



Ohio

Ohio's Model Curriculum | Mathematics with Instructional Supports Grade 7

Mathematics Model Curriculum

with Instructional Supports

Grade 7

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

Standards for Mathematical Practices—Grade 7

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.

In Grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?” When students compare arithmetic and algebraic solutions to the same problem, they identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

In Grade 7, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

MP.3 Construct viable arguments and critique the reasoning of others.

In Grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. For example, as students notice when geometric conditions determine a unique triangle, more than one triangle, or no triangle, they have an opportunity to construct viable arguments and critique the reasoning of others. Students should be encouraged to answer questions such as these: “How did you get that?”, “Why is that true?”, or “Does that always work?” They explain their thinking to others and respond to others’ thinking.

MP.4 Model with mathematics.

In Grade 7, students model problem situations symbolically, graphically, in tables, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use experiments or simulations to generate data sets and create probability models. Proportional relationships present opportunities for modeling. For example, for modeling purposes, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building. Students should be encouraged to answer questions such as “What are some ways to represent the quantities?” or “How might it help to create a table, chart, or graph?”

Continued on next page

Standards for Mathematical Practice, continued

MP.5 Use appropriate tools strategically.

Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in Grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms. Teachers might ask, “What approach are you considering?” or “Why was it helpful to use ___?”

MP.6 Attend to precision.

In Grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations, or inequalities. Teachers might ask, “What mathematical language, definitions, or properties can you use to explain ___?”

MP.7 Look for and make use of structure.

Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (e.g., $6 + 2n = 2(3 + n)$ by distributive property) and solve equations (e.g., $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real-world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities. Solving an equation such as $8 = 4(n - \frac{1}{2})$ is easier if students can see and make use of structure, temporarily viewing $(n - \frac{1}{2})$ as a single entity.

MP.8 Look for and express regularity in repeated reasoning.

In Grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$ and construct other examples and models that confirm their generalization. Students should be encouraged to answer questions such as “How would we prove that ___?” or “How is this situation both similar to and different from other situations using these operations?”

Mathematics Model Curriculum

with Instructional Supports

Grade 7

STANDARDS	MODEL CURRICULUM
<p>RATIOS AND PROPORTIONAL RELATIONSHIPS Analyze proportional relationships and use them to solve real-world and mathematical problems.</p> <p>7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. <i>For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.</i></p> <p>7.RP.2 Recognize and represent proportional relationships between quantities.</p> <p>a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p> <p><i>Continued on next page</i></p>	<p>Expectations for Learning</p> <p>In Grade 6, students begin reasoning about ratios, rates, and percents using models. In Grade 7, students extend this reasoning to proportions, direct variation equations, and more advanced percent problems. They identify unit rates in representations of proportional relationships. Students work with equations in two variables to represent and analyze proportional relationships. Also, they solve multi-step mathematical and real-world ratio and percent problems, such as problems involving percent increase and decrease. They also extend their learning of ratios to those specified by rational numbers. The study of proportional relationships is a foundation for the study of functions, which continues through high school and beyond.</p> <p>ESSENTIAL UNDERSTANDINGS</p> <p>Percents</p> <ul style="list-style-type: none"> • A percent is a specific kind of ratio with a whole of 100. • All percent problems involve a part, a whole measured in some unit, and the same part and whole measured in hundredths. • Percents can be bigger than 100% and less than 1%. • Percent problems can be represented with a proportion or an equation. • Percent increase or percent decrease problems require careful attention to the referent whole by determining to what the whole (or 100% amount) a percentage refers. • Percent of change that involves an increase includes the following: tax, markups, gratuities, commissions, fees, etc. • Percent of change that involves a decrease includes the following: markdowns, discounts, etc. • Percent error is the difference between the approximate and exact value divided by the exact value.

- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.*
- d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
- 7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Unit Rate & Proportional Relationship

- A unit rate is a comparison of two quantities where the second quantity (denominator) is one.
- A rate can be written as a complex fraction which can be used to find the unit rate.
- A proportional relationship is a relationship between quantities.
- Proportions involve vertical and horizontal multiplicative relationships.
- In a table that represents a proportional relationship between y and x , $\frac{y}{x}$ is constant.
- The unit rate, which is the constant of proportionality, can be identified through models.
- Proportional relationships can be written as equations using the constant of proportionality, e.g., $y = kx$; $y = mx$; $t = pn$.
- The constant of proportionality is not always rational, e.g., π .
- The unit rate is the amount of change in y as x increases by one unit, e.g., in a table or graph.
- Graphs that represent proportional relationships are linear and go through the origin.

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to describe mathematical reasoning.
- Attend to precision in recording mathematical statements.
- Pay attention to and make sense of quantities.
- Consider mathematical units involved in a problem.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Recognize and use a pattern or structure.

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Expectations for Learning, continued**INSTRUCTIONAL FOCUS****Unit Rate and Proportional Relationships**

- Identify the constant of proportionality (unit rate).
- Compute and compare unit rates of two rational numbers, including complex fractions, in a real-world context.
- Represent proportional relationships in various formats including tables, graphs, diagrams, verbal descriptions, and equations.
- Align units when writing a proportion.
- Determine if a relationship is proportional by using graphing, unit rates, scale factors, and other strategies.
- Use proportional relationships to solve real-world ratio problems with multiple steps.
- Explain the meaning of a point (x, y) on a proportional graph, with special attention to the points $(0, 0)$ and $(1, r)$, where r is the unit rate.

Percents

- Use proportional relationships and equations to solve mathematical and real-world percent problems.
- Use benchmark percents to estimate and determine the reasonableness of a solution.
- Attend closely to the wording of percent problems to determine what the whole (or 100% amount) to which a percentage refers.
- Solve problems where the unknown is the part, whole, or percent.
- Solve problems when the percent is greater than 100% or less than 1%.
- Find the percent change (increase or decrease) using proportions and/or equations.
- Recognize that the total cost of an item discounted (20% off) is equivalent to the difference of the value and 100% ($100\% - 20\% = 80\%$).
- Recognize that the total cost of an item with sales tax or commission (5%) is equivalent to the sum of the value and 100% ($100\% + 5\% = 105\%$).
- Calculate simple interest by multiplying the principal by the rate of interest by the number of years of the loan or investment ($I = prt$).
- Calculate the percent error by finding the difference between the approximate and exact value divided by the exact value.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43](#)
- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45](#)
- [Ohio's K-8 Learning Progressions, Ratios and Proportional Relationships, page 15](#)

CONNECTIONS ACROSS STANDARDS

- Represent proportional relationships within and between similar figures (7.G.1).
- Use proportional reasoning in examining a sample of a population (7.SP.1).
- Use proportional reasoning when predicting the probability of an event (7.SP.6-8).
- Connect proportional reasoning to write and solve equivalent expressions and equations involving percent (7.EE.2-4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>THE NUMBER SYSTEM Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</p> <p>7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite quantities combine to make 0. <i>For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</i></p> <p>b. Understand $p + q$ as the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p>c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</p> <p><i>Continued on next page</i></p>	<p>Expectations for Learning</p> <p>In Grade 6 students learned to locate rational numbers on the number line. In Grade 7 they extend their understanding of operations with fractions to operations with rational numbers. This is the first time students will add, subtract, multiply, and divide negative rational numbers. Time should be spent developing understanding through the use of models and manipulatives to discover the rules of negative integers and become fluent in applying them. At this point, it is essential that students understand the relationship among all four operations and their properties. In future grades, students will apply their understanding of operations with rational numbers to expressions and equations.</p> <p>ESSENTIAL UNDERSTANDINGS</p> <p>Rational Numbers</p> <ul style="list-style-type: none"> The set of integers consists of positive whole numbers, their opposites, and 0. A rational number is any number that can be written as the quotient or fraction $\frac{p}{q}$ of two integers, a numerator p, and non-zero denominator q. A rational number can be converted to a decimal using long division; the decimal form of a rational number terminates in 0s or repeats. In a fraction the negative sign can be written in the numerator, the denominator, or out front, e.g., $\frac{-3}{4} = \frac{3}{-4} = -\frac{3}{4}$. <p>Addition and Subtraction</p> <ul style="list-style-type: none"> When modeling operations with integers on a number line, the sign of the number indicates the direction and the number indicates the amount of spaces moved. In a number line model, the subtraction sign means to change directions. A number and its opposite are additive inverses; they have a sum of 0, i.e., $a + (-a) = 0$. Subtraction of rational numbers is adding the additive inverse, i.e., $p - q = p + (-q)$ The absolute value of $p - q$ is just the distance from p to q, regardless of direction. <p><i>Continued on next page</i></p>

