following: tax, markups, gratuities, commissions, fees, etc. • Percent of change that involves a decrease includes the following: markdowns, discounts, etc. • Percent error is the difference between the approximate and exact value divided by the exact value. *Continued on next page*

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.RP.1-3)</th></tr>
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b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

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Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Unit Rate & Proportional Relationship

- A unit rate is a comparison of two quantities where the second quantity (denominator) is one.
- A rate can be written as a complex fraction which can be used to find the unit rate.
- A proportional relationship is a relationship between quantities.
- Proportions involve vertical and horizontal multiplicative relationships.
- In a table that represents a proportional relationship between y and x, $\frac{y}{x}$ is constant.
- The unit rate, which is the constant of proportionality, can be identified through models.
- Proportional relationships can be written as equations using the constant of proportionality, e.g., $y = kx$; $y = mx$; $t = pn$.
- The constant of proportionality is not always rational, e.g., π.
- The unit rate is the amount of change in y as x increases by one unit, e.g., in a table or graph.
- Graphs that represent proportional relationships are linear and go through the origin.

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to describe mathematical reasoning.
- Attend to precision in recording mathematical statements.
- Pay attention to and make sense of quantities.
- Consider mathematical units involved in a problem.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Recognize and use a pattern or structure.

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STANDARDS	MODEL CURRICULUM (7.RP.1-3)
7.RP.1-3, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** **Unit Rate and Proportional Relationships** • Identify the constant of proportionality (unit rate). • Compute and compare unit rates of two rational numbers, including complex fractions, in a real-world context. • Represent proportional relationships in various formats including tables, graphs, diagrams, verbal descriptions, and equations. • Align units when writing a proportion. • Determine if a relationship is proportional by using graphing, unit rates, scale factors, and other strategies. • Use proportional relationships to solve real-world ratio problems with multiple steps. • Explain the meaning of a point (x, y) on a proportional graph, with special attention to the points (0, 0) and (1, r), where r is the unit rate. **Percents** • Use proportional relationships and equations to solve mathematical and real-world percent problems. • Use benchmark percents to estimate and determine the reasonableness of a solution. • Attend closely to the wording of percent problems to determine what the whole (or 100% amount) to which a percentage refers. • Solve problems where the unknown is the part, whole, or percent. • Solve problems when the percent is greater than 100% or less than 1%. • Find the percent change (increase or decrease) using proportions and/or equations. • Recognize that the total cost of an item discounted (20% off) is equivalent to the difference of the value and 100% (100% – 20% = 80%). • Recognize that the total cost of an item with sales tax or commission (5%) is equivalent to the sum of the value and 100% (100% + 5% = 105%). • Calculate simple interest by multiplying the principal by the rate of interest by the number of years of the loan or investment ($I = prt$) • Calculate the percent error by finding the difference between the approximate and exact value divided by the exact value.

STANDARDS	MODEL CURRICULUM (7.RP.1-3)
7.RP.1-3, continued	**Content Elaborations** • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43 • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45 • Ohio's K-8 Learning Progressions, Ratio and Proportional Relationships, page 15 **CONNECTIONS ACROSS STANDARDS** • Represent proportional relationships within and between similar figures (7.G.1). • Use proportional reasoning in examining a sample of a population (7.SP.1). • Use proportional reasoning when predicting the probability of an event (7.SP.6-8). • Connect proportional reasoning to write and solve equivalent expressions and equations involving percent (7.EE.2-4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.1 *Make sense of problems and persevere in solving them.*
MP.3 *Construct viable arguments and critique the reasoning of others.*
MP.5 *Use appropriate tools strategically.*
MP.6 *Attend to precision.*

Proportional Reasoning means understanding and being able to reason about the structural relationship between the four quantities in a proportion. Students need to realize that the quantities are linked and change together (covary) but the constant ratio remains fixed. They need to not only be able to discern the multiplicative relationship but to extend the relationship to new ratios.

In Grade 6 students reasoned about ratios using models such as tables, double number lines, tape diagrams, and graphs. They avoided using fraction notation for ratios and did not set up nor explicitly solve proportions. Now in Grade 7, students should be able to set up proportions using fraction notation. *Note: Solving problems using cross products should be avoided.*

Building from the development of rate and unit concepts in Grade 6, applications should now focus on solving unit-rate problems with more sophisticated numbers. Entries in tables and unit rates can be rational numbers including complex fractions. For scaffolding ideas and more information about ratios and rates see Model Curriculum Grade 6.RP.1-3.

EXAMPLE

The ratio of flour to sugar in a cookie recipe is $2\frac{1}{4}$ cups of flour to $1\frac{1}{2}$ cups of sugar. Mikayla is making cookies for a fundraiser. If she uses $15\frac{3}{4}$ cups of sugar, how much flour will she need?

Providing opportunities to solve problems based within contexts that are relevant to seventh graders will help them to create meaning about rates, ratios, and proportions. Examples include researching newspaper ads and constructing their own question(s); keeping a log of prices (particularly sales) and determining savings by purchasing items on sale; timing students as they walk a lap on the track and figuring their rates; creating open-ended problem scenarios with and without numbers to give students the opportunity to demonstrate conceptual understanding; inviting students to create a similar problem to a given problem and explain their reasoning. Missing-value problems and comparison problems are necessary but not a prerequisite to understanding proportionality.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

PROPORTIONAL RELATIONSHIPS

The emphasis in this cluster is on understanding proportional relationships not on performing isolated computations with proportions. A proportional relationship is a relationship between two quantities. Students obtain proportional reasoning when they understand that the ratio of the two quantities remains constant even though the corresponding values of the quantities may change ($y = kx$). In other words, the relationship of the first quantity compared to the amount of the second quantity is always the same regardless if the quantities increase or decrease.

A proportion is a statement of equality between two ratios. However, the standards define a proportion in terms of a rate and emphasize the relationship between quantities. In a proportion there must also exist a constant ratio such that each measure in the first quantity multiplied by the constant ratio gives the corresponding measure in the second quantity. Proportional relationships are introduced and developed through the analysis of graphs, tables, equations, and diagrams. This is not the time for students to learn to cross multiply to solve problems. Ratio tables serve a valuable purpose in the solution of proportional problems.

Recognizing Proportional Relationships

It is important that students are able to differentiate between situations that are directly proportional and those that are not. Otherwise, they may haphazardly apply proportional techniques to nonproportional situations. That means they need to carefully attend to the relationships in the problem.

EXAMPLE

Identify whether each situation is proportional or nonproportional. Explain why. Then solve.

- **a.** A candle is burning at a constant rate. It has burned 12 mm after 20 minutes. How many millimeters has the candle burned after 50 minutes?
- **b.** A candle is burning at a constant rate. When it has burned 30 mm, its height is 75 mm. When it has burned 60 mm, what will be the candle's height?
- **c.** An altar in a church needs to be lit up continuously for a weeklong festival. It uses one special candle at a time, If the church uses special candles that last 7 hours each, then 24 such candles will be needed. If the church uses candles that last 8 hours each. How many 8-hour candles will they need?
- **d.** Two identical candles A and B, lit at different times are burning at the same constant rate, how many millimeters will candle A have burned?
- **e.** Two different candles, P and Q, lit at the same time, are burning at different but constant rates. When candle P has burned 16mm, candle Q has burned 10mm. When candle Q has burned 35 mm, how many millimeters will candle P have burned?

Taken from "Burning the Candle at Just One End" by Kien H. Lim published in *Mathematics Teaching* in the Middle School, Volume 14, No 8, April 2009.

Discussion: Part **a.** is proportional since the burning rate is constant. Part **b.** is additive in nature. Part **c.** is inversely proportional not directly proportional. Part **d.** has a constant difference not a constant ratio, and Part **e**. is proportional because it has a common ratio. This task could be extended by having students write equations for the situations.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Although students are not required to be proficient in inverse variations, situations that are inversely proportional can be used to make a contrast between those that are directly proportional and other types of situations.

Students should be given missing value problems, graphs, and tables that are both directly proportional and not directly proportional, so they get in the habit of checking for proportionality. They should not just test whether they are proportional or not, but they should be able to explain why they are proportional or not using language such as "for every," "for each," "per," "constant ratio," or "unit rate." They should also be able to convey the understanding that something is proportional because the constant relationship is a ratio.

EXAMPLE

Which tables are directly proportional? How do you know? Write an equation for each situation.

meters	seconds
10	3
6	5
15	2

meters	seconds
15	20
6	11
10	15

meters	seconds
15	30
10	20
6	12

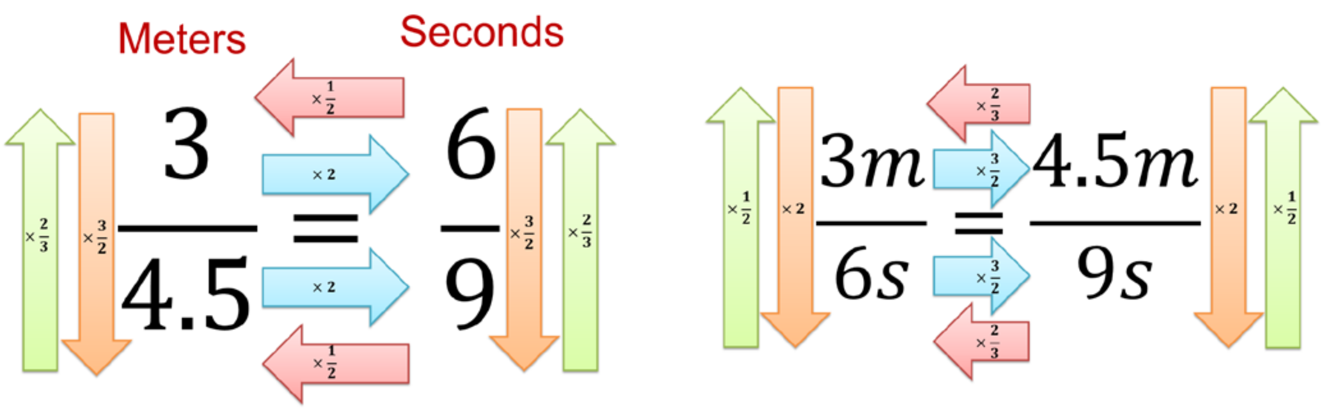
After some initial exploration of direct proportions vs non-direct proportions, it may be helpful to use 3-column tables where the third column is the unit rate. This also helps draw attention to the unit rate/the constant of proportionality.

EXAMPLE

Have students restate "y is directly proportional to x if $y = kx$ for some constant k" in their own words and give an example. Taken from Proportionality, an Illustrative Mathematics task.

Within and Between Relationships/Vertical and Horizontal Multiplicative Relationships

One way to view and reason with proportions is to use within and between relationships. Within relationships focus on making comparisons within the same units/measure-space such as 180 miles: 60 miles = 6 gallons: 2 gallons. Whereas between relationships focus on making comparisons between different units/measure-space such as 180 miles: 6 gallons = 60 miles: 2 gallons. Notice the that the two within relationships are equal as are the two between relationships. Different researchers define the within and between relationships using opposite meanings, so the terminology can be confusing. To differentiate between the relationships the Essential Understanding statement just lists the concept as "vertical and horizontal multiplicative relationships" which of course only holds true if the proportion is written in fractional form, but is also useful terminology if units are not included in the proportion.



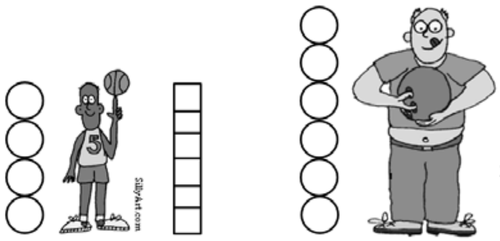
Students should use these relationships to help them view the structure of a proportion. They should come to the realization that proportions can be written in four different but equivalent ways: $\frac{4m}{5s} = \frac{16m}{20s}$; $\frac{5s}{4m} = \frac{20s}{16m}$; $\frac{4m}{16m} = \frac{5s}{20s}$; $\frac{16m}{4m} = \frac{20s}{5s}$.

To help students see these relationships, start with simple problems where the multiples are easy and evident. Then move to more difficult problems including using those with rational numbers. As students discover these relationships, they can use them as strategies in solving problems with proportions. This method is a good alternative to cross multiplying as it builds students' reasoning about the relationships within a proportion.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

A basketball player and a bowler are measured in both balls and blocks. The basketball player is four balls high and 6 blocks high. If the bowler is 6 balls high, what is his height in blocks?



EXAMPLE

The dosage of a children's medicine states that the child should receive 0.625 mL for every 10 lbs of weight. How many mL should a 35 lb child receive?

Horizontal Relationship **Vertical Relationship**

$$\frac{0.625\ mL}{10\ lbs} \cdot \left(\frac{3.5}{3.5}\right) = \frac{x\ mL}{35\ lbs}$$ or $$3.5 \times \frac{0.625\ mL}{x\ mL} = \frac{10\ lbs}{35\ lbs} \times 3.5$$

Discussion: After the student sets up the proportion he or she can determine the vertical or horizontal relationship (which may vary depending on how he or she sets up the proportion.) Two examples are shown above. A connection in the first example can also be made to the Identity Property of Multiplication since $\left(\frac{3.5}{3.5}\right) = \mathbf{1}$, and anything multiplied by 1 retains the same value. Draw attention to the unit rate.

TIP!

Labeling units helps students organize the quantities when writing proportions.

$$\frac{3\ eggs}{2\ cups\ of\ flour} = \frac{z\ \ eggs}{8\ cups\ of\ flour}$$

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Avoid Cross-Multiplying

The issue with the cross-multiplying rule is that it circumvents reasoning about proportions. Students may be able to solve the problem, but then never develop the understanding needed for higher mathematics. Instead they should be focusing on the constant of proportionality (unit rate).

An alternative of cross products is to use common denominators to find the missing element in a proportion. This method still allows students to make sense of proportions. Students may also desire to use vertical and horizontal relationships, unit rates, or other strategies.

EXAMPLE

Fruit punch uses a ratio of 2.5 cups of syrup to 5.5 cups of water. If 6.5 cups of syrup are used to make punch, how many cups of water are needed?

Common Denominator Method

$$\frac{\textit{cups of syrup}}{\textit{cups of water}} = \frac{2.5}{5.5} = \frac{6.5}{w}$$

TIP!

Another way to label units to help students organize the quantities when writing proportions is shown above.

$$\frac{\textit{cups of syrup}}{\textit{cups of water}} = \left(\frac{w}{w}\right) \cdot \frac{2.5}{5.5} = \frac{6.5}{w} \cdot \left(\frac{5.5}{5.5}\right)$$

Note: A connection can be made to the Identity Property of Multiplication.

$$\frac{\textit{cups of syrup}}{\textit{cups of water}} = \frac{2.5w}{5.5w} = \frac{6.5(5.5)}{5.5w}$$

Since the denominators are the same $\mathbf{2.5w = 35.75}$ and students can solve the equation by dividing each side by 2.5 to get 14.3 cups.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Vertical and Horizontal Relationship Method

$$\frac{\text{cups of syrup}}{\text{cups of water}} = \frac{2.5}{5.5} = \frac{6.5}{w}$$

$6.5 \div 2.5 = 2.6$ $\quad\quad$ $5.5 \div 2.5 = 2.2$

$$\frac{\text{cups of syrup}}{\text{cups of water}} = \frac{2.5}{5.5} \cdot \left(\frac{2.6}{2.6}\right) = \frac{6.5}{w} \quad \text{or} \quad \frac{\text{cups of syrup}}{\text{cups of water}} = 2.2 \times \frac{2.5}{5.5} = \frac{6.5}{w} \times 2.2$$

This also results in 14.3 cups.

Discussion: This method is a good alternative to cross multiplying as it builds upon students' reasoning about relationships within a proportion. It is often much easier to do mentally. To understand this method, student should start with integers where mental operations are easy to perform, and then move to more difficult problems including those with rational numbers.

Comparing Proportions

The common denominator method can be used to compare proportions.

EXAMPLE

Which is the better buy? An 18.7 oz box of Raisin Bran for $3.39 or 43.3 oz box for $8.18

Unit Rate Method

A student could compare prices by converting to a unit rate. (See the next section for more information on unit rates.)

$$\frac{\text{price}}{\text{ounces}} = \frac{\$3.39}{18.7} \text{ or } \frac{\$8.18}{43.3}$$

$$\frac{\text{price}}{\text{ounces}} = \frac{\$3.39}{18.7} = \frac{\$0.181}{1} \text{ or } \frac{\text{price}}{\text{ounces}} = \frac{\$8.18}{43.3} = \frac{\$0.189}{1}$$

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Common Denominator Method

$$\frac{\boldsymbol{price}}{\boldsymbol{ounces}} = \frac{\$\mathbf{3.39}}{\mathbf{18.7}}\ \boldsymbol{or}\ \frac{\$\mathbf{8.18}}{\mathbf{43.3}}$$

$$\frac{\boldsymbol{price}}{\boldsymbol{ounces}} = \left(\frac{\mathbf{43.3}}{\mathbf{43.3}}\right)\cdot\frac{\$\mathbf{3.39}}{\mathbf{18.7}}\ \boldsymbol{or}\ \frac{\$\mathbf{8.18}}{\mathbf{43.3}}\cdot\left(\frac{\mathbf{18.7}}{\mathbf{18.7}}\right)$$

Note: A connection can be made to the Identity Property of Multiplication.

$$\frac{\boldsymbol{price}}{\boldsymbol{ounces}} = \frac{\$\mathbf{146.787}}{\mathbf{809.71}}\ \boldsymbol{or}\ \frac{\$\mathbf{152.966}}{\mathbf{809.71}}$$

The student can clearly see that when the ounces are the same (809.71 oz) the smaller box at $146.787 would be cheaper.

CONSTANT OF PROPORTIONALITY (UNIT RATES)

The constant of proportionality, *k*, is a structural element that appears in a variety of contexts. However, in many cases it seems hidden. It can be the following:

- the unit rate;
- the slope in a linear graph that goes through the origin;
- the difference between table entries;
- the rate of one quantity changed compared to the other;
- the scale factor in similar figures;
- theoretical probability;
- a scale on a map;
- a percentage in a sales tax, mark-up, or discount problem; or
- k (which is equal to $\frac{y}{x}$) in an equation in the form $y = kx$.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

A fixed ratio (unit rate) is associated with a quality such as steepness, flavor, or speed that remains fixed as the variables change together. Ask students questions such as "How can qualities stay fixed as quantities change?" Help students see the unit rate (the constant of proportionality) by using 3-column tables with the unit rate as the third column. Building on Grade 6, it is important that students see the unit rate as one quantity such as speed, taste, unit price, mpg, etc instead of a comparison of two quantities. They need to isolate the attribute associated with the fixed ratio from other measurable attributes. After students have many experiences using the 3-column unit-rate tables, have students discover that the pattern of finding the unit rate is $\frac{y}{x}$.

EXAMPLE

Distance (miles)	Time	Speed (mph) (Unit Rate)
248	4 hours	
	7 hours 30 min	
589		

A car is driving at a constant rate. Use the table to find the following:

- How far does it go in 7 hours and 30 minutes?
- How long will it take a car to go 589 miles?
- How can the unit rate (speed) help in answering the questions?
- What pattern do you notice about the unit rate?
- Write an equation describing the situation.
- What connection is there between the unit rate and the equation?

EXAMPLE

Have students graph a line such as $y = \frac{2}{3}x$. Using graphing technology and/or tables have students explore that as (x, y) moves along the line, its coordinates change together but y is always 2 parts and x is always 3 parts even as the parts change in size, so the common ratio (constant of proportionality) is $\frac{2}{3}$.

Students should realize that the constant of proportionality can also be irrational like π.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

Part 1

Jason is building a circular patio. He is trying to decide the size of the patio, so he experiments with various diameters.

- **a.** Fill out a table to represent various combinations of area and diameter.
- **b.** Is the relationship between area and diameter proportional? Explain.
- **c.** If it is proportional, state the constant of proportionality.
- **d.** Write an equation describing the situation.

Part 2

Jason decides to build a fence around his circular patio.

- **a.** Fill out a table to represent various combination of perimeter and diameter.
- **b.** Is the relationship between perimeter and diameter proportional? Explain.
- **c.** If it is proportional, state the constant of proportionality.
- **d.** Write an equation describing the situation.

GRAPHING PROPORTIONS

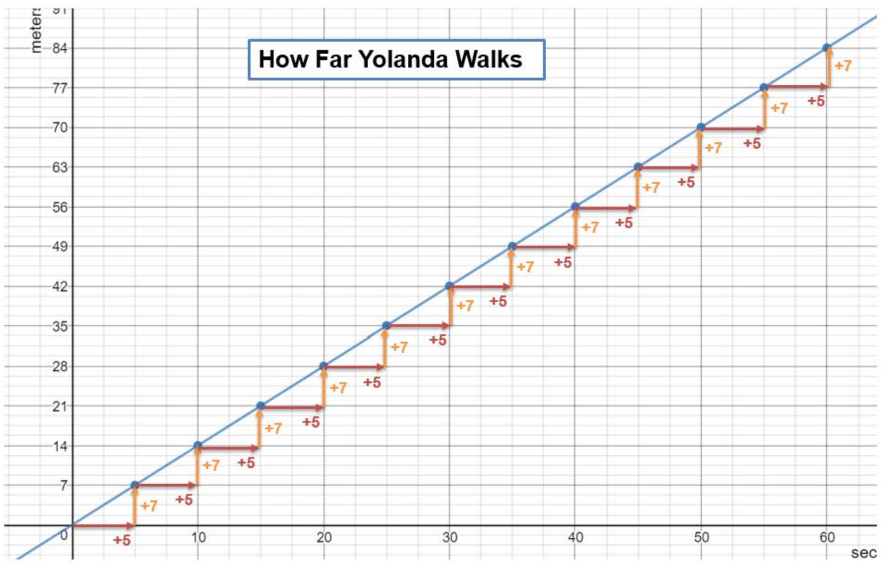
This is the time to push for a deep understanding of what a representation of a proportional relationship looks like and its characteristics: a straight line through the origin on a graph; a "rule" that applies for all ordered pairs; an equivalent ratio or an expression that describes the situation, etc. In Grade 6 the emphasis is on graphing equivalent ratios, the emphasis in Grade 7 is on the idea of rate.

Have students explore graphs that are proportions and those that are not. Given various graphs, they may make tables using three points on the graph and decide whether they are proportional or not. Ask students what all proportional graphs have in common. Students should come to the conclusion that a proportional graph is a straight line that goes through the origin. Draw special attention to the points (0, 0) and (1, r) with (1, r) being the unit rate/constant of proportionality.

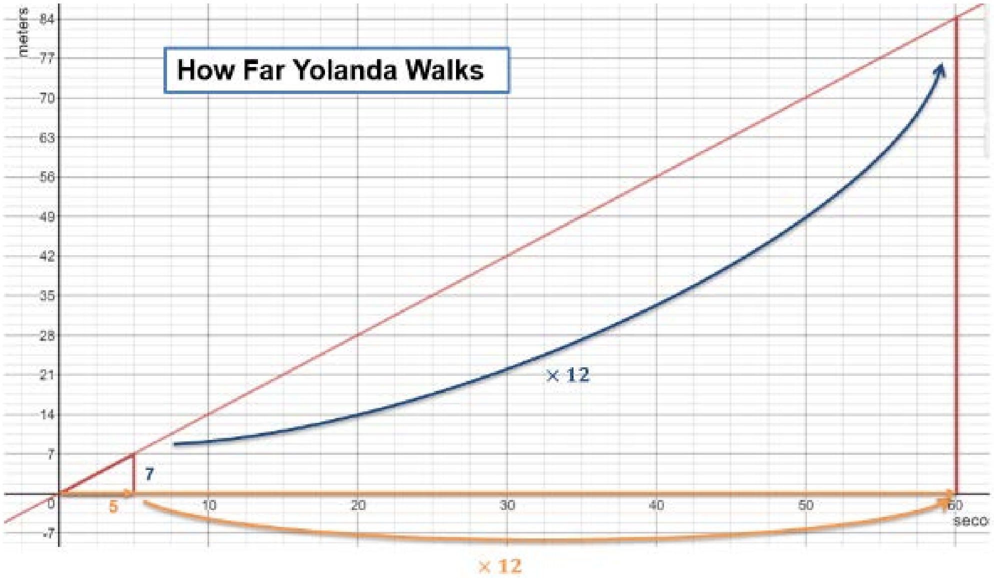
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

In Grade 6 students initially looked at proportional graphs as repeated addition and moved toward scalar multiplication. See Grade 6 Model Curriculum 6.RP.1-3 for more information.

Repeated Addition



Scalar Multiplication



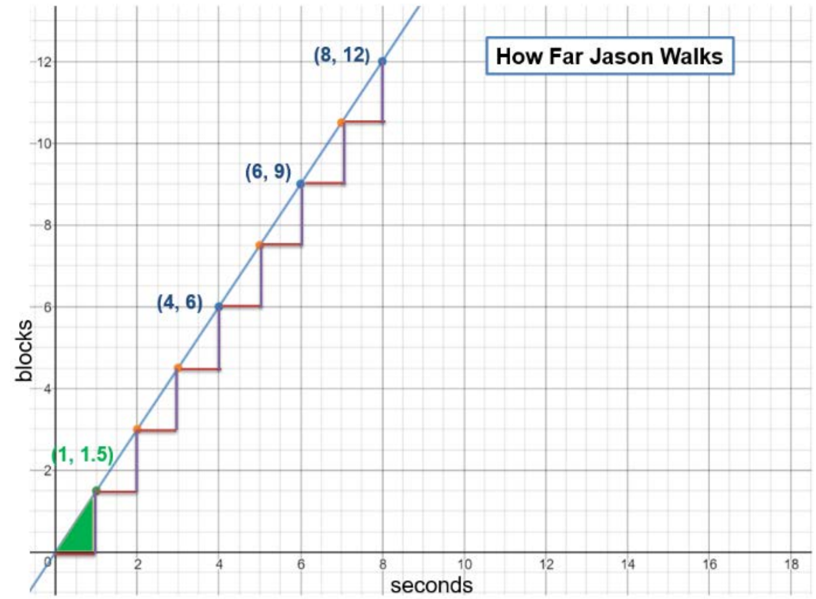
Using the 3-column unit-rate ratio tables, students should see the connection between the unit rate and the slant of the line. (Students are not required to use the term slope until Grade 8.) This is especially evident in tables where each entry in the table increases by the same amount (repeated addition). This connection could also help them visualize that the unit rate is $\frac{y}{x}$. At this point it would be a good idea to review the terms independent and dependent variable from Grade 6 (6.EE.9). When graphing a proportion they should realize that the constant ratio is equivalent to the unit rate, but oftentimes the constant ratio is easier to visualize and graph.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

Jason walked to a friend's house at a constant rate. Complete the table and graph the coordinates (time, distance) on a coordinate plane. (Sample graphs are shown below.)

Time (seconds)	Distance (blocks)	Speed (spb) (Unit Rate)
4	6	
6	9	
8	12	



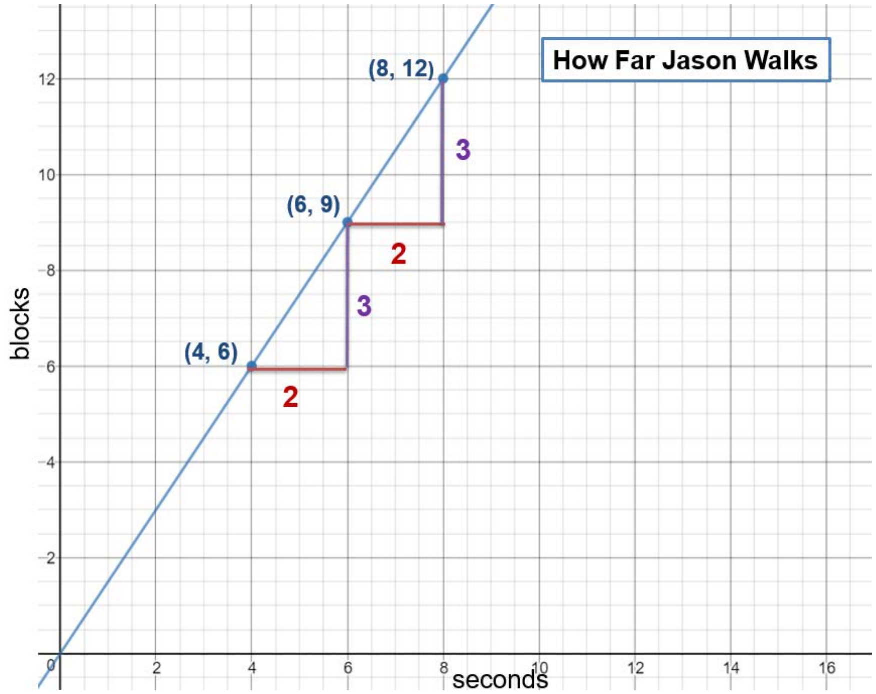
Part 1

a. What connection is there between the unit rate and the graph?
b. What do the coordinates on the graph represent?
c. For every 1 unit you move to the right, you move up ______units?
d. For every 2 units you move to the right, you go up 2 • ______ units?
e. For every 3 units you move to the right, you go up 3 • ______ units?
f. For every 4 units you move to the right, you go up 4 • ______ units?
g. For every *x* units you move to the right, you go up x • ______ units?
h. Write an equation to model the situation.
i. How does the equation correspond to the graph?

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Part 2

a. Is there another common ratio $(\frac{y}{x})$ that is easier to use than the unit rate? Explain.
b. Graph 2 more points using your new common ratio.
c. To get from point to point on the line, for every _____ units you go the *right*, you go *up* _____ units.
d. To get from point to point on the line for every _____ units you go *up*, you go _____ units to the *right*.
e. Does it matter if you go *up* and to the *right* or to the *right* and *up*? Explain.
f. Write an equation to model the situation?
g. How does this equation compare to the equation in Part 1?
h. Which equation do you prefer? Explain.



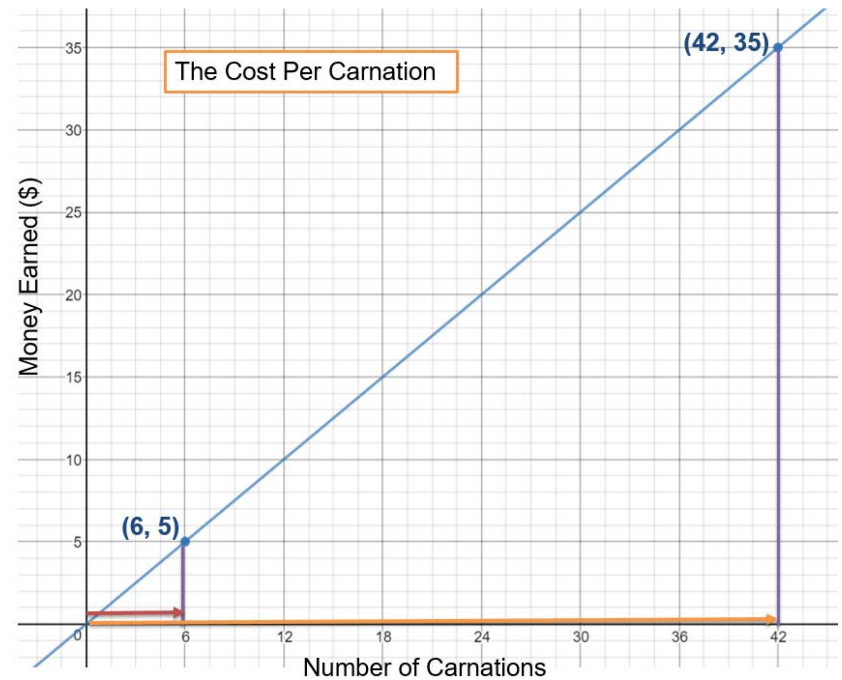
As in Grade 6, teachers should move students from repeated addition toward scalar multiplication to develop multiplicative reasoning. Understanding graphs illustrating scalar multiplication will aid students in their understanding of similar figures and dilations. Students should be given a unit rate and be able to graph a proportional relationship and determine a point on the line. They also should be able to be given a coordinate and use it to graph a proportion and determine the unit rate.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

Ronnita sold carnations for a fundraiser where they had bundled pricing. The first day she sold a bundle of 6 carnations for \$5. The next day she made \$35 selling bundles of carnations for the same price.

- **a.** Is the situation proportional? Explain.
- **b.** Use a graph to show how many carnations she sold on Day 2.
- **c.** Write an equation to model the situation.
- **d.** Solve the problem using the equation and compare your solution to part **b.**
- **e.** Explain why your solutions match or do not match.

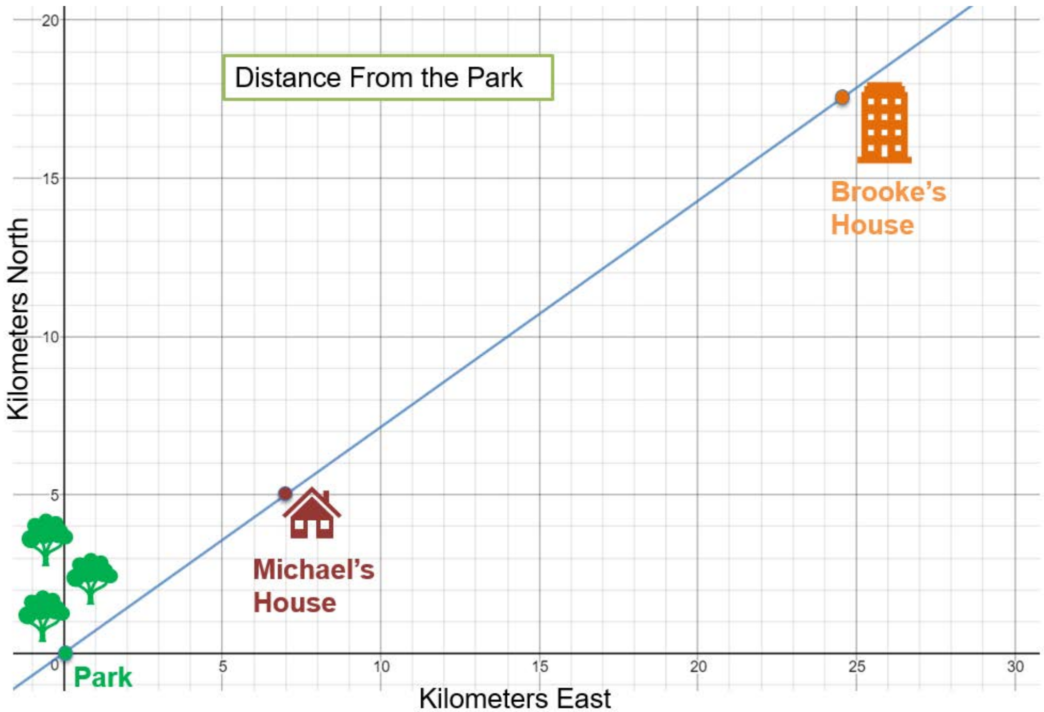


Discussion: Students should be able to identify the independent and dependent variable from Day 1 and plot (6, 5) on a coordinate plane. Since each carnation costs the same amount the situation is proportional. Therefore, the student can create a graph going through point (6, 5) and the origin. They then know that \$35 is the dependent variable (y-value), so they have to find where $y = 35$ on the line. The corresponding x-value to $y = 35$ is 42, so they sold 42 carnations. The price per bundle is 6 carnations for \$5. Therefore, the equation of the line $y = \frac{5}{6}x$, where $\frac{5}{6}$ is the constant of proportionality. Draw attention to the fact that (1, r) is the unit rate, which is sometimes hard to see on a graph. Students could also use the equation to solve the problem by substituting in \$35 for y to get $35 = \frac{5}{6}y$ which is 42. A connection could also be made from the graph to similar triangles. *Note: the unit rate is still $\frac{5}{6}$ even though there is a 6 in the denominator for it can be rewritten as the complex fraction $\frac{\left(\frac{5}{6}\right)}{1}$, where 1 carnation would cost $\frac{5}{6}$ of a dollar (or approximately \$0.83). However, since the cost of a single carnation is an irrational number, the bundle price is more accurate and may make more sense to students than the unit rate.* Draw students' attention to the within and between relationships of the figures and tie it back to proportional relationships of similar figures. For example there is a relationship *within* the coordinates that lie on the same line but there is also a relationship *between* the corresponding coordinates of the two figures.

Within Ordered Pairs on the Same Line	**Between Corresponding Coordinates on the Same Line**
(10, 6) and $y = {}^3/_5x$; (30, 18) and $y = {}^3/_5x$	(10, 6) **x 3** = (30, 18)

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE



The shortest distance between any two points is a straight line. How much farther from the park is Brooke's house than Michael's house?

Discussion: Students should notice that the distance from Michael's house to Brooke's house is proportional. Therefore, they can set up a proportion based on the location of Michael's house (7,5) and Brooke's house (24.5, 17.5) as $\frac{7}{5} = \frac{24.5}{17.5}$. They should see that the horizontal relationship is 3.5 units, so Brooke's house is 3.5 times as far. This type of reasoning will aid students in dilations in Grade 8.

PERCENTS

Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on those between 0 and 100, now percents above 100 and less than 1 are introduced.

Students interchange 15%, 0.15%, $\frac{15}{100}$ as fifteen hundredths percent. This is a difficult concept to comprehend. Emphasize that a percent means per 100, so 0.15% means 0.15 per 100.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Misleading Percents

Discuss how percents can be used to mislead people. They can make big numbers look small and small numbers look big. Which sounds better, a company laid off 5% of its work force or a company laid off 2,000 people? Another example of deceptive use of percents could be choosing to use an increase instead of percent change or vice versa. For example a shirt that used to sell for $25 dollars now sells for $35, which is only a change of $10, but it is a 40% increase in price.

Percent as a Rate

Many students who have a part/whole understanding of percent make mistakes when they need to apply percents to situations that are not part/whole such as percent change. For example when solving the problem 60 = __% of 30, students will oftentimes give 50% for the answer. They just jump to dividing the larger number by the smaller number instead of reasoning through the process. Emphasizing percents as a multiple or a rate will help this situation. See Model Curriculum 6.RP.1-3 for more information on percent as a rate.

TIP!

In real-life when percents are used, the two referent quantities are often implied. Give students situations where they have to infer the referent quantities and have them rewrite the situation using comparisons. For example, the media may state that the unemployment rate is 4.1%. Readers need to infer what is being compared: the number of eligible workers who are not working compared to the total number of eligible workers." One way students could rewrite the phrase is "The unemployment rate is 4.1% for the number of eligible workers who are not working compared to the number of eligible workers. It will also be helpful for students to learn to think critically about what the source is, so the 4.1% is not a percent of all adults who do not have a job. In other words it may include retirees and those who have chosen to stay at home, etc.

Multi-step Problems

Students should solve a variety of problems involving percent including tax, interest, tip, mark-ups, mark-downs, commissions, fees, discounts etc. Since the students in Grade 6 do not formally set-up and solve proportions, they were limited to solving percent problems using ratio reasoning with models such as tables and double number line diagrams and 100 grids. Now in Grade 7 students can solve percents more formally using proportions and equations (7.EE.3-4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

Jonathan bought a used car for $14,130 which was 75% of the original price. How much did the car originally cost?

Discussion: The proportion bar allows students to set up the proportion and create meaning. If they so choose, they can set up their proportion as an equivalency of two fractions, create a ratio table, or use any of the other methods available to them such as tape diagrams, double number lines or graphing. A student may also choose to use an equation. Each student should choose whichever method is makes sense to him or her.

Percent Bar Model

$$\frac{14{,}130}{w} = \frac{75}{100}$$

Equation Model

$$Part = \% \times Whole$$
$$14{,}130 = \frac{75}{100} \cdot w$$

TIP!

For students who have trouble with decimal conversion, encourage them to write the percent as a fraction over 100.

TIP!