- d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- c. Apply properties of operations as strategies to multiply and divide rational numbers.
- d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

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Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Multiplication and Divisions

- Multiplication of rational numbers can be modeled on the number line.
- A positive product is the result of multiplying two numbers with the same sign.
- A negative product is the result of multiplying two numbers with different signs.
- Division is the inverse of multiplication, so the same rules for rational numbers apply.
- Division can be written using a fraction bar.
- Every quotient of integers (with a nonzero divisor) is a rational number.
- A repeating quotient has a line of the repeating numerals.

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to describe mathematical reasoning.
- Apply and justify mathematical concepts, terms, and their properties.
- Draw a picture or create a model to make sense of a problem.
- Compute using strategies or models.
- Determine reasonableness of results.
- Use different properties of operations flexibly.

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7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Note: Although, rote memorization of the names of the properties is not encouraged, it is expected for teachers to use formal language so that students gain familiarity and are able to recognize and apply the correct terminology.

Addition and Subtraction

- Recognize that opposite numbers are additive inverses and have a sum of 0.
- Use models to represent and solve addition and subtraction of rational numbers, i.e., number line or chips.
- Solve mathematical and real-world problems using addition and subtraction of rational numbers.
- Interpret the sums and differences of rational numbers in real-world contexts.
- Show that the distance between two rational numbers on a number line is the absolute value of their difference.
- Apply the Associative Property of Addition, Commutative Property of Addition, Additive Identity Property, and Additive Inverse Property when solving problems.

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Expectations for Learning, continued**INSTRUCTIONAL FOCUS, CONTINUED****Multiplication and Division**

- Use properties of operations to connect previous understanding of multiplication and division to rational numbers.
- Connect multiplication of integers to the distributive property.
- Multiply and divide integers.
- Solve mathematical and real-world problems using multiplication and division of rational numbers.
- Interpret the product and quotient of rational numbers in real-world contexts.
- Convert a rational number to a decimal, and understand that the decimal form of a rational number will repeat or terminate.
- Apply the Associative Property of Multiplication, Commutative Property of Multiplication, Identity Property of Multiplication, Inverse Property of Multiplication, Distributive Property, and Zero Product Property when solving problems.
- Use patterns to recognize the sign of the product when a string of numbers is multiplied together.
- Recognize that the number line can be divided into an infinite number of segments which can be represented by decimals or fractions.
- Explain the difference between a terminating and repeating decimal.

Operations with Rational Numbers

- Solve real-world and mathematical problems involving the four operations with rational numbers.
- Solve problems involving complex fractions.

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Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45](#)
- [Ohio's K-8 Learning Progressions, The Number System, pages 16-17](#)