Students should not rely exclusively on the word "of" to indicate the whole, since "of" has many meanings in English language and does not always signal the whole. In addition there are many times where the word "of" is not even present. If they depend upon using "of" to signal the whole, they will be at a loss when it is missing from a problem.

Percent Change

Traditionally students have difficulty with percent change. Many students have struggle envisioning a percent greater than 100 due to instruction that is limited to part-whole scenarios. Another reason for difficulty is that percent change uses additive language such as more than, less than, increase, and decrease for a multiplicative concept. Language also creates confusion as many times problems state "more than" when it means "proportionally more than." Students have to infer whether it is an additive or multiplicative process from the text.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Proportion bars, tape diagrams, and double number lines can be useful to visualize percent increase and decrease problems. Discuss that there are two perspectives in solving percent increase and percent decrease problems. The first perspective is finding the increase or decrease and the adding/subtracting it to/from the original to get the new price. The second perspective is to combine the percents first (20% off would be 80% of the original price or a meal with a 15% tip would be 115% of the original bill), and then set up and solve the proportion or equation. Students should be able to explain why both situations work.

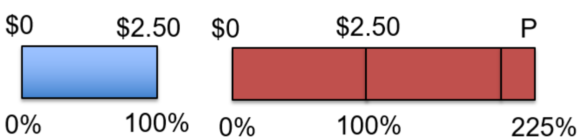
In percent problems, especially in percent change problems, it is vital to be able to correctly identify the whole (100% amount).

Many students will incorrectly interpret 120% as $\frac{120\%}{100}$ instead of $\frac{120}{100}$. Emphasize that a percent is always over 100, and that percents can be greater than 100 or less than 1.

EXAMPLE

A pound of coffee costs \$2.50 at wholesale. A grocery store marks the coffee up 225% before selling to the customers. What is the selling price of the coffee?

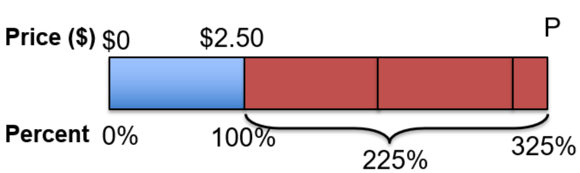
Method 1: Find the part, then add the percentages



$$\frac{P}{2.50} = \frac{225}{100}$$
$$P = \$5.63$$

$$100\% + 225\% = 325\%$$
$$\$2.50 + \$5.63 = \$8.13$$

Method 2: Add the percentages, then find the part

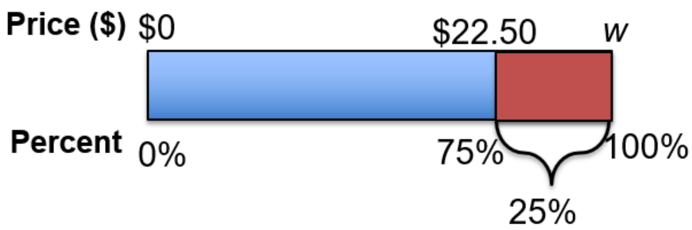


$$100\% + 225\% = 325\%$$
$$\frac{P}{2.50} = \frac{325}{100}$$
$$P = \$8.13$$

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

Ali bought a shirt that was on sale for \$22.50. It was 25% off. How much was the original price?



$$100\% - 25\% = 75\%$$
$$\frac{\$22.50}{w} = \frac{75}{100}$$
$$w = \$30$$

Note: Without first converting this example to 75%, writing the equation in this situation becomes very complex for seventh graders: 0.25x = x – \$22.50. Discuss when taking the part first and then subtracting it from the total is more efficient than combining the percents first.

EXAMPLE

Fatima found the same shirt on sale at two stores. The original price is \$25.99 at both stores. At Store A it was on clearance for 40% off, and there was an additional sale for 20% off clearance items. At Store B, it was 60% off the original price. What store should Fatima buy the shirt from? Explain.

EXAMPLE

If I take any amount of money and increase it by 30% and then decrease the new amount by 30%, is the final amount greater than, less than, or equal to the original. Justify your decision using models. Does it matter which amount of money you chose to start with? Explain.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

A deep discount store has a continual discount of 75% off all original prices.

a. Graph the situation on a coordinate plane. Define your independent and dependent variables.
b. Write an equation to model the situation and then graph.
c. Explain the connection between the graph and the equation.
d. Using the graph, how much would a dress that originally costs $40 be at the discount store?
e. If a sweater at the discount store is $5, how much was the original price?

Discussion: It may be helpful to switch part **a.** and part **b.** in this problem. Doing so will emphasize different skills. It is beneficial for students to know both how to create an equation from a situation and then graph it and take a situation, graph it, and then write an equation.

EXAMPLE

The dimensions of a 3 inch by 5 inch photograph was enlarged by 40%, what are the new dimensions of the photograph?

Simple Interest

Students should solve simple interest problems in terms of a mark-up proportion problem in order to create understanding. Percents may be fractions or decimals. Although, simple interest can be taught using the equation *I* = Prt, where *I* stands for interest, *P* stands for principle, *r* stands for rate as a percent, and *t* for time in years; this does not always promote understanding.

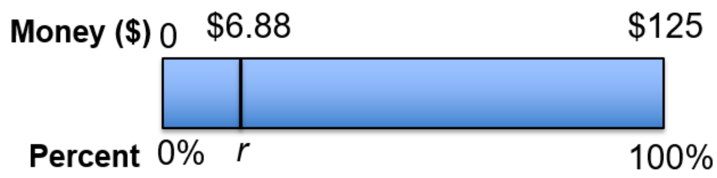
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

Bakari deposits $125 in the bank and leaves it there untouched for 3 years. At the end of the 3 years he earned $20.63 in interest. What rate did the bank offer him? Round to the nearest percent.

Years	Interest
3	$20.63
1	I

$\times 3$ $\times \frac{1}{3}$



$$\frac{3}{\$20.63} = \frac{1}{I}$$
$$I = \$6.88$$

$$\frac{\$6.88}{125} = \frac{r}{100}$$
$$r = 5.5\%$$

Percent Error

Absolute error is $|approximate\ value - exact\ value|$. Percent error is $\frac{|approximate\ value - exact\ value|}{|exact\ value|} \cdot 100\%$. Discuss the difference between absolute error and percent error. Explain that the percent error is useful for comparing the error to the original amount. The smaller the object is, the more precision is needed. An error of an inch is a significant amount when measuring something like a book, but it is much less significant when measuring the distance of a cross country course. The situation also affects the significance of the percent error. In medicine being imprecise may have serious consequences, but small errors in cooking a roast may be inconsequential.

EXAMPLE

- **a.** Enrique and Ming measured the length of a book that was $12\frac{5}{8}$ inches long. Enrique measured $12\frac{3}{4}$ inches and Ming measured $12\frac{1}{2}$ inches. Whose measurement had the greatest amount of error? Explain.
- **b.** Sheng measured a $16\frac{5}{8}$ window at $16\frac{1}{2}$ inches. Elena measured the length of a $\frac{5}{8}$ inch nail at $\frac{3}{4}$ inches. Whose measurement had the greatest amount of error? Explain.

Discussion: In Part **a.,** each student had the same absolute error of $\frac{1}{8}$ inch and percent error of about 0.99%, so both had a very small error or measurement. In Part **b.,** both students again had an absolute error of $\frac{1}{8}$ of an inch. However, the percent errors differ as Sheng had a percent error of about 0.75% which is a very small error in measurement and Elena had a percent error of 20% which is not accurate. Emphasize that the smaller the object being measured the more precision is needed.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

EXAMPLE

A scientist was allowed a 0.5% error for weighing objects in her research project. If the goal is to get to the actual weight of 16.2 grams, what is the range of acceptable weights?

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Play money - act out a problem with play money
- Advertisements in newspapers
- Unlimited manipulatives or tools (do not restrict the tools to one or two, give students many options)
- Literature such as *Holes*, *Alice in Wonderland*, *If You Hopped Like a Frog*

Proportions

- Cinder Versus Juice—Variation 2 by Illustrative Mathematics is a task where students apply proportional reasoning to determine the better deal.
- The Mr. Tall/Mr. Short Puzzle by University of Nebraska is an activity where students can apply proportions using nonstandard measurement units.
- Holes by Dan Meyer is a 3-act task where students apply proportions to digging holes.
- Write and Solve Proportions to Calculate Sale Price by Mobius Math is a worksheet that has students write and solve proportions using tables, tape diagrams, and graphs.

Unit Rates

- Big Bucks is a lesson by Yummy Math where students find the unit rate of pay for a collection of athletes.

Graphing Proportions

- Albert Pujols…On Pace? 2013 is a lesson by Yummy Math that has students graph proportions.
- Cheesy Goldfish Crackers by Yummy Math is a lesson that uses ratio tables to graph proportions.

Percents

- Dueling Discounts by Dan Meyer is a 3-act task where students determine which coupon gives the best deal.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

Curriculum and Lessons from Other Sources

- Georgia Standards of Excellence Curriculum Frameworks, Unit 3: Ratio and Proportional Relationships has many tasks that pertain to this cluster.
- Illustrative Mathematics, Grade 7, Unit 2: Introducing Proportional Relationships has many tasks the pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- Illustrative Mathematics, Grade 7, Unit 4: Proportional Relationships and Percentages has many tasks the pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- EngageNY, Grade 7, Module 1, Topic A, Lesson 1: An Experience in Relationships as Measuring Rate, Lesson 2: Proportional Relationships, Lesson 3: Identifying Proportional Relationships in Tables, Lesson 4: Identifying Proportional Relationships in Tables, Lesson 5: Identifying Proportional Relationships in Graphs, Lesson 6: Identifying Proportional Relationships in Graphs are lessons that address this cluster.
- EngageNY, Grade 7, Module 1, Topic B, Lesson 7: Unit Rates as the Constant of Proportionality, Lesson 8: Representing Proportional Relationships with Equations, Lesson 9: Representing Proportional Relationships with Equations, Lesson 10: Interpreting Graphs of Proportional Relationships are lessons that address this cluster.
- EngageNY, Grade 7, Module C, Lesson 11: Ratios of Fractions and their Unit Rates, Lesson 12: Ratios of Fractions and their Unit Rates, Lesson 13: Finding Equivalent Ratios Given the Total Quantity, Lesson 14: Multi-step Ratio Problems, Lesson 15: Equations and Graphs of Proportional Relationships Involving Fractions are lessons that address this cluster.
- EngageNY, Grade 7, Module 4, Topic A, Lesson 1: Percent, Lesson 2: Part of a Whole as a Percent, Lesson 3: Comparing Quantities with Percent, Lesson 4: Percent Increase and Decrease, Lesson 5: Finding One Hundred Percent Given Another Percent, Lesson 6: Fluency with Percents are lessons that address this cluster.
- EngageNY, Grade 7, Module 4, Topic B, Lesson 7: Markup and Markdown Problems, Lesson 8: Percent Error Problems, Lesson 9: Problem Solving When the Percent Changes, Lesson 10: Simple Interest, Lesson 11: Tax, Commissions, Fees, and Other Real-World Percent Problems are lessons that address this cluster.
- The Utah Middle School Math Project is an open source textbook and workbook.

General Resources

- Arizona Progressions on 6-7 Ratios and Proportional Relationships
  This cluster is addressed on pages 8-11.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

References

- Baltus, C. (April 2010). Connected representations from proportion to linear functions. *Mathematics Teacher*, *103(8),* 590-596.
- Beckmann, C., Thompson, D., & Austin, R. (2004). Exploring proportional reasoning through movies and literature. *Mathematics Teaching in the Middle School*, *9(5),* 256-261.
- Beckmann, S. & Izsák, A. (2015). Two perspectives on proportional relationships: Extending complementary origins of multiplication in terms of quantities, *Journal for Research in Mathematics Education*, *46(1),* 17-38.
- Bostic, J. (February 2015). A blizzard of a value. *Mathematics Teaching in the Middle School*, *20(6),* 350-357.
- Bush, S., Karp, K., Nadler, J., & Gibbons, K. (November 2016). Using artwork to explore proportional reasoning. *Mathematics Teaching in the Middle School*, *22(4),* 216-223.
- Carlson, M., Jacobs, S., Coe, E, Larsen, S., & Hsu, E. (2002). *Journal for Research in Mathematics Education*, *33(5),* 352-378.
- Cohen, J. (May 2013). Strip diagrams: Illuminating proportions. *Mathematics Teaching in the Middle School*, *18(9),* 536-542.
- de la Cruz, J. & Garney, S. (May 2016). Saving money using proportional reasoning. *Mathematics Teaching in Middle School*, *21(9),* 552-561.
- Ebby, C. & Petit, M. (February 2017). Using learning trajectories to enhance formative assessment. *Mathematics Teaching in the Middle School*, *22(6),* 368-372.
- Heinz, K. & Sterba-Boatwright, B. (March 2008). The when and why of using proportions. *Mathematics Teacher*, *101(7),* 528-532.
- Ercole, L., Frantz, M., & Ashline, G. (April 2011). Multiple ways to solve proportions. *Mathematics Teaching in Middle School*, *16(8),* 482-490.
- Howe, C. & Nunes, T. (April 2010). Rational number and proportional reasoning: Using intensive quantities to promote achievement in mathematics and science. *International Journal of Science and Mathematics Education*, *9(2),* 39-417. doi: 10.1007/s10763-010-9249-9. Lobato, J. Orrill, C., Druken, B., & Jacobsen, E. (2011, March). Middle school teachers' knowledge of proportional reasoning for teaching. In Author Unknown, *Extending, expanding, and applying the construct of mathematical knowledge for teaching.* Symposium conducted at the Annual Meeting of the American Educational Research Association, New Orleans, LA.
- Lamon, S. (2005). *More In-Depth Discussion of the Reasoning Activities in "Teaching Fractions and Ratios for Understanding."* (2nd ed.). Mahwah, NJ, Lawrence Elbaum Associates Inc.
- Lamon, S. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 629-668). Charlotte, NC: Information Age Publishing.
- Langrall, C. & Swafford, J. (December 2000). Two balloons for two dollars: Developing proportional reasoning. *Mathematics Teaching in the Middle School*, *6(4*), 254-261.
- Lim, K. (April 2009). Burning the candles at just one end. *Mathematics Teaching in the Middle School*, *14(8),* 492-500.
- Lo, J., Watanabe, T, & Cai, J. (2004). Developing ratio concepts: An Asian perspective. *Mathematics Teaching in the Middle School*, *9(7),* 362-367.
- Olson, M., Zenigami, F., & Slovin, H. (March 2010). The percent predicament. *Mathematics Teaching in the Middle School*, *15(7),* 374-376.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.RP.1-3)

References, continued

- Parker, M. & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research*, *65(4),* 421-481.
- Parker, M. (2004). Reasoning and working proportionally with percent. *Mathematics Teaching in the Middle School*, *9(6),* 326-330.
- Riehl, S. & Steinthorsdottir, O. (November 2014). Revising Mr. Tall and Mr. Short. *Mathematics Teaching in the Middle School*, *20(4),* 220-228.
- Steinthorsdottir, O. Sriraman, B. (2009). Icelandic 5th-grade girls' developmental trajectories in proportional reasoning. *Mathematics Education Research Journal*, *21(1),* 6-30.
- The Mr. Short / Mr. Tall puzzle (October 2007). *Module 1 - How People Reason*. 3. Retrieved from https://digitalcommons.unl.edu/cgi/viewcontent.cgi?referer=https://www.google.com/&httpsredir=1&article=1002&context=adaptworkshopmodule1.
- Van De Walle, J., Karp, K., Bay-Williams, J. (2010). *Elementary and Middle School Mathematics* (7th ed.). Boston, MA: Pearson Education, Inc.
- Watanabe, T. (October 2015). Visual reasoning tools in action. *Mathematics Teaching in Middle School*, *21(3),* 152-160.
- Weber, M., Pierone, A., & Strom, A. Obstacles in developing robust proportional reasoning structures: A story of teachers' thinking about the shape task. Retrieved from http://sigmaa.maa.org/rume/crume2016/Papers/RUME_19_paper_107.pdf.
- White, P. & Mitchelmore, M. (2005). Teaching percentage as a multiplicative relationship. Philip Clarkson et al (team from Flagship of ACU). 783-790. Melbourne, Australia: Deakin University press. Retrieved from http://www.merga.net.au/documents/RP912005.pdf
- Yopp, D. (March 2016). Dilations and the equation of a line. *Mathematics Teaching in Middle School*, *21(7),* 406-414.

STANDARDS	MODEL CURRICULUM (7.NS.1-3)
THE NUMBER SYSTEM **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **b.** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. *Continued on next page*	**Expectations for Learning** In Grade 6 students learned to locate rational numbers on the number line. In Grade 7 they extend their understanding of operations with fractions to operations with rational numbers. This is the first time students will add, subtract, multiply, and divide negative rational numbers. Time should be spent developing understanding through the use of models and manipulatives to discover the rules of negative integers and become fluent in applying them. At this point, it is essential that students understand the relationship among all four operations and their properties. In future grades, students will apply their understanding of operations with rational numbers to expressions and equations. **ESSENTIAL UNDERSTANDINGS** **Rational Numbers** • The set of integers consists of positive whole numbers, their opposites, and 0. • A rational number is any number that can be written as the quotient or fraction $\frac{p}{q}$ of two integers, a numerator p, and non-zero denominator q. • A rational number can be converted to a decimal using long division; the decimal form of a rational number terminates in 0s or repeats. • In a fraction the negative sign can be written in the numerator, the denominator, or out front, e.g., $\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$. **Addition and Subtraction** • When modeling operations with integers on a number line, the sign of the number indicates the direction and the number indicates the amount of spaces moved. • In a number line model, the subtraction sign means to change directions. • A number and its opposite are additive inverses; they have a sum of 0, i.e., $a + (-a) = 0$. • Subtraction of rational numbers is adding the additive inverse, i.e., $p - q = p + (-q)$ • The absolute value of $p - q$ is just the distance from p to q, regardless of direction. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.NS.1-3)
d. Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. **c.** Apply properties of operations as strategies to multiply and divide rational numbers. **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. *Continued on next page*	**Expectations for Learning, continued** **ESSENTIAL UNDERSTANDINGS, CONTINUED** **Multiplication and Divisions** • Multiplication of rational numbers can be modeled on the number line. • A positive product is the result of multiplying two numbers with the same sign. • A negative product is the result of multiplying two numbers with different signs. • Division is the inverse of multiplication, so the same rules for rational numbers apply. • Division can be written using a fraction bar. • Every quotient of integers (with a nonzero divisor) is a rational number. • A repeating quotient has a line of the repeating numerals. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to describe mathematical reasoning. • Apply and justify mathematical concepts, terms, and their properties. • Draw a picture or create a model to make sense of a problem. • Compute using strategies or models. • Determine reasonableness of results. • Use different properties of operations flexibly. *Continued on next page*

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.NS.1-3)</th></tr>
<tr><td>7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.</td><td>
<h2>Expectations for Learning, continued</h2>
<h3>INSTRUCTIONAL FOCUS</h3>
Note: Although, rote memorization of the names of the properties is not encouraged, it is expected for teachers to use formal language so that students gain familiarity and are able to recognize and apply the correct terminology.
Addition and Subtraction

Recognize that opposite numbers are additive inverses and have a sum of 0.
Use models to represent and solve addition and subtraction of rational numbers, i.e., number line or chips.
Solve mathematical and real-world problems using addition and subtraction of rational numbers.
Interpret the sums and differences of rational numbers in real-world contexts.
Show that the distance between two rational numbers on a number line is the absolute value of their difference.
Apply the Associative Property of Addition, Commutative Property of Addition, Additive Identity Property, and Additive Inverse Property when solving problems.

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STANDARDS	MODEL CURRICULUM (7.NS.1-3)
7.NS.1-3, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS, CONTINUED** **Multiplication and Division** • Use properties of operations to connect previous understanding of multiplication and division to rational numbers. • Connect multiplication of integers to the distributive property. • Multiply and divide integers. • Solve mathematical and real-world problems using multiplication and division of rational numbers. • Interpret the product and quotient of rational numbers in real-world contexts. • Convert a rational number to a decimal, and understand that the decimal form of a rational number will repeat or terminate. • Apply the Associative Property of Multiplication, Commutative Property of Multiplication, Identity Property of Multiplication, Inverse Property of Multiplication, Distributive Property, and Zero Product Property when solving problems. • Use patterns to recognize the sign of the product when a string of numbers is multiplied together. • Recognize that the number line can be divided into an infinite number of segments which can be represented by decimals or fractions. • Explain the difference between a terminating and repeating decimal. **Operations with Rational Numbers** • Solve real-world and mathematical problems involving the four operations with rational numbers. • Solve problems involving complex fractions. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.NS.1-3)
7.NS.1-3, continued	**Content Elaborations** • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45 • Ohio's K-8 Learning Progressions, The Number System, pages 16-17 **CONNECTIONS ACROSS STANDARDS** • Use properties of operations to generate equivalent expressions (7.EE.1-2). • Solve multi-step, real-world numerical and algebraic equations and/or inequalities with rational numbers (7.EE.3-4). • Use proportional relationships to solve real-world and mathematical problems (7.RP.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.3 *Construct viable arguments and critique the reasoning of others.*
MP.6 *Attend to precision.*
MP.7 *Look for and make use of structure.*

Students in Grade 6 represented negative numbers and worked with absolute values for the first time. In Grade 7, learning now progresses toward exploring and applying the rules for operations (addition, subtraction, multiplication and division) with integers. Although instruction may start with integers, ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with positive and negative fractions and decimals.

THE NEGATIVE SIGN

Many students struggle with the negative sign. Historically, the concept of the negative sign that we use today did not even emerge until 1867 because of its conceptual difficulties. One reason for confusion is that students think that the idea of a negative number is impossible since they cannot imagine negative quantities. Therefore, they think that problems such as $x - 5 = 2$ are impossible, for in their minds one cannot subtract a larger number from a smaller number. So, when a negative presents itself, many students prefer to change the structure of an equation. One way to help students is to talk about values, order, and direction instead of quantities. For example, positive 5 is greater than positive 4, but –4 is greater than –5.

Another reason for confusion is that the negative sign can mean several things:

- A sign attached to a number to form negative numbers;
- A subtraction; or
- An indication to take the opposite of.

For example in the problem $3 - (-6)$, the first negative sign could mean *subtraction* or the *opposite of* and the second negative could mean *negative* 6. Not only are there several meanings, but the meaning of the negative sign can change when solving the same equation. For example in $5 - y = 7$, the negative sign initially means *subtraction*. However, when –5 is subtracted from both sides resulting in $-y = 2$, the negative sign indicates *the opposite of*. Then in the final form of $y = -2$, the negative sign represents a *sign* of a number. Because of the confusion around the negative sign it may be helpful for students to understand the different meanings of the negative sign and identify which meaning is used when in a problem including the meaning shifts.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

EXAMPLE

- Which is greater $--5$ or 5?
- Which is greater $-x$ or x?
- Which is greater -6 or x?
- Which is greater $x + x$ or x?

Discussion: Students typically have difficulties with these types of problems because they have difficulty viewing negative signs as "the opposite of." In parts **b.-d.**, students have a hard time recognizing that they do not have enough information. For example, in part **b**. if x is a positive number, x would be greater. However, if x is a negative number, $-x$ is greater, and if x were 0, the two statements would be equal. To help counter these difficulties, it may be helpful during instruction to read $-x$ as "the opposite of x" instead of "negative x." That way students will not incorrectly infer from the language that "negative x" could indeed represent a positive number. For part **c**., you could challenge students to place the two numbers on a number line and allow them to discover how *x* could appear on either side of the number line regardless if it had a negative sign attached to it.

EXAMPLE

Solve $-x = 3$ and justify your steps.

Discussion: Some students may just move the minus sign from one side of the equation to the other side without knowing why. Move students toward understanding by using questions such as "If the opposite of *x* is three, what is *x*?" or by pointing students toward rewriting the equation using the Multiplicative Identify Property such as $-1 \cdot x = 3$. Students may also have difficulty substituting –3 back into the original equation in order to justify their solution because they may have difficulty understanding the concept of having two negative signs next to each other such as – –3. This is especially true if they do not use parenthesis because they may be seeing both negative signs as a sign attached to a negative number instead of seeing the first sign means "opposite of" and the second sign means the sign attached to the negative number. Using parenthesis helps them make this distinction.

EXAMPLE

Solve $7 - x = 3$ and justify your steps.

Discussion: Although, this example aligns more closely with cluster 7.EE.3-4, its purpose here is to highlight the negative sign. Students will often omit the minus sign to incorrectly get $x = 3 - 7$ instead of $-x = 3 - 7$. Although some students may simply have forgotten the minus sign, many will not understand that the negative sign changes types: it moves from meaning *subtraction* to an indication of *the opposite of* or a *negative sign attached to the variable* using the Multiplicative Identity Property $-1x = 3 - 7$. Therefore, it may appear that some students just forgot, they may have dropped the negative sign intentionally because they did not know what to do with it.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

Another issue that is confusing to students is that a negative sign appears at different heights or lengths depending on fonts (-3 or -3 or $^{-}3$ or -3) or on different parts of a fraction. Students should understand that $-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$ since $-1(2 \div 3)$ and $-2 \div 3$ and $2 \div -3$ are all equivalent to $-0.\overline{6}$. Another confusing thing is that sometimes parenthesis are included when there are two negative signs and sometimes they are not. Using parenthesis helps students differentiate between two different meanings of negative signs. Also, sometimes students think that all parenthesis mean multiplication.

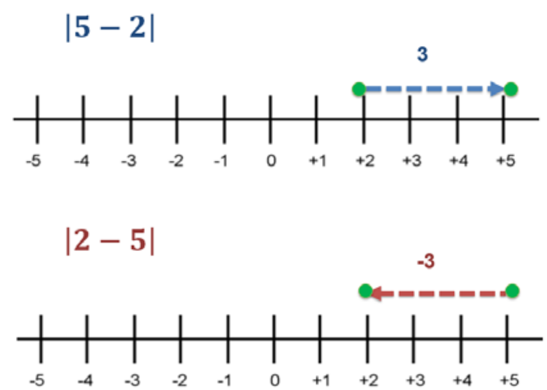
PROPERTIES OF OPERATIONS

It is important when performing operations that students are able to justify their steps using the properties. Although, the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion, so students are able to gain familiarity with and recognize the correct terminology. Students are still expected to use the thinking of the properties even if they are not able to recall their formal names. For example students should be able to recognize and apply that –3 + 3 = 0 even if they forget that the official name is the Additive Inverse Property. If they forget, the teachers or their classmates should be able to reiterate the correct property during discussion.

Students need to become fluent in using operations and properties of operations with all rational numbers, not just with integers. Although all properties of operations should be addressed, this cluster should especially emphasize the Additive Inverse Property, the Multiplicative Inverse Property, and the Distributive Property.

ABSOLUTE VALUE

In Grade 6, students should have learned that the absolute value of a number does not take into account sign or direction; it only is a measure of distance (magnitude) from 0. Discourage students from saying that the "answer is always positive or 0" since that will lead to misconceptions when students encounter problems such as $|4x - 2| = 18$ in high school. Instead emphasize that it is the "distance from 0." This is why the value of something like $|x| = -5$ has no solution, since distance cannot be negative. Used in the context of a number line the absolute value of an expression such as $|5 - 2|$ is the length of the arrow from 2 to 5. This is equivalent to the length of the arrow from 5 to 2 which is represented by $|2 - 5|$. Therefore $|5 - 2| = |2 - 5|$. Notice this is different than the values of 5 – 2 and 2 – 5 which take into account the sign (direction).



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

EXAMPLE

Name an equivalent expression to $\left|13\frac{8}{17} - 5.369\right|$. Explain your reasoning.

Discussion: The numbers in this problem are intentionally difficult to calculate in order to push students to rewrite the expression as $\left|5.369 - 13\frac{8}{17}\right|$. They should be able to explain that the distance between the two points is the same regardless of the starting point. However, draw attention to the fact that the values of $13\frac{8}{17} - 5.369$ and $5.369 - 13\frac{8}{17}$ are different because then the direction(sign) becomes a factor.

EXAMPLE

Compare $|a|$ and $|-a|$.

Discussion: Students should see that these are equivalent expressions because a and $-a$ are both the same distance from zero. Also draw attention that a could be positive or negative number in both situations. For example,

if $a = -17$,
then $|a| = |-17| = 17$,
and $|-a| = |-(-17)| = |17| = 17$.

Note that in this example, $-a$ is positive. Highlight examples such as this to address a common misconception that $-a$ is negative

To support understanding absolute value as distance, students should sometimes solve and explain absolute value problems using number lines.

EXAMPLE

Find the solution to the following equations using a number line for each item.

a. $|x| = 2$
b. $|x + 1| = 2$
c. $|x + 2| = 2$
d. $|x + 3| = 2$
e. $|x - 1| = 2$
f. $|x - 2| = 2$
g. $|x - 3| = 2$

Discussion: This task is suited as an instructional tool in understanding absolute value and preventing misconceptions; it is not an appropriate assessment item. Students should be encouraged to solve the equations <u>mentally</u> (not algebraically) and then graph the solutions on a number line. They should see that each absolute value equation has two solution points that make the equation true but one distance (2). Also, the solutions are not always the number to the right of the equal sign. In addition they will see the solutions do not always have to have both a positive and negative solution; it could have two positive solutions and two negative solutions. Moreover, they should be able to see that although the center can shift, the distance from the center remains the same.

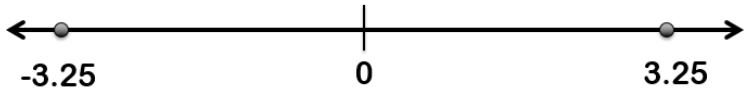
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

ZERO PAIRS

Students can make sense of the zero pair (Additive Inverse Property) in the context of real-life situations. For example if the elevator goes up 3 floors (+3) and then goes down 3 floors (–3), the resulting change is 0. If Jean walks 2.3 miles away from home (2.3) and then 2.3 miles toward home (–2.3), she ends up at 0 miles from home. Through the exploration of absolute values, students can discover the zero pair.

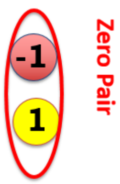
EXAMPLE

- Compare the distance between –3.25 and 0 and 3.25 and 0. Explain.
- What is the distance between the two points? Why?
- What is the value of the sum of the two points? Why?
- Write an equation to express the distance between the two points?
- Write an equation to express the value of the sum of the two points?
- Compare the two equations.
- Describe the relationship between the two points.

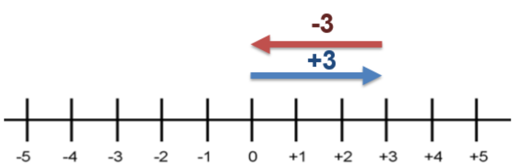


The zero pair can be illustrated using integer chips by adding both a positive and a negative chip simultaneously. It can also be illustrated on a number line by overlapping line segments.

Integer Chips



Number Line



ADDING AND SUBTRACTING RATIONAL NUMBERS

Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined. Repeated opportunities over time with models will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules.

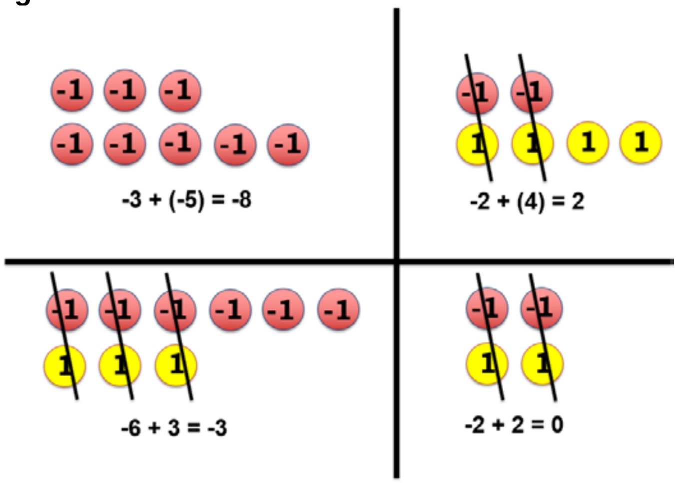
Fractional rational numbers, decimals, and whole numbers should also be used in computations and explorations. Students should be able to give contextual examples of integer operations, write and solve equations for real-world problems, and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Integer Chips

Two-color counters or colored chips can be used as a physical model for adding and subtracting integers. Although integer chips allow the idea of the zero pair (Additive Inverse Property) to be apparent, they are not useful in representing non-integer rational numbers for the numbers must be integers. *Note: Algebra tiles can be used in the place of integer chips for consistency across concepts.*

With one color designated to represent positives and a second color for negatives, addition/subtraction can be represented by placing the appropriate numbers of chips for the addends and their signs on a board. Using the notion of opposites (or the Zero Pair/Additive Inverse Property), the board is simplified by removing pairs of opposite colored chips. The answer is represented by the total of the remaining chips with the sign representing the appropriate color.

Figure 1: Addition



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

One of the difficulties students have when subtracting rational numbers is transitioning from an arithmetic view of subtraction (where the subtrahend is smaller than the minuend) to an algebraic view of subtraction (adding the opposite). Using integer chips should illustrate this concept.

Figure 2: Subtraction

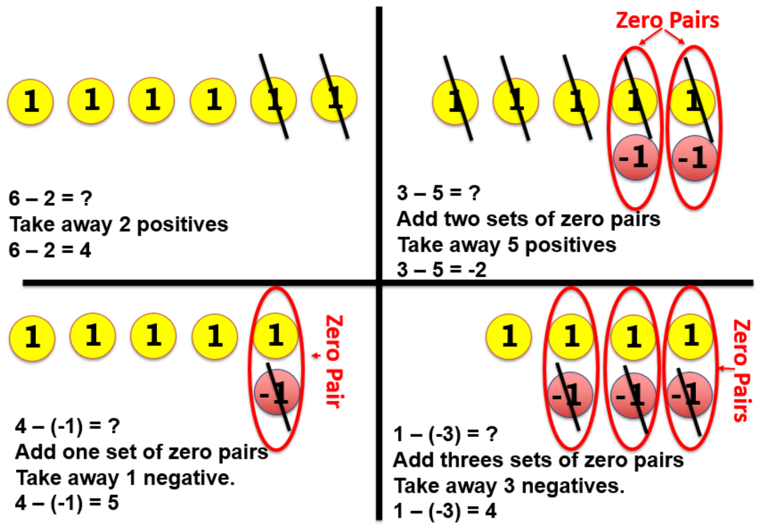
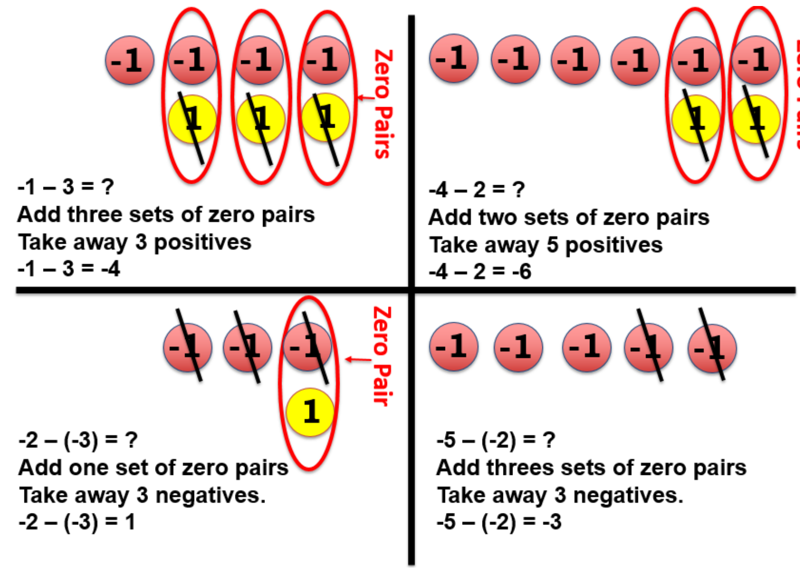


Figure 3: Subtraction



Number Lines

Number lines present a visual image for students to explore and record addition and subtraction results. One of the positive aspects about using a number line model is that it is not limited to integers; it also lends itself toward connections on the coordinate plane. Students can use number lines with arrows and hops. When using number lines, establishing which factor will represent the length, number, and direction of the hops will facilitate understanding.

In previous grades students used number lines to model operations. Now in Grade 7, the arrows on the number line have directions. If the second line segment overlaps the first in an opposite direction, it will backtrack and "undoes" part or parts of the first segment. After allowing students time working with the models, have students generalize rules for addition and subtraction.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

Figure 1: Adding Using the Same Sign

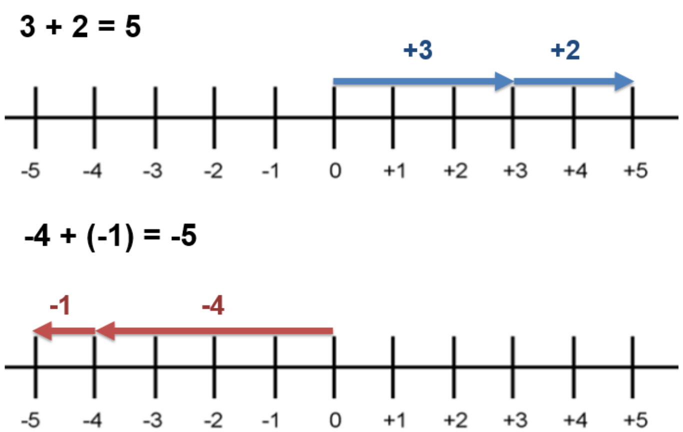
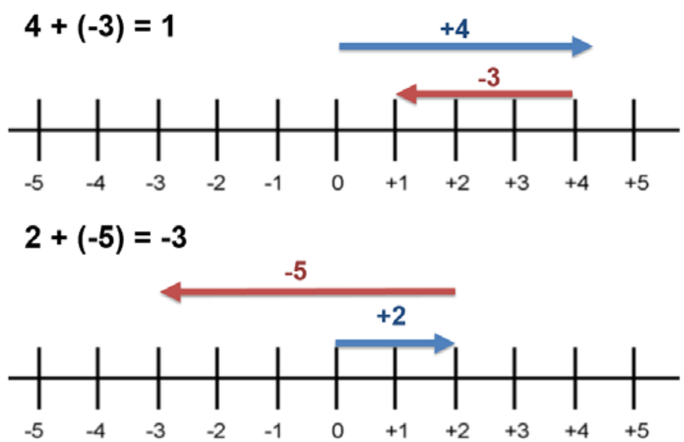


Figure 2: Adding Using Different Signs



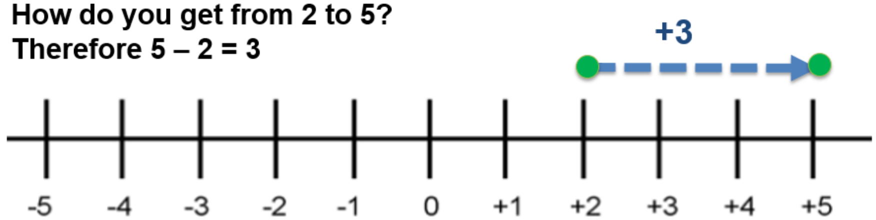
There are several ways to solve subtraction problems using a number line. Some methods use the subtraction sign to mean "change directions." Lessons such as Elevator Arithmetic and Flipping for Integers, and Walking the Number Line use similar methods. However, another way to look at subtraction is to view it as a missing addend problem which is used in previous grades. For example, 2 – 4 would be the same as –4 + ? = 2 and –3 – (–6) would be the same as –6 + ? = –3. Using this method there needs to be a distinction between the distance (absolute value) which is always positive and "how to get there" which includes both a magnitude (distance) and a direction. Think of subtraction as "[directed] difference" that is an arrow from a point to a point or "point + arrow = point". Students should eventually make the connection that $p - q$ is the same as $p + (-q)$. In other words, subtraction is just adding the opposite value.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

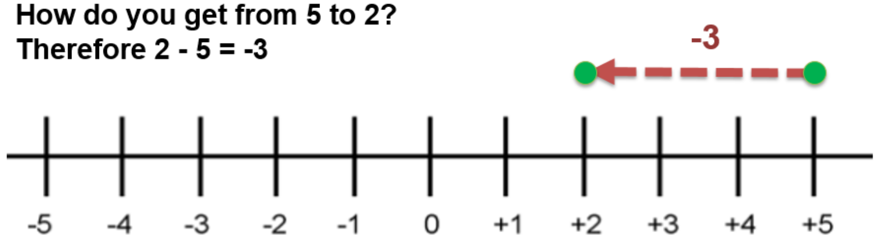
EXAMPLE:

Part A:

5 - 2 = ?
Think: 2 + ? = 5
How do you get from 2 to 5?
Therefore 5 – 2 = 3



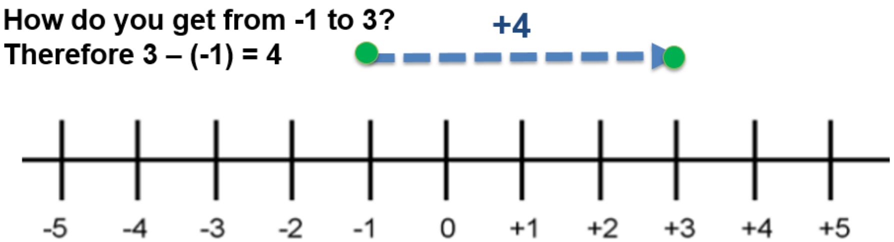
2 - 5 = ?
Think: 5 + ? = 2
How do you get from 5 to 2?
Therefore 2 - 5 = -3



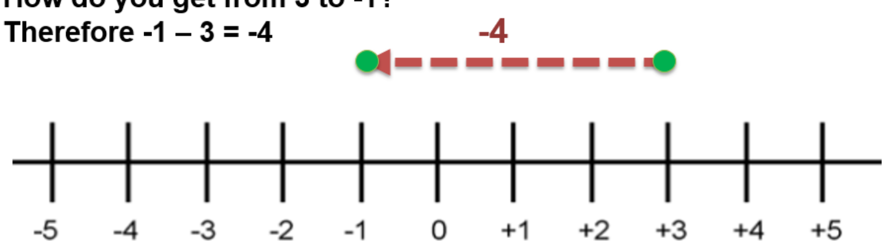
- What is similar between the two problems?
- What is different?

Part B:

3 – (-1) = ?
Think: -1 + ? = 3
How do you get from -1 to 3?
Therefore 3 – (-1) = 4



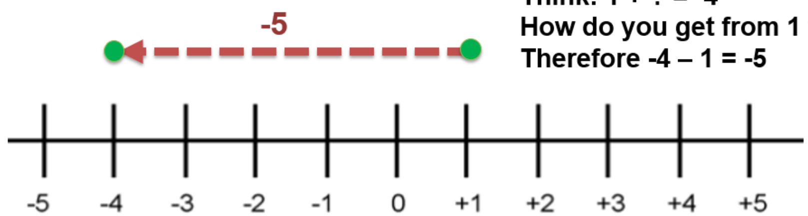
-1 – 3 = ?
Think: 3 + ? = -1
How do you get from 3 to -1?
Therefore -1 – 3 = -4



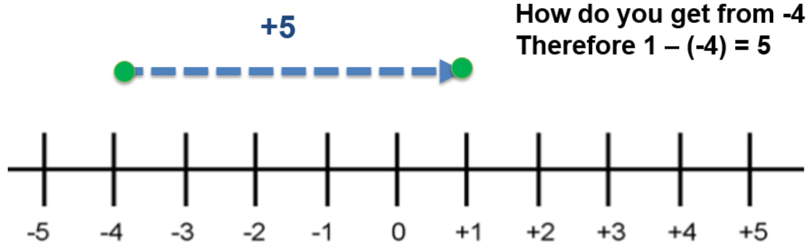
- What is similar between the two problems?
- What is different?

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

Part C



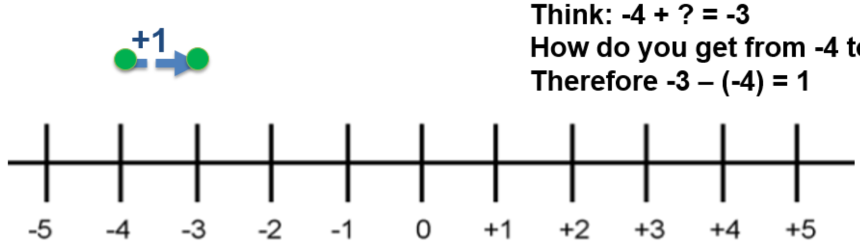
-4 – 1 = ?
Think: 1 + ? = -4
How do you get from 1 to -4?
Therefore -4 – 1 = -5



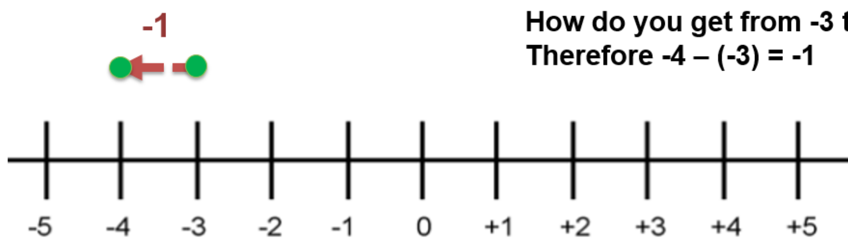
1 – (-4) = ?
Think: -4 + ? = 1
How do you get from -4 to 1?
Therefore 1 – (-4) = 5

- What is similar between the two problems?
- What is different?

Part D



-3 – (-4) = ?
Think: -4 + ? = -3
How do you get from -4 to -3?
Therefore -3 – (-4) = 1



-4 – (-3) = ?
Think: -3 + ? = -4
How do you get from -3 to -4?
Therefore -4 – (-3) = -1

- What is similar between the two problems?
- What is different?

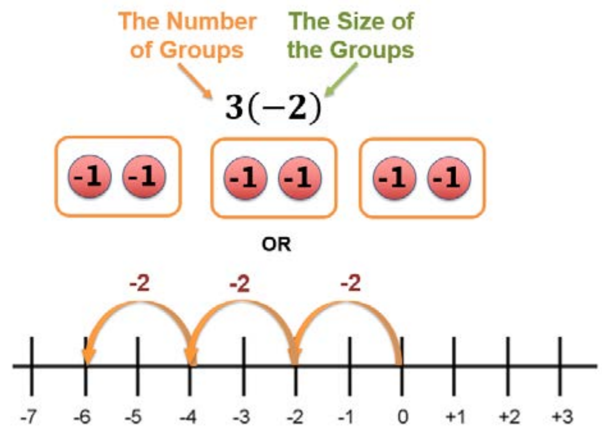
Part E

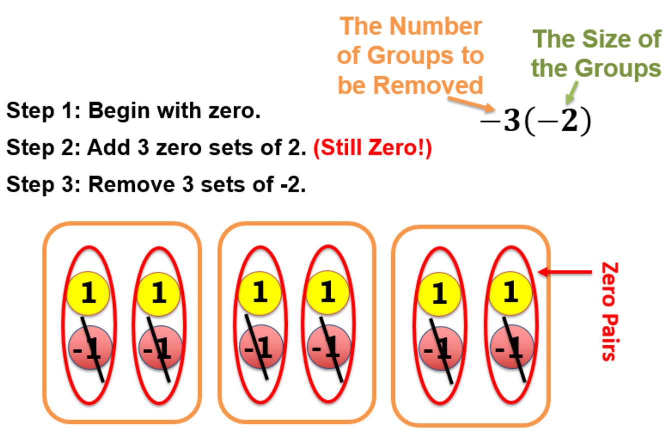
- Based on what on the what you noticed in Parts **A-D**, create a rule or rules for subtracting integers. Explain why your rule(s) work.
- How does that compare to your rules for adding integers?

MULTIPLYING RATIONAL NUMBERS

Multiplying and dividing integers should be thought of as an extension of adding and subtracting integers. Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers.

In multiplication, the first factor indicates the number of sets, and the second factor indicates the size of the set. This can be easily modeled with situations involving a positive times a positive or a positive times a negative. A negative times a positive can be inferred using the Commutative Property of Multiplication.





However, a negative times a negative can be problematic, for students want to know "How can we have a negative group of something?" One way to view it is as repeated subtraction. If the first factor being positive indicates repeated addition, then the first factor being negative indicates repeated subtraction. Therefore "negative 3 sets of negative –2" or $-3(-2)$ means to remove 3 sets of –2 from zero.

Another method of understanding multiplication of negative numbers is fast forwarding and rewinding students walking backwards and forwards. Student can then see that when you rewind (–) and walk backwards (–), the video shows them walking forward. An app called Reverse Vid for iPhone allows students to rewind videos. See also the lesson Multiplying Integers Using Videotape for a similar idea.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

Students will discover that they can multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. They should then analyze and solve problems leading to the generalization of the rules for operations with integers.

Another method for learning multiplication/division rules is to use patterns. Beginning with known facts, students can predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2, and 3 below).

Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \cdot -4 = 16$, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products, and commutativity of multiplication.

EXAMPLE

Table 1	Table 2	Table 3
$4 \cdot 4 = 16$	$4 \cdot 4 = 16$	$-4 \cdot -4 = 16$
$4 \cdot 3 = 12$	$4 \cdot 3 = 12$	$-4 \cdot -3 = 12$
$4 \cdot 2 = 8$	$4 \cdot 2 = 8$	$-4 \cdot -2 = 8$
$4 \cdot 1 = 4$	$4 \cdot 1 = 4$	$-4 \cdot -1 = 4$
$4 \cdot 0 = 0$	$4 \cdot 0 = 0$	$-4 \cdot 0 = 0$
$4 \cdot -1 =$	$-4 \cdot 1 =$	$-1 \cdot -4 =$
$4 \cdot -2 =$	$-4 \cdot 2 =$	$-2 \cdot -4 =$
$4 \cdot -3 =$	$-4 \cdot 3 =$	$-3 \cdot -4 =$
$4 \cdot -4 =$	$-4 \cdot 4 =$	$-4 \cdot -4 =$

- Is it always true that multiplying a negative factor by a positive factor results in a negative product? Explain.
- Does a positive factor times a positive factor always result in a positive product? Explain.
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?
- How is the numerical value of the product of any two numbers found?

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

EXAMPLE

- Fill in the blanks

$$(-1) = (-1)^{\boxed{1}} = \_\_\_\_$$
$$(-1)(-1) = (-1)^{\square} = \_\_\_\_$$
$$(-1)(-1)(-1) = (-1)^{\square} = \_\_\_\_$$
$$(-1)(-1)(-1)(-1) = (-1)^{\square} = \_\_\_\_$$
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$$(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1) = (-1)^{\square} = \_\_\_\_$$

- What patterns do you notice?
- What do you think $(-1)^{20}$ equals? Explain.
- What do you think $(-1)^{25}$ equals? Explain.
- How does what you found relate to the rules for multiplying integers?

Division of integers is best understood by relating division to multiplication and applying the rules. The Inverse Property of Multiplication should be used to connect division to multiplication. Since $8 \div (-2)$ is the same as $8\left(-\frac{1}{2}\right)$, the rules for multiplication apply to division. In time, students will transfer the rules to division situations. *(Note: In standard 7.NS.2b, the algebraic language $(-(p/q) = (-p)/q = p/(-q))$ is written for the teacher's information, not as an expectation for all students.)*

CHANGING FORMS OF RATIONAL NUMBERS

In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. They can do this by making equivalent fractions with denominators using powers of ten or by using long division.

Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalent fractions can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected for the students. Terminating decimals fall exactly on some tick mark on a number line; however repeating decimals do not. For example $\frac{1}{3}$ is always subdividing an interval of smaller and smaller sizes. Students can also explore patterns to determine which fractions repeat and which fractions terminate. Technology can be used to aid students in recognizing patterns.

Students can also use their knowledge of proportions and unit rates to help them understand why they can convert a fraction to a decimal by dividing.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

EXAMPLE

If 4 cups of sugar are needed for every 9 cups of flour, how many cups of sugar are needed for every 1 cup of flour.

÷ 9

sugar (cups)	4	c
flour (cups)	9	1

÷ 9

$$\frac{4 \div 9}{9 \div 9} = \frac{c}{1}$$

- Use proportional reasoning to solve your problem.
- How could you use division to solve your problem?
- What is the connection between solving a unit rate proportion and converting a rational number such as $\frac{4}{9}$ to a decimal?

Complex Fractions

Students should become familiar with solving problems involving complex fractions. Draw attention to the Multiplicative Identity Property and the Multiplicative Inverse Property for solving expressions with complex fractions. This concept connects nicely with cluster 7.RP.1-3.

EXAMPLE

If Miguel walks $\frac{2}{3}$ of a mile in $\frac{3}{4}$ of an hour, what speed in miles per hour did he walk?

Step 1: $\dfrac{2/3}{3/4}$

Step 2: $\dfrac{2/3}{3/4} = \dfrac{2/3}{3/4} \cdot \dfrac{4/3}{4/3}$ ← Multiplicative Identity Property

Step 3: $\dfrac{2/3}{3/4} = \dfrac{2/3}{3/4} \cdot \dfrac{4/3}{4/3} = \dfrac{2/3 \cdot 4/3}{1}$

Multiplicative Inverse Property

Step 4: $\dfrac{2/3 \cdot 4/3}{1} = \dfrac{8}{9}$ miles per hour

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Two-color counters or Algebra tiles
- Calculators
- Integer Chips is an applet by Holt Winston Rinehart.

Opposites and Zero Pairs

- Distances on the Number Line 2 is an Illustrative Mathematics task that reinforces the understanding of opposites.

Adding and Subtracting Integers

- Card Sort: Integer Chips by Michael Fenton is a Desmos activity that uses integer chips to explore addition and subtraction of integers.
- Adding and Subtracting Integers is an applet by Geogebra that has students use a number line to add and subtract integers.
- Integer Operations by Daivd Cox is a Desmos activity that explores patterns in addition and subtraction of integers.
- Differences and Distances is an Illustrative Mathematics task that connects distances on a number line with subtraction using rational numbers.
- Operations on the Number Line is an Illustrative Mathematics task that helps solidify students' understanding of signed numbers. One has to assume the number line diagram is drawn to scale.
- Rounding and Subtracting is an Illustrative Mathematics task where students have to attend to precision when subtracting.

Multiplying and Dividing Integers

- Multiplying Integers Investigation by Greta is a Desmos activity that explores multiplication of integers.
- Why is a Negative Times a Negative Always Positive? is an Illustrative Mathematics task that explores multiplying negative numbers using the Distributive Property.
- Temperature Change is an Illustrative Mathematics task where students interpret the idea $-(p/q) = (-p)/q = p/(-q)$.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

Applying Integer Operations

- Weather Extremes is an activity by YummyMath that applies integer operations.
- The Vortex is an activity by YummyMath that applies integer operations.
- How Much Did the Temperature Change in Boston? is a Yummy Math activity that explores integers.

Approximating Decimals

- Repeating Decimals as Approximations is an Illustrative Mathematics task where students reflect on the meaning of approximating repeating decimals.

Curriculum and Lessons from Other Sources

- Illustrative Mathematics Curriculum, Unit 7.5: Rational Number Arithmetic has Lessons 1-14 that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- EngageNY, Grade 7, Module 2, Topic A, Lesson 1: Opposite Quantities Combine to Make Zero, Lesson 2: Using the Number Line to Model the Addition of Integers, Lesson 3: Understanding Addition of Integers, Lesson 4: Efficiently Adding Integers and Other Rational Numbers, Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers, Lesson 6: The Distance Between Two Rational Numbers, Lesson 7: Addition and Subtraction of Rational Numbers are lessons that pertains to this cluster.
- EngageNY, Grade 7, Module 2, Topic B, Lesson 10: Understanding Multiplication of Integers, Lesson 11: Develop Rules for Multiplying Signed Numbers, Lesson 12: Division of Integers, Lesson 13: Converting Between Fractions and Decimals Using Equivalent Fractions, Lesson 14: Converting Rational Numbers to Decimals Using Long Division, Lesson 15: Multiplication and Division of Rational Numbers are lessons that pertain to this cluster.
- Georgia Standards of Excellence Curriculum Frameworks, Grade 7, Unit 1: Operations with Rational Numbers contains many tasks pertaining to this cluster.
- The Utah Middle School Math Project is an open source textbook and workbook.

General Resources

- Arizona 6-8 and High School Progression on the Number System
  This cluster is addressed on pages 9-13.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

General Resources, continued

- Arizona 6-8 and High School Progression on the Number System
  This cluster is addressed on pages 9-13.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

References

- Bishop, J., Lamb, L., Phillip, R., Whitacre, I. & Schappelle, B. (September 2016). Unlocking the structure of positive and negative numbers. *Mathematics Teaching in the Middle School*, *22(2),* 84-91.
- Chiu, M. (2002). Metaphorical reasoning: Novices and experts solving and understanding negative number problems. *Educational Research Journal*, *17(1),* 19-41.
- Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6–8, The Number System; High School, Number*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Dickman, K. & Bofferding, L. (December 2016/January 2017). Zero pairs: Learning additive inverses. *Mathematics Teaching in the Middle School*, *22(5),* 308-311.
- Ellis, M. & Bryson, J. (April 2011). A conceptual approach to absolute value equations and inequalities, *Mathematics Teacher*, *104(8),* 592-598.
- Gallardo, A. (2002). The extension of the natural number domain to the integers in the transition from arithmetic to algebra. *Educational Studies in Mathematics*, *49*, 171-192.
- Gregg, J. & Gregg, D. (August 2007). A context for integer computation. *Mathematics Teaching in the Middle School*, *13(1),* 46-50.
- Kidd, M. (May 207). Be resolute about absolute value. *Mathematics Teaching in the Middle School*, *12(9),* 518-521.
- Lamb, L., Bishop, J., Phillipp, R., Schappelle, B., Whitacre, I., & Lewis, M. (August 2012). Developing symbol sense for the minus sign. *Mathematics Teaching in the Middle School*, *18(2),* 5-9.
- Lamb, L., Bishop, J., Phillipp, R., Schappelle, B., Whitacre, I., & Lewis, M. (2014). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics, *Journal for Research in Mathematics Education*, *45(1),* 19-61.
- Peil, Timothy. Section 5.1—integers: addition and subtraction. *Math 303—Foundations of Number Systems*. Retrieved from http://web.mnstate.edu/peil/M303/Math303.html Minnesota State University.
- Ponce, G. (November 2008). Using, seeing, feeling, and doing absolute value for deeper understanding. *Mathematics Teaching in the Middle School*, *14(4),* 234-240.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.NS.1-3)

References, continued

- Stephan, M. & Akyuz, D. (2012). A proposed instructional theory for integer addition and subtraction. *Journal for Research in Mathematics Education. 43(4),* 428-464.
- Taylor, S. & Mittag, K. (August 2015). Easy absolute value? absolutely. *Mathematics Teaching in the Middle School*, *21(1),* 49-52.
- Van De Walle, J., Karp, K., Bay-Williams, J. (2010). *Elementary and Middle School Mathematics* (7th ed.). Boston, MA: Pearson Education, Inc.
- Vlassis, J. (August 2008). The Role of mathematical symbols in the development of number conceptualization: The case of the minus sign. *Philosophical Psychology*, *21(4),* 555-570.
- Yakes, C. (2017). Rational number operations on the number line. *The Mathematics Enthusiast*, *14(1),* 309-324. Retrieved from https://scholarworks.umt.edu/cgi/viewcontent.cgi?article=1400&context=tme

STANDARDS	MODEL CURRICULUM (7.EE.1-2)
EXPRESSIONS AND EQUATIONS **Use properties of operations to generate equivalent expressions.** **7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.2** In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p − 0.15p) is equivalent to (1 − 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.*	**Expectations for Learning** In prior grades, students created equivalent expressions using the properties of operations with positive rational numbers and coefficients. They used the algebraic order of operations to simplify numerical expressions and evaluate algebraic expressions. Students in Grade 7 extend their knowledge to include properties of operations with positive and negative rational numbers. Students gain experience writing expressions in multiple ways. These expressions can serve different purposes and provide different ways of seeing a problem. This provides the foundation for analyzing and solving more complicated linear equations in Grade 8. **ESSENTIAL UNDERSTANDINGS** • Equivalent expressions always have the same value even if written in different forms. • Equivalent expressions can be generated using properties of operations (Distributive Property, Associative Properties of Multiplication, Associative Property of Addition, Commutative Property of Multiplication, Commutative Property of Addition, and Identity Property of Multiplication). • The order of operations is used to generate equivalent algebraic expressions. • The coefficient of a single variable is 1 even if it is not written. *For example, –x = –1x* and *x = 1x.* • A fractional coefficient can be written in two ways, e.g., $\frac{x}{3} = \frac{1}{3}x$. • Negative rational terms can be written in three ways, e.g., $\frac{1}{-3} = \frac{-1}{3} = -\frac{1}{3}$. • In problems involving percentages, 100% of the variable *x* can be written as *x* = *1x.* • Factoring a GCF can be used to write an equivalent expression. • Writing expressions in equivalent forms can serve different purposes and provide different ways of seeing a problem in context. *Continued on next page*

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Expectations for Learning, continued

MATHEMATICAL THINKING

- Use different properties of operations flexibly.
- Pay attention to and make sense of quantities.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Solve multi-step problems accurately.
- Compute accurately and efficiently with grade-level numbers.

INSTRUCTIONAL FOCUS

- Recognize simple equivalent expressions.
- Apply properties of operations to factor a GCF and expand linear expressions with rational coefficients.
- Utilize the order of operations with rational number coefficients to create an equivalent expression.
- Use the distributive property with positive and negative rational numbers. (Variables are limited to the power of one.)
- Use factoring of a GCF to rewrite equivalent expressions with positive and negative rational numbers. (Variables are limited to the power of one.)
- Write and interpret equivalent expressions that represent a real-world problem.
- Use the properties of rational numbers to combine like terms.

Content Elaborations

- Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45
- Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19

CONNECTIONS ACROSS STANDARDS

- Apply and extend previous understandings of operation with fractions to add, subtract, multiply, and divide rational numbers (7.NS.1-3).
- Use proportional relationships to solve multi-step ratio and percent problems (7.RP.3).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.2 *Reason abstractly and quantitatively.*
MP.6 *Attend to precision.*
MP.7 *Look for and make use of structure.*
MP.8 *Look for and express regularity in repeated reasoning.*

Students build on their previous understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions. Now in Grade 7, they extend the properties that were initially used with whole numbers to understand that properties hold for integers and rational numbers (positive and negative) as well. This cluster should not be taught in isolation but should be integrated into other clusters whenever appropriate.

PROPERTIES OF OPERATIONS

It is important that students are able to justify their thinking using the properties. Although, the focus should not be on identifying the properties of operation, teachers should be using their formal names in classroom discussion so students are able to gain familiarity with and recognize the correct terminology. Students are still expected to use the thinking of the properties even if they are not able to recall their formal names. For example, a student should be able to recognize and apply that –3 + 3 = 0 even if he or she forgets that the official name is the Additive Inverse Property. The teacher or classmate could reiterate the correct terminology to reinforce vocabulary.

Provide opportunities for students to use and understand the properties of operations. These include the Commutative, Associative, Identity, and Inverse Properties of Addition and of Multiplication, the Zero Property of Multiplication, and the Distributive Property. One method students can use to become convinced that expressions are equivalent is to justify each step using properties when simplifying an expression. They need to become fluent in using operations and properties of operations with all rational numbers, not just with integers.

Students should extend their understanding of properties of operations to expressions containing variables.

EXAMPLE

Find the equivalent expressions.

- $b - 2(2)$
- $2b - 4$
- $4 - 2b$
- $2b - 2$

e. $-4 + 2b$
f. $b - 2 + b - 2$
g. $2(b - 2)$
h. $b + b + b + b$

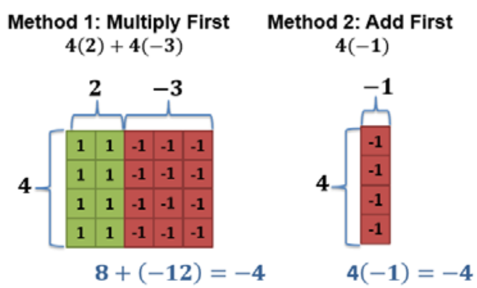
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

Distributive Property

Although students should be fluent using all the properties, special attention should be paid to the Distributive Property. Previously students have used area models to represent the Distributive Property, they can now extend this thinking using models to rational numbers and variables.

EXAMPLE

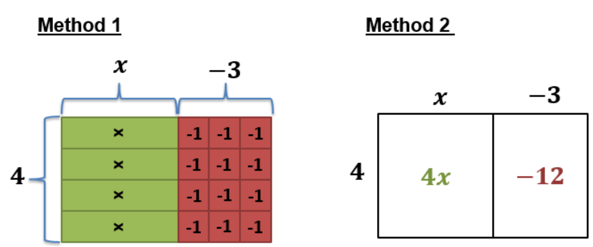
Evaluate $4(2-3)$



Students can transition to area models with variables using strategies such as Algebra tiles or boxes. The Algebra tiles allow students to see the area more conceptually, but the box allows for rational numbers besides integers. These area models will be useful in high school Algebra 1/Math 1 for multiplying and factoring polynomial expressions.

EXAMPLE

Simplify $4(x-3)$

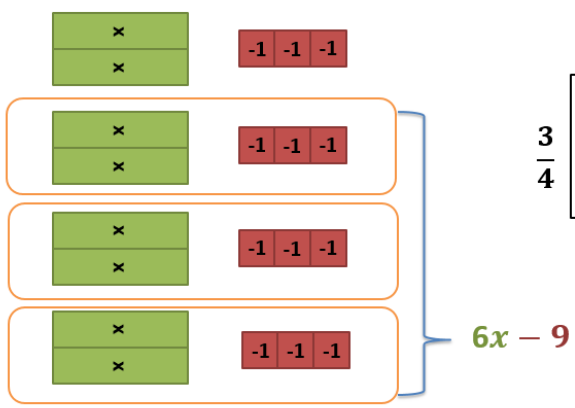


INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

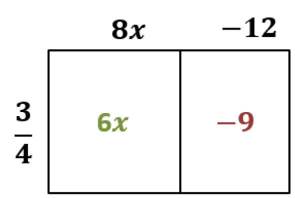
EXAMPLE

Simplify $\frac{3}{4}(8x - 12)$

Method 1



Method 2

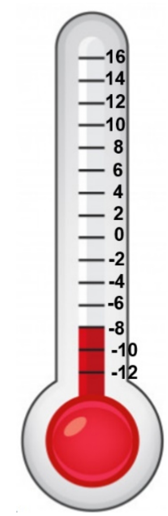


EQUIVALENT EXPRESSIONS

Provide opportunities to build upon the experience of writing expressions using variables in order to represent situations. Have students apply the properties of operations to generate equivalent expressions. These expressions may look different and use different numbers, but the values of the expressions are the same.

EXAMPLE

- Suppose the temperature is –4°F and the temperature drops by 6°F. Explain how to use the number line to find the new temperature.
- Using the context of a thermometer find three equivalent expressions that results in the same expression as –4 – 6.



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

Provide opportunities for students to experience expressions for amounts of increase and decrease. In 7.EE.2, when the expression is rewritten, the variable has a different coefficient. Within a context, the coefficient aids in the understanding of the situation. Another example is this situation which represents a 10% decrease: $b - 0.10b = 1.00b - 0.10b$ which equals $0.90b$ or 90% of the amount.

One method that students can use to become convinced that expressions are equivalent is by substituting a numerical value for the variable and evaluating the expression.

EXAMPLE

5.2(3 + 2x) is equal to 5.2 • 3 + 5.2 • 2x. Let x = –6 and substitute –6 for x in both equations.

1st Expression	**2nd Expression**
5.2(3 + 2 • –6)	5.2 • 3 + 5.2 • 2 • (–6)
5.2(3 – 12)	15.6 + (–62.4)
5.2(–9)	–46.8
–46.8	

Algebraic Order of Operations and Equivalent Expressions

In Grade 6 students wrote equivalent expressions using the algebraic order of operations. Now they extend that knowledge to negative numbers and place a greater emphasis on rational numbers including decimals and fractions. In Grade 7 subtraction should be thought of as the opposite of addition, and division should be thought of as the opposite of multiplication. *Note:* Avoid PEMDAS as it leads to many misconceptions and errors in computation. See 6.EE.1-4 for more information on using algebraic order of operations and strategies for avoiding PEMDAS.

EXAMPLE

$-\frac{2}{3}(6 + 3b) - (-2)^3$

Common errors for equations such as $5 - 3(4 - 2y)$ could include the following:

- Simplifying parenthesis first even though it is impossible to simplify, $5 - 3(2y)$;
- Detaching a coefficient from a parenthesis that indicates multiplication, $2(4 - 2y)$;
- Omitting multiplying the coefficient to all the numbers in the parenthesis, $5 - 12 - 2y$; and/or
- Losing the negative sign attached to the coefficient, $5 - 12 - 6y$.

Using models to represent the distributive property, rewriting the equation to an equivalent expression that adds the opposite, and drawing attention to the various meanings of the negative signs (See 7.NS.1-3) can help alleviate some of these errors.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

EXAMPLE

Jack, Jill, Jimmy, and Julie all simplified the problem $5 - 2(4x + 2)$ and got different answers. Evaluate each student's thinking and explain a way they could avoid future mistakes.

Jack	Jill	Jimmy	Juli
$5 - 2(4x + 2)$	$5 - 2(4x + 2)$	$5 - 2(4x + 2)$	$5 - 2(4x + 2)$
$3(4x + 2)$	$5 - 8x + 2$	$5 - 2(6x)$	$5 - 8x + 4$
$12x + 6$	$7 - 8x$	$5 - 12x$	$9 - 8x$

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. For example, having a student simplify an expression like 8 – 4(2*m* – 5) + 3*m* can bring to light several misconceptions. Do the students immediately subtract the 4 from the 8 before distributing the –4? Do they only multiply the –4 and the 2*m* and forget to distribute the –4 to both terms in the parenthesis? Do they collect all like terms 8 + (–4) + (–5) and 2*m* + 3*m* before distributing? Do they combine unlike terms in the parenthesis such as 2*m* and –5 first to make –3*m*. Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations which may be due to the misapplication of PEMDAS.

Combining Like Terms

Students started combing like terms in Grade 6 (6.EE.3-4). Now they will extend this concept to negative numbers. It may be helpful to use Algebra tiles to illustrate this process as it will prevent future misconceptions from forming such as trying to combine $2x$ and $3x^2$. Students should then extend that concept to other rational numbers besides integers.

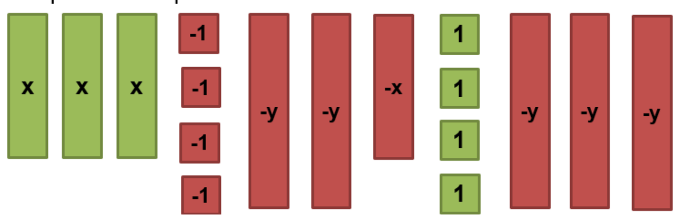
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

EXAMPLE

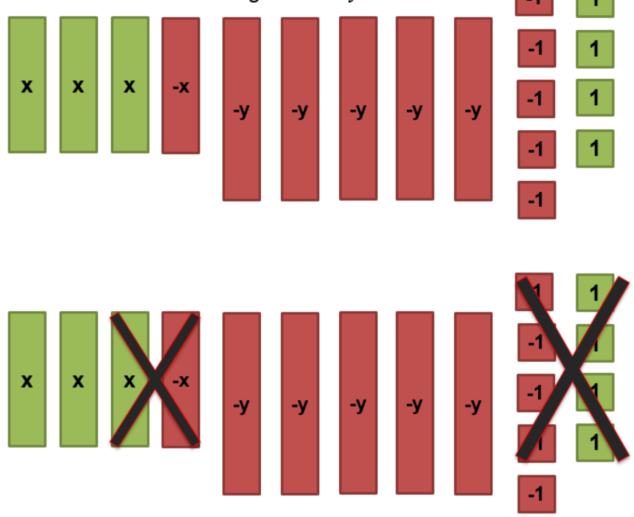
Simplify $3x - 5 - 2y - x + 4 - 3y$ using Algebra tiles.

Step 1: Rewrite $3x - 5 - 2y - x + 4 - 3y$ as $3x + (-5) + (-2y) + (-x) + 4 + (-3y)$.

Step 2: Represent the problem.



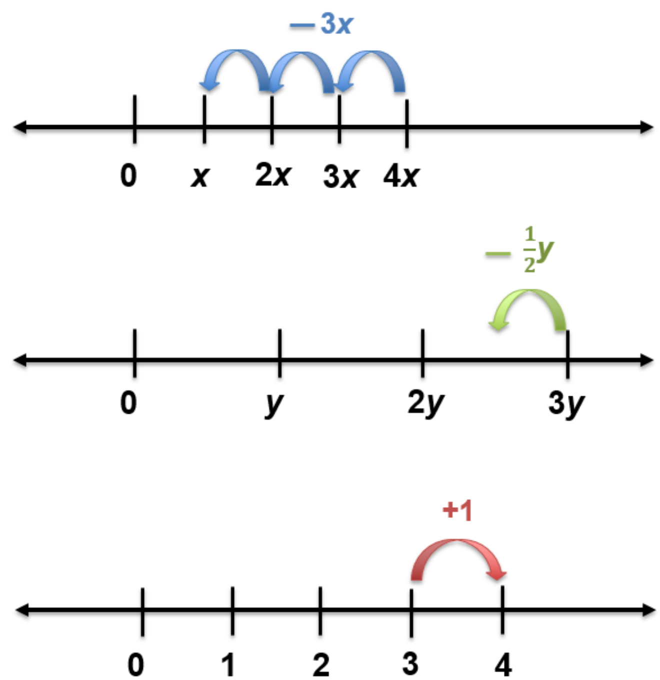
Step 3: Combine like terms to get $2x - 5y - 1$.



EXAMPLE

Simplify: $4x + 3 - 3x + 3y + 1 - \frac{1}{2}y$

$(4x - 3x) + (3y - \frac{1}{2}y) + (3 + 1)$



Discussion: The benefit of the number line model over the Algebra tiles model is that it can show fractional values of variables. An arithmetic number line contains 0, and 1 is the unit of reference. An Algebra number line contains 0, but *x* is the unit of reference. Each variable has its own number line with its own interval since variables typically can have different amounts. It would be useful to have a discussion about how although the *x*'s seem smaller in the diagram, they could have a bigger value or even the same value as *y*. They can apply the same strategies to add and subtract on a number line in previous grades. Therefore the solution is $x - 2\frac{1}{2}y + 4$.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

EXAMPLE
Simplify $(3.2a - 6b) - (2 + 4.1a - 7b)$.

Discussion: Students oftentimes have difficulty distributing the negative sign. It may be helpful to draw attention to the invisible one using the Multiplicative Identity Property and have students rewrite the problem as $(3.2a - 6b) + (-1)(2 + 4.1a - 7b)$.

Students should also solve real-world problems that require combining like terms.

EXAMPLE
Malcolm is putting a fence around his patio. The length needs to be 3 feet longer than twice its width. One side of the patio is attached to his house.

- **a.** Write at least 3 equivalent expression representing the perimeter.
- **b.** If the width of the patio is 4.25 feet, how much fencing will he need?
- **c.** Which expression is the easiest to answer the question in part b.? Explain.
- **d.** Now Malcom decides he wants to put 1 ft^2 decorative tiles around his patio while still keeping the interior dimensions the same. Write 3 expressions representing the exterior perimeter of the patio.
- **e.** How much fencing is needed for the exterior perimeter of the patio?

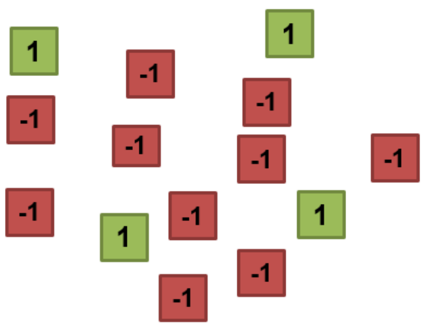
Discussion: Discuss with students how fencing is only needed for the three sides of the rectangular patio. Students may come to the realization that a gate is needed for the patio, which may affect the fencing. Different students may decide on different sized gates, which is an opportunity of discussion.

Distributive Property and Factoring
Writing equivalent expressions includes simple cases of factoring out a GCF. This can be illustrated using the area model that students are already familiar with. In this model the students are given the areas, and they are asked to find the lengths and widths. It might be wise to point out that, although in real-life length and width cannot be negative, our model allows for lengths and widths to be negative to illustrate the concept.

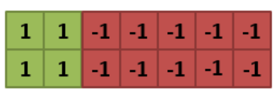
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

EXAMPLE

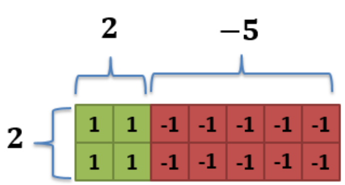
Create a rectangle of 4 + (-10) keeping the same color tiles together.



Find the length and width.



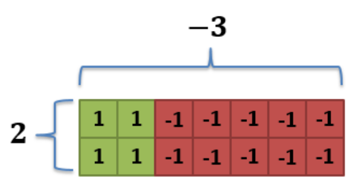
Method 1: Find each section first



$2 \text{ by } (2 + (-5))$

$2 \text{ by } -3$

Method 2: Add First

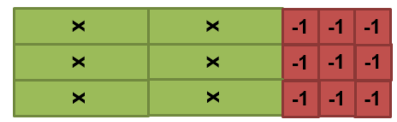


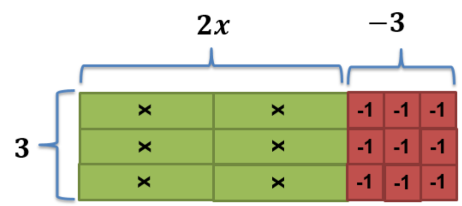
$2 \text{ by } -3$

EXAMPLE

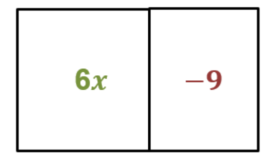
Write an expression to represent the possible dimensions of a rectangle with an area of $6x - 9$.

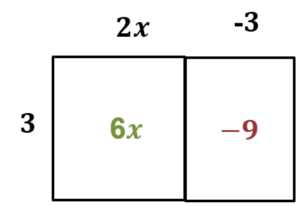
Method 1





Method 2





Solution: $6x - 9 = 3(2x - 3)$

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

EXAMPLE

Factor out a GCF: $-24m + 6n - 18$.

-24*m*	**6*n***	**-18**

Discussion: Students may factor out a 2, 3, or 6. Use this as an opportunity to discuss the benefits of the GCF.

EXAMPLE

Write the sum as a product of two factors.

- $(x - 2y) + (x - 2y) + (x - 2y) + (x - 2y)$.
- $3a + 2b - 6a + 7b$

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Algebra Tiles

- CPM Tiles is an applet from the CPM Educational Program that allows students to use Algebra tiles. The benefit of this applet is that students can change the dimensionality of *x* and *y*. However, it is limited by not allowing for a negative representation of the tiles.
- Algebra Tiles Applet by NCTM Illuminations is a link to a virtual algebra tiles applet.
- Algebra Tile Templates on Geogebra has interactive algebra tile models that students can use to model expressions.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

Equivalent Expressions

- Writing Expressions is a task by Illustrative Mathematics where students write an expression for the sequence of operations.
- Ticket to Ride is an Illustrative Mathematics instructional task to illustrate how different, but equivalent, algebraic expressions can reveal different information about a situation represented by those expressions.
- Equivalent Expressions is an Illustrative Mathematics task that directly addresses a common misconception held by many students who are learning to solve equations.