CONNECTIONS ACROSS STANDARDS

- Use properties of operations to generate equivalent expressions (7.EE.1-2).
- Solve multi-step, real-world numerical and algebraic equations and/or inequalities with rational numbers (7.EE.3-4).
- Use proportional relationships to solve real-world and mathematical problems (7.RP.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>EXPRESSIONS AND EQUATIONS Use properties of operations to generate equivalent expressions.</p> <p>7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</p> <p>7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. <i>For example, a discount of 15% (represented by $p - 0.15p$) is equivalent to $(1 - 0.15)p$, which is equivalent to $0.85p$ or finding 85% of the original price.</i></p>	<p>Expectations for Learning</p> <p>In prior grades, students created equivalent expressions using the properties of operations with positive rational numbers and coefficients. They used the algebraic order of operations to simplify numerical expressions and evaluate algebraic expressions. Students in Grade 7 extend their knowledge to include properties of operations with positive and negative rational numbers. Students gain experience writing expressions in multiple ways. These expressions can serve different purposes and provide different ways of seeing a problem. This provides the foundation for analyzing and solving more complicated linear equations in Grade 8.</p> <p>ESSENTIAL UNDERSTANDINGS</p> <ul style="list-style-type: none"> • Equivalent expressions always have the same value even if written in different forms. • Equivalent expressions can be generated using properties of operations (Distributive Property, Associative Properties of Multiplication, Associative Property of Addition, Commutative Property of Multiplication, Commutative Property of Addition, and Identity Property of Multiplication). • The order of operations is used to generate equivalent algebraic expressions. • The coefficient of a single variable is 1 even if it is not written. <i>For example, $-x = -1x$ and $x = 1x$.</i> • A fractional coefficient can be written in two ways, e.g., $\frac{x}{3} = \frac{1}{3}x$. • Negative rational terms can be written in three ways, e.g., $\frac{1}{-3} = \frac{-1}{3} = -\frac{1}{3}$. • In problems involving percentages, 100% of the variable x can be written as $x = 1x$. • Factoring a GCF can be used to write an equivalent expression. • Writing expressions in equivalent forms can serve different purposes and provide different ways of seeing a problem in context. <p><i>Continued on next page</i></p>

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use different properties of operations flexibly.
- Pay attention to and make sense of quantities.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Solve multi-step problems accurately.
- Compute accurately and efficiently with grade-level numbers.

INSTRUCTIONAL FOCUS

- Recognize simple equivalent expressions.
- Apply properties of operations to factor a GCF and expand linear expressions with rational coefficients.
- Utilize the order of operations with rational number coefficients to create an equivalent expression.
- Use the distributive property with positive and negative rational numbers. (Variables are limited to the power of one.)
- Use factoring of a GCF to rewrite equivalent expressions with positive and negative rational numbers. (Variables are limited to the power of one.)
- Write and interpret equivalent expressions that represent a real-world problem.
- Use the properties of rational numbers to combine like terms.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

CONNECTIONS ACROSS STANDARDS

- Apply and extend previous understandings of operation with fractions to add, subtract, multiply, and divide rational numbers (7.NS.1-3).
- Use proportional relationships to solve multi-step ratio and percent problems (7.RP.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>EXPRESSIONS AND EQUATIONS Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <i>For example, if a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i> <i>Continued on next page</i></p>	<p>Expectations for Learning In Grade 6, students write and solve one-step equations and graph inequalities on a number line. In Grade 7, students will solve more complex equations by applying properties of operations with rational numbers. In addition, students will solve, graph, and interpret the solutions of inequalities. Students will make use of prior knowledge of rational numbers to solve multi-step numerical problems with positive and negative numbers and estimate the reasonableness of their answers. They will continue to gain fluency with positive and negative numbers as they solve more complex equations in 8th grade.</p> <p>ESSENTIAL UNDERSTANDINGS Real-life and Mathematical Problems</p> <ul style="list-style-type: none"> • Variables are used to represent a quantity. • The order of operations is used to write and solve equations given within a context of a word problem. • A solution is a value that makes an equation or an inequality true. • Inverse operations may be used to solve equations and inequalities. • Equivalent expressions always have the same value even if written in different forms. • Equivalent expressions can be generated by using properties of operations (distributive property, associative, commutative, identity and inverse properties of multiplication and addition). • A term includes the operational sign in front of it. <p><i>Continued on next page</i></p>

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
- b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Inequalities

- Inequalities have infinitely many solutions.
- Solutions to inequalities can be represented on number line diagrams.
- Point c is not included in the graphical solution to $x > c$ or $x < c$; the number line diagram represents this with an open circle around point c .
- Point c is included in the graphical solution to $x \geq c$ or $x \leq c$; the number line diagram represents this with a closed circle at point c .
- All of the solutions to an inequality are represented with a shaded region on a number line diagram.
- The inequality $x > c$ is equivalent to $c < x$, and $x \geq c$ is equivalent to $c \leq x$.
- When multiplying or dividing both sides of an inequality by a negative number, the order of the comparison it represents is reversed.

MATHEMATICAL THINKING

- Represent real-world problems mathematically.
- Compute accurately and efficiently with grade-level numbers.
- Determine reasonableness of results.
- Use accurate mathematical vocabulary to describe mathematical reasoning.
- Use estimation and mental computation strategies.
- Recognize and use a pattern or structure.
- Use informal reasoning.