The Distributive Property

- Distributing and Factoring Using Area is a lesson by NCTM Illuminations that uses the area model to solve problems involving the Distributive Property and factoring. *NCTM now requires a membership to view their lessons.*

Curriculum and Lessons from Other Sources

- The Utah Middle School Math Project is an open source textbook and workbook. Chapter 3 pertains to this cluster.
- EngageNY, Grade 7, Module 2, Topic B, Lesson 16: Apply the Properties of Operations to Multiply and Divide Rational Numbers is a lesson that pertains to this cluster.
- EngageNY, Grade 7, Module 2, Topic A, Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers, Lesson 9: Applying the Properties of Operations to Add and Subtract Rational Numbers are lessons that pertains to this cluster.
- EngageNY, Grade 7, Module 3, Topic A, Lesson 1: Generating Equivalent Expressions, Lesson 2: Generating Equivalent Expressions, Lesson 3: Writing Products as Sums and Sums as Products, Lesson 4: Writing Products as Sums and Sums as Products, Lesson 5: Using the Identity and Inverse to Write Equivalent Expressions, are lessons that pertains to this cluster.
- The Utah Middle School Math Project is an open source textbook and workbook. Chapters 3 and 6 pertains to this cluster.

General Resources

- Arizona 6-8 Progressions on Expressions and Equations
  This cluster is addressed on page 8.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.1-2)

Research

- Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). 6-8, Expressions and Equations*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Parker, F. & Treviño, V. (November 2016). Burgers and fries: exploring equivalent expressions, *Mathematics Teaching in Middle School*, *22(4),* 242-246.

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.EE.3-4)</th></tr>
<tr><td>

EXPRESSIONS AND EQUATIONS

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example, if a woman making $25 an hour gets a 10% raise, she will make an additional $^1/_{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 ¾ inches long in the center of a door that is 27 $^1/_2$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

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Expectations for Learning

In Grade 6, students write and solve one-step equations and graph inequalities on a number line. In Grade 7, students will solve more complex equations by applying properties of operations with rational numbers. In addition, students will solve, graph, and interpret the solutions of inequalities. Students will make use of prior knowledge of rational numbers to solve multi-step numerical problems with positive and negative numbers and estimate the reasonableness of their answers. They will continue to gain fluency with positive and negative numbers as they solve more complex equations in 8th grade.

ESSENTIAL UNDERSTANDINGS

Real-life and Mathematical Problems

- Variables are used to represent a quantity.
- The order of operations is used to write and solve equations given within a context of a word problem.
- A solution is a value that makes an equation or an inequality true.
- Inverse operations may be used to solve equations and inequalities.
- Equivalent expressions always have the same value even if written in different forms.
- Equivalent expressions can be generated by using properties of operations (distributive property, associative, commutative, identity and inverse properties of multiplication and addition).
- A term includes the operational sign in front of it.

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STANDARDS	MODEL CURRICULUM (7.EE.3-4)
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* *Continued on next page*	**Expectations for Learning, continued** **ESSENTIAL UNDERSTANDINGS, CONTINUED** **Inequalities** • Inequalities have infinitely many solutions. • Solutions to inequalities can be represented on number line diagrams. • Point c is not included in the graphical solution to $x > c$ or $x < c$; the number line diagram represents this with an open circle around point c. • Point c is included in the graphical solution to $x \geq c$ or $x \leq c$; the number line diagram represents this with a closed circle at point c. • All of the solutions to an inequality are represented with a shaded region on a number line diagram. • The inequality $x > c$ is equivalent to $c < x$, and $x \geq c$ is equivalent to $c \leq x$. • When multiplying or dividing both sides of an inequality by a negative number, the order of the comparison it represents is reversed. **MATHEMATICAL THINKING** • Represent real-world problems mathematically. • Compute accurately and efficiently with grade-level numbers. • Determine reasonableness of results. • Use accurate mathematical vocabulary to describe mathematical reasoning. • Use estimation and mental computation strategies. • Recognize and use a pattern or structure. • Use informal reasoning. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.EE.3-4)
b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** **Real-life and Mathematical Problems** • Define variables in context using appropriate units. • Solve multi-step problems with positive and negative rational numbers including whole numbers, decimals, and fractions, and determine the reasonableness of solution(s). • Construct and solve multi-step equations and inequalities with variables on one side, and determine the reasonableness of solution(s). • Compare an algebraic solution to an arithmetic solution, identifying the sequence of operations used in each. **Inequalities** • Develop an understanding of the impact a negative coefficient has on the solution set of an inequality in context. • Graph solutions to inequalities and interpret solutions within a context with positive and negative coefficients. • Discover that to maintain a true statement, the inequality symbol must be reversed when multiplying or dividing by a negative number. **Content Elaborations** • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45 • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 3, pages 46-47 • Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19 **CONNECTIONS ACROSS STANDARDS** • Solve real-world and mathematical problems involving the four operations with rational numbers (7.NS.3). • Use properties of operations to generate equivalent expressions (7.EE.1-2). • Recognize and represent proportional relationships between quantities (7.RP.2). • Use facts about supplementary, complementary, vertical, and adjacent angles to solve simple equations for an unknown angle in the figure (7.G.5). • Convert rational numbers to decimals (7.NS.2d).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.2 *Reason abstractly and quantitatively.*
MP.6 *Attend to precision.*
MP.7 *Look for and make use of structure.*
MP.8 *Look for and express regularity in repeated reasoning.*

In Grade 6 students solved one-step equations focusing on inverse operations. Now that students are learning integer operations, they can use these concepts when solving equations and inequalities. The focus has now shifted from one-step to two-step equations and inequalities (or three-step if the Distributive Property is used). Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to problem situations that require multi-step equations and inequalities.

This is also the context for students to practice using rational numbers including integers and positive and negative fractions and decimals. It is appropriate to expect students to show their steps in their work. *Students should be able to move toward explaining their thinking using correct terminology.*

Tip: To assist students' assessment of the reasonableness of their answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation.

PROPERTIES OF OPERATIONS

In connection with 7.EE.1 students should apply the properties of operations and equality found in Table 3 and Table 4 of Ohio's Learning Standards. Teachers should be using the correct terminology to justify steps when performing operations and solving equations. Although, Grade 7 students should not be required to know the formal names of the properties, they should be encouraged to recognize them and use them to justify their steps when solving equations. For example, students may say "change order" for commutative property or "rearranging groups" for associative property which acceptable at this level, but teachers and/or classmates should reiterate the correct vocabulary during discussions. Students should not be assessed in situations where they have to recall the formal property names, but they should be able to recognize them from a given list. *Note: The Addition Property of Equality and the Subtraction Property of Equality can be used interchangeably since subtracting a number is the same as adding its opposite. The same is true for the Multiplication Property of Equality and the Division Property of Equality.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

VARIABLES

Letters are used in mathematics in a variety of ways: labels (*m* for meters), constants, (π or e), unknowns ($3x + 1 = 10$), universal statements ($a + b = b + a$), varying quantities ($y = 3x – 5$), parameters such as *m* and *b* in $y = mx + b$, and quantities in a formula ($A = lw$). This leads to confusion when students encounter letters that are used as variables.

In Grade 6 a variable is defined as—

- a letter that represents an unknown (6.EE.6); or
- a letter that represents any number in a specified set (6.EE.6); or
- two quantities that change (covary) in relationship to one another (6.EE.9).

Grade 7 continues to build on that understanding as equations, proportions, and unit rates become more sophisticated. Unfortunately, many students incorrectly view the variable as a label; they have a hard time distinguishing the name of an object from the name of the attribute from a quantity of measure. The expression 0.77*b* could mean 0.77 bananas instead of the cost of *b* number of bananas at $0.77 per pound. Incorrectly viewing variables as labels can be difficult misconception to undo, so it may be wise to avoid using mnemonic variables.

Ask students if $a = a$ is always, sometimes, or never true. Then ask them if $a = b$ is sometimes, always, or never true. Many students will say that $a = b$ is always false because they do not recognize that two variables can equal the same number. They incorrectly think that different letters must represent different values. Give them situations where both *a* and *b* could be the same number. Another example could be "Does $a + m + k$ sometimes/always/never equal $a + d + k$?" Connect these ideas to a graph, where *x* can indeed equal *y* on the graph $y = x$.

EXPRESSIONS

Expressions can act as an operator where $x + 2$ can mean to "add 2 to a number," or they can be an answer where $x + 2$ is "a number that is two greater than *x*."

EXAMPLE

If *y* is any number, write a number that is—

- 5 more than *y*
- 7 less than *y*
- Twice as great as *y*
- 10% of *y*

The concept of viewing expressions as operators is also applied when defining a sequence of odd and even numbers or problems involving area and perimeter. If the first even number is *a*, then the next even number is $a + 2$. This again holds true for odd numbers because each odd number (like even numbers) are two numbers apart.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

Many students view expressions operationally instead of structurally. An example of operational thinking would be viewing the expression $3x + 6y$ as two separate entities joined by an operation such as "Three times the cost of a pencil plus 6 times the cost of a pen" or "(3 pencils times the cost of 1 pencil) + (6 pens times the cost of 1 pen)." An example of structural thinking would be viewing the expression 3x + 6y as a single entity ($3x + 6y$) such as "the amount of money that is needed to buy 3 pencils and 6 pens" or "the total amount of money you spent." Although both views are correct, viewing an expression structurally enables students to understand more advanced algebraic concepts.

EXAMPLE

Which is larger y or x in the expression $y + 2x – 7$?

Discussion: Some students think that x must be larger because they judge the size of the term by the coefficient and do not realize that the variables can "vary." They also may only tend to only think of the positive domain.

EQUATIONS

Experiences in solving equations should move through Bruner's stages of concrete, pictorial, and algebraic/abstract representation. Utilize experiences with the pan balance model, hangers, tape diagrams, or Algebra tiles as a visual tool for maintaining equality (balance):

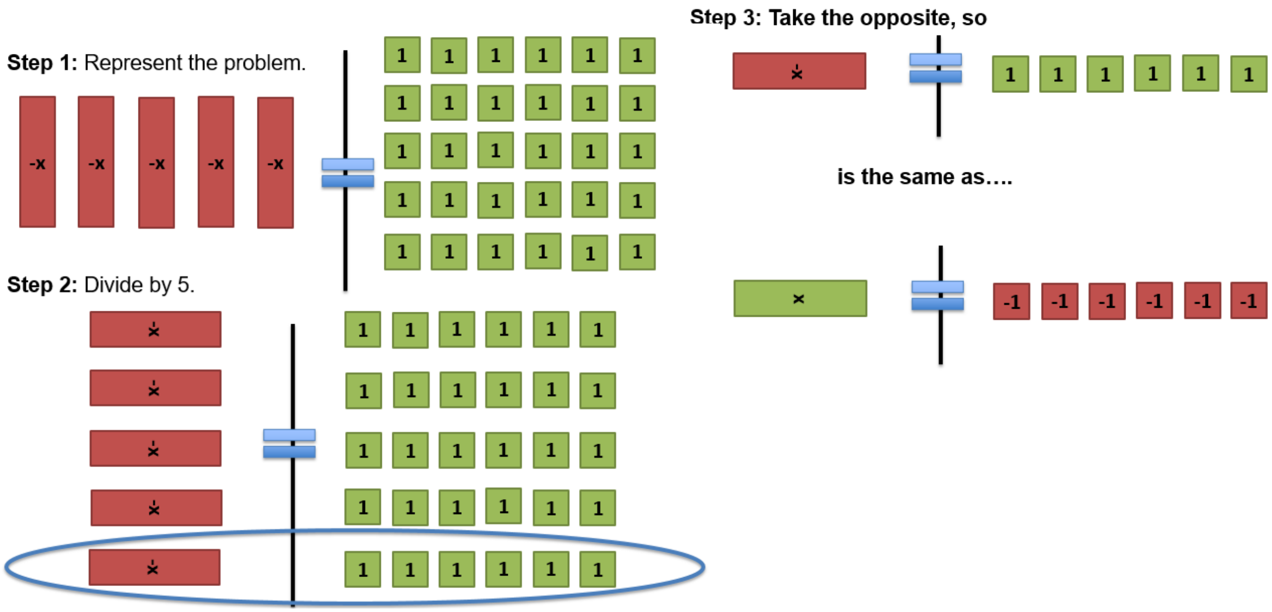
- First with simple numbers;
- Then with pictures symbolizing relationships; and
- Finally, with rational numbers.

This allows understanding to develop as the complexity of the problems increases.

EXAMPLE

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

Solve $-5x = 30$ and justify your steps.



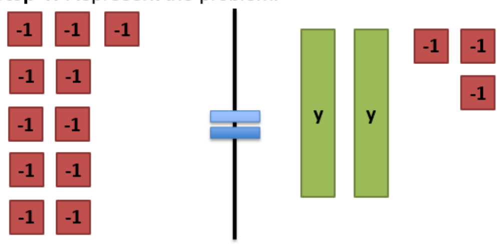
Discussion: Some students have difficulty with this type of question because they cannot conceptualize multiplying by a negative number. However other students misinterpret the multiplicative structure and think of the equation as $-5 + x = 30$ or $5 - x = 30$. (See 7.NS.1-3 for common errors involving negative numbers.) Using Algebra tiles, having students describe the usage of the negative sign, and/or having students justify their steps can help alleviate this misconception.

EXAMPLE

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

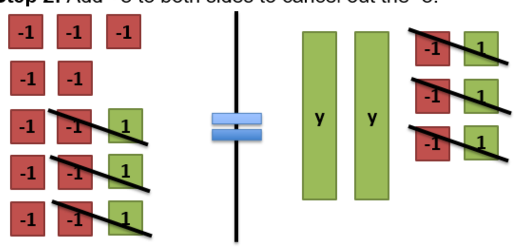
Solve –11 = 2y – 3 using Algebra tiles and justify your steps.

Step 1: Represent the problem.

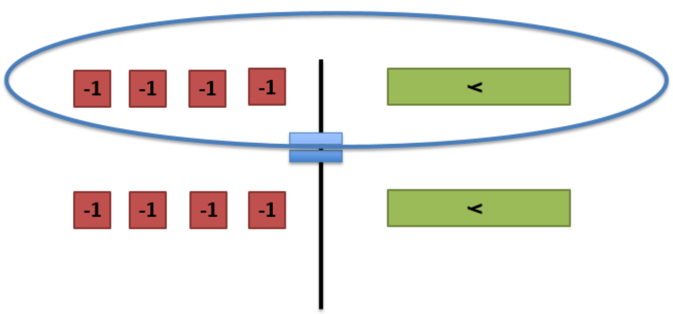


Note: As teachers initially model these steps they could use the properties to justify the movement of the Algebra tiles. For example, the teacher could say: We can add 3 positive numbers to both sides by the Addition Property of Equality. This will allow us to make 3 zero pairs by the Additive Inverse Property.

Step 2: Add +3 to both sides to cancel out the -3.



Step 3: Divide both sides by 2.



So y = -4.

Note: For example, the teacher could say: We could divide each side by 2 by applying the Division Property of Equality. Although notice that dividing by 2 is the same thing as multiplying $\frac{1}{2}$, so we could also use the Multiplicative Property of equality to multiply each side by $\frac{1}{2}$. The Multiplicative Inverse Property allows us to multiply 2y by $\frac{1}{2}$ to get 1y. Then we multiply 8 by $\frac{1}{2}$ to get –4, so –4 = 1y. Because of the Multiplicative Identity Property, we can just say that –4 = y. I know that some of you prefer the y on the left side of the equation, so we can flip the equation so that y = –4 by the Symmetric Property of Equality.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

TIP! Some students see equations as a procedural process rather than something useful to answer meaningful questions. Try saying "Which number or numbers can be substituted in the place of *x* to make the left side of the equation equal to the right side?" instead of just saying "Solve for x," to highlight the underlying significance.

EQUAL SIGN

Many students see the equal sign as a signal to compute in order to find an answer instead of a viewing it as a relational symbol. They need to see the equal sign as meaning "the same as" or as a balance beam. One method that may be helpful to introduce the equal sign is by writing equivalent expressions in conjunction with writing expressions with inequalities. Another method could be covering up a number on one or both sides of arithmetic identities and having student find the missing number or numbers. For example, 3 + 8 – 2 = 3 + □ – 2, students could find many combinations of numbers that equal 8 such as 3 + 5 or 2(4) or –16 ÷ (–2) or 10 – 3 + 1. Another strategy is using true and false equations. Create examples that highlight relational thinking.

EXAMPLE

Determine if each equation is true or false.

- 783 + 346 = 773 + 356
- 24 – 27 = 25 – 26
- 24 – 27 = 25 – 28
- 5(–2) = 2.5(–4)
- –18(0) = –18 + 0
- $-\frac{3}{4}+\frac{3}{8}=-\frac{3}{8}+\frac{3}{4}$
- –24.3 = –6.1 + 18.2

Discussion: Instead of computing, encourage students to determine the truth of the statement by focusing on relational thinking. Move toward replacing one or two of the terms with a variable(s), and have students find the solution by using relational thinking. After students become familiar with the process have them write their own true/false statements and open sentences.

Fractions

Some studies have shown that a students' fractional knowledge correlates with their ability to write equations. Therefore encourage students to solve equations with fractions by using diagrams instead of just using inverse operations. This may aid in creating understanding to alleviate the misuse of fraction rules in later grades/courses. Numbers that are easily modeled should be used initially until students internalize the process.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

EXAMPLE

Solve $\frac{3}{4}x + 8 = 14$.

Step 1

$\frac{3}{4}x + 8 = 14$

$\quad -8 \quad -8$

Step 2

$\frac{3}{4}x = 6$

				= 6

Step 3

$\frac{3}{4}x = 6$

2	2	2		= 6

Since ¾ of x equals 6, 6 has to be divided by 3, so ¼ = 2.

Step 4

$\frac{3}{4}x = 6$

2	2	2	2	= 6

To find the whole, the last box has to be filled in, and since each part of a fraction is equal, the last box should also equal 2 which makes the whole 8. After doing many problems like this, students should be able to justify why the Multiplicative Inverse Property works and that one could also alternatively divide each side by 3 and multiply each side by 4. Make the connection between their invented procedure (÷3, then ×4) and multiplying both sides by the inverse.

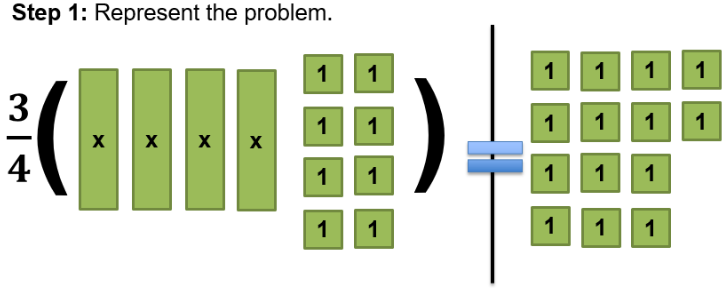
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

EXAMPLE

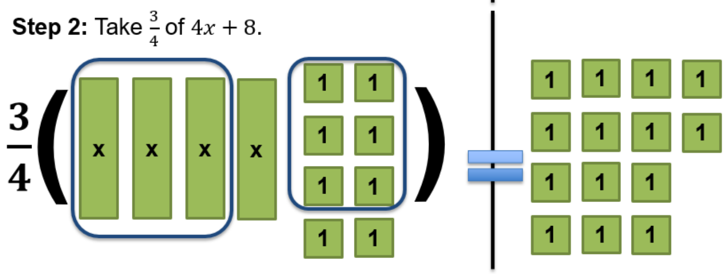
Solve $\frac{3}{4}(4x + 8) = 14$.

This problem should be contrasted to the previous example, and the difference of fraction operations should be discussed.

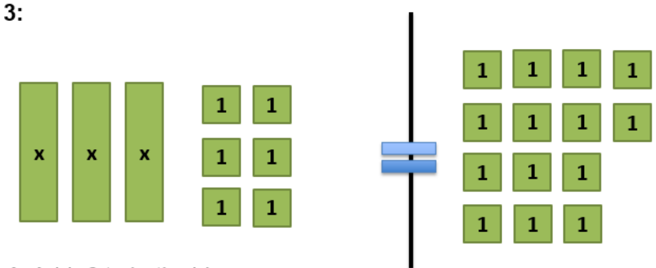
Step 1: Represent the problem.



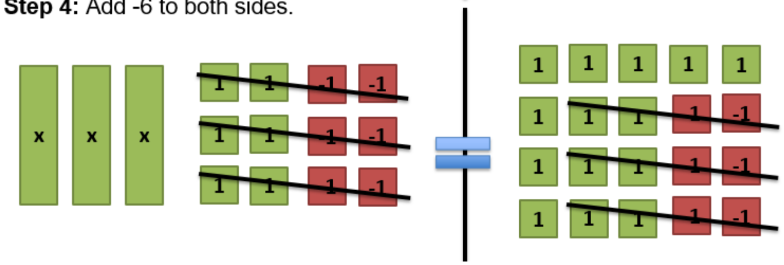
Step 2: Take $\frac{3}{4}$ of $4x + 8$.



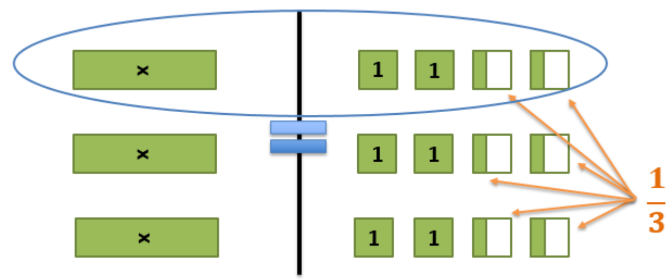
Step 3:



Step 4: Add -6 to both sides.



Step 5: Divide each side by 3.



So $x = 2\frac{2}{3}$.

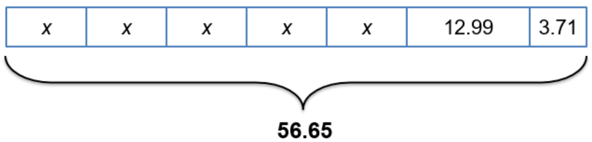
Tape diagrams may also be a useful tool for modeling equations. Unlike Algebra tiles, students are not limited to integers.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

EXAMPLE

Stephanie had $56.65 to spend at Target. She spent $12.99 on a pair of sunglasses, and spent the rest on 5 bottles of nail polish that were priced the same. Tax came to $3.71. How much did each bottle cost?

Tape Diagram:



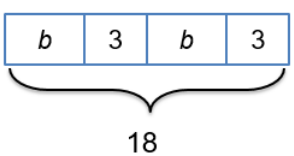
Equation:

$$
\begin{aligned}
5x + 12.99 + 3.71 &= 56.65 \\
5x + 16.70 &= 56.65 \\
-16.70 \quad & \quad -16.70 \\
\frac{5x}{5} &= \frac{39.95}{5} \\
x &= \$7.99
\end{aligned}
$$

Discussion: The tape diagram builds nicely on students' previous strategies with proportional reasoning. It also leads to inverse operations as students will notice that it is best to subtract 12.99 and 3.71 from 56.65 before dividing by 5. Draw connections between the tape diagram and the steps of the equation to move students from arithmetic reasoning to algebraic reasoning. Again, the properties of equality and operations should still be emphasized.

EXAMPLE

Write a story representing the situation. Then write a corresponding equation.



INEQUALITIES

In Grade 6 students wrote inequalities in the forms of $x > c$ and $c < x$. Notice that they only addressed greater than and less than symbols. Now

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

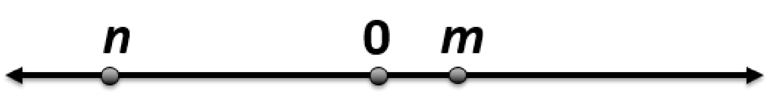
in Grade 7 students use ≥ and ≤. Discuss why teachers should, when graphing on a number line, a closed circle represents ≥ and ≤ and an open circle represents > and <. Students should also have practice solving one and two-step inequalities with rational numbers.

Understanding the placement of integers on the number line is an important foundational concept for understanding inequalities. Students have a difficult time understanding that $-x$ can be positive if $x < 0$. Explain why the terminology "x is smaller than y" creates confusion when x and y are both negative as students confuse it with the idea that "x is closer to 0 than y" (absolute value). The examples below can help confront those problems. These concepts can also be used to start a discussion about why multiplying or dividing by –1 will change inequalities. *Note: Present situations where the variable is both on the left and the right side of the equality. Students need to be fluent solving inequalities where the variable is on the left and right of the inequality for later algebraic concepts using compound inequalities. Therefore, discourage students from always writing the variable on the left side of an inequality.*

TIP! The words "greater than," "less than," "more than," and "less than" can cause confusion because sometimes they refer to inequalities and at other times they refer to addition or subtraction. Help students distinguish between the different types of situations.

EXAMPLE

Plot and label the points $-m$, $-(-m)$, $-n$, and $-(-n)$ on the number line.



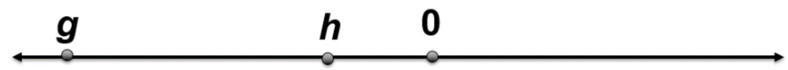
- What do you notice.
- If m is a positive number, then $3m > m$. Also m is closer to 0 than $3m$. What happens if m is a negative number?
- What effect does have on the inequality? Explain.
- What effect does m being a negative number that have on the m's distance from 0, $3m$, and $-3m$? Explain.

Discussion: Students oftentimes think that n must be positive and $-n$ must be negative, but n can be any number such as –2. Reinforce the fact that $-n$ is the opposite of n, so if n is –2 then $-n$ is $-(-2)$.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

EXAMPLE

a. Given that $h > g$, plot the locations of $-g$ and $-h$. Write an inequality comparing –g and –h. Justify your thinking. (See the number lines to the right for three different possibilities.)



b. If $h < g$, write an inequality that relates $-g$ and $-h$. Justify your thinking using the number lines similar to those in part **a.**

EXAMPLE

- Choose a pair of numbers for x and y for which both inequalities statements are true: $x < y$ and $x + y < x$. Then represent your situation on the number line.

- Choose a pair of numbers for x and y for which both inequalities statements are true: $x < y$ and $x < x + y < y$. Then illustrate your situation on the number line.

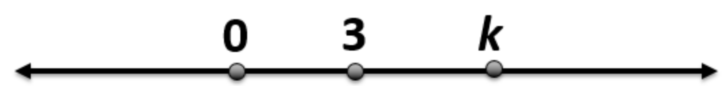
EXAMPLE

Part 1

Given that k is a number greater than 3, draw a number line with the points 0, k, and 3 on it where 3 and k are positive side of the number line. *(Shown in Line 1.)*

- What type of number is k? Positive? Negative? Rational? Integer? Explain.
- Write an inequality describing the relationship between 3 and k.
- Write an inequality describing the relationship between 3 and $-k$.

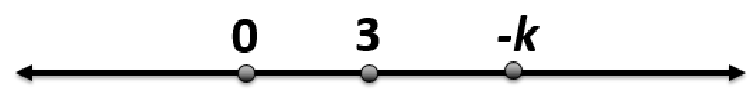
Line 1



Given that $-k$ is a number greater than 3, draw a number line with the points 0, $-k$, and 3 where 3 and –k are on the negative side of the number line. *(Shown in Line 2.)*

- What type of number is k? Positive? Negative? Rational? Integer? Explain.
- Write an inequality describing the relationship between 3 and $-k$.
- Write an inequality describing the relationship between 3 and k.

Line 2



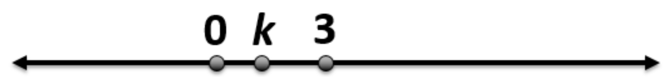
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

Part 2

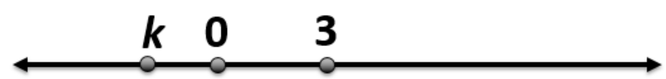
Given that $3 > k$, draw a number line with the points 0, k, and 3 that represent a possible situation.

- What types of numbers could k be? Positive? Negative? Rational? Integer? Explain.
- If k is a positive number, write an inequality comparing the situation. *(Shown in Line 3.)*
- If k is a negative number write an inequality comparing the situation. *(Shown in Line 4.)*
- Does it matter in this case if the value of k is positive or negative?
- Plot $–k$ in Lines 5-7. What do you notice about –k?
- If we still hold to $3 > k$, can we write an inequality comparing 3 to $–k$? Explain.
- However, if we still hold to $3 > k$, what do we know for sure about the value of $–k$ value? Explain.
- Write an equivalent inequality to $3 > k$ using $–k$. Explain why it is true.
- Using what you learned write an equivalent inequality using $–p$ for the following:
- $p < 4$
- $2.5 > p$
- $–3 < p$
- $p > -\frac{1}{4}$
- $p \geq 7$
- Using what you learned write an equivalent inequality using b for the following:
- $–b > 5$
- $–b < 2.4$
- $–3 > –b$
- $\frac{5}{6} < –b$
- $–b < –6.1$
- Write a rule that explains to a friend how to change any inequality with a negative variable to an equivalent one with a positive variable. Make sure to explain why it works.

Line 3



Line 4



Lines 5-7

*From "Algebra on the Number Line" *Mathematics Teacher,* December 2010/January 2011.

Graphing

When graphing, students should be given problems where the variable is both on the left and right side of the inequality sign. Teachers should **avoid** telling students that the inequality points the same direction on the number line as the arrow; this creates a misconception that is hard to break when students work on compound inequalities in high school. An alternative strategy is to ask students to name 3 points that make an inequality true, and then draw the arrow in that direction.

EXAMPLE

Graph $4 \leq k$.

Teacher: What are 3 solutions to the inequality? Is 4 a solution to the inequality?
Student: 4, 7, 100
Teacher: Plot those numbers on a number line.

Teacher: Now draw an arrow going in that direction.

Teacher: According to the number line, what are some solutions to the inequality? Name some rational numbers that are also solutions. Do those solutions match our original inequality statement?

Students should also have practice writing inequalities based on a number line.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

EXAMPLE

Write the inequality modeled below.

REAL-WORLD PROBLEMS WITH EQUATIONS AND INEQUALITIES

Students should be able to create equations and inequalities from real-world situations where they always precisely define the variable(s).

EXAMPLE

A gym membership has a \$99.00 initiation fee and a \$29.99 monthly fee. Mason has paid \$308.93 so far. How many months has she been a member?

- Define the variable.
- Write an equation representing the situation.
- Solve the equation.

EXAMPLE

Margot bought some apples and pears for \$6.40. If she bought \$2.80 worth of pears and 6 apples. How much does each apple cost?

- Define the variable.
- Write an equation representing the situation.
- Solve the equation.

TIP! When writing and solving equations, students need to be precise in defining the variables. They should not say x = apples, but rather x = the cost of an apple or x = the quantity of apples. This may help alleviate students confusing variables with labels. It may also be helpful to avoid assigning the mnemonic letter *a* to apples in classroom discussions.

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

EXAMPLE

Adrian's grandma gave him $30 for his birthday. He wanted to buy a hat for $12.00 and some candy bars that cost $1.48. He lives in a state that does not have sales tax on food or clothing. How many candy bars did he buy?

- Write an inequality to represent the situation. Don't forget to define the variable.
- Solve and graph the inequality.
- Describe the possible solutions to the inequality.

Discussion: Students should also practice writing inequalities from real-world situations where the words "less than" or "greater than" is absent from the context. They should notice that only whole numbers make sense in terms of buying candy bars; therefore their graphs should only include closed circles on the whole numbers less than 12.

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Algebra tiles
- Balances

Algebra Tiles

- Algebra Tiles Applet by NCTM Illuminations is a link to a virtual algebra tiles applet.
- CPM Tiles is an applet from the CPM Educational Program that allows students to use Algebra tiles. The benefit of this applet is that students can change the dimensionality of *x* and *y*. However, it is limited by not allowing for a negative representation of the tiles.
- Algebra Tiles by Geogebra is an applet that uses Algebra tiles.

Solve Multi-Step Problems

- Who is the Better Batter? is a task from Illustrative Mathematics that has students use a real-world task where it is useful to convert fractions to decimals or percents.
- Gotham City Taxis is a task from Illustrative Mathematics that allows students to solve a problem using a variety of methods including equations.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

Solve Multi-Step Problems, continued

- Discounted Books is a task from Illustrative Mathematics that has students use a real-world task involving percents.
- Guess My Number is a task from Illustrative Mathematics that allows students to visualize mathematical operations.
- How many pennies? is a YummyMath 3-act task about collecting pennies.

Expressions

- Pool Border Problem is a Desmos activity where student construct expressions with variables to model tiling a pool.

Equations

- Central Park is a Desmos activity where student construct equations with variables to model area of spaces in a parking lot.
- Algebra Equations by Math Playground uses a balance beam to model equations.
- Geology Rocks Equations is a lesson by NCTM Illuminations where students explore equations with manipulatives such as blocks and counters. *NCTM now requires a membership to view their lessons.*
- Solving Linear Equations by Mathematics Assessment Project is a task where students form and solve equations using factoring and the distributive property.
- Fencing by Mathematics Assessment Project is a task where students must figure out the cost of building a fence.
- Math playground: Modelling Algebraic Equations has a balance beam that allows students to model one- and two-step equations using a balance. It also allows students to make their own equations, so teachers can give students equations to use that fit within the constraint of the 6th grade curriculum.
- Macy's Star Rewards is a YummyMath task that explores different Macy's discounts.

Inequalities

- Inequality Wars in an interactive game by XP Math where students fly the Millennium Falcon through an Asteroid Field Destroy Asteroids that satisfy solutions to the Target Inequality.
- One Step Inequality Jeopardy is an online jeopardy game to practice solving one-step equations.

Curriculum and Lessons from Other Sources

- EngageNY's Grade 7, Module 3, Topic B, Lesson 7: Understanding Equations, Lesson 8: Using If-Then Moves in Solving Equations, Lesson 9: Using If-Then Moves in Solving Equations, Lesson 10: Angle Problems and Solving Equations, Lesson 11: Angle Problems and Solving Equations, Lesson 12: Properties of Inequalities, Lesson 13: Inequalities, Lesson 14: Solving Inequalities, Lesson 15: Graphing Solutions to Inequalities are lessons that pertain to this cluster.
- Illustrative Mathematics, Grade 7, Unit 6: Expressions, Equations, and Inequalities, has many lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- The Utah Middle School Math Project is an open source textbook and workbook. Chapters 3 and 6 pertains to this cluster.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.EE.3-4)

General Resources

- Arizona 6-8 Progressions on Expressions and Equations
  This cluster is addressed on pages 8-10.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

References

- Akgün, L. & Özdemir, M. (2006) Students' understanding of the variable as general number and unknown: A case study. *The Teaching of Mathematics*, *9(1),* 45-51.
- Capraro, M. & Joffrion, H. (2006). Algebraic equations: Can middle school students meaningfully translate from words to mathematical symbols. *Reading Psychology*, *27*, 147-164.
- Clement, J., Narode, R., & Rosnick, P. (October 1981). Intuitive misconceptions in Algebra as a source of math anxiety. *Focus on Learning and Problems in Mathematics*, *3(4),* 36-45.
- Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). 6-8, Expressions and Equations*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Hackenberg, A. & Lee, M. (March 2015). Relationships between students' fractional knowledge and equation writing. *Journal for Research in Mathematics Education*, *46(2),*196-243.
- Johanning, D. (2008). Learning to use fractions: Examining middle school students' emerging fraction literacy, *Journal for Research in Mathematics Education*, *39(3),* 281-310.
- Knuth, E., Stephens, A., McNeil, N., & Alibali, M. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, *37(4),* 297-312.
- MacGregor, S. & Stacey, K. (1997). Students' understandings of algebraic notation. *Educational Studies in Mathematics*, *33*, 1-19.
- Store, J., Richardson, K., & Carter, T. (March 2016). Fostering understanding of variables with patterns. *Teaching Children Mathematics*, *22(7),* 420-427.
- McCabe, T., Sorto, M., & White, A. (December 2010/January 2011). Algebra on the number line. *Mathematics Teacher*, *104(5),* 379-382.
- Turner, E., Sugimoto, A., Stoeher, K., & Kurz, E. (November 2016). Creating inequalities from real-world experiences. *Mathematics Teaching in the Middle School*, *22(4),* 236-240.

STANDARDS	MODEL CURRICULUM (7.G.1-3)
GEOMETRY **Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. **b.** Represent proportional relationships within and between similar figures. **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions. **7.G.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	**Expectations for Learning** The investigation of drawing, constructing, and describing geometric figures in Grade 7 leads to development of the concepts of similarity, congruence, cross sections, and interior angle sums of triangles and quadrilaterals. Students will understand that in similar figures corresponding angles are congruent and corresponding sides are proportional. Students will apply this knowledge of similar figures to solve real-world problems including those with scale drawings. It is imperative students explore relationships by using multiple models including technology to develop geometric concepts in congruence and similarity. Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence in Grade 8 and high school. *Note: The congruence criteria for triangles will be formalized in high school geometry.* The student understanding of this cluster aligns with van Hiele Level 1 (Analysis) with certain aspects of this cluster moving toward van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** **Similar Figures** • Angles are congruent if they are equal in measure. *Note: 7th grade students may use the term "equal in measure" in place of congruent.* • Similar figures have corresponding angles that are congruent and corresponding side lengths that are proportional. • Applying a scale factor greater than one results in a bigger image. • Applying a scale factor of 1 results in a congruent image. *Note: Students are not required to understand congruency of two figures until 8th grade.* • Applying a scale factor less than 1 but greater than zero results in a smaller image. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.G.1-3)
7.G.1-3, continued	**Expectations for Learning, continued** **ESSENTIAL UNDERSTANDINGS, CONTINUED** **Drawing Geometric Figures** • The sum of all three angles in any triangle equals 180 degrees. • The sum of all four angles in any quadrilateral equals 360 degrees. • Three possible outcomes exist when constructing triangles with given measurements of sides and/or angles: a unique triangle, more than one triangle, or no triangle. • Some quadrilaterals have more specific names based on relationships such as pairs of parallel sides, congruent sides, and angle relationships. **Slicing Three-Dimensional Figures** • Slicing a three-dimensional figure results in a two-dimensional shape. • Slicing a three-dimensional figure in different ways could result in different shapes. **MATHEMATICAL THINKING** • Explain mathematical reasoning. • Make and modify a model to represent mathematical thinking. • Solve real-world problems accurately. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Use technology strategically to deepen understanding. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.G.1-3)
7.G.1-3, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** **Similar Figures** • Draw or create a model to make sense of a problem involving similar figures. • Identify corresponding sides and angles of similar figures. • Compare and contrast the relationship between the angle measures and side lengths in a scale drawing and its original figure. • Apply proportional understanding to the relationship between side lengths in similar figures. • Investigate the relationship between the areas of similar figures. • Identify the impact of scale on length and area. • Compute actual lengths and areas from a scale drawing. • Reproduce a scale drawing using a different scale. • Draw scaled figures with proper figure labels, scale, and dimensions. **Drawing Geometric Figures** • Draw a picture or create a model of triangles or quadrilaterals with given conditions. • Investigate whether a given set of side lengths and angle measures determines a unique triangle, creates multiple triangles, or does not create a triangle. • Investigate quadrilaterals with a given set of side lengths, angle measures, and relationship between sides (parallel, perpendicular, neither) to observe types and properties. • Investigate whether it is possible to construct more than one quadrilateral with the same given set of side lengths, angle measures, and relationship between sides. • Discover the sum of the interior angles of triangles and quadrilaterals. **Slicing Three-Dimensional Figures** • Investigate the various outcomes of slicing three-dimensional figures and the resulting two-dimensional face. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.G.1-3)
7.G.1-3, continued	**Content Elaborations** • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43 • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 3, pages 46-47 • Ohio's K-8 Learning Progression, Geometry, page 21 **CONNECTIONS ACROSS STANDARDS** • Analyze proportional relationships, and use them to solve real-world and mathematical problems (7.RP. 1 – 3). • Construct simple equations to solve for unknown dimensions in similar figures (7.EE.4.a).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems. After much work is done on paper, Geometry software can aid in students understanding of Geometry.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.1 *Make sense of problems and persevere in solving them.*
MP.3 *Construct viable arguments and critique the reasoning of others.*
MP.4 *Model with mathematics.*
MP.5 *Use appropriate tools strategically.*
MP.7 *Look for and make use of structure.*

VAN HIELE CONNECTION

Shapes

The student understanding of this cluster with respect to shapes aligns with a van Hiele Level 1 (Analysis) and moves toward Level 2 (Informal Deduction/Abstraction).