Continued on next page

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Real-life and Mathematical Problems

- Define variables in context using appropriate units.
- Solve multi-step problems with positive and negative rational numbers including whole numbers, decimals, and fractions, and determine the reasonableness of solution(s).
- Construct and solve multi-step equations and inequalities with variables on one side, and determine the reasonableness of solution(s).
- Compare an algebraic solution to an arithmetic solution, identifying the sequence of operations used in each.

Inequalities

- Develop an understanding of the impact a negative coefficient has on the solution set of an inequality in context.
- Graph solutions to inequalities and interpret solutions within a context with positive and negative coefficients.
- Discover that to maintain a true statement, the inequality symbol must be reversed when multiplying or dividing by a negative number.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 2, pages 44-45](#)
- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 3, pages 46-47](#)
- [Ohio's K-8 Learning Progressions, Expressions and Equations, pages 18-19](#)

CONNECTIONS ACROSS STANDARDS

- Solve real-world and mathematical problems involving the four operations with rational numbers (7.NS.3).
- Use properties of operations to generate equivalent expressions (7.EE.1-2).
- Recognize and represent proportional relationships between quantities (7.RP.2).
- Use facts about supplementary, complementary, vertical, and adjacent angles to solve simple equations for an unknown angle in the figure (7.G.5).
- Convert rational numbers to decimals (7.NS.2d).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>GEOMETRY Draw, construct, and describe geometrical figures and describe the relationships between them.</p> <p>7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.</p> <ol style="list-style-type: none"> Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. Represent proportional relationships within and between similar figures. <p>7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions.</p> <ol style="list-style-type: none"> Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions. <p>7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</p>	<p>Expectations for Learning</p> <p>The investigation of drawing, constructing, and describing geometric figures in Grade 7 leads to development of the concepts of similarity, congruence, cross sections, and interior angle sums of triangles and quadrilaterals. Students will understand that in similar figures corresponding angles are congruent and corresponding sides are proportional. Students will apply this knowledge of similar figures to solve real-world problems including those with scale drawings. It is imperative students explore relationships by using multiple models including technology to develop geometric concepts in congruence and similarity. Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence in Grade 8 and high school. <i>Note: The congruence criteria for triangles will be formalized in high school geometry.</i></p> <p>The student understanding of this cluster aligns with van Hiele Level 1 (Analysis) with certain aspects of this cluster moving toward van Hiele Level 2 (Informal Deduction/Abstraction).</p> <p>ESSENTIAL UNDERSTANDINGS</p> <p>Similar Figures</p> <ul style="list-style-type: none"> Angles are congruent if they are equal in measure. <i>Note: 7th grade students may use the term “equal in measure” in place of congruent.</i> Similar figures have corresponding angles that are congruent and corresponding side lengths that are proportional. Applying a scale factor greater than one results in a bigger image. Applying a scale factor of 1 results in a congruent image. <i>Note: Students are not required to understand congruency of two figures until 8th grade.</i> Applying a scale factor less than 1 but greater than zero results in a smaller image. <p><i>Continued on next page</i></p>

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Drawing Geometric Figures

- The sum of angles in a triangle equals 180 degrees.
- The sum of angles in a quadrilateral equals 360 degrees.
- Three possible outcomes exist when constructing triangles with given measurements of sides and/or angles: a unique triangle, more than one triangle, or no triangle.
- Some quadrilaterals have more specific names based on relationships such as pairs of parallel sides, congruent sides, and angle relationships.

Slicing Three-Dimensional Figures

- Slicing a three-dimensional figure results in a two-dimensional shape.
- Slicing a three-dimensional figure in different ways could result in different shapes.

MATHEMATICAL THINKING

- Explain mathematical reasoning.
- Make and modify a model to represent mathematical thinking.
- Solve real-world problems accurately.
- Make connections between concepts, terms, and properties within the grade level and with previous grade levels.
- Use technology strategically to deepen understanding.

Continued on next page

Expectations for Learning, continued**INSTRUCTIONAL FOCUS****Similar Figures**

- Draw