Level 1 can be characterized by the student doing some or all of the following:

- identifying shapes as visual-wholes;
- recognizing geometric figures by their appearance alone;
- not identifying the properties of figures; and/or
- grouping figures that look "alike."

Level 2 can be characterized by the student doing some or all of the following:

- noticing relationships within and between properties and figures;
- forming meaningful definitions;
- justifying reasoning using informal arguments;
- drawing logical maps and diagrams; and/or
- not yet understanding formal deduction.

Visualization

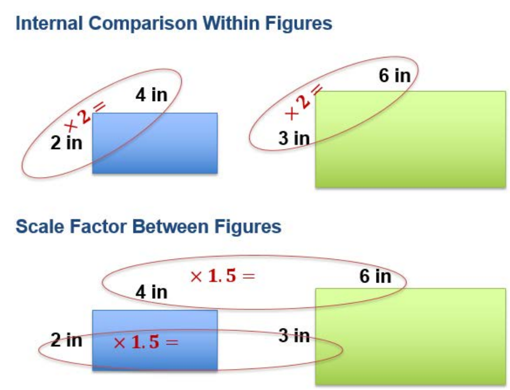
The student understanding of this cluster with respect to visualization aligns with a van Hiele Level 1 (Analysis). Level 1 can be characterized by the student doing some or all of the following:

- showing a greater degree of attention to properties of shapes and solids;
- building 3D figures from 2D images and 2D drawings from 3D figures;
- viewing a figure from front, back, left, and right positions of solids;
- visualizing cross-sections when slicing solids;
- comparing solids based on properties;
- using observation as a basis for explanations; and/or
- understanding that a movement is made, then after observing the result, the next movement is selected, etc.

See the van Hiele pdf on ODE's website for more information about van Hiele levels.

SIMILAR FIGURES

In Grade 7 students build on proportional reasoning including the constant of proportionality to develop concepts of similarity. This will be extended to dilations in Grade 8 and lead to future work in similarity and congruence.



Similarity is an increase or decrease that is multiplicative in nature instead of additive; this is a new concept for students. One aspect of similarity is that two figures are similar if there is the same constant ratio applied to all dimensions. To maintain similarity no stretching or shrinking can be applied to a figure unless it is applied to all its sides.

Similar figures have a proportional relationship within and between corresponding sides. The proportional relationship between figures is called the scale factor. There are also internal proportional relationships that are static within each of the similar figures. For example, the ratio of height to width in a similar rectangle will always be the same for similar figures no matter what the scale. See 7.RP.1-3 for more information about within and between multiplicative relationships. Although the focus of this cluster is on rectangles and triangles, it may be useful to discuss why all circles are similar.

TIP

Although this standard is limited to triangles and quadrilaterals, it may be helpful to introduce more complex images such as an L-shaped figure to challenge students who try to scale using additive instead of multiplicative strategies.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

TIP

Regularly provide students with figures that are **not** similar to ensure that students are continually checking for similarity.

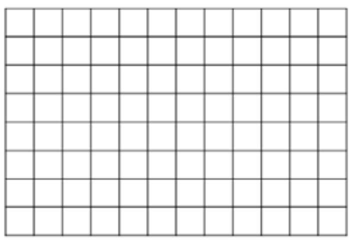
Internal Comparisons Within Figures

EXAMPLE

Part 1

Which of the figures is the "more square"? Explain why and justify your explanation using math.

Rectangle A



Rectangle B

Discussion: Although neither one of these rectangles is a square, students should be given the opportunity to reason about what determines "squareness"—4 equal sides. Therefore, the question remains, which rectangle has sides that are more equal. Both rectangles have a width that is 4 units shorter than the length, so the difference in addition cannot determine "squareness." That leaves multiplication; in Rectangle A the length is 1.5 times the width, and in Rectangle B the length is 3 times the width. Since the ratio of length to width of Rectangle A is closer to 1 (The ratio of length to width in a square is 1) than the ratio of length to width of Rectangle B, Rectangle A is "more square." Discuss how all squares are similar—they have the same shape even if they are different sizes. Discuss the meaning of "same shape" (similar). The number of sides? The lengths of the sizes? The angles?

Part 2

Give students several sets of similar rectangles. For example, one set could have sides with a ratio of 3 to 4. The second set could have sides with a ratio of 1 to 3. The third set could have a ratio of 2 to 3. The fourth set could be a set of squares. Identify each rectangle with a letter. (Sets do not have to have an equal amount of rectangles.)

- Sort the rectangles into piles of similar rectangles.
- Explain why the rectangles in each group are similar.
- Make a table for each set of similar rectangles with the following headings: rectangle name, length of long side, length of short side, observations.
- What do you notice about the rectangles?
- Will that be true for any set of similar rectangles? Explain.
- Would that be true for other shapes such as similar triangles? Explain.

Set 1

Rectangle Name							
Length of Long Side							
Length of Short Side							
Observations							

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Part 3

Find the missing side length of the similar triangles.

Figure 1

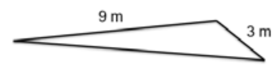
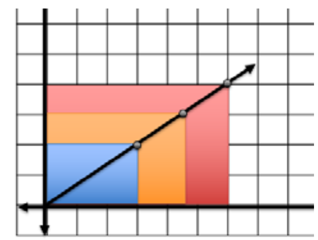


Figure 2

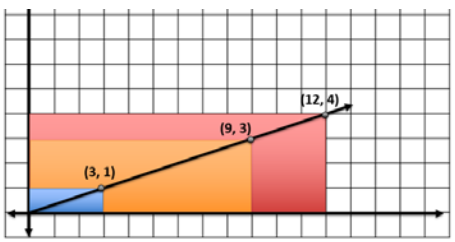
Part 4

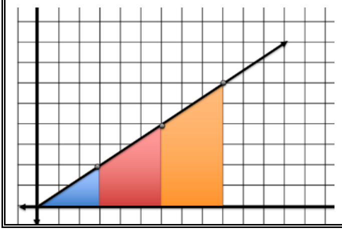
- Using your similar rectangle sets in Part 2, write an equation that represents the relationship between length and width for each set.
- Could you find the missing length for the triangle in Part 3 by using an equation? Explain.
- Will your equation in Part 4**b**. work to find the length of the longest side of the triangle? Explain.

Part 5



- Take each set of similar rectangles from Part 2. and stack them where the rectangle aligns in one corner. Notice how all the far corners also line up. Draw a line connecting the far corners of each set. This is called the nesting method.
- How does this connect to the graph of a proportion? Explain.
- Use the nesting method to find which of the following rectangles are similar:
    - o 3 cm × 4.5 cm, 1.5 cm × 2 cm, 6 cm × 9 cm, 4 cm × 6 cm
- Draw a new set of similar rectangles on a coordinate plane using the nesting method. Label each point where the line crosses the corner of the rectangle. What connection can you make between the common ratio in Part 2 and the line connecting the outside vertices of the rectangles?





- Draw a new set of similar right triangles on a coordinate plane so that the hypotenuses lie on a line that goes through the origin. Label each point where the line intersects the vertex of one of the triangles. What connection can you make between the height and the base of the triangle and the line connecting the vertices of the triangles?

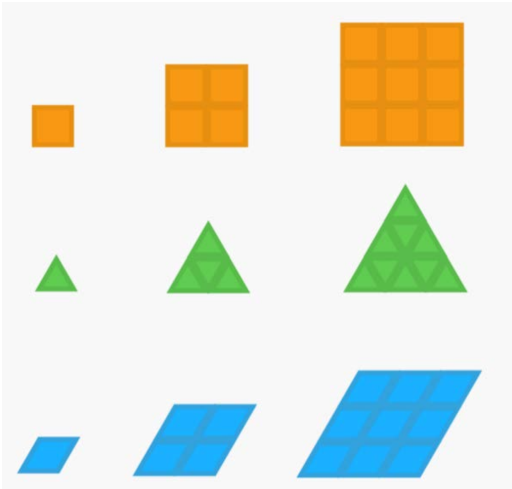
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Scale Factor

The special factor or constant ratio that compares the image to the original figure is called the scale factor. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of times you multiply the side measure of one figure to obtain the corresponding side measure of a similar figure. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. After students have time to use the shapes concretely, they should also practice drawing them.

EXAMPLE

Ask and record questions about the relationship of the original block to the created shape. A sample of a recording sheet is shown.



Shape	Original Side Length	1st Created Side Length	2nd Created Side Length	Scale Relationship of 1st Created to Original	Scale Relationship of 2nd Created to Original
Square	1 unit	2 units	3 units	2:1	3:1
Equilateral Triangle	1 unit				
Rhombus	1 unit				

Discussion: This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. In other words, if the original side can be stated to represent 2.5 inches, what would be the new lengths and what would be the scale? Have students follow Bruner's stages of representation: concrete, pictorial, abstract.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

EXAMPLE

Shape	Original Side Length	Created Side Length	Scale
Square	2.5 inches		
Parallelogram	3.25 cm		
Trapezoid	Base A: 1/4 in Base B: 3/4 in	Base A: Base B:	

Provide opportunities for students to use scale drawings of geometric figures with a given scale. The opportunities should require them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

As students work with scale factor, they should come to the realization that a scale factor greater than 1 enlarges an image, a scale factor less than 1 shrinks the image, and a scale factor of exactly 1 produces a congruent image (Note: Students do not need to use the word *congruent* until Grade 8.)

After students have had several explorations using a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Start with simple shapes and whole-number side lengths. This allows all students to have access to discover and understand the relationships. An interesting discovery is that the relationship of the scale of the side lengths to the scale of the respective perimeters are the same scale and the areas are the scale squared. Students should move on to drawing scaled figures on grid paper with proper figure labels, scale, and dimensions.

EXAMPLE

Draw shapes on grid paper and a corresponding scale image. Use a table to explore how scaling affects the perimeter and area of a figure. (See an example table below.)

Shape	Side Lengths	Scale	Original Perimeter	Scaled Perimeter	Perimeter Scale	Original Area	Scaled Area	Area Scale
Rectangle	2 x 3 in	2	10 in	20 in	2	6 in^2	24 in^2	4
Right Triangle	3cm-4cm-5cm	3	12cm	36cm	3	6 cm^2	54 cm^2	9
Square	20 in	1/4	80 in	20 in	1/4	400 in^2	25 in^2	1/16

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Provide word problems that require finding missing side lengths, perimeters, or areas. In addition, allow students to design their own word problems asking for missing side lengths, perimeters, and/or areas.

EXAMPLE

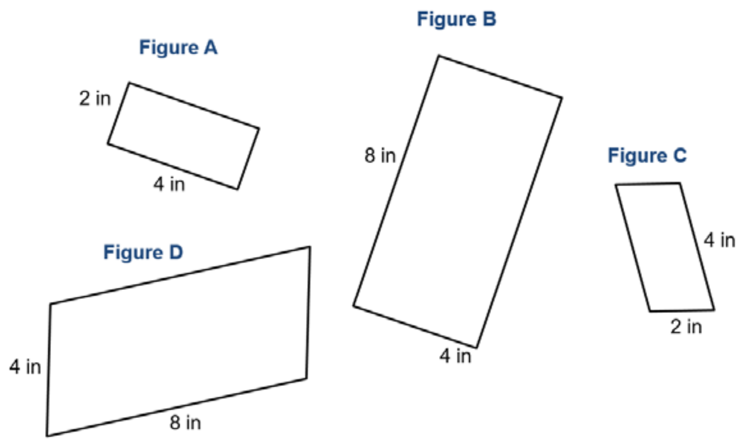
- If a 4 by 4.5 cm rectangle is drawn in a scale of 3, what is the measure of the area in the scaled image?
- If a 4 by 4.5 cm rectangle is drawn in a scale of 6, what will the measure of the side lengths be in the scaled image? What is the measure of the area in the scaled image?
- If a 4 by 4.5 cm rectangle drawn in a scale of $\frac{1}{2}$, what will be the measure of the perimeter of the scaled image? What is the measure of the area in the scaled image?
- Suppose the area of one triangle is 16 square units, and the scale factor between this triangle and a new triangle is 2.5, what is the measure of the area in the scaled image?

ANGLES

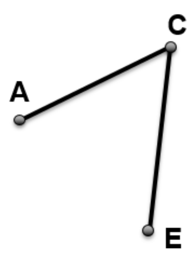
Students need to come to the realization that in similar figures, although side lengths change together proportionally, the angles measures remain the same.

EXAMPLE

Which figures are similar? Explain.



EXAMPLE



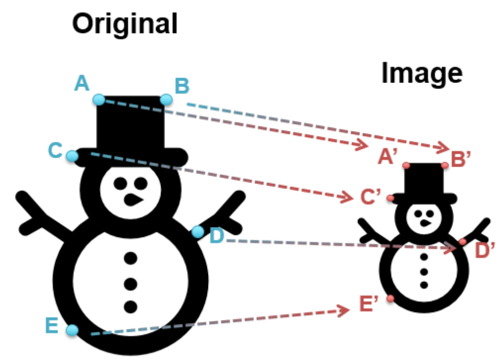
- Draw a similar figure to the figure above with a scale factor of 3.
- What do you notice about the measure of the angle?

Discussion: If students try to triple the size of the angle, ask them if it is still the same shape. Then lead them to triple the length of each of the line segments instead of the angle opening. A real-world application of this is when you e.g., fly and get off course a few degrees, the longer time you fly in that direction the further from you goal you will get This can be connected to Social Studies when Emilia Earhart's flew to "Paris" in 1932, and she landed in Northern Ireland instead.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

SCALE DRAWINGS

In a scale drawing there is a precise pairing of each individual point on the original figure to its corresponding point on the image. Such pairing is an example of one-to-one correspondence. Every reduced or enlarged distance in the drawing is proportional to the original distance. Therefore $\frac{\overline{A'B'}}{\overline{AB}} = \frac{\overline{B'D'}}{\overline{BD}} = \frac{\overline{D'E'}}{\overline{DE}}$....., etc, which is the scale factor. Artists use this method of measuring different distances in their work and comparing it to the original to ensure proportionality.



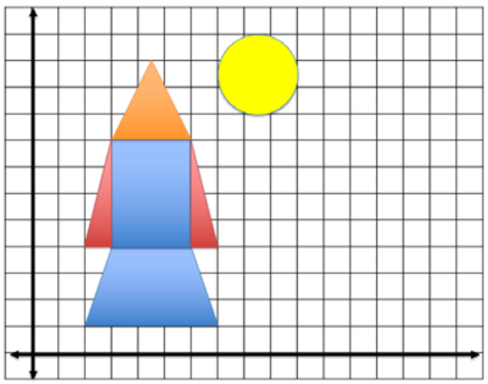
Real-life examples of scale drawings could be blueprints, photocopies, and maps. Some maps distort distances and are not drawn to scale. Also, technical drawings and some photographs may have distorted scales. Reading scales on maps and determining the actual distance (length) is an appropriate contextual situation. Scale drawings can include shapes other than triangles and quadrilaterals such as circles. Using objects that create similar figures from a light source such as an overhead projector or flash light can be useful in providing applications for similar figures and scale drawings.

Students can also use grids and/or the coordinate plane to help them create scale drawings.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

EXAMPLE

Create a scale drawing that has a scale factor that is ½ of the original.



Discussion: This activity will set the stage for dilations in Grade 8. In Grade 7, they can draw their image anywhere on the grid paper.

Scales drawings can be connected to percent increase in 7.RP.3.

EXAMPLE

Mary has a photo that is 4 inches by 5 inches.

- If she wants a new image that is 75% of the original, what are the new dimensions of the image?
- If she wants to reduce the original by 75%, what are the dimensions of the new image?

DRAWING, DESCRIBING, AND CONSTRUCTING GEOMETRIC FIGURES

Drawing

The standard,7.G.2 is related to the following Grade 7 cluster "Solve real-life and mathematical problems involving angle measure, area, surface area, and volume."

TIP!

To avoid misconceptions students should regularly be exposed to shapes and figures from many perspectives and orientations, not just the prototypical example.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Many careers and everyday activities require spatial reasoning. Some research suggests that 7th grade is the optimal time for developing spatial visualization. Sketching figures can help students develop an intuitive understanding of geometry. Although drawings should become precise over time, informal free-hand sketches can help develop spatial reasoning.

EXAMPLE

- Locate (approximately) the midpoint of each side of a given obtuse triangle. Use a ruler to check how good your guesses are.
- Locate the midpoints of three sides of a given triangle. Now join each vertex to the midpoint of the opposite side to obtain three segments. Do this for many triangles. *Question:* What do you notice about these segments?
- Again, draw a triangle and locate the midpoints of the three sides. This time, join these midpoints to each other to get four smaller triangles inside the original one. Do this for many triangles. *Question:* What do you notice about these smaller triangles?
- Draw a triangle, and from each vertex, drop a line perpendicular to the (lines containing the) opposite side. Do this for many triangles, some obtuse and some acute: *Question:* What do you notice about the three perpendiculars?
- Draw a circle with a given center and a given radius.
- Draw a circle passing through the three vertices of a given triangle. Try acute, obtuse, and right triangles. *Question:* What do you notice about the center of a circle?

Taken from H. Wu's Prealgebra Draft, page 190 revised October 26, 2011, follow the link for more examples.

Discussion: Although, these examples do not align specifically to Grade 7 content, they can be helpful in the process of developing spatial reasoning.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles and quadrilaterals with straws, sticks, or geometry apps. This should be done prior to using rulers, compasses, and/or protractors. Have students discover and justify the side and angle conditions that will form triangles or quadrilaterals.

Use Geometry software to allow students to manipulate figures while constructing.

This would also be a great place to introduce using a compass and straightedge to copy segments with the same length (congruent).

To prepare students for 7.G.3 and 7.G.6, teach students how to draw three-dimensional figures such as right prisms. One way to help students draw three-dimensional prisms is to have them draw the two congruent bases first, and then connect the bases to form the rest of the prism. Another way is by drawing a base and a vanishing point. Discuss how lines that are parallel in real-life do not always appear parallel in drawings because of perspective.

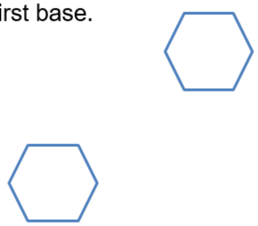
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

EXAMPLE

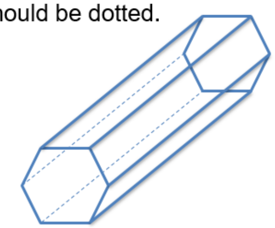
Draw a hexagonal prism.

Method 1: Using Two Bases

Step 1: Draw 2 congruent bases. The second base's position should be shifted vertically and horizontally from the first base.



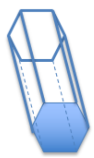
Step 2: Draw lines connecting the corresponding vertices. The connecting lines that intersect the initial base should be dotted.



Note: Have students practice varying the positions of the base to display different perspectives. Notice how changing the initial base changes the orientation. Students may want to color in initial base, erase portions of the dotted lines that intersect the initial base, and/or make the nonvisible edges of the far base dotted.

Positioned vertically; Most visible face viewed from top

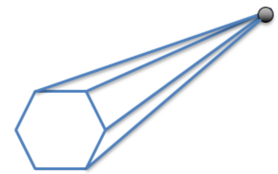
Positioned horizontally; Most visible face viewed from front



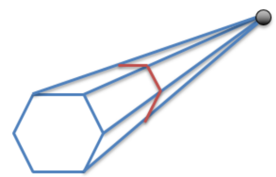
Method 2: Using a Vanishing Point

Step 1: Draw a base.

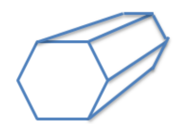
Step 2: Create lines that go to a vanishing point. Do not make lines that go through the base.



Step 3: Create parallel lines to the edges of the base between the lines that go towards the vanishing points



Step 4: Erase the lines from the edge of the far base to the vanishing point.



Note: Although this figure does not use dotted lines to show other sides, they can be easily incorporated using the same principle as Method 1.

To develop spatial reasoning, students also need to be able to manipulate figures in their mind. They should have practice using perspectives to draw figures from different views: top view, front view, side view.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

EXAMPLE

Draw the figure below from the front view, top view, and right view.

Figure

Top View

Front View

Right View

Describing Figures

Students need to describe three-dimensional figures with correct mathematical terminology such as vertices, faces, edges, right, parallel, bases, and perpendicular.

Triangles

Triangle explorations should involve giving students the following to determine if a unique triangle, no triangle, or an infinite set of triangles results:

- three side measures;
- three angle measures;
- two side measures and an included angle measure; and
- two angles and an included side measure.

After discussing the results obtained from their explorations, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. Another driving question should be "Which conditions form a unique triangle?" This question should lead students to informally come to the conclusions that ASA, SAS, AAS, SSS are conditions for unique triangles but AAA and ASS are not. They are not expected to have these conditions memorized or even know the abbreviations.

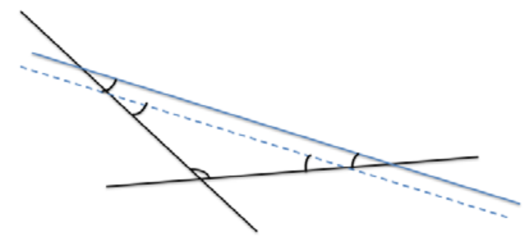
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

EXAMPLE

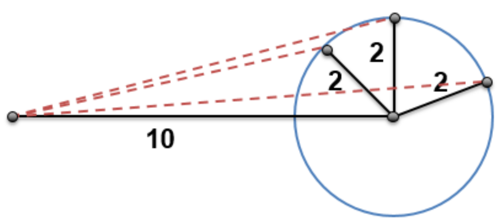
- Cut your straws into the following lengths
  - Set A: 3 in, 4 in, and 6 in
  - Set B: 2 cm, 4 cm, and 9 cm
  - Set C: 1 in, 4 in, and 5 in
  - Set D: 1 in, 3 in, and 5 in
  - Set E: 3 cm, 4 cm, and 5 cm
  - Set F: 4 cm, 3cm, and 12 cm
- Determine which set can form triangles.
- What should the criteria be to form a triangle from any three side lengths?

EXAMPLE

Using Geometry software or a pencil and paper. Build a triangle from 3 intersecting lines. Measure the angles. Move one line outward keeping it parallel to its original position. Measure the angles. What do you notice about the triangles?



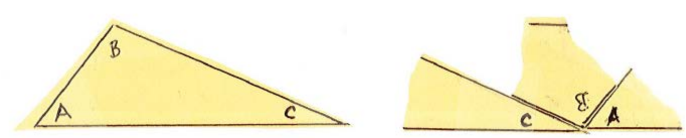
Discussion: The student should come to the realization that although the second triangle is larger (or smaller) than the original, the angles are the same. Therefore, the two triangles are similar.



EXAMPLE

A circle has a radius of 2 units. Its radius is one side of a triangle. The other side of a triangle has a length of 10. How many different triangles can you make?

Students can discover informally that the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees by rearranging the angles of triangles. This will be further formalized in Grade 8. Encourage students not to make conclusions after one example, but explain that they need many examples of different types (acute, obtuse, right, scalene, equilateral, and isosceles) until they can make a generalization that can apply to all triangles.



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Quadrilaterals

Quadrilateral explorations should involve given conditions:

- four sides;
- four angles;
- two sides and two angles;
- pairs of parallel lines;
- pairs of perpendicular lines; and
- noticing types and properties of the resulting quadrilaterals.

Through discussion students will determine if certain conditions create a unique quadrilateral or multiple quadrilaterals. Students should also see that quadrilaterals are "floppy" compared to "rigidity" of triangles. In a quadrilateral it is possible to construct many different quadrilaterals with the same side lengths compared to quadrilaterals where it is not.

EXAMPLE

Using polystrips (or straws) to create quadrilaterals with the following lengths.

- Figure A: 4, 4, 7, 7
- Figure B: 6, 6, 6, 6
- Figure C: 5, 6, 8, 8
- Figure D: 2, 3, 4, 12
- Figure E: 4, 5, 6, 7

Name	Quadrilateral	Trapezoid	Parallelogram	Rhombus	Square	Kite (optional)
A						
B						
C						
D						
E						

- Fill out the chart to determine if a figure makes a quadrilateral and if so which type(s)?
- What qualities do the side lengths have to have in order to create a quadrilateral?
- Can you make more than one type of quadrilateral with the same side lengths? Explain.
- What is the minimum requirement needed to make a trapezoid? A parallelogram? A rhombus? A square?
- Which quadrilaterals have four congruent angles?
- Which quadrilaterals have opposite angles that are congruent?

Discussion: This activity can be extended where students use polystrips to create quadrilaterals using diagonals. They can then explore the properties of diagonals in quadrilaterals such as equivalent diagonals, mutually bisecting diagonals, perpendicular diagonals, or perpendicular bisecting diagonals.

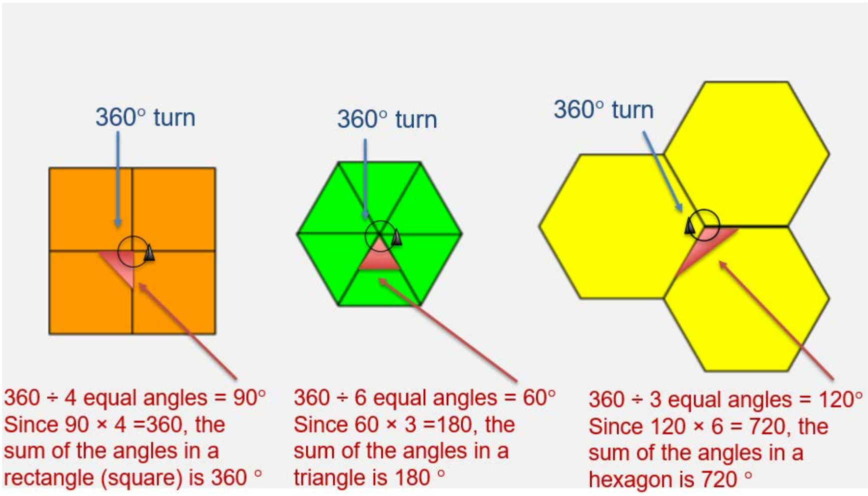
Constructing using a compass and straight edge will prepare students for formal constructions in Geometry (G.CO.12-13).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

EXAMPLE

Construct a parallelogram using a compass and ruler.

Another way to find the sum of the angles of a quadrilateral (or any polygon for that matter) is to take equilateral figures such as pattern blocks and make a shape that shows a 360° turn. Students can then find one of the angles, and then multiply that by the number of angles in the figure.



SLICING THREE-DIMENSIONAL FIGURES

Slicing three-dimensional figures to observe the cross sections formed helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what they discover. For example, use clay to form a cube, then pull string through it in different angles and record the shape(s) of the slices found. Challenges can be given: "See how many different two-dimensional cross sections you can create by slicing a cube."

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

For a cube, the most obvious cross sections are squares. Slightly harder to see are the non-square rectangles that can be achieved by angling the cross section slightly away from parallel to a face. Starting from a vertex, triangular cross sections can result from "slicing off a corner." And what is most amazing is the hexagonal cross section that results from slicing parallel to the ground halfway between top and bottom vertices when the cube is hung from a vertex. For a cylinder, the most obvious cross sections are circles. Students should also be able to find rectangular and oval-shaped (elliptical) cross sections. Some other cross sections have both curved and straight "edges" and would be hard to describe.

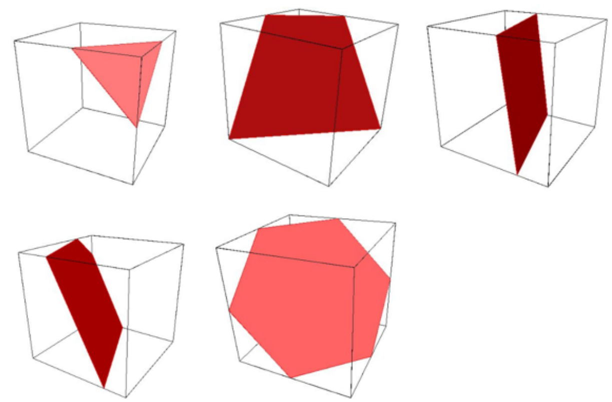


Image taken from: https://ckrao.wordpress.com/2015/02/27/cross-sections-of-a-cube/

TIP! Try out materials ahead of time. Some clay will not slice with string, and some attempts to slice might deform the solid. Play-Doh or Jello molds may work.

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Straws, clay, angle rulers, protractors, rulers, grid paper
- Road Maps - convert to actual miles
- Desmos is a free graphing calculator that is available to students as website or an app.
- Geogebra is a free graphing calculator that is available to students as website.
- Wolframalpha is dynamic computing tool.

Drawing Three-Dimensional Figures

- Using Cubes and Isometric Drawings by NCTM Illuminations is a series of lessons that have students analyze three-dimensional figures. *NCTM now requires a membership to view their lessons.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Similar Figures

- Scaling Angles and Polygons is a task by Illustrative Mathematics where students draw a scaled drawing of figures.
- Scale Drawing 7.G.1 by Geogebra is an applet that allows students to explore similar figures using scale factor.
- Glowing Rectangles by Yummy Math has students find the missing dimension of similar television screens.

Scale Drawings

- Map Distance is a task by Illustrative Mathematics where students have to translate between information provided on a map that is drawn to scale and the distance between two cities represented on a map.
- Rescaling Washington Park is a task by Illustrative Mathematics that get students to think critically about the effect of changing scales on an image.
- Photographs is a task by the Mathematics Assessment Project that has students find dimensions of scaled photographs.
- Scale Model of the Solar System is a video that puts the scale of the universe into perspective.
- Investigating Connections between Measurements and Scale Factors of Similar Figures is an applet by NCTM Illuminations that allows students to explore similar shapes and figures connecting their areas and perimeters to a graph. *NCTM now requires a membership to view their lessons.*
- Sphero Draw & Drive is a 3-Act Task by Mike Wiernicki that has students draw a scale drawing of a ball that rolls.
- Creating a Two-Dimension Model and Constructing a Three-Dimensional Model are lessons by NCTM Illuminations that explore scale models. *NCTM now requires a membership to view their lessons.*
- Drawing to Scale: A Garden is a task by the Mathematics Assessment Project that has students use and interpret scale drawing to plan a garden layout.

Construct Geometrical Figures

- Constructing Triangles by Geogebra is an applet that allows students to change angles and side lengths to see what combinations make a triangle.
- Triangle Maker by Geogebra is an applet that allows students to construct triangles given different lengths sides.
- Properties of Quadrilaterals by Heide McCarty and Luci Howard from Geogebra created is a series of applets that allow students to figure out the properties of quadrilaterals by moving figures.

Slicing Three-Dimensional Figures

- Sections of 3D Shapes by Anthony C.M. from Geogebra created is an applet that allows students to slice different figures.
- Virtual Polystrips by Connect Mathematics 2 is an applet involving virtual polystrips.
- Math Shorts Episode 8-Slicing Three Dimension Figures by Planet Nutshell is a short video about a ninja slicing geometric solids.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Curriculum and Lessons from Other Sources

- EngageNY, Grade 7, Module 1, Topic D, Lesson 16: Relating Scale Drawings to Ratios and Rates, Lesson 17: the Unit Rate as the Scale Factor, Lesson 18: Computing Actual Lengths from a Scale Drawing, Lesson 19: Computing Actual Areas from a Scale drawing, Lesson 20: An Exercise in Creating a Scale Drawing, Lesson 21: An Exercise in Changing Scales, Lesson 22: An Exercise in Changing Scales are lessons that pertain to this cluster.
- EnageNY, Grade 7, Module 4, Topic C, Lesson 12: The Scale Factor as a Percent for Scale Drawings, Lesson 13: Changing Scales, Lesson 14: Computing Actual Lengths from a Scale Drawing, Lesson 15: Solving Area Problems Using Scale Drawings are lessons that pertain to this cluster.
- EngageNY, Grade 7, Module 6, Topic B, Lesson 5: Identical Triangles, Lesson 6, Drawing Geometric Shapes, Lesson 7: Drawing Parallelograms, Lesson 8: Drawing Triangles, Lesson 9: Conditions for a Unique Triangles—Three Sides and Two Sides and the Included Angles, Lesson 10: Conditions for a Unique Triangle—Two Angles and a Given Side, Lesson 11: Conditions on Measurements that Determine a Triangle, Lesson 12: Unique Triangles-Two Sides and Non-Included Angle, Lessons 13: Checking for Identical Triangles, Lesson 14: Checking for Identical Triangles, Lesson 15: Using Unique Triangles to Solve Real-World and Mathematical Problems are lessons that pertain to this cluster.
- EngageNY, Grade 7, Module 6, Topic C, Lesson 16: Slicing a Right Rectangular Prism with a Plane, Lesson 17: Slicing a Right Rectangular Pyramid with a Plane, Lesson 18: Slicing on an Angle, Lesson 19: Understanding Three-Dimensional Figures.
- Illustrative Mathematics, Grade 7, Unit 1: Scale Drawings has many lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- Illustrative Mathematics, Grade 7, Unit 7: Angles, Triangles, and Prisms, Lesson 6: Building Polygons (Part 1), Lesson 7: Building Polygons (Part 2), Lesson 8: Triangles with 3 Common Measures, Lesson 9: Drawing Triangles (Part 1), Lesson 10: Drawing Triangles (Part 2), Lesson 11: Slicing Solids are some lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- The Utah Middle School Math Project is an open source textbook and workbook.

General Resources

- Arizona 7-12 High School Progressions on Geometry
  This cluster is addressed on pages 3-4, 6-7.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- van Hiele Model of Geometric Thinking is a pdf created by ODE that summarized the van Hiele levels.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.1-3)

Research

- Battista, M. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 629-668).
- Battista, M. (November 1999). The importance of spatial reasoning. *Teaching Children Mathematics*, *6(3),* 171-177.
- Charlotte, NC: Information Age Publishing.
- Chen, Y. & Mix, K. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development*, *15(1),* 2-11. doi: 10.1080/15248372.2012.725186
- Clements, D. & Battista, M., (1992) Geometry and spatial reasoning. In D.A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp.420-464). New York: Macmillan.
- Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 7–8, HS, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Cox, D. & Lo, J. (August 2012). Discuss similarity using visual intuition. *Mathematics Teaching in the Middle School*, *18(1),* 30-36.
- DeJarnette, A., Lausell, R., and González, G. (August 2015). Shadow puppets: Exploring a context for similarity and dilations. *Mathematics Teacher*, *109(1),* 20-27.
- Frost, J., & Dornoo, M. (2006). Similar shapes and ratios. *Mathematics Teaching in the Middle School*, *12(4),* 222-224.
- Kelly, G., Ewers, T., & Proctor, L. (December 2002). Developing spatial sense: Comparing appearance with reality. *Mathematics Teacher, 95(9),* 702-704.
- Kinach, B. (March 2012). Fostering spatial vs. metric understanding in geometry. *Mathematics Teacher*, *105(7),* 534-540.
- Ontario Ministry of Education. (2014). *Paying attention to spatial reasoning: Support document for paying attention to mathematics education.* Retrieved from: http://www.edu.gov.on.ca/eng/literacynumeracy/LNSPayingAttention.pdf
- Pugalee, D., Frykholm, J., Johnson, A, Slovin, H, Malloy, C., & Preston, R. (2002). *Navigating Through Geometry in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.
- Van De Walle, J., Karp, K., Bay-Williams, J. (2010). *Elementary and Middle School Mathematics* (7th ed.). Boston, MA: Pearson Education, Inc.
- Wu, H. (2013). Teaching Geometry According the Common Cores Standards. From https://math.berkeley.edu/~wu/Progressions_Geometry.pdf
- Wu, H. (2011). Pre-Algebra. From https://math.berkeley.edu/~wu/Pre-Algebra.pdf
- Yopp, D. (March 2016). Dilations and the equation of a line. *Mathematics Teaching in Middle School*, *21(7),* 406-414.

STANDARDS	MODEL CURRICULUM (7.G.4-6)
GEOMETRY **Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.6** Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**Expectations for Learning** Grade 7 is the first experience that students have with measuring radius, diameter, and circumference of circles and exploring their relationships. Students build on earlier experiences with angle measurement to solve problems that involve special angle pairs. Students will apply their understanding of area, surface area, and volume to solve real-world problems using a variety of strategies. *Note: The volume of a pyramid and surface area and volume of a cylinder are not addressed at this grade level.* This cluster could also be extended for some students to include the surface area of cylinders. In Grade 8, students will apply their knowledge of circles and expand their understanding of volume to cones, cylinders, and spheres. Although, students' formal understanding of rational and irrational numbers is not defined until Grade 8, at this grade, they are expected to just have an intuitive understanding of pi and its approximation. In Grade 8, students will extend their knowledge of special angle pairs to relationships when parallel lines are cut by a transversal. The student understanding of circles aligns with van Hiele Level 2 (Informal Deduction/Abstraction). The student understanding of special angle pairs, area, surface area, and volume aligns with van Hiele Level 1 (Analysis). **ESSENTIAL UNDERSTANDINGS** **Circles** • Points on a circle are the same distance from the center. • A circle is created by connecting all the points equidistant from the center point • The radius is the distance from any point on the circle to the center. • The diameter is a straight line that passes through a circle and its center point where both end points lie on the circle. • The diameter is twice the radius. • The circumference is the distance around a circle. • Pi is a constant ratio of a circle's circumference divided by its diameter. • The symbol for pi is π. *Continued on next page*

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.G.4-6)</th></tr>
<tr><td>7.G.4-6, continued</td><td>

<h2>Expectations for Learning, continued</h2>
<h3>ESSENTIAL UNDERSTANDINGS, CONTINUED</h3>
Special Angle Pairs

Two angles are supplementary when their angle measures have a sum of 180 degrees.
Two angles are complementary when their angle measures have a sum of 90 degrees.
Vertical angles are the angles opposite each other when two lines intersect; the angles are congruent.
Vertical angles are congruent because they are both supplementary to the same angle.
Two angles are adjacent when they have a common side and a common vertex and do not overlap.

Applications of Area, Surface Area, and Volume

A right prism is a prism whose bases are parallel to one another and whose lateral faces are rectangles.
In a right prism the bases are perpendicular to the vertical sides.
Prisms can have bases other than rectangles.
The two bases of a prism are the same shape and size.

<h3>MATHEMATICAL THINKING</h3>

Explain mathematical reasoning.
Use accurate mathematical vocabulary to describe mathematical reasoning.
Make and modify a model to represent mathematical thinking.
Solve real-world mathematical problems.

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<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.G.4-6)</th></tr>
<tr><td>7.G.4-6, continued</td><td>

<h2>Expectations for Learning, continued</h2>
<h3>INSTRUCTIONAL FOCUS</h3>
<h4>Circles</h4>

Identify and define characteristics of circles: radius, diameter, circumference (perimeter), and area.
Explore radius and diameter relationships.
Investigate the relationship between a circles' radius or diameter and its circumference and area.
Explain the relationship between the radius (and/or diameter) and the length measure of the circle (circumference).
Discover an approximate value of pi.
Know the formulas for the area and circumference of a circle.
Apply the formulas for the area and circumference of a circle to solve real-world problems.

<h4>Special Angle Pairs</h4>

Investigate the relationships of special angle pairs through modeling.
Identify and define special angle pairs: supplementary, complementary, vertical, and adjacent angles.
Use special angle pairs to write and solve equations for multi-step problems.
Measure and find patterns among the angles of intersecting lines including cases where angles lie within polygons.

<h4>Applications of Area, Surface Area, and Volume</h4>

Draw a picture or create a model to compose and decompose two- and three-dimensional figures to find area, surface area, and volume.
Differentiate between linear, square, and cubic units and when to use them.
Solve real-world problems involving area, surface area, and volume.

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<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.G.4-6)</th></tr>
<tr><td>7.G.4-6, continued</td><td>
<h2>Content Elaborations</h2>

Ohio's K-8 Critical Area of Focus Grade 7, Number 3, pages 46-47
Ohio's K-8 Learning Progressions, Geometry, page 21

<h3>CONNECTIONS ACROSS STANDARDS</h3>

Write and solve equations to solve geometric problems (7.EE.4.a).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice

This cluster focuses on but is not limited to the following practices:

***MP.1** Make sense of problems and persevere in solving them.*

***MP.3** Construct viable arguments and critique the reasoning of others.*

***MP.4** Model with mathematics.*

***MP.5** Use appropriate tools strategically.*

***MP.6** Attend to precision.*

This cluster builds on Grade 6 topics of area, surface area, and volume and on Grade 4's work with angles. This is the students' initial work with circles. Their work with area extends to composite figures and volume extends to right prisms besides rectangular prisms. The goal of this cluster is for students to justify their thinking mathematically and apply the geometric concepts to real-world problems. *Note: It is appropriate in Grade 7 (although not required) to explore the surface area of a cylinder and/or volume of a pyramid. There is no specific mention of surface area of cylinders or volume of pyramids in the standards, so it is up to a district to decide the best place to teach these concepts*

VAN HIELE CONNECTION

In Grade 7 the student understanding of circles aligns with van Hiele Level 2 (Informal Deduction/Abstraction).The student understanding of special angle pairs, area, surface area, and volume aligns with van Hiele Level 1 (Analysis).

Visualization

The van Hiele Level 1 (Analysis) can be characterized by the student doing some or all of the following:

- showing a greater degree of attention to properties of shapes and solids;
- building 3D figures from 2D images and 2D drawings from 3D figures;
- viewing a figure from front, back, left, and right positions of solids;
- visualizing cross-sections when slicing solids;
- comparing solids based on properties;
- using observation as a basis for explanations; and/or
- understanding that a movement is made, then after observing the result, the next movement is selected, etc.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

Geometric Measurement

The van Hiele Level 1 (Analysis) can be characterized by the student doing some or all of the following:

- comparing length, area, or volume by manipulating and matching parts;
- visually comparing shapes by composing/decomposing;
- visualizing structured iteration of length, area, or volume units;
- organizing area and volume units into (2D, 3D) array structure without gaps or overlaps.
- using a single unit, row, or layer repeatedly (iterating) to correctly measure or construct length, area, or volume respectively.
- determining measurement without having to show every unit instead of using only numbers (no visible units or repeated units); and/or
- creating composite units, columns, rows, or layers to find length, area, or volume.

Shapes

The van Hiele Level 2 (Informal Deduction/Abstraction) can be characterized by the student doing some or all of the following:

- noticing relationships within and between properties and figures;
- forming meaningful definitions;
- justifying reasoning using informal arguments;
- drawing logical maps and diagrams; and/or
- not yet understanding formal deduction.

See the van Hiele pdf on ODE's website for more information about van Hiele levels.

CIRCLES

Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, π, and area. Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius.

EXAMPLE

Part 1

- Give students a sheet of paper with a rectangle, a triangle, and a circle on it.
- Tell them to "color the circle green." *Most students will color the area of the circle.*

Example continued on next page

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

Part 2

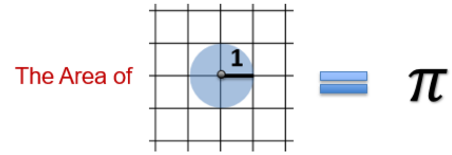
- Give each student a small object such as a piece of candy or binder clip to hold.
- Stand in one place as the teacher. Take a rope with a knot on each end, and hold one knot to your belly-button.
- Tell each student to take the knot on the other end of the rope, hold it to his or her belly-button, and stretch the rope its full length.
- Have each student remain in place.
- Continue with each student until all students are placed a rope-length's distance from the teacher. (The teacher needs to turn 360-degrees, but stay on the same spot.)
- Have each student place their small object on the floor and remain in their spot.
- Remove the rope, when all students are equal distance from the teacher.
- Have each student take two steps back.

Discussion: Use this example to help students discover the "circle." Discuss that a circle is actually all the points (objects) that make a circle and that each of the points are an equal distance from the center (the teacher). After the discussion have students take a new piece of paper with the rectangle, triangle, and circle printed on it and direct them to "color the circle red." Hopefully this time when they are instructed to "color the circle red," they will only color the circle, not the area of the circle.

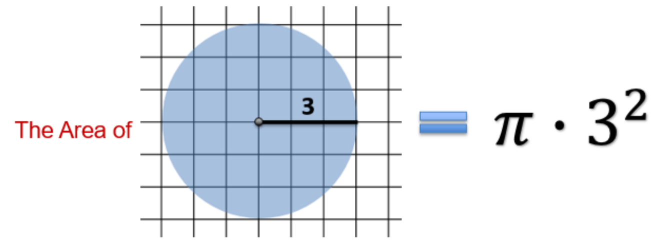
Students are to explore relationships between area, circumference, diameter, and radius, recognizing the constant of proportionality between each of these elements without formally defining the irrational nature of π.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

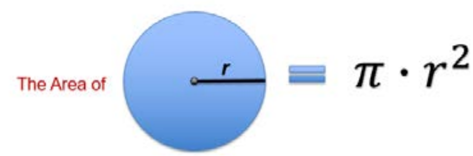
Finding the Area of a Circle using Similar Figures
Establish that all circles are similar (7.G.1), and remind students that when a figure is scaled by a factor of *x*, the area changes by a factor of x^2. Define π as the area of the region inside a circle when the radius is 1 unit. (Defining π in this manner will help students in high school Geometry.)



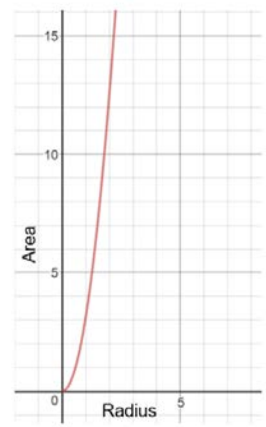
Now if you enlarge a circle by a scale factor of 3, that means that the radius changes by a factor of 3 and the area changes by a factor of 3^2.



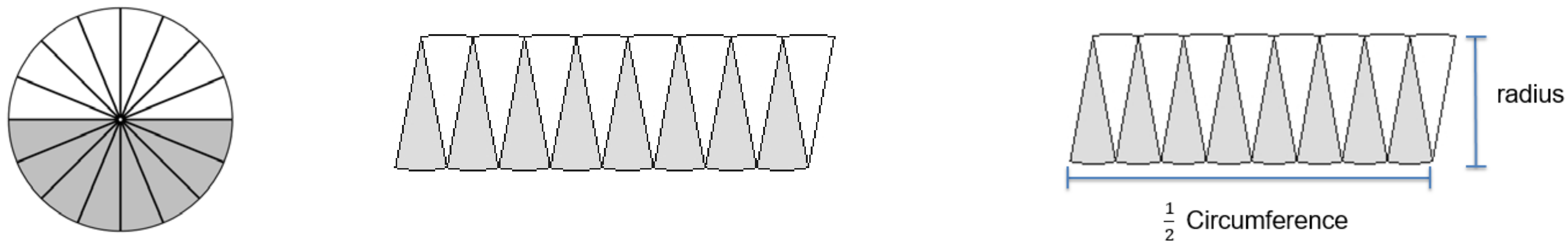
So if you enlarge a circle by a scale factor of *r*, that means that the radius changes by a scale factor of *r*, and the area changes by a scale factor of r^2; hence the formula for the area of a circle is $A = \pi r^2$.



Draw attention that although there is a relationship between the area and radius, it is not directly proportional because the graph is not a straight line.

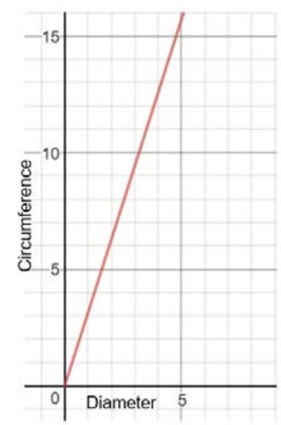


Using the Area of a Circle to Find the Circumference
Another visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. Explain to students that the perimeter of the circle is the circumference.



Ask students to identify where the circumference and the radius is in their new shape. Identifying the radius gives cause for conversations. They should come to the conclusion that the height is the radius and the length is ½ of the circumference.

Therefore another formula for area is $A = \frac{1}{2}Cr$. Since students already found the area of circle as $A = \pi r^2$ or $A = \pi rr$, students should realize that $\frac{1}{2}Cr = \pi rr$. If they divide each side by r, then can simplify the formula to $\frac{1}{2}C = \pi r$. Now if they solve for C by multiplying each side by 2, they will get $C = 2\pi r$, and they may see that $2r = d$ and change the formula to get $C = d\pi$.

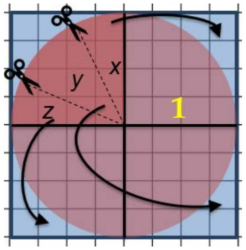


Finding an Approximation of π
Given multiple-sized circles, students should then explore the relationship between the radius or diameter and the length measure of the circle (circumference) finding an approximation of π and connecting it to the formula for circumference they already found. String or yarn laid over the circle and compared to a ruler is an adequate estimate of the circumference. As students discover there is a proportional relationship between the circumference and the diameter (or radius), draw attention to the fact that the constant of proportionality is π. It is still the constant of proportionality even though it never repeats or end, it is still a constant number.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

This same process can be followed to find the relationship between the diameter and the area of a circle by using grid paper to estimate the area. In figuring area of a circle, the squaring of the radius can also be explained by showing a circle inside a square. Again, the formula is derived and then learned. After explorations, students should then solve problems, set in relevant contexts, using the formulas for area and circumference.

EXAMPLE



Discussion: Have students draw a circle inscribed in a square on grid paper where the radius is 1. They could count the squares inside the quarter circle segment and compare that to the non-covered square where the other three segments are. They should be able to see that they can cover three squares totally and then have a tiny part left. Another way is to have students literally cut apart ¼ of the segment of a circle and fit it into the rest of the square. They should see that the area is about 3 square units with a little bit left over. Then the radius could be changed. They should come to the conclusion that ¼ sector of the circle still fits into three other squares with a little left over.

Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering). Make a connection to units of measure. For example the length of a rectangle can be measured in centimeters because centimeters can surround the rectangle, and the area of the rectangle is measure in squared centimeters because squares cover the rectangle.

Students may incorrectly believe that π is an exact number such as 3.14 or $\frac{22}{7}$ rather than understanding that 3.14 and $\frac{22}{7}$ are just approximations of π.

ANGLES

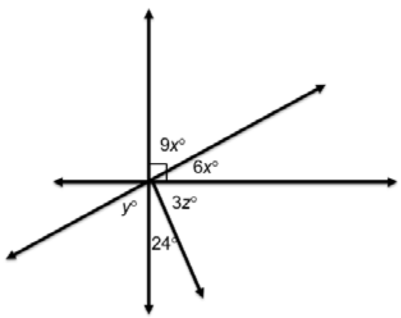
In previous grades, students have studied angles by type according to size: acute, obtuse and right. Their role as an attribute in polygons were also studied. Now angles are studied based upon the special relationships that exist among them: supplementary, complementary, vertical, and adjacent. Two angles are considered to be "the same" at this grade level if they can be superimposed upon each other. This standard also ties in with 7.EE.4.

Provide students the opportunities to explore these relationships. At first they can measure and find patterns among the angles of intersecting lines or within polygons. Then they can utilize the relationships to write and solve equations for multi-step problems.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

EXAMPLE

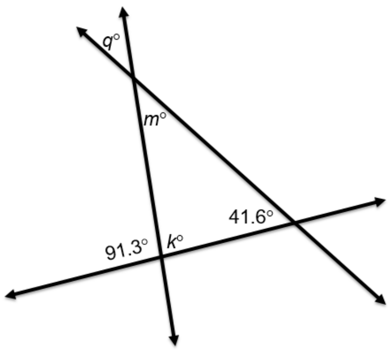
Find the measure of *x*, *y*, and *z*.



A student often incorrectly thinks that a wide angle with short sides is smaller than a narrow angle with long sides. To confront this problem have students compare angles with different side lengths.

EXAMPLE

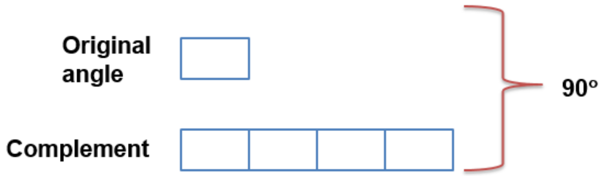
Find the measure of *q*, *m*, and *k*.



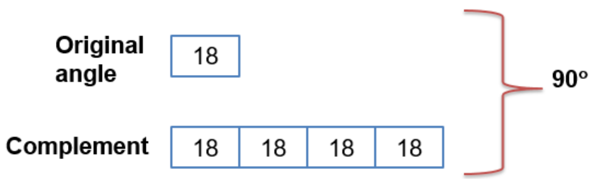
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

EXAMPLE

The measurement of an angle is ¼ the measurement of its complement. Find the measurement of an angle and its complement.



Method 1



Method 2

$$\frac{1}{4}x + \frac{4}{4}x = 90$$
$$\frac{5}{4}x = 90$$
$$\left(\frac{4}{5}\right)\frac{5}{4}x = 90\left(\frac{4}{5}\right)$$
$$x = 72$$

Therefore the original angle equals 18° and the complement equals 72°.

Discussion: A student can start by drawing a diagram to represent the original angle and its complement. In Method 1 a student can divide 90 by 5 (since there are 5 total boxes) and get 18 for each box. They can then see the original angle equals 18° and the complement is 18(4) or 72°. Students need to do many examples with Method 1 to reach the sophistication of Method 2.

APPLICATIONS OF AREA, SURFACE AREA, AND VOLUME

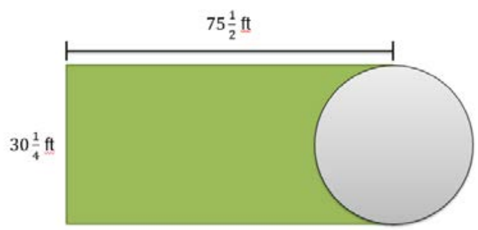
Real-world and mathematical multi-step problems that require finding area, perimeter, volume, and surface area of figures composed of triangles, quadrilaterals, polygons, cubes, and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios, and various units of measure with same system conversions.

Area

In Grade 6 students found the area for triangles, special quadrilaterals, and polygons. Grade 7 extends this concept to include composite shapes including circles, semi-circles, and quarter-circles. Most area problems should be given in a real-world context. Students can also find the area of regular polygons using triangles if the apothem is given. (Students do not need to use the term apothem.)

EXAMPLE

Melissa wants to plant grass next to her circular patio. How big is the grassy area?



Surface Area

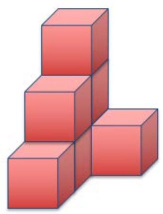
In Grade 6 students used nets to find surface area of three-dimensional figures when nets were composed of rectangles and triangles. Grade 7 extends this concept to finding the surface area of pyramids and more complex prisms. Instead of being restricted to using nets to find surface area, students may prefer to draw the different views of a structure (front, right, top) The use of formulas to find surface area should be discouraged.

To develop their spatial reasoning skills, have students build structures with unit cubes and find the surface are of their structure. They can solve these problems by using a net or by drawing the different views of the figure (front, right, top). Also give students a net, and have them build the structure from the net.

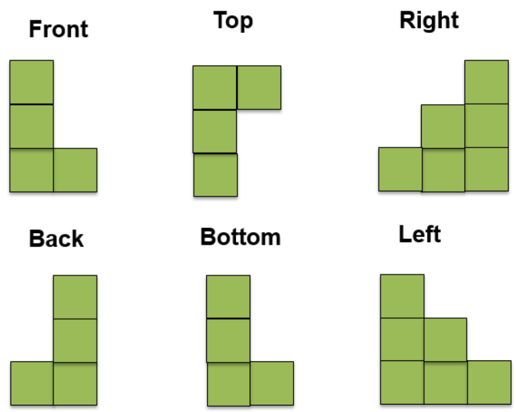
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

EXAMPLE

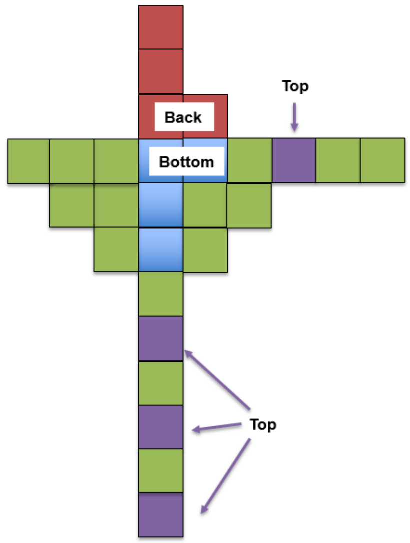
Find the surface area of the figure below made of inch cubes.



Method 1: Draw the 6 different view of the figures



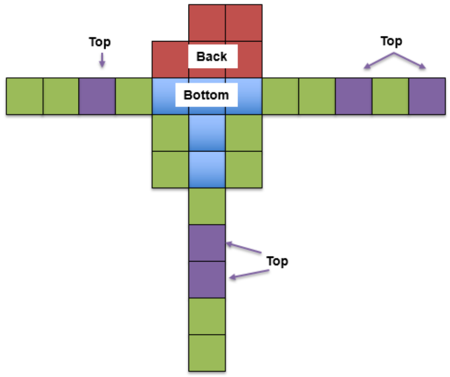
Method 2: Draw a Net
(Note: There are several correct nets that can be drawn.)



Discussion: Some students will prefer drawing nets and others will prefer drawing the six different views. After practice with both methods, let students use their preferred method. Once students are comfortable finding the surface area of unit cubes, tell students that the cubes lengths are rational numbers such as ¼ inch and have them calculate the surface area.

EXAMPLE

A figure has a surface area of 28in^2. Its net is shown to the right, build the corresponding block structure.



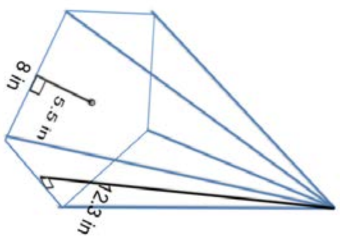
Discussion: For scaffolding purposes you can add or remove labels to help or challenge students. Color coding is another strategy that can help. The same activity can also be done using the six different views of a figure.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

Most of the work students do involving surface area should be in the context of real-world problems. Draw a connection between nets and the top, bottom, right, left, front, and back view of the figures. Draw attention to congruent faces in a figure and encourage students to use that knowledge to be more efficient in their work. When a base is a regular polygon such as a hexagon, the apothem (students do not need to know this term) or other pertinent information needs to be given, so students can use triangles to figure out the area of the base(s). Even though students are not necessarily finding the volume of a pyramid at this level, distinguishing between the slant height and the height of the shape is important for students to find the surface area of a pyramid. Students may also find the surface area of prisms and pyramids where the bases are irregularly shaped polygons that can be easily divided into familiar shapes that students already know how to calculate.

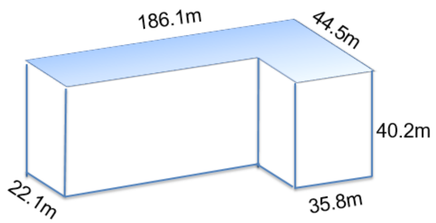
EXAMPLE

An artist is making a sculpture using steel plates which includes the base in the shape of a pentagonal pyramid. How much steel will he need?



EXAMPLE

Mark needs to put sealant on an underground L-shaped building. He needs to seal the top and sides of the building. How much sealant does he need?

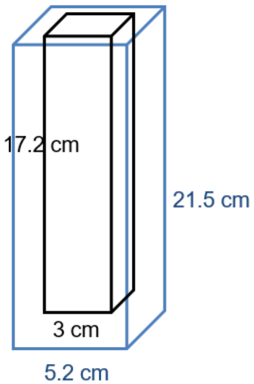


Volume

Connect the concept of volume of prisms to slicing geometric solids (7.G.3). Have students build solids using layers in the shape of the base. They can also build three-dimensional prisms and pyramids by using coffee stirrers with twist ties, modeling clay, plastic drinking straws, or rods created out of rolled newspaper. Connect triangular prisms with rectangular prisms by letting students discover that half of a rectangular prism could be a triangular prism. Again, emphasize volume in the context of real-world problems. Use problems that connect volume to surface area.

EXAMPLE

A glass vase is in the shape of a square prism. It has an opening that is also a square prism. Calculate the amount of glass in the vase.



Some students incorrectly think that the top and bottom of a prism are always the bases of a prism not realizing that a prism can be rotated. Reinforce that the bases must be two parallel faces.

A common misconception among middle grade students is that "volume" is a "number" that results from "substituting" other numbers into a formula. For these students there is no recognition that "volume" is a measure—related to the amount of space occupied. If a teacher discovers that students still do not have an understanding of volume as a measure of space, it is important to provide opportunities for hands on experiences where students "fill" three dimensional objects. Begin with right-rectangular prisms, and fill them with cubes. This will help students understand why the units for volume are cubed. See Cubes from NCTM Illuminations.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- circular objects of several different sizes
- string or yarn
- tape measures, rulers
- grid paper
- paper plates

Circles

- Square Circles by NCTM Illuminations is lesson that features two creative twists on the standard lesson of having students measure several circles to discover that the ratio of the circumference to the diameter seems always to be a little more than 3. *NCTM now requires a membership to view their lessons.*
- Apple Pi by NCTM Illuminations is a lesson using estimation and measurement skills, students will determine the ratio of circumference to diameter and explore the meaning of π. Students will discover the circumference and area formulas based on their investigations. *NCTM now requires a membership to view their lessons.*
- Circle Tool by NCTM Illuminations is a three-part online applet where students can explore with graphic and numeric displays how the circumference and area of a circle compare to its radius and diameter. Students can collect data points by dragging the radius to various lengths and clicking the "Add to Table" button to record the data in the table.
- Geometry of Circles by NCTM Illuminations is a lesson that uses a MIRA™ geometry tool, students determine the relationships between radius, diameter, circumference, and area of a circle. *NCTM now requires a membership to view their lessons.*
- Finding Areas of Circles is a task by Mathematics Assessment Project where students apply their knowledge of the relationship between the radius and the area of the circle.
- How Can We Water All of the Grass? is a modeling problem by Robert Kaplinsky where students have to apply their knowledge of finding the area of circles.
- Rolling a Circle to Find Pi is an applet from Geogebra where students explore circumference.
- Would You Rather #15 Pizza Slices from wouldyourathermath has students compare areas of different slices of pizza. It contains a link to Geogebra for scaffolding: Would You Rather #15 Pizza Slices.
- The Circle's Measure is a lesson by Utah's Education Network where students make a connection between the circumference and diameter of circles.
- Approximating the Area of a Circle is a task by Illustrative Mathematics where students explore how the ratio Area of a Circle (radius of circle)^2 does not depend on the radius.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

Circles, continued

- Pi Day Is Coming! by YummyMath has many activities relating to Pi Day.
- Area of a Circle by Annenberg Learner has students explore the formula for area of a circle.
- Circles and Pi by Annenberg Learner contains two lessons where students investigate the circumference and area of a circle.

Angles

- Complementary and Supplementary Angles Exploration is an applet by Geogebra that explores complementary and supplementary angles.

Area of Composite Figures and Surface Area

- Finding the Area of Irregular Figures by NCTM Illuminations is a lesson where students estimate the areas of highly irregular shapes by using decomposition. *NCTM now requires a membership to view their lessons.*
- Dandy Candies is a 3-act task by Dan Meyers where students apply surface area and volume to a situation involving wrapping cube-shaped candy pieces.
- Maximizing Area: Gold Rush is a task by Mathematics Assessment Project where students explore the effects on area while keeping the perimeter constant.

Volume

- Sand Under the Swing Set is a task by Illustrative Mathematics where students explore volume as well as applying scale factor in a real-world context.

Curriculum and Lessons from Other Sources

- Illustrative Mathematics, Grade 7, Unit 3: Measuring Circles has many lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- Georgia Standards of Excellence Curriculum Frameworks, Grade 7, Unit 4: Geometry has many tasks that align to this cluster. These tasks can be found on pages 42-122
- Illustrative Mathematics, Grade 7, Unit 7, Lesson 1: Relationships of Angles, Lesson 2: Adjacent Angles, Lesson 3: Nonadjacent Angles, Lesson 4: Solving for Unknown Angles, Lesson 5: Using Equations to Solve for Unknown Angles, Lesson 12: Volume of Right Prisms, Lesson 13: Decomposing Bases for Area, Lesson 14: Surface Area of Right Prisms, Lesson 15: Distinguishing Volume and Surface Area, Lesson 16: Applying Volume and Surface Area are many lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- EngageNY, Grade 7, Module 3, Topic C, Lesson 16: The Most Famous Ratio of All, Lesson 17: The Area of Circle, Lesson 18: More Problems on Area and Circumference, Lesson 19: Unknown Area Problems on the Coordinate Plane, Lesson 20: Composite Area Problems, Lesson 21: Surface Area, Lesson 22: Surface Area, Lesson 23: The Volume of a Right Prism, Lesson 24: The Volume of a Right Prism, Lesson 25: Volume and Surface Area, Lesson 26: Volume and Surface Area are lesson that pertain to this cluster.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

Curriculum and Lessons from Other Sources, continued

- EngageNY, Grade 7, Module 6, Topic D, Lesson 20: Real-World Area Problems, Lesson 22: Area Problems with Circular Regions, Lesson 23: Surface Area, Lesson 24: Surface Area are lessons that pertain to this cluster.
- EngageNY, Grade 7, Module 6, Topic E, Lesson 25: Volume of Right Prisms, Lesson 26: Volume of Composite Three-Dimensional Shapes, Lesson 27: Real-World Volume Problems are lessons that pertain to this cluster.
- EngageNY, Grade 7, Module 3, Topic B, Lesson 10: Angle Problems and Solving Equations, Lesson 11: Angle Problems and Solving Equations are lessons that pertain to this cluster.
- EngageNY, Grade 7, Module 6, Topic A, Lesson 1: Complementary and Supplementary Angles, Lesson 2: Solving for Unknown Angles Using Equations, Lesson 3: Solving for Unknown Angles Using Equations, Lesson 4: Solving for Unknown Angles Using Equations, are lessons that pertain to this cluster.
- Utah's Education Network, Mathematics, Grade 7 has many links to lessons and activities organized by standard.

General Resources

- Arizona 7-High School Progression on Geometry
  This cluster is addressed on pages 4, 7-8.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.

References

- Battista, M. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 629-668). Charlotte, NC: Information Age Publishing.
- Battista, M. (November 1999). The importance of spatial reasoning. *Teaching Children Mathematics*, *6(3),* 171-177.
- Chen, Y. & Mix, K. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development*, *15(1),* 2-11. doi: 10.1080/15248372.2012.725186
- Clements, D. & Battista, M., (1992) Geometry and spatial reasoning. In D.A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp.420-464). New York: Macmillan.
- Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 7–8, HS, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Flores, A. & Regis, T. (March 2003). How many times does a radius square fit into a circle? *Mathematics Teaching in the Middle School*, *8(7),* 363-457.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.G.4-6)

References, continued

- Kelly, G., Ewers, T., & Proctor, L. (December 2002). Developing spatial sense: Comparing appearance with reality. *Mathematics Teacher, 95(9),* 702-704.
- Miles, V. (November 2014). Maximizing volume with solids and nets. *Mathematics Teaching in the Middle School*, *20(4),* 247-251.
- Neel-Romine, L, Paul, S., & Shafer, K. (November 2012). Get to know a circle. *Mathematics Teaching in the Middle School*, *18(4),* 222-227.
- Ontario Ministry of Education. (2014). *Paying attention to spatial reasoning: Support document for paying attention to mathematics education.* Retrieved from: http://www.edu.gov.on.ca/eng/literacynumeracy/LNSPayingAttention.pdf
- Tent, M. (April 2001). Circles and the number π. *Mathematics Teaching in the Middle School*, *6(8),* 452-457.
- Van De Walle, J., Karp, K., Bay-Williams, J. (2010). *Elementary and Middle School Mathematics* (7th ed.). Boston, MA: Pearson Education, Inc.
- Wheatley, J. (August 2011). An investigation of three-dimensional problem solving and levels of thinking among high school geometry students: A project report presented to the graduate faculty of Central Washington University. Retrieved from: https://pdfs.semanticscholar.org/994a/d70f32b0dd2b38342da230b6275e08f9032e.pdf
- Wu, H. (2013). Teaching Geometry According the Common Cores Standards. From https://math.berkeley.edu/~wu/Progressions_Geometry.pdf
- Wu, H. (2011). Pre-Algebra. From https://math.berkeley.edu/~wu/Pre-Algebra.pdf

STANDARDS	MODEL CURRICULUM (7.SP.1)
STATISTICS AND PROBABILITY **Use sampling to draw conclusions about a population.** **7.SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population. **a.** Differentiate between a sample and a population. **b.** Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.	**Expectations for Learning** In earlier grades students have been using data, both categorical and numerical, to answer simple statistical questions, but have paid little attention to how the data were selected. In 6th grade students use center and variability to describe data. In 7th grade, students will begin to investigate populations, sampling, and bias. Students move from Level A to Level B in the GAISE model by sampling more than one group. Using these concepts, students will begin to draw their own conclusions and support the conclusions of others. In high school, students will further develop their understanding using experimental designs through random sampling. **ESSENTIAL UNDERSTANDINGS** • Statistics is the name for the science of collecting, analyzing, and interpreting data. • A population consists of everyone in a specific group and a sample is a subset from a specific group. • Results from a sample can be generalized for a much larger population. • Sampling variability exists because the sample proportion varies from sample to sample. • Bias, a systematic favoritism in the data collection process, can occur in the way the sample is selected or in the way data are collected. **MATHEMATICAL THINKING** • Construct valid conclusions. • Critique reasoning used to draw conclusions. • Formally explain mathematical reasoning. • Use formal and precise mathematical language. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.SP.1)
7.SP.1, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** • Differentiate between a sample and a population. • Describe what makes a sample an accurate representation of a population. • Describe how sample size affects inferences made about the population. • Develop informal understanding of bias. • Determine what factors create bias such as wording, length, timing of questions, and the choice of individuals. **Content Elaborations** • Ohio's K-8 Critical Areas of Focus, Grade 7, Number 4, page 48 • Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23 • GAISE Model, pages 14 – 15 o Focus of 7th grade is Level A – B, pages 22-59 **CONNECTIONS ACROSS STANDARDS** • Broaden the understanding of the framework of the GAISE Model (7.SP.2). • Describe and analyze distributions (7.SP.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.1)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice

This cluster focuses on but is not limited to the following practices:

***MP.1** Make sense of problems and persevere in solving them.*

***MP.2** Reason abstractly and quantitatively.*

***MP.6** Attend to precision.*

***MP.8** Look for and express regularity in repeated reasoning.*

"Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write." is a quote by H.G. Wells in *How to Lie with Statistics* (1954). Initial understanding of statistics, specifically variability and the measures of center and spread begins in Grade 6. This cluster focuses on understanding what statistics is and its importance. Statistics is the science of collecting, analyzing, and interpreting data. Many people present statistical information as facts, but upon a closer look it is clear that the data is being presented with a certain bias. Without being able to critically read and evaluate statistical numbers and graphs, people can easily be misled.

Although mathematics and statistics both use numbers and have many similarities, there are also differences. In mathematics many people oftentimes remove the context from the problem, do the math or look for patterns, and then reinsert the context to solve the problem. This is impossible in statistics because the meaning of the patterns depends upon the context. Without the context, statistics would not exist.

Note: One of the changes to this cluster was the deletion of the word "random" from the cluster statements. Students should informally learn about what a random sample is and how it is useful in statistics. Students are not required to actually use true random sampling when collecting data. Instead they should discuss which samples are the best representative of a population. However, a teacher may wish to extend to more sophisticated ideas of random sampling depending on the makeup of his or her class.

PURPOSE OF STATISTICS

The purpose of statistics is to gain information about a population. This is done by analyzing the data, making inferences based on the data, and communicating that data to others which could be done verbally, mathematically, or graphically.

SAMPLES

A population refers to the entirety of data in a set, whereas a sample is a smaller subset of the population. *Note: The population refers to the entirety of data which may or may not include people.* A sample is used, because in most circumstances it is difficult, costly, or impossible to gain information from an entire population.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.1)

Samples are used to make predictions about an entire population and judge the possible discrepancies of the predictions. The more closely a sample represents the population, the more accurately the data can be used to describe the population; and therefore the inferences one can make about the population are stronger.

Students should start to recognize that variability not only occurs between individuals in the same group, but also between two groups. A representative sample should take the difference between groups into account.

EXAMPLE

Discuss the pros and cons of the following groups to use as a sample to finding the average G.P.A of the students at your school.

- Honors Algebra
- Drama
- Homerooms that are organized by first period classes
- Homerooms that are organized alphabetically
- The class that you are in
- The class with most of your friends
- The basketball team
- An 8^{th} grade English class
- After school tutoring
- The first 25 people you see at school in the morning
- Your lunch table
- The first 25 people who approach you sitting in a booth at lunch

Providing opportunities for students to use real-life situations. This shows the purpose for using sampling to make inferences about a population. Students need to develop the skills and understandings required to produce a representative sample of the general population. Understanding validity of a representative sample recognizes that each element of the population has an equal opportunity to be selected in the sample. The sample size also affects the validity of the sample. In addition, students should explore the role probability has in selecting a sample.

One sample is not representative of the entire population. Many samples must be taken in order to make an inference that is valid. By comparing the results of one sample with the results of multiple samples, students can correct this misconception.

Provide students with samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.1)

EXAMPLE

- What might have caused the data to look like this?
- What conclusions can you make based on your data?
- How strong are your conclusions? Justify using your data.
- Are there any issues with your sample? If so, explain how that could affect your outcome?
- Could your findings "scale up" to a larger group? If so, explain how.

Sample Mean

The population mean is the mean of the population. It is often represented by mew (μ). The sample mean $(\bar{x})$ is the "average" of all the means from the various samples. It is an estimate of the population mean. The more samples that are used in calculating the sample mean, the closer the sample mean approaches the population mean. For example, a student is going to get a more accurate sample mean if he or she "averages" all the means in the entire class than if he or she just "averages" the means of his or her table. Students do not need to know the Greek letters, but they may need to recognize them depending on the type of technology available in their classroom.

Random Sampling

Random sampling is a way to remove bias. Although students at this level may not be actually using true random sampling procedures when collecting data, the benefits of a random sample should be discussed.

EXAMPLE

- Take about 15-20 seconds and determine the average diameter for the 80 circles.
- Take about 15 seconds and select the 5 circles that best represent the sizes of the 80 circles.
- By measuring find the average diameter of your 5 circles.
- Number each circle from 1-80. Then use a random number generator to select 5 circles.
- Find the average diameter of the circles chosen by the random number generator.
- Compare the means of your personal sample to the sample created by the random number generator.
- Take the mean of all your classmates means found by using their personal sample. Then take the mean of all the means generated by the random number generator. Compare these two numbers. What do you find?

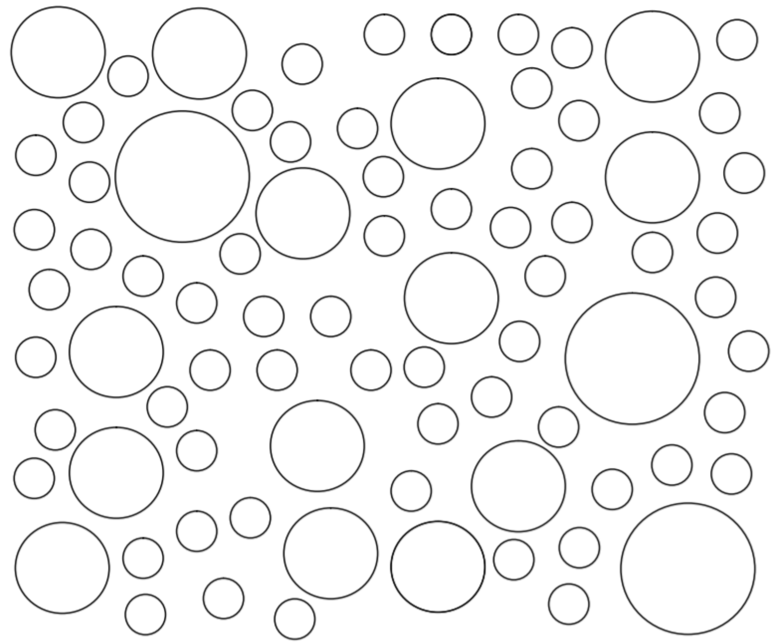


Figure 22: Eighty circles

Taken from page 52-54 of the GAISE Report.

Discussion: Personal selection usually tends to have sample means that are larger than the true mean diameter. This is because personal selection tend to be bias and people tend to favor larger circles. Random selection tends to have some samples that overestimate the mean and some that underestimate the mean. The more and more randomly generated sample means one finds, the closer the data will represent the population. *Note: This activity could help students at this grade level understand bias and the purpose of random sampling. This concept will also tie in nicely to long-run relative frequencies and the idea of the sample mean used for Mean Absolute Deviation.*

Census at School can be used as an example of a random sample.

BIAS

Bias is favoritism that is present in the process of collecting data. Sometimes bias is intentional and sometimes its unintentional, but it can lead to misleading results. Bias can occur from—

- a non-representative sample of the population
- wording of questions
    - confusing
    - leading questions
    - using loaded words
    - discrete responses
- unwilling participants
- self-selected participants.

Increasing the sample size reduces sample error, but it does not reduce bias. When students decide to select a sample from a specific group of people (friends), use the situation as an opportunity to discuss bias.

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Graphing calculator
- Desmos is a free online graphing utility and app.
- Census at School is an international classroom project that provides real data for classrooms. (Note: On some computer platforms this data can be copied from their Excel spreadsheet right into Desmos.)

Randomness

- Peeps is a task by YummyMath where students estimate the number of Peeps sold each Easter season. They consider reasonability by making guesses they think might be a little too high and a little too low. Students determine necessary info, problem solve, and improve their original estimates. They conduct random samplings of their estimates and compare the mean of their estimates to the actual number of peeps sold each season.
- How Random is the iPod's Shuffle? by Statistics Education Web has students informally explore randomness using iPod playlists.

Ohio Department of Education

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.1)

Representative Sample

- Bias in Sampling is a lesson from Annenberg Learner. This content resource addresses statistics topics that teachers may be uncomfortable teaching due to limited exposure to statistical content and vocabulary. This resource focuses a four-component statistical problem-solving process and the meaning of variation and bias in statistics; it investigates how data vary.
- Explore Sampling Variability—Higher Education Attainment Across the United States by the United States Census Bureau explores the sampling variability in sample percentages of those who hold a bachelor's degree.
- Counting Trees is a task by Assessment Mathematics Project that informally introduces the usefulness of samples.

Curriculum and Lessons from Other Sources

- EngageNY, Grade 7, Module 5, Topic C, Lesson 13: Populations, Samples, and Generalizing from a Sample to a Population, Lesson 14: Selecting a Sample, Lesson 15: Random Sampling are lessons that pertain to this cluster.
- Illustrative Mathematics, Grade 7, Unit 8: Probability and Sampling, Lesson 12: Larger Populations, Lesson 13: What Makes a Good Sample?, Lesson 14: Sampling in a Fair Way, Lesson 20: Memory Test are lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- The Utah Middle School Math Project is an open source textbook and workbook.

General Resources

- Arizona 6-8 Progression on Statistics and Probability
  This cluster is addressed on pages 8-9.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- LOCUS is an NSF Funded project focused on developing assessments of statistical understanding. *Note: Teachers must create an account to access the assessments.*
- *K-12 Statistics Education Resources* is a collection of websites put together by the American Statistical Association for teachers.
- A Sequence of Activities for Developing Statistical Concepts by Christine Franklin & Gary Kader is an article published in *The Statistics Teacher Network*, Number 68, Winter 2006. It has an overview of the GAISE model and its levels, and it includes activities at each level.
- Statistics Teacher is an online journal published by the American Statistical Association (ASA) and NCTM. It includes articles, lesson plans, and professional development opportunities.
- Significance is a magazine that demonstrates the practical use of statistics and shows how statistics benefits society.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.1)

References

- Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8 Statistics and Probability*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Franklin, C., Kader, G., Mewborn, D, Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. Alexandria, VA: American Statistical Association.

STANDARDS	MODEL CURRICULUM (7.SP.2)
STATISTICS AND PROBABILITY **Broaden understanding of statistical problem solving.** **7.SP.2** Broaden statistical reasoning by using the GAISE model: **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?"* (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)	**Expectations for Learning** Students build on their previous work from 6th grade (6.SP.1-3) to broaden their statistical reasoning through the use of the GAISE model framework. Most of the expectations for students in 6th grade were at Level A. In 7th grade, students should be progressing from Level A to Level B, where the student directed process of selecting a sample and understanding its connection to the population. Students at Level B become more aware that some questions have distinct answers whereas others have answers that can vary. Whereas in Level A, the teachers posed most of the statistical questions, in Level B students begin to pose their own questions. The questions students form are no longer limited to the classroom. They begin to develop an awareness of design differences. Students at Level B start comparing two populations (two classrooms) and start informally comparing a sample to a population (compare classroom to whole school). The GAISE model framework will continue through high school. **ESSENTIAL UNDERSTANDINGS** • Statistics is the name for the science of collecting, analyzing, and interpreting data. • The GAISE model framework is used to analyze and interpret data and has four steps: Formulate the Question; Collect Data to Answer the Question; Analyze the Data; and Interpret Results. • Data are not just numbers; they are numbers generated with respect to a particular context and situation. • There are two types of data: categorical and numerical. • Categorical data are sorted into groups and categories. • Numerical data are measurable. • A statistical question anticipates a response that varies, from one individual to the next, and this variability is described in terms of spread and overall shape. • A distribution shows all values of data and how often they occur. • A set of data has a distribution which can be described by its center, spread, and overall shape. • A measure of variation is a single number that describes the extent to which data vary in a distribution. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.SP.2)
7.SP.2, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** **Broaden understanding of the GAISE model** **Step 1 – Formulate the Question:** • Begin to pose student generated statistical questions with variability that go beyond the classroom. • Recognize the distinction between a population, census, and a sample. **Step 2 - Collect Data to Answer the Question:** • Design a collection method to answer a statistical question. • Conduct sample surveys of two or more groups or comparative experiments. **Step 3 - Analyze the Data:** • Use properties of distributions (center, spread, shape) as tools of analysis. • Determine variability (spread) within a group. • Compare individual to individual, individual to group, group to group. • Summarize the numerical data sets in relation to the context using graphical displays: o histograms o interquartile range (IQR) and mean absolute deviation (MAD) o boxplots and five number summaries: lower extreme (min), upper extreme (max), median (Q2), lower quartile (Q1), and upper quartile (Q3) • Show distribution as all values of data and how often they occur. **Step 4 - Interpret Results:** • Draw conclusions and make generalizations from the analysis of the data between two groups. o Describe differences between two or more groups using center, spread, and shape. • Acknowledge whether a sample may or may not be representative of a larger population. *Continued on next page*

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.SP.2)</th></tr>
<tr><td>7.SP.2, continued</td><td>
<h2>Expectations for Learning, continued</h2>
<h3>MATHEMATICAL THINKING</h3>

Make sense of and create statistical problems.
Formally explain mathematical reasoning.
Use formal and precise mathematical language.
Analyze student created questions.
Pay attention to and make sense of quantities.

<h2>Content Elaborations</h2>

Ohio's K-8 Critical Areas of Focus, Grade 7, Number 4, page 48
Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23
GAISE Model pages 14 – 15
Focus of 7th grade is Levels A – B, pages 22-59

<h3>CONNECTIONS ACROSS STANDARDS</h3>

Extend the GAISE Model to address questions involving two populations versus one population (7.SP.1).
Describe and analyze distributions (7.SP.3).
Data may also be generated using a chance process (7.SP.5-8).

</td></tr>
</table>

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.2)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice

This cluster focuses on but is not limited to the following practices:

MP.1 *Make sense of problems and persevere in solving them.*

MP.2 *Reason abstractly and quantitatively.*

MP.3 *Construct viable arguments and critique the reasoning of others.*

MP.6 *Attend to precision.*

MP.8 *Look for and express regularity in repeated reasoning.*

In Grade 6, students are introduced to the GAISE model, where they use measures of center and variability to describe data. Students continue to use this knowledge in Grade 7 as they progress from working with a single group to working making comparisons between two or more groups. In Grade 7, students broaden understanding of all four parts of the GAISE model by advancing into Level B to pose their own questions, look beyond the data, and consider samples as representations of a whole population. See GAISE model Framework pages 14 and 15.

The GAISE model serves as a framework for pre-k-12 teachers. It describes what is meant by a statistically literate high school graduate. The framework provides steps and levels to achieve this goal. This framework helps to demonstrate that statistics is an investigative problem-solving process immersed in a context and not a set of fancy tools, graphs, and procedures for their own sake isolated from a context. It can be thought of as the scientific method for statistics. For scaffolding ideas see Grade 6 Model Curriculum 6.SP.1-3.

GAISE MODEL FRAMEWORK (for examples at each step see pages 16-21 in GAISE model framework)

Step 1: **Formulate Questions** - *Anticipating Variability—Making the Statistics Question Distinction*

- Clarify the problem at hand.
- Formulate one (or more) questions that can be answered with data.

Step 2: Collect Data - *Acknowledging Variability—Designing for Differences*

- Design a plan to collect appropriate data.
- Employ the plan to collect the data.

Step 3: Analyze Data - *Accounting of Variability—Using Distributions*

- Select appropriate graphical and numerical methods.
- Use these methods to analyze the data.

Step 4: Interpret Results- *Allowing for Variability—Looking beyond the Data*

- Interpret the analysis.
- Relate the interpretation to the original question.

Because the other statistics standards in Grade 7 focus on analyzing data, the focus of this cluster is—
- understanding and identifying the four steps of the GAISE model;
- asking statistical questions; and
- collecting data.

At times teachers should be providing questions, collections of data, and graphs to generate rich discussions about general conclusions and the GAISE model process. However, students should also have opportunities to do all fours steps of the process.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.2)

GAISE LEVELS

There are four steps in the GAISE model and all four steps of the statistical process are used at all three levels A, B, C.

The depth of understanding and sophistication of methods used increases across the levels. Grade 6 was the first-time students will be introduced to the GAISE model, so they began at Level A. In Grade 7, students are moving toward Level B. Hands-on learning is predominant through every level of the GAISE model.

- In Level A the learning is more teacher driven. For example, a class may collect data to answer teacher provided questions about their classroom.
- In Level B the learning becomes more student centered. For example, a class may collect data to answer student-created questions about their school.
- In Level C the learning is highly student driven. For example, a class may collect data to answer student-created questions about their community and model the relationship between data sets collected.

STATISTICAL QUESTIONS

A statistical question has to have data that varies—not just a single answer. For example, "How many pounds does my dog weigh?" is not a statistical question, but "What is the average weight of beagles?" is a statistical question. In Grade 6 students were required to differentiate between a statistical and non-statistical question. Now as students in Grade 7 are becoming more aware of the distinction between the two types of questions, they begin to pose their own questions. They should start making decisions about what variables to measure and how to measure them. In addition, they should start to ask questions that involve groups bigger than their classroom. Students need to be able to informally distinguish between a sample, a population, and a census.

COLLECTING DATA

In Grade 7, students should be given more opportunities to collect their data than in Grade 6. Now that students have an informal understanding between a sample and population (7.SP.1), they need to be more discerning when collecting data. Although at this grade level, they will not be formally using random samples, they should strive to make them as random as possible using common sense approaches. For example, an Honors Algebra class might not be the best sample to use in representing the entire school. They can also try to use random selection as much as possible such as putting all the names of the people in their class into a hat.

At this level students can collect data using sample surveys, simple comparative experiments, observations, or simulations (7.SP.8c). Census at School is a good resource to obtain real student data.

When creating a sample survey, students should consider the wording of questions. They must be unambiguous and easy to understand. Discuss loaded words and how to avoid bias. If questions are asked orally, tone of voice can also create bias. Multiple choice questions should include all possible answers and avoid overlap.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.2)

EXAMPLE

Describe the flaw in this survey question.

How many hours of sleep do you get every night?

6 hours   7 hours   More than 8 hours

A simple comparative experiment is like a science experiment where there is a comparison of two or more conditions. Discuss with students the importance of experimental design when conducting simple comparison experiments.

Students should be able to differentiate between variation and error. Teachers can use errors that occur in the classroom when collecting data and discuss how the impact of these errors impact the final results. Errors can sometimes affect outliers, clusters, and gaps in the graphical representation of the data. Students should ask themselves if the outlier is legitimate or is it due to an error, and therefore should be discarded?

ANALYZING DATA

Students need multiple opportunities to look at data in order to determine and word statistical questions. Data should be analyzed from many sources, such as organized lists, box-plots, dot plots, histograms, bar graphs, and stem-and-leaf plots. After students have analyzed the data, it is important to think about what may have caused the data to look like it does. This should lead to a discussion on variability, and students should be able to start to quantify the variability (IQR). They also start to compare two or more distributions. As students move toward Level B, they should apply proportional reasoning to statistics; they should be able to summarize and interpret data in terms of percents and fractions. Specifics with respect to analyzing data will be discussed more in-depth in the next cluster: 7.SP.3.

INTERPRETING DATA

The most important part of statistics is writing a conclusion. A statistical conclusion has to be based on data. Once students make a conclusion, ask them "How can you support this with the data?" They can use numerical values or descriptions of graphs to back up their conclusion. Encourage students to avoid causation statements. At Level B, students describe the difference between two or more groups and acknowledge that a sample may not be representative of a larger population.

MISLEADING DATA

This is a great opportunity to have students examine how different inferences can be made based on the same two sets of data. Have students investigate how advertising agencies use data to persuade customers to use their products. Additionally, provide students with two populations and have them use the data to persuade both sides of an argument such as displaying data using different scales. Other misleading graphs such as circle graphs, pictographs, and line charts should also be analyzed.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.2)

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Graphing calculator
- Desmos is a free online graphing utility and app.
- Census at School is an international classroom project that provides real data for classrooms. (Note: On certain computer platforms this data can be copied from their Excel spreadsheet right into Desmos.)

Curriculum and Lessons from Other Sources

- ASA Recommended Sources of Lesson Plans and Other Resources for Teaching Common Core Statistics and Probability Topics is a list of resources for teachers.

General Resources

- Arizona 6–8 Progression on Statistics and Probability
  This document serves as teacher background for 7.SP.2. This cluster is addressed on—
  Pages 2-3:
    - Bullet points on page 2 refer to GAISE model.
    - Random sampling is addressed at high school level NOT 7th grade.
  Pages 5-6:
    - Although mean absolute deviation, or MAD, is listed in this document as 6th grade content, in Ohio's standards, it is not addressed until 7th grade. MAD sets the stage for introducing standard deviation in high school.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards. *Note: Ohio's Learning Standards 7.SP.1 does not include random sampling as indicated on 7.SP.1*
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- LOCUS is an NSF Funded project focused on developing assessments of statistical understanding. Teachers must create an account to access the assessments.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.2)

General Resources, continued

- K-12 Statistics Education Resources is a collection of websites put together by the American Statistical Association for teachers.
- A Sequence of Activities for Developing Statistical Concepts by Christine Franklin & Gary Kader is an article published in *The Statistics Teacher Network*, Number 68, Winter 2006. It has an overview of the GAISE model and its levels and it includes activities at each level.
- Statistics Teacher is an online journal published by the American Statistical Association (ASA) and NCTM. It includes articles, lesson plans, and professional development opportunities.
- Significance is a magazine that demonstrates the practical use of statistics and shows how statistics benefits society.
- Statistical Education of Teachers (SET), Chapter 5: Preparing Middle-School Teachers to Teach Statistics provides background for middle school teachers teaching statistics.

References

- Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8, Statistics and Probability*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Franklin, C., Kader, G., Mewborn, D, Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. Alexandria, VA: American Statistical Association.
- Groth, R. & Bargagliotti, A. (August 2012). GAISEing Into the common core of statistics. *Mathematics Teaching in the Middle School*, *18(1),* 38-45.

STANDARDS	MODEL CURRICULUM (7.SP.3)

STANDARDS

STATISTICS AND PROBABILITY

Summarize and describe distributions representing one population and draw informal comparisons between two populations.

7.SP.3 Describe and analyze distributions.

- **a.** Summarize quantitative data sets in relation to their context by using mean absolute deviation[G] (MAD), interpreting mean as a balance point.
- **b.** Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot[G] (line plot), the separation between the two distributions of heights is noticeable.*

MODEL CURRICULUM (7.SP.3)

Expectations for Learning

In 6th grade, students analyzed distributions (center, spread, and shape) in a single population. In 7th grade, they will begin to draw comparisons between two populations. For the first time, students will interpret mean as a balance point and use the mean absolute deviation (MAD) as the measure of variability from the mean. It is imperative students develop the conceptual understanding of the MAD. Students will take their conceptual understanding of MAD in 7th grade and apply it to standard deviation in high school.

ESSENTIAL UNDERSTANDINGS

- Descriptive statistics may include measures of center and spread.
- There is variability between groups.
- Data can be represented in different ways to persuade people.
- The important purpose of a measure of center is not the value itself, but the interpretation it provides for the variation of the data.
- The sum of the distances from each data point below the mean to the mean equals the sum of the distances from each data point above the mean to the mean.
- Mean absolute deviation (MAD) is one way to measure the extent to which a distribution is stretched or squeezed about its mean.
- The mean absolute deviation (MAD) is the average distance that each data value is from the mean.

MATHEMATICAL THINKING

- Formally explain mathematical reasoning.
- Use formal and precise mathematical language.
- Pay attention to and make sense of quantities.
- Solve real-world problems accurately.
- Determine the reasonableness of results.
- Analyze visual models.

Continued on next page

STANDARDS	MODEL CURRICULUM (7.SP.3)
7.SP.3, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** • Interpret mean as a balance point. • Explore, explain, and calculate the mean absolute deviation (MAD). • Summarize data using MAD within a context. • Summarize and describe distributions representing one population. • Informally compare distributions representing two populations using MAD, histograms, dot plots, and/or boxplots. **Content Elaborations** • Ohio's K-8 Critical Area of Focus, Grade 7, Number 4, page 48 • Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23 • GAISE Model, pages 14 – 15 o Focus of 7th grade is Level A – B, pages 22-59 **CONNECTIONS ACROSS STANDARDS** • A sample allows results to be generalized to a much larger set of data, the population from which the sample was selected (7.SP.1). • Broaden understanding of the framework of GAISE Model (7.SP.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, and possible connections between topics in addition to highlighting some misconceptions. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Also, examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice
This cluster focuses on but is not limited to the following practices:
MP.1 *Make sense of problems and persevere in solving them.*
MP.2 *Reason abstractly and quantitatively.*
MP.6 *Attend to precision.*

Cluster 7.SP.3 is basically two separate clusters combined into one, so that the numbering of the standards would stay the same to enable teachers to easily find resources that are aligned to the national standards.

Part **a.** of this standard addressing Mean Absolute Deviation was formerly embedded in 6.SP.5. In Ohio it has been moved to 7th grade.

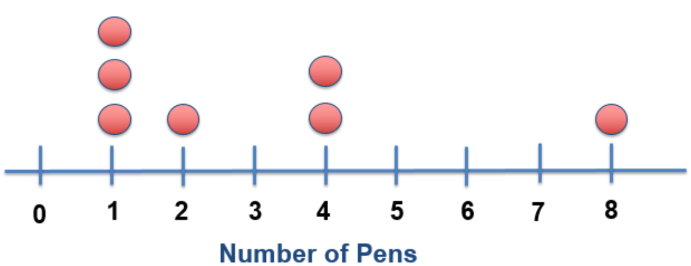
MEAN AS A BALANCE POINT

The important purpose of the measure of center is not the value itself, but the interpretation that it provides for the variation of the data. In Grade 6, the mean was interpreted as a "fair share." Now in Grade 7, the mean should be interpreted as a balance point or fulcrum to help students transition toward mean absolute deviation. It is important for students to interpret the mean as a balance point, where the data positioned on a lever are balanced with respect to the fulcrum. Discuss how the mean is affected by extreme values.

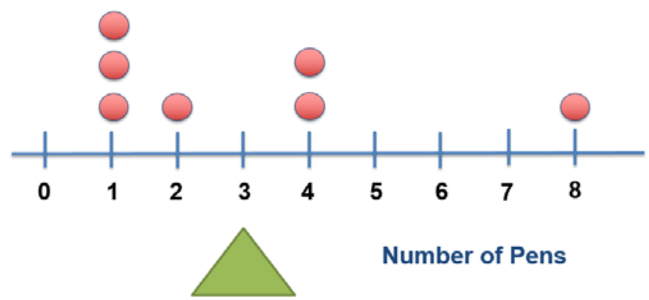
EXAMPLE

Joan wanted to find out the "average" number of pens in a person's purse. She surveyed 7 people and she collected the following data: 1, 2, 8, 4, 4, 1, 1.

Step 1: Make a dot plot of the number of pens that each person in your group had in their bag.



Step 2:
Decide where the fulcrum or balance point would be.



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

MEAN ABSOLUTE DEVIATION (MAD)

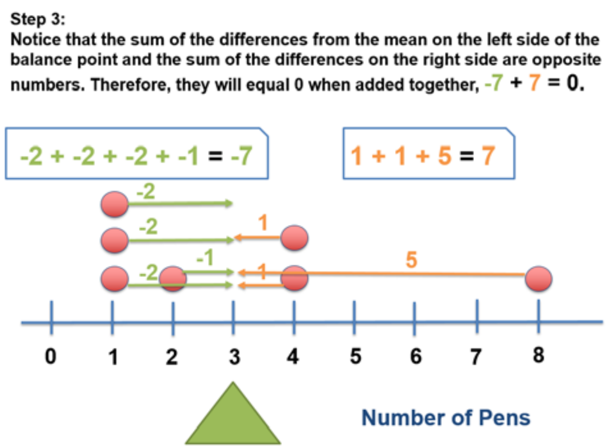
Mean absolute deviation (MAD) is a method of measuring spread which builds upon the concept of the mean being a balance point. Deviation measures the difference each data point is from a secondary data point, which in this case is the mean. Basically, MAD calculates the mean distance of each data value from the mean. It is the "average" distance of all the data points to the mean. Because distance is positive, the absolute deviation is taken. Without taking the absolute deviation, the sum of the data points on the left side of the balance point would equal zero when added to the sum of the data points on the right side of the balance point. It is important for students to see that connection.

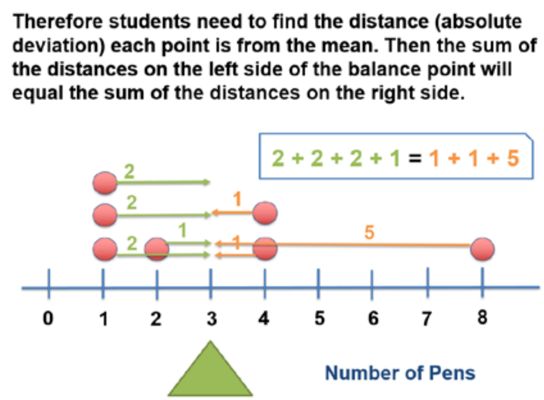
To calculate the MAD:

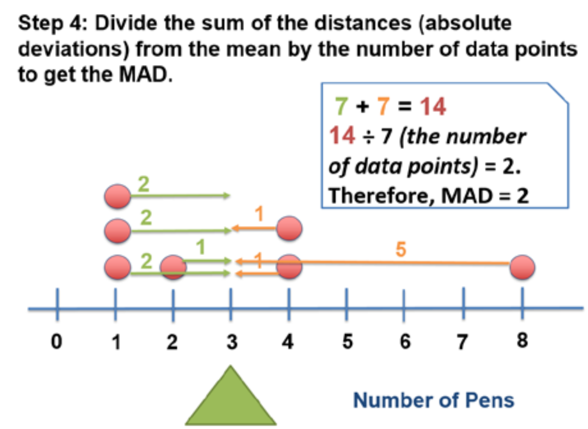
1.) Find the mean.

2.) Find the absolute value of the difference between each data value and the mean.

3.) Find the mean (average) of those differences.







The closer the MAD is to 0 the more consistent the data are. Although the mean provides useful information, it does not give a picture of spread. MAD provides more information than the mean alone. Understanding mean absolute deviation conceptually will lay the foundation for calculating standard deviation in high school.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Use MAD during discussions of shape (clustering, outliers) and variability. Students need to be able to state the MAD in words in the context of the problem. Although, it is important that they become proficient in calculating the MAD, it is of greater importance that they are able to explain what MAD is and what it does.

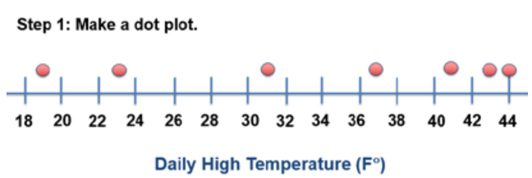
TIP! It may be helpful to have students create a dot plot of the data before calculating the mean absolute deviation.

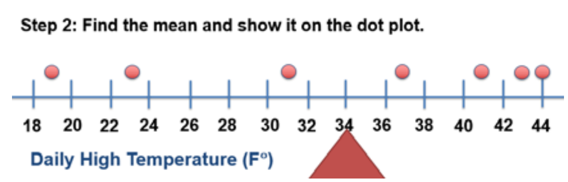
This is an introduction of the MAD. Most of the misconceptions will be struggles finding the mean and finding the absolute value. Watch for students who identify distance as a negative number. Distance must always be a positive value because it is the absolute value from zero. Students also need to recognize the mean as the balance point. The focus of this standard is to give students a deep conceptual understanding of the mean absolute deviation as opposed to simply being able to calculate it.

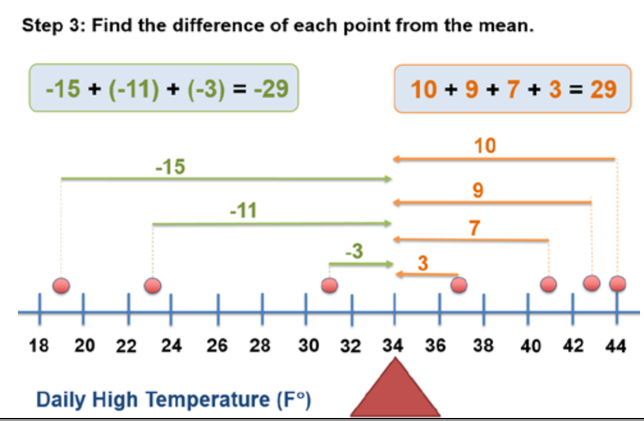
EXAMPLE

The daily high temperatures in Fahrenheit for the first week of February are as follows: 41°, 31°, 23°, 43°, 44°, 19°, 37°.

- Create a dot plot of the data.
- Find the MAD.
- Interpret the meaning of MAD in this context in words.
- What conclusions can you make about these data?







INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Step 4: Take the absolute value of the distance (or absolute deviation) from the mean and find the sums.

Method 1

$|-29| + |29| = 58$

Method 2

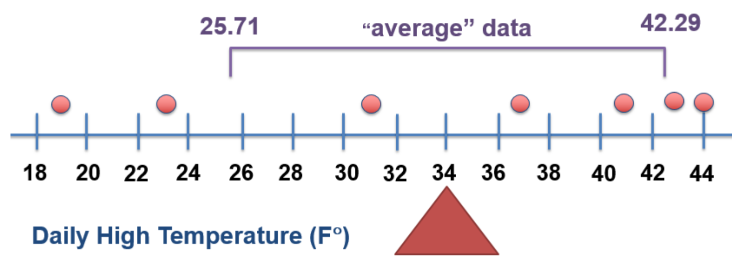
$|7| + |-3| + |-11| + |9| + |10| + |-15| + |3| = 58$

Step 5: Divide the total by the number of data points

$58 \div 7 \approx 8.29$, so the MAD is approximately 8.29 which is the "average" distance from the mean.

Step 6: Therefore an "average" data point falls between 25.71 and 42.29. So, the "average" daily high for the first week of February fell between 25.71°F and 42.29°F.

$34 - 8.29 = 25.71$
and
$34 + 8.29 = 42.29$



Discussion: This is the importance of MAD. Discuss how a MAD of 8.29 shows that the temperature is not very consistent, rather it fluctuated a lot. It is essential that they are able to explain what the MAD is and what it does in the context of the problem. *Note: In Grade 8 a data point that is more than twice MAD distance from the mean can be informally stated as a very large difference.*

EXAMPLE

Create two different scenarios where the mean is 5 and the MAD is 3.

EXAMPLE

Give students a set of data. Have students find mean absolute deviation (MAD) with all data points and then select a data point to remove and discuss how that affects the MAD. Do this with several sets of data.

DRAW INFORMAL COMPARISONS BETWEEN TWO POPULATIONS

The focus in Grade 6 was on creating graphical representations of the data. In Grade 7, the emphasis is on summarizing, describing, and analyzing graphical displays. Seventh grade is the first time that two populations will be analyzed. Students will compare two populations using means, medians, mean absolute deviations (MADs), histograms, dot plots, and boxplots. See Model Curriculum 6.SP.4-5 for more information on constructing graphical displays.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Patterns in the graphical displays should be observed, as should any outliers in the data set. Students should identify the attributes of the data and know the appropriate use of the attributes when describing the data. Pairing contextual situations with data and its corresponding graphical display is essential. Given a distribution, students should determine which measure of center is the best for the data set. It is important for students to see data distributions with visual overlap.

TIP!

To extend these concepts students can use samples to make inferences about a population. See cluster 7.SP.1. This cluster can also be connected with simulations from 7.SP.8c.

The tool Census at School provides a lot of real-data that can be used to construct graphical displays to compare data.

EXAMPLE

Question of Interest: Do male and female students differ in the amount of time spent on homework?

- Make two box plots using a sample size of 10 females and 10 males from Census at School data.
- Compare the distributions using measures of center and measures of variability.
- What conclusions can you make from the data?
- Do you think your sample is big enough to answer the question? Explain.

It is important to use the interquartile range in box plots when describing the variation of the data.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

EXAMPLE

Question of Interest: How do female's life expectancies differ in Africa and Europe?

- Draw two different types of graphical displays that compare the data from the two continents.
- Which graph do you think best represents the data? Explain.

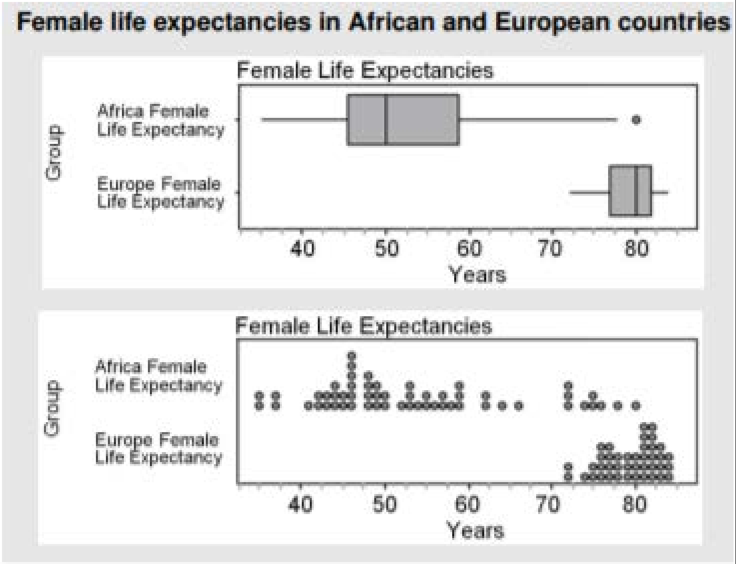


Image taken from Arizona's 6-8 Progression on Statistics and Probability page 10

Using graphing calculators to explore box plots (box-and-whisker plots) removes the time intensity from their creation and permits more time to be spent on the meaning.

Histograms

There are two types of histograms: Frequency Histograms and Relative Frequency Histograms. Grade 6 primarily addressed Frequency Histograms. As students do more work with percents in Grade 7, Relative Frequency Histograms can be introduced. They allow students to view the same data in terms of percentages (or their decimal approximations). This means that all the relative frequencies in the histogram should add up to 1 or 100%. Students should recognize that when using relative frequencies the shape of a histogram is the same as the shape of the histogram constructed using frequencies (assuming the intervals are the same).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

EXAMPLE

Question of Interest: How do girls' heights and boys' heights in 7th grade differ?

- Use Census at School to obtain the data. (An example of data is shown below.)

  Boys: 160, 151, 160, 154, 156, 145, 157, 181, 172, 160, 150, 157, 164, 172, 163, 152, 165, 180, 161, 153, 151, 180, 163, 168, 164

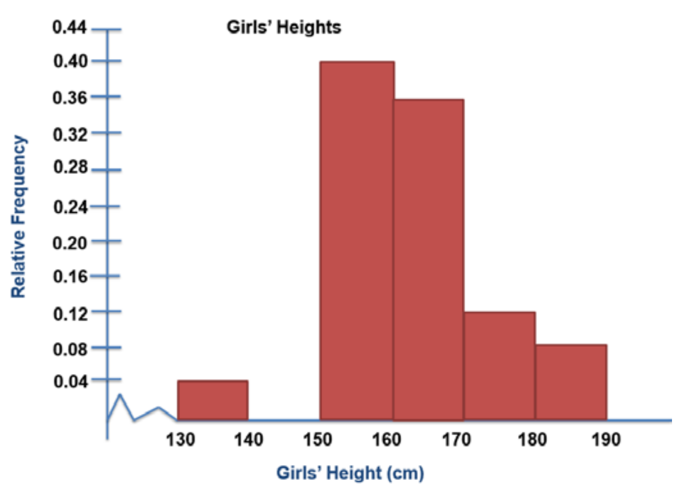
  Girls: 156, 164, 161, 135, 152, 157, 187, 170, 161, 155, 170, 161, 155, 170, 154, 152, 162, 162, 157, 153, 181, 154, 169, 165, 160
- Make a stem-and-leaf plot to organize the data.
- Make a histogram to represent the data.
- Describe and compare the variability and measure of center of the two data sets.
- What conclusions can you make about 7th grade girls' and boys' heights.
- Do you feel your sample is representative of the population? Explain.

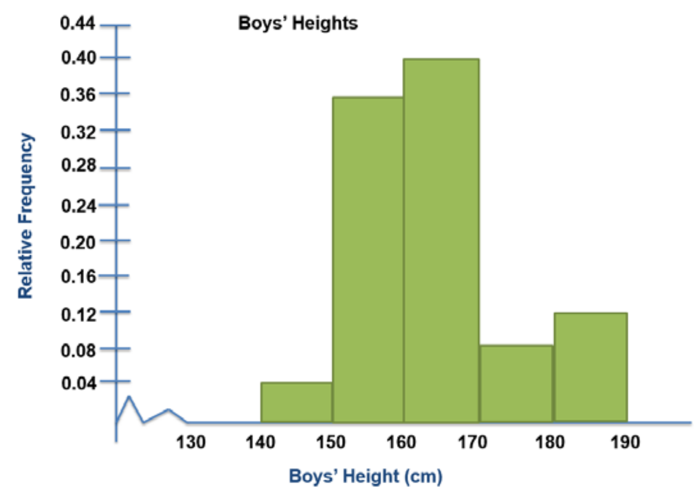
Student Heights (cm)

Girls		Boys
5	13	
	14	5
7 7 6 5 5 4 4 3 2 2	15	0 1 1 2 3 4 6 7 7
9 5 4 2 2 1 1 1 0	16	0 0 0 1 3 3 4 4 5 8
0 0 0	17	2 2
7 1	18	0 0 1

Key: 15|3 = 153

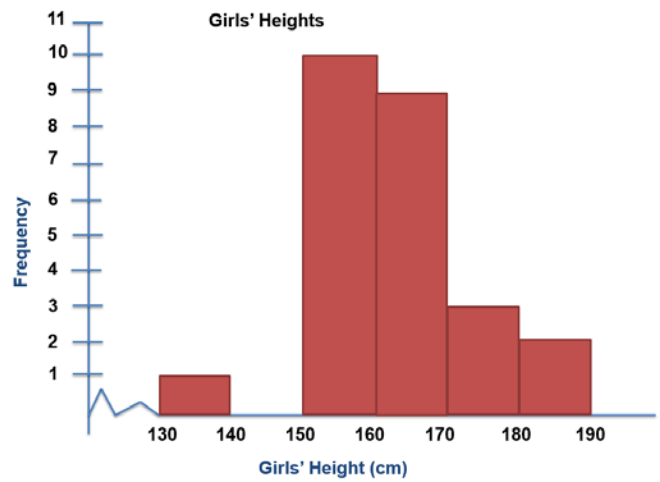
Relative Frequency Histograms

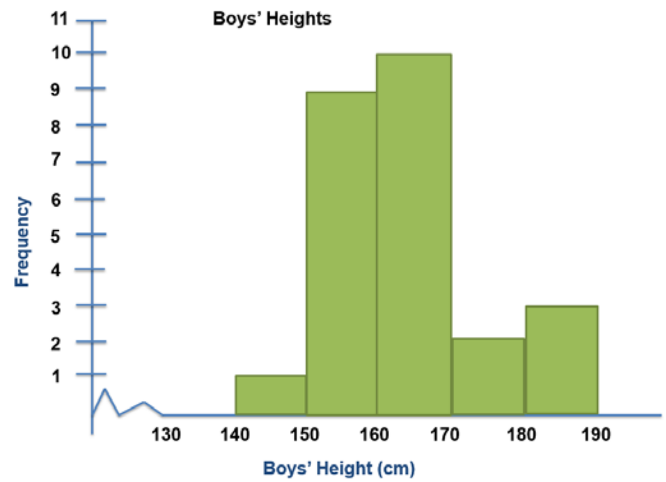




INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Frequency Histograms





Discussion: Encourage some students to make frequency histograms and other students to make relative frequency histograms. Compare the shapes of the two types of histograms using the same data sets. Discuss why they would be the same.

Variability

Analyzing Data will help students begin to understand that responses to a statistical question will vary, and that this variability is described in terms of spread and overall shape. One example of a measure of spread is the range. Like the measures of center, a single number for measures of spread is used to summarize the data. Students also describe the shape of the distribution. Are the data symmetric or skewed? Are there any clusters? Are there any outliers? Why does the distribution take this shape?

Students should understand the difference between measurement variability, natural variability, and induced variability.

- Measurement variability occurs when the measurement tool is imprecise or produces unreliable results.
- Natural variability is the idea that variability occurs in nature. People and things are different.
- Induced variability is something that occurs based on how an experiment or survey is designed or implemented. Sometimes it produces errors or unforeseen consequences, but oftentimes induced variability is helpful as it allows for control groups such as in a medical experiment.

Note: Students do not need to know these words, but have a basic understanding of the different causes of variability.

In Grade 6 students looked for variability within a group. Now, in Grade 7 students are analyzing variability between groups (populations). Mean absolute deviation and IQR are ways to quantify this variability.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Graphing calculator
- Desmos is a free online graphing utility and app.
- Census at School is an international classroom project that provides real data for classrooms. (Note: On some computer platforms this data can be copied from their Excel spreadsheet right into Desmos.)
- A Little Stats: Adventures in Teaching Statistics created by Amy Hogan is a list of free internet sources for real data and datasets for public use.

Dot Plots

- How Fast Are You? Published by the Statistics Education Web is a lesson where students collect 20 reaction times and make a graphical representation of the data.

Histograms

- How are Single-Parent Household Distributed Across the United States? by the United States Census Bureau is a lesson where students create and compare dot and box plots and use key features of histograms and box plots.
- Which Is Better … Original Movies or their Sequels? is an activity by Yummy Math. Students see what they can conclude from various types of graphs and consider what size sampling of movies and their sequels is an adequate, representative population of movies. This is a very open-ended activity that will allow 6th or 7th graders to conduct data analysis at one level: measures of central tendency & variability, box plots, histograms etc.

Box Plots

- Bubble Trouble published by the Statistics Education Web is a lesson where students perform an experiment involving different types of water to make bubbles and then analyze their results using comparative boxplots.
- Armspans by Statistics Education Web is a lesson where students use box plots to compare the measurements of boys and girls armspans.
- Step Into Statistics by the Statistics Education Web is a lesson where students use box plots to compare the foot sizes of boys and girls.
- Don't Spill the Beans by the Statistics Education Web is a lesson where students use box plots to compare how many beans students can hold in their dominant versus nondominant hands.
- How Long is 30 Seconds? by the Statistics Education Web is a lesson where students use box plots to compare how successful boys and girls are at predicting how long is 30 seconds.
- Colors Challenge by the Statistics Education Web is a lesson where students use box plots to explore the Stroop effect.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Box Plots, continued

- Describing and Comparing Data Distributions by the United States Census Bureau is a lesson where students compare and contrast data distributions and use key features of histograms and box plots.
- Does the Percentage of People Who Walk to Work in Cities Vary with Population Size? by the United States Census Bureau is a lesson where students create boxplots to make inferences.
- Temperatures by the Mathematics Assessment Project is a task where students will use line graphs and box plots to compare temperatures in California and Washington.
- Pixar versus DreamWorks is an activity by Yummy Math (6.SP.1-5) (7.SP.3) Depending on grade level, students might compare the two companies using box plots or bar graphs (this is a golden opportunity to get into box plots). For example, middle students can use the data to compare movie ratings, box office profits, or box office gross earnings with box plots.
- Sticks to the Roof of Your Mouth by Susan N. Friel and William T. O'Connor is an activity that uses different types of peanut butter to explore distributions. Here is another version of the activity from Pearson that also uses histograms. Additional Practice has other questions that supplement this lesson.
- Boxplot Template from Desmos allows students to create a box plot.
- Statistics: Boxplots by Rachel Fruin from Desmos is an activity that has students compare two boxplots.
- Mean and Median is an applet by NCTM Illuminations that allows students to investigate the mean, median, and box plot for a data set up to 15 integers.
- Season 12, American Idol is an activity by Yummy Math where students compare historical season premier and season finale audience size in several different ways to determine which event has had higher viewership. They consider outliers and judge the reliability of the mean and median.

Mean Absolute Deviation

- Mean Absolute Deviation is a video from Khan Academy which can be helpful if a teacher needs to build background information for himself or herself on mean absolute deviation.
- Mean Deviation by Math is Fun has background information and a few examples. The formula and the Greek letters should not be used with students.
- Lesson 12 Mean Absolute Deviation are student pages from a Glencoe textbook.
- Mean Absolute Deviation by Open Middle is an enrichment Problem for exploring MAD with students.

Curriculum and Lessons from Other Sources

- EngageNY, Grade 6, Module 6, Topic A Lesson 5: Describing a Distribution Displayed in a Histogram is a lesson that pertains to this cluster.
- EngageNY, Grade 6, Module 6, Topic B, Lesson 7, The Mean as a Balance Point, Lesson 9: The Mean Absolute Deviations (MAD), Lesson 10: Describing Distributions Using the Mean and MAD, Lesson 11: Describing Distributions Using the Mean and MAD are lessons that pertain to this cluster.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

Curriculum and Lessons from Other Sources, continued

- EngageNY, Grade 6, Module 6, Topic D, Lesson 19: Comparing Data Distributions, Lesson 20, Describing Center, Variability, and Shape of a Data Distribution from a Graphical Representation, Lesson 21: Summarizing a Data Distribution by Describing Center, Variability, and Shape,
- EngageNY, Algebra 1, Module 2, Topic B, Lesson 8: Comparing Distributions is a lesson that pertains to this cluster.
- Illustrative Mathematics, Grade 6, Unit 8, Lesson 10: Finding and Interpreting the Mean as a Balance Point, Lesson 11: Deviation from the Mean, Lesson 12: Using Mean and MAD to Make Comparisons are lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- Illustrative Mathematics, Grade 7, Unit 8, Lesson 11: Comparing Groups, Lesson 15: Estimating Population Measures of Center are lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- Georgia Standards of Excellence Curriculum Frameworks, Grade 7, Unit 5: Inferences has many lessons that pertain to this cluster.
- The Utah Middle School Math Project is an open source textbook and workbook.

General Resources

- Arizona's 6-8 Progression on Statistics and Probability
This cluster is addressed on pages 9-10.
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards. *Note: Ohio's Learning Standards have included Mean Absolute Deviation as part of this standard, having moved it from Grade 6.SP.5Bc.*
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- LOCUS is an NSF Funded project focused on developing assessments of statistical understanding. Teachers must create an account to access the assessments.
- K-12 Statistics Education Resources is a collection of websites put together by the American Statistical Association for teachers.
- A Sequence of Activities for Developing Statistical Concepts by Christine Franklin & Gary Kader is an article published in The Statistics Teacher Network, Number 68, Winter 2006. It has an overview of the GAISE model and its levels and it includes activities at each level.
- Statistics Teacher is an online journal published by the American Statistical Association (ASA) and NCTM. It includes articles, lesson plans, and professional development opportunities.
- Significance is a magazine that demonstrates the practical use of statistics and shows how statistics benefits society.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.3)

References

- Casey, S. & Bostic, J. (September 2016). Structurally sound statistics instruction. *Mathematics Teaching in Middle School*, *22(2),* 101-107.
- Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8, Statistics and Probability*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Franklin, C., Kader, G., Mewborn, D, Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. Alexandria, VA: American Statistical Association.
- Groth, R. & Bargagliotti, A. (August 2012). GAISEing into the common core of statistics. *Mathematics Teaching in the Middle School*, *18(1),* 38-45.
- Peters, S., Bennett, V., Young, M, and Watkins, J. (February 2016). A fair and balanced approach to the mean. *Mathematics Teaching in Middle School*, *21(6)* 364-372.

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (7.SP.5-8)</th></tr>
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STATISTICS AND PROBABILITY

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around $^{1}/_{2}$ indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.7 Develop a probability model[G] and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

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Expectations for Learning

Grade 7 introduces the formal study of probability. Students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions. It is imperative students use simulations to model concepts of probability as a foundation for further work in high school.

ESSENTIAL UNDERSTANDINGS

- Probability is the study of the chance (likelihood) that a particular event will occur.
- The theoretical probability of an event describes how often the event will occur in an infinite number of repetitions of a chance process. It also is the long run ratio of the number of times the event occurs divided by the number of times that the chance process is repeated.
- Probability is a number between 0 and 1 that has no units.
    - Near 1 is most likely; near 0 is least likely; and ½ is neither likely nor unlikely.
- An outcome is a possible result of an event.
- A probability model provides a probability for each possible non-overlapping outcome for a chance process.
    - A sample space is the collection of all possible individual outcomes.
    - An event is an outcome or set of outcomes in an experiment; it is a subset of the sample space.
        - A simple event has one outcome.
        - A compound event has more than one outcome.
    - The total probability of all such outcomes is 1.
- Frequency (absolute frequency) is a quantity that has no units represented by a real number greater than or equal to zero. It is the number of items occurring in a given set; it is a count. *Note: Frequency has a different meaning in statistics than is used in common usage, mathematics, and physics.*
- Relative (observed or experimental) frequency is the ratio of times an event occurs to the number of occasions which it might occur in the same period. e.g., if a coin is flipped 1,000 times and heads occurs 498 times, the relative frequency is 498/1000 or 0.498.

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STANDARDS	MODEL CURRICULUM (7.SP.5-8)
a. Develop a uniform probability model[G] by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations. **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space[G] for which the compound event occurs. **b.** Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event.	**Expectations for Learning, continued** **ESSENTIAL UNDERSTANDINGS, CONTINUED** • The relative frequency of an event is based on the outcomes of collected data. o A simulation is the use of a probability model to imitate a real situation. o The simulation is supposed to give similar results to the real situation and predicts what should occur. o In the long-run (increased trials), the relative frequency approaches theoretical probability of the event. *Note: Middle school students may use experimental probability interchangeable with relative frequency.* • In cases when it is difficult or impossible to compute the theoretical probability, the relative frequency can be used to estimate the theoretical probability of the event by repeated trials of an experiment and counting the number of outcomes. **MATHEMATICAL THINKING** • Use precise mathematical vocabulary to describe mathematical reasoning. • Develop and use probability models. • Make and analyze conjectures. • Use technology strategically to deepen understanding. *Continued on next page*

STANDARDS	MODEL CURRICULUM (7.SP.5-8)

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	**Expectations for Learning, continued**
	INSTRUCTIONAL FOCUS
	• Identify a question to explore using probability.
	• Design a probability model (uniform and non-uniform) based on observed frequencies to answer the question.
	  o Collect data (through simulations or experiments).
	  o Organize data (lists, tables, tree diagrams, etc.) into an appropriate sample space.
	  o Define events as subsets of the sample space.
	  o Assign a probability to an event as a ratio (number of times the event occurs to the total number of trials).
	  o Discuss the likelihood of an event as a number (fractions, decimals, percents).
	• Analyze results and explain possible discrepancies between observed (experimental) and theoretical outcomes.
	• Use probability from a repeated chance process to predict the likelihood of a long-run event.
	
	Content Elaborations
	• Ohio's K-8 Critical Areas of Focus, Grade 7, Number 5, page 49
	• Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22 – 23
	• Glossary – see Probability Model and Uniform Probability Model
	
	CONNECTIONS ACROSS STANDARDS
	• Extend the idea of a population to the context of probability (7.SP.1).
	• Use proportional relationships to describe probability (7.RP.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

Instructional Strategies

Note: The model curriculum is not a collection of lessons nor a full curriculum; it does not suggest pace, sequence, or amount of time spent on topics. It provides information about a topic related to the standards including ideas for examples, strategies for teaching, common misconceptions, and possible connections between topics. In addition, the emphasis of the model curriculum is on instruction not on assessment; therefore, the examples in the Instructional Strategies section are written for instructional purposes not necessarily as assessment items. Examples may need to be rewritten to accommodate the needs of each individual classroom.

Standards for Mathematical Practice

This cluster focuses on but is not limited to the following practices:

***MP.4** Model with mathematics.*

***MP.5** Use appropriate tools strategically.*

***MP.6** Attend to precision.*

Grade 7 is the introduction to the formal study of probability. Through multiple hands-on experiences, students begin to understand probability (of simple and compound events), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, use probability from a repeated chance process to predict the likelihood of a long-run event, and use information from simulations to make predictions.

UNDERSTANDING PROBABILITY

Build the concept of expressing probability as a number between 0 and 1, inclusive.

- Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as $\frac{1}{2}$.
- Then advance to situations in which the probability is somewhere between any two of these benchmark values.
- Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen.

EXAMPLE

Determine if the following situations are certain, likely, neither likely nor unlikely, unlikely, or impossible:

Part A

- Drawing a card with a red suit from a standard deck of 52 number cards
- Rolling a 1 with two standard number cubes simultaneously
- Picking a yellow marble from a bag that contains 12 yellow marbles and 4 blue marbles
- Rolling a 7 with two standard number cubes simultaneously
- Not rolling a 2 on one number cube
- Drawing the suit of hearts from a standard deck of 52 number cards
- Flipping heads on a two-sided coin
- Drawing a card with a black or red suit from a standard deck of 52 number cards
- Picking a green marble from a bag that contains 12 yellow marbles and 4 blue marbles
- Not picking a yellow marble from a bag that contains 12 yellow marbles and 4 blue marbles
- Not rolling an even number on two number cubes

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

Part B

- Snow falling in July in Ohio
- Not getting homework all week
- The sun rising tomorrow
- Rain falling in April in Ohio
- Rain falling every day, all summer in Ohio
- (Insert classmate's name) making the basket for every single shot in basketball
- (Insert teacher's name) forgetting a student's name
- Falling and breaking your leg
- No one coming to the opening showing of Star Wars new movie
- (Student's name) not using technology all day

Here are some guiding questions for probability:

- How do different displays help you interpret the results?
- How can probability be used to predict the outcome of future events?
- How does understanding probability help you make decisions?
- Why is it important for you to understand how probability is calculated and presented in real-world situations?
- How do people use probability to influence others?

Source: Sample Essential Questions for Math from Springboard, College Board on Folsom Cordova Unified School District website.

Students may misunderstand that evaluating the "fairness" of a game does not depend on the skill or strategy used by the player.

APPROXIMATING PROBABILITY

A chance experiment is an experiment where the outcome is not certain such as flipping a coin or rolling a number cube. Students can use chance experiments to collect data. They need to come to the understanding that as they increase the number of trials in their chance experiment the relative frequency (observed or experimental) over the long-run approaches the theoretical probability. Therefore if they have no way of knowing the theoretical probability (e.g., the number and kinds of tiles in a bag are hidden), they can do many, many trials to figure out an approximation of the theoretical probability. Then they could use that information to make further conclusions. Students can also use the theoretical probability to estimate the relative frequency, keeping in mind that what "should happen" does not always happen and that oftentimes the event will be at least close in value if not exact to the theoretical probability. These concepts can be tied into sampling (7.SP.1). They may also want to compare their probability distributions using graphical representation to help them analyze the situation (7.SP.3).

Note: When the structure of a situation is known, it leads toward probability. When the structure of a situation is unknown, it leads to statistics.

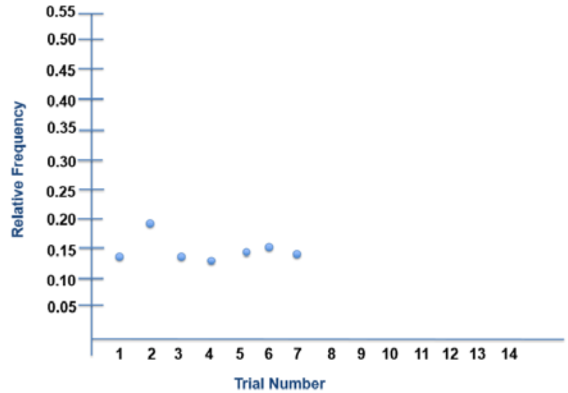
INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

EXAMPLE

Question of Interest: What can you say about the population of blue gills in the pond based on drawing a sample of fish?

- Give students a cup with about 30 black beans and 200 white beans, but do not tell them how many beans are in the cup. Explain to students that the black beans represent blue gills and the white beans represent other fish. Tell students that they need to figure out what percentage of blue gills are in a pond. Have students use a spoon to "go fishing" by pulling the beans out of the cup without looking. Explain to them that after each trial or sample, they have to replace the "fish" and mix up the beans. Have them record their data in a table similar to the one below.
- Have them graph their combined percent of blue gills to total fish on a chart where the *y*-axis states the relative frequency.
- Ask students what they notice about the relative frequency as the number of trials increase.
- Ask them questions such as "If you caught 32 fish, how many do you predict are bluegills? Explain." Or "If there are 450 fish in the pond, how many do you predict are blue gills? Explain." If 5 blue gills are caught in a net, how many other fish do you predict are also in the net? Explain." "Are your predictions guaranteed to be accurate? Explain."
- When will situations like this be used in real-life?

Trial	Number of Blue Gill	Number of Total Fish	Ratio of Blue Gill to Total Fish	Percent of Blue Gill to Total Fish	Combined Ratio of Blue Gill to Total Fish	Combined Percent of Blue Gill to Total Fish
1	3	22	3/22	13.6%	3/22	13.6%
2	5	19	5/19	26.3%	8/41	19.5%
3	1	24	1/24	4.1%	9/65	13.8%
4	2	23	2/23	8.7%	11/88	12.5%
5	5	20	5/20	25%	16/108	14.8%
6	4	23	4/23	17.4%	20/131	15.2%
7	3	24	3/24	12.5%	23/155	14.8%
8						
9						
10						



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

EXAMPLE

Part 1

James

Color	Frequency	Probability
Red	0	
Orange	5	
Yellow	3	
Green	4	
Blue	3	
Brown	0	

Mindy

Color	Frequency	Probability
Red	3	
Orange	2	
Yellow	2	
Green	4	
Blue	2	
Brown	2	

- James had a snack-sized bag of M&Ms that he got trick-or-treating. He wrote down the frequency of each color in the table above. He did not have any red or brown M&Ms in his bag, so he decided that getting a red or brown M&M is very rare. Do you agree with his conclusion? Explain.
- Calculate the probabilities of the colors of M&Ms in James's bag.
- Mindy did not believe James, so she brought in a snack-sized bag of M&Ms from her Halloween candy (shown in the above table). Calculate the probabilities of the colors of M&Ms in Mindy's bag.
- James still thought he was right since the probability of getting a red M&M ($\frac{3}{30}$) or getting a brown M&M ($\frac{2}{30}$) was still the lowest probability once they combined bags. Do you agree with James? Explain.

Part 2

Mindy is still convinced that getting a brown or a red M&M in a snack-sized bag is not rare. She gets 10 other students to bring in their M&M bags from Halloween. Combined they get the following data: 24 red, 39 orange, 24 yellow, 8 green, 38 blue, 23 brown. Based on this information, do you agree with Mindy or James?

Part 3

The proportion of M&Ms in bags has changed over time. Apparently, the proportion even differs depending on the manufacturing plant. Here are the probabilities of colors from the Cleveland plant: Red = 0.131, Orange = 0.205, Yellow = 0.135, Green = 0.198, Blue = 0.207, and Brown = 0.124. Based on Cleveland's probabilities, what is the estimated probabilities of getting a red M&M in 15-piece snack-sized bag? A brown M&M?

For more information on M&M Distributions of color see The Distribution of Colors for Plain M&M Candies published by Rick Wicklin on February 20, 2017.
https://joshmadison.com/2007/12/02/mms-color-distribution-analysis/

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

PROBABILITY MODELS

Probability models are used to predict how likely outcomes of a chance process would be if not known in advance.

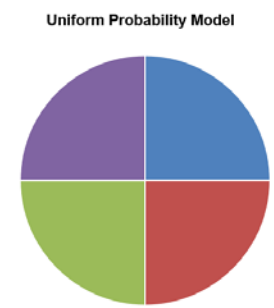
- A probability model consists of a set of all possible non-overlapping outcomes of a chance process and an assignment of probabilities to the individual outcomes so that the total probability over all such outcomes is one.
- The collection of all possible individual outcomes is called the *sample space* for the model. Subsets of a sample space are called *events*.
- The probabilities of the model can be either *theoretical* (based on the structure of the process, for example a fair number cube has the probability of $\frac{1}{6}$ of observing each face in a single roll) or *experimental* (based on observed data, for example the estimated probability of observing a head for a coin that produces 7 heads in 10 flips would be $\frac{7}{10}$).

Provide students with models of equally likely outcomes and models of not equally likely outcomes-and have students determine probabilities. These outcomes are called *simple events*

Students may expect the theoretical and experimental probabilities of the same data to match in the short term. However, it is only in the long-run, the experimental probability will get closer to the theoretical probability.

Uniform Probability Models

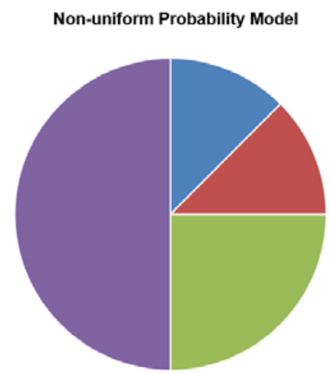
A model whose outcomes are equally likely is called a *uniform probability model*. For example, a spinner with four equal different colored sectors is a uniform probability model with a probability of ¼ of the spinner landing in any sector (outcome).



Students may think that "equally likely" always means a 50% probability. Explain to students that each outcome is equally likely if the probability has an equal chance of happening. In other words, the probability does not change if you change the question to a different outcome in the sample space.

Non-uniform Probability Models

A model whose outcomes are not equally likely is called a *non-uniform probability model.* For example, putting the letters "p-r-o-b-a-b-i-l-i-t-y" individually in a hat and choosing one letter without looking is a non-uniform probability model because the different letters (outcomes) do not have the same chance of being chosen—choosing the letter "b" is twice as likely as choosing the letter "y."



EXAMPLE

A circle with a diameter of 6 is inscribed in a square.

- If a point in the square is selected at random, what is the probability that it will be inside the circle?
- If a point in the square is selected at random, what is the probability that it will **not** be inside the circle?

COMPOUND EVENTS

Students should begin to expand the knowledge and understanding of finding the probabilities of simple events to finding the probabilities of compound events by creating organized lists, tables, and tree diagrams. This helps students create a visual representation of the data. From each sample space, students determine the probability (fraction, decimal, percent) of each possible outcome.

A *compound event* is an event consisting of more than one outcome.

- One occurrence of a device with multiple successful outcomes. For example, obtaining an outcome of an even number on one roll of a number cube {2, 4, 6}.
- More than one occurrence of a similar device happening together. For example, obtaining a sum of 7 when rolling two number cubes {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}
- More than one occurrence of a similar device happening together. For example, obtaining a head and a tail when flipping a coin twice {(H,T), (T,H)}
- More than one occurrence of different devices happening together. For example, obtaining a head and an even number when flipping a coin and rolling a number cube {(H,2), (H,4), (H,6)}.

SAMPLE SPACE

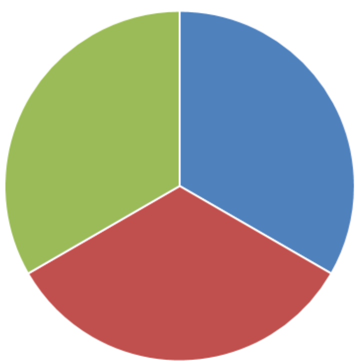
A sample space can be represented by tree diagrams, lists, and tables. Ask guiding questions to help students create methods for creating organized lists and tree diagrams for situations with more elements such as "How many outcomes are possible?", " What does each branch of the tree diagram represent?", or "How can you use your list to find the probability of the event?"

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models and tying the simulation to a real-world situation.

TIP!

Students may have difficulty determining the sample space. Students should investigate with a variety of organizers.

Uniform Probability Model

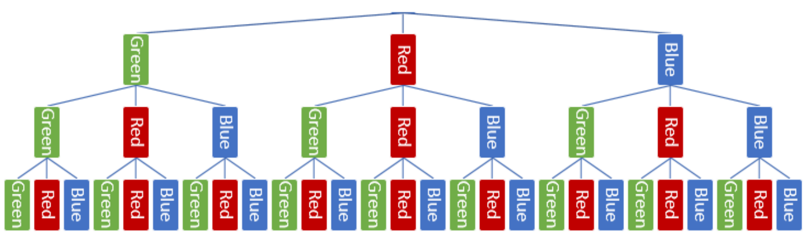


EXAMPLE

Represent the sample space with a tree diagram and answer the following questions:

- What is the probability of spinning 3 reds in a row?
- What is the probability of spinning two reds and a blue?
- What is the probability of spinning red, blue, and red in order?
- What is the probability of spinning at least 2 reds?
- What is the probability of spinning at most 1 red?

For those who struggle making tree diagrams, Microsoft Word Smart Art feature allows students to make tree diagrams in a more organized fashion.



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

EXAMPLE

Represent the sample space with a tree diagram and answer the following questions:

- What is the probability of spinning 3 reds in a row?
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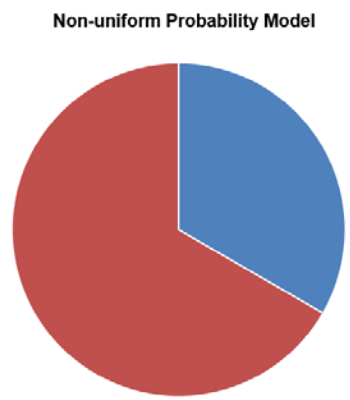
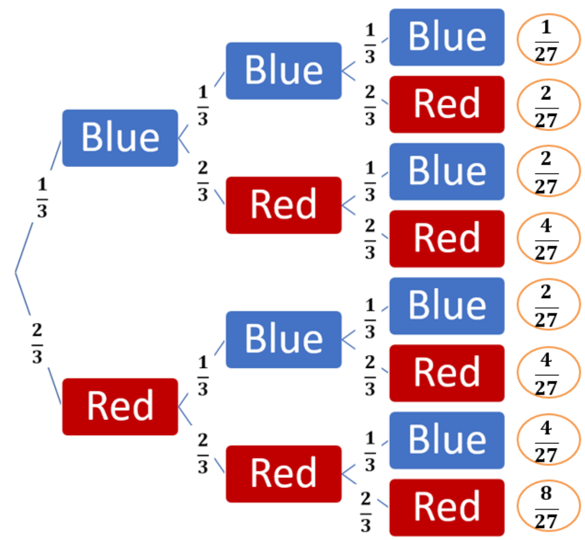
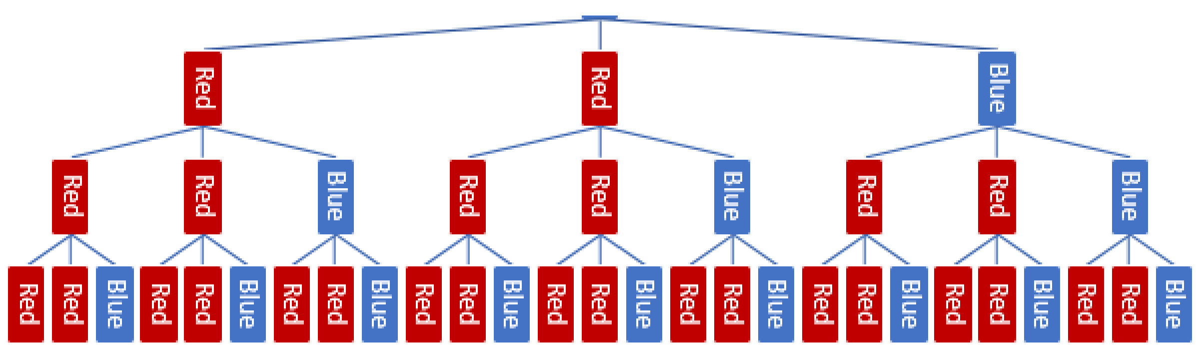




For students who struggle with using fractions and tree diagrams have them start by drawing out the entire sample space and then move to using fractional representations.



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

EXAMPLE

Part 1

Pair up students to play a game. Give each group a pair of number cubes and a playing board. Each student gets 12 playing pieces. Tell students that their goal is to be the first one to get all their playing pieces across the river by rolling the number cubes. Before they begin, they must place all 12 of their playing pieces by a number on their side of the river. Not all numbers need to be assigned a playing piece. Numbers may have more than 1 playing pieces.

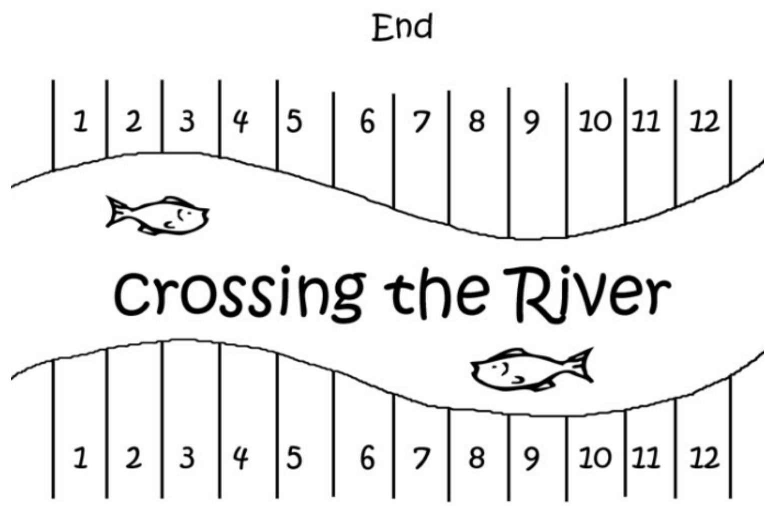
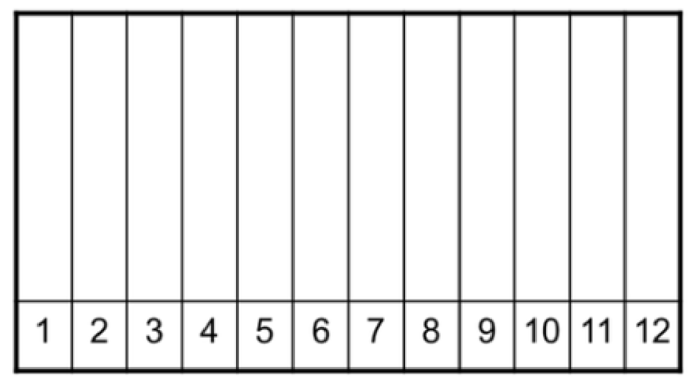


Image taken from Andrea Vollenwyder's Portfolio
https://vollenwyder.wordpress.com/category/teaching/

Discussion: Some students will initially put a playing piece on 1, which is impossible to roll. Use guiding questions to help students discover that rolling a 1 is impossible. A question may be "I will give you a million dollars if you win." After playing several games, students should start to notice that the middle numbers such as 6, 7, and 8 get rolled more often. Challenge students to explain why.

Part 2

After students have played a few games, have students make a line plot to keep track of their rolls. Then have them find the experimental probability of rolling each number.



INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

Part 3
Have students create a sample space representing the sum of rolling two dice. Then have them find the theoretical probability of rolling each number.

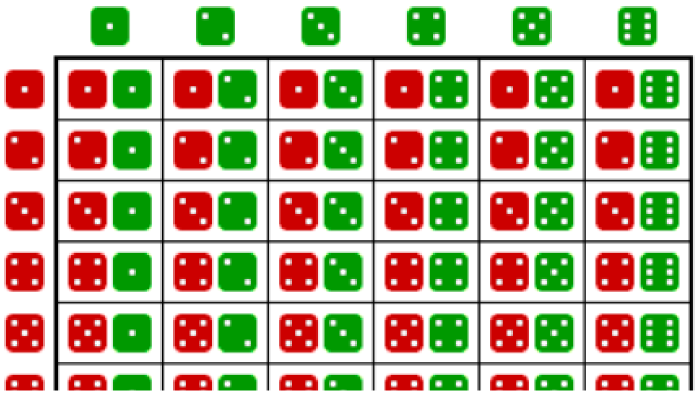


Image taken from https://socratic.org/questions/what-is-the-probability-of-rolling-doubles-on-a-pair-of-dice

Ask them the following questions:

- What is the probability of rolling a 1? Explain.
- What is the probability of rolling a sum that is a prime number? Explain.
- Which sums are you most like to roll? Explain.
- Why might people say that 7 is a lucky number? Explain.

SIMULATIONS

It is often important to know the probabilities of real-life events that may not have known theoretical probabilities. Scientists, engineers, and mathematicians design simulations to answer questions that involve topics such as diseases, water flow, climate changes, or functions of an engine. Results from the simulations are used to estimate probabilities that help researchers understand problems and provide possible solutions to these problems.

After the basics of probability are understood, students should experience setting up a model and using simulation (by hand or with technology) to collect data and estimate probabilities for a real situation that is sufficiently complex that the theoretical probabilities are not obvious. Simulation is a procedure that will allow students to answer questions about real problems by running experiments that closely resemble the real situation. Simulation uses devices such as coins, number cubes, cups full of paper clips or legos, or cards to generate outcomes that represent real outcomes. Students may find it difficult to make the connection between device outcomes and the real outcomes of the experiment.

- For example, suppose that, on average, a basketball player makes about three out of every four foul shots. This problem requires a device that produces a 75% chance of success. A fair coin toss cannot be used because its probability of success is 50%. Instead, a 6-sided number cube could be used by specifying that the numbers 1, 2, or 3 represent a hit, the number 4 represents a miss, and the numbers 5 and 6 would be ignored. A simple option could be a 4-sided number cube. A nonuniform spinner that has ¾ of a color shaded would also work in this circumstance as would a uniform spinner divided into 4 sections where 3 of the colors are identified as successes. Also, a bag of 4 marbles where 3 were the same color could also be used. Give students opportunities to choose the devices they will use to collect data
- For example, if 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? With 40% taken as the probability that a donor has type A blood, a random digit generator would be a good device to use. For example, 1, 2, 3, 4 could represent type A blood, and 0, 5, 6, 7, 8, 9 could represent non-type A blood. It may be helpful to define a successful trial and a failed trial before beginning. For a trial to be considered a success, it takes three **non-**type A blood types to occur (source: EngageNY Grade 7 Module 5, p. 120).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

TIP!

Use a percent grid (a 10 by 10 grid) to record data from a simulation.

Students also develop the understanding that the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

- For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
- Use a non-uniform process to determine probability, e.g., "Match, No Match" Game (Is It Fair? by Marilyn Burns, 1995):

Students may expect that simulations will produce all of the possible outcomes of the device. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does include all possibilities.

Instructional Tools/Resources

These tools and resources may be helpful for instruction. Before use, they should be evaluated at the local level to determine their appropriateness for instruction.

Manipulatives/Technology

- Coins
- Number cubes
- Number cards
- Other small objects such as marbles, Legos, or colored paper clips
- Spinners
- Adjustable Spinner by NCTM Illuminations is an applet that lets you create different types of spinners.
- Coin Toss by Digital First is an applet that involves graphing a flipped coin.

Randomness

- Evaluating Statements about Probability is a task by Mathematics Assessment Project where students explore randomness and equally likely events.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

Law of Large Numbers

- Probability Graphing by NCTM Illuminations has students explore the idea the Law of Large Numbers.

Theoretical Probability

- Probability: What are the Chances? by PBS Learning is a lesson with an accompanying video about exploring whether chance events are likely or unlikely.
- Stay or Switch? by Illustrative Mathematics allows students to explore theoretical probability when someone on a game show chooses to stay or switch based upon knowing one of the prize boxes.
- Rolling Dice by Illustrative Mathematics has students explore the theoretical probability of rolling dice.
- How Many Buttons? by Illustrative Mathematics has students use generated data to explore theoretical probability.
- Probability with Marbles by Open Middle has challenging math problems that are worth solving.
- Probability with Spinner by Open Middle has challenging math problems that are worth solving.
- Rolling with the Same Probability by Open Middle has challenging math problems that are worth solving.
- Probability of Rolling Two Six-Sided Dice by Open Middle has challenging math problems that are worth solving.
- Yellow Starbursts is a 3-act task by Dan Meyer where students have to guess how may packs will have yellow starbursts.

Experimental Probability

- Tossing Cylinders by Illustrative Mathematics has student determine experimental probabilities by collecting data.

Theoretical versus Experimental Probability

- What are my Chances? by NCTM Illuminations has students do experiments to compare theoretical and experimental probability. *NCTM now requires a membership to view their lessons.*
- Exploring Geometric Probabilities with Buffon's Coin Problem by Statistics Education Web is a lesson that connects experimental and theoretical probability to geometry with respect to area.

Sample Space

- Tetrahedral Dice by Illustrative Mathematics has students create an organized list or table to determine all the possible outcomes of a chance experiment.
- Probability Space by PBS Learning is a lesson with an accompanying video about sample space.
- Thinkport: Probability and Tree Diagrams by PBS Learning is a lesson with an accompanying interactive video about using a tree diagram to find the probability that a set of triplets will be all girls.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

Compound Events

- Red, Green, or Blue? by Illustrative Mathematics is a task that has students find compound events using organized lists, tables, or tree diagrams.
- Rolling Twice by Illustrative Mathematics has students compute the theoretical probability of a compound event.
- Sitting Across from Each Other by Illustrative Mathematics has students compute the theoretical probability of a compound event based on a seating configuration.
- Stick or Switch? by NCTM Illuminations has students compute probabilities for simple compound events. *NCTM now requires a membership to view their lessons.*
- Compound Probability by PBS Learning is a lesson with an accompanying video about finding the probability of compound events.

Simulations and Probability Models

- Waiting Times by Illustrative Mathematics has students find an approximate answer to compound even using a simulation.
- Thinkport: Hoops and Probability by PBS Learning is a lesson with an accompanying video about probability, relative frequency, and percentages using Michael Jackson's career statistics.

Games of Chance

- Designing: A Game of Chance is a task by Mathematics Assessment Project that helps students design and carryout a probability experiment.
- Analyzing Game of Chance is a task by Mathematics Assessment Project that has students confront common probability misconceptions and discuss the relationship between theoretical probabilities, observed outcomes, and sample sizes.
- The Game of SKUNK by NCTM Illuminations has students gain a basic understanding of probability by playing a game. *NCTM now requires a membership to view their lessons.*
- Performance Assessment Task: Counters from Inside Mathematics has students use theoretical and experimental probability to analyze a game.
- Performance Assessment Task: Fair Game? from Inside Mathematics has students use probabilities to represent sample space for simple and compound events and use that information to analyze a game.
- How Many Royal Flushes Do You Expect to Get? is a task by Robert Kaplinsky that has students explore the probabilities in an online poker game.
- Problem of the Month: Game Show from Inside Mathematics' Problems of the Month has students use game theory, probability, and expected value. They are non-routine math problems designed to be used schoolwide to promote a problem-solving theme at your school with different levels that allow scaffolding. They were developed by the Silicon Valley Mathematics Initiative and are aligned to the Common Core standards.
- What is the Probability of "Pigging Out"? by Statistics Education Web where students estimate the probability of the game Pass the Pigs by designing a simulation.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

Games of Chance, continued

- Problem of the Month: Fair Games from Inside Mathematics' Problems of the Month has students use probability and expected value. They are non-routine math problems designed to be used schoolwide to promote a problem-solving theme at your school with different levels that allow scaffolding.
- Probability Risk Analysis by PBS Learning is a lesson with an accompanying video about developing probability models.
- Prime Time Probability by NCTM Illuminations has students use a computer game to calculate the probability of independent events. *NCTM now requires a membership to view their lessons.*

Counting Methods

- Ice Cream Scoop is a task by You Cubed that has students use counting methods to discover the different combinations of ice cream.
- Leo the Rabbit is a task by You Cubed that has students use counting methods to discover the different ways Leo can hop up or down a flight of stairs.
- Problem of the Month: Party Time from Inside Mathematics' Problems of the Month has students use logic, reasoning, and counting methods. They are non-routine math problems designed to be used schoolwide to promote a problem-solving theme at your school with different levels that allow scaffolding.
- Problem of the Month: Rod Trains from Inside Mathematics' Problems of the Month has students use combinations and number theory. They are non-routine math problems designed to be used schoolwide to promote a problem-solving theme at your school with different levels that allow scaffolding.
- Problem of the Month: Squirreling It Away from Inside Mathematics' Problems of the Month has students use number operations, organized lists, and counting methods. They are non-routine math problems designed to be used schoolwide to promote a problem-solving theme at your school with different levels that allow scaffolding.
- Problem of the Month: Diminishing Return from Inside Mathematics' Problems of the Month has students use number operations, organized lists, and probability. They are non-routine math problems designed to be used schoolwide to promote a problem-solving theme at your school with different levels that allow scaffolding.
- Problem of the Month: Friends You Can Count On from Inside Mathematics' Problems of the Month has students use probability and expected value. They are non-routine math problems designed to be used schoolwide to promote a problem-solving theme at your school with different levels that allow scaffolding.

Curriculum and Lessons from Other Sources

- EngageNY, Grade 7, Module 5, Topic A, Lesson 1: Chance Experiments, Lesson 2: Estimating Probabilities by Collecting Data, Lesson 3: Chance Experiments with Equally Likely Outcomes, Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes, Lesson 5: Chance Experiments with Outcomes that are Not Equally Likely, Lesson 6: Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities, Lesson 7: Calculating Probabilities of a Compound Event are lessons that pertain to

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

this cluster.

Curriculum and Lessons from Other Sources, continued

- EngageNY, Grade 7, Module 5, Topic B, Lesson 8: The Difference Between Theoretical and Estimated Probabilities, Lesson 9: Comparing Estimated Probabilities to Probabilities Predicted by a Model, Lesson 10: Conducting a Simulation to Estimate the Probability of an Event, Lesson 11: Conducting a Simulation to Estimate the Probability of an Event, Lesson 12: Applying Probability to Make Informed Decisions are lessons that pertain to this cluster.
- Illustrative Mathematics, Grade 7, Unit 8, Lesson 1: Mystery Bags, Lesson 2: Chance Experiments, Lesson 3: What are Probabilities?, Lesson 4: Estimating Probabilities Through Repeated Experiments, Lesson 5: More Estimating Probabilities, Lesson 6: Estimating Probabilities Using Simulation, Lesson 7: Simulating Multi-Step Experiments, Lesson 8: Keeping Track of All Possible Outcomes, Lesson 9: Multi-step Experiments, Lesson 10: Designing Simulations are lessons that pertain to this cluster. *You must create an account to view Illustrative Mathematics lessons on OpenUp Resources.*
- Georgia's Standards of Excellence Framework, Grade 7, Unit 6: Probability has many activities that pertain to this cluster.
- Probability Through Data: Interpreting Results from Frequency Tables by Hopfensperger, Kranendonk, and Scheaffer is the teacher's edition of the textbook series Data-Driven Mathematics published by Dale Seymour Publications. The student edition can be found here.
- The Utah Middle School Math Project is an open source textbook and workbook.

General Resources

- Arizona 6-8 Progressions on Statistics and Probability.
  This cluster is addressed on the following pages:
  - Page 2 (2nd to last paragraph)-Their introduction to probability is based on seeing probabilities of chance events as long-run relative frequencies of their occurrence, and many opportunities to develop the connection between theoretical probability models and empirical probability approximations. This connection forms the basis of statistical inference.
  - Pages 7-8 *Chance processes and probability models* section
- Coherence Map by Achieve the Core is an interactive website that shows the progression and interconnectedness of the Common Core standards. To be noted, there are some differences between Ohio's Learning Standards and the Common Core, so although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- High School Coherence Map by UnboundED is a map of the relationships among high school mathematics standards. Please note: This resource is aligned to the Common Core not Ohio's Learning Standards. Although the tool is useful, it needs to be checked with respect to Ohio's Learning Standards.
- Statistics Teacher is an online journal published by the American Statistical Association (ASA) and NCTM. It includes articles, lesson plans, and professional development opportunities.
- Significance is a magazine that demonstrate the practical use of statistics and shows how statistics benefits society.
- Chance is a magazine about statistics for anyone who is interested in using data to advance science, education, and society.
- Estimation 180's Carnival of Probability is a blog about how to host a "Carnival of Probability."

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (7.SP.5-8)

References

- Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6-8, Statistics and Probability*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.
- Young, L. (September 2016). Toy pigs teach statistics. *Mathematics Teaching in the Middle School*, *22(2),* 114-117.

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*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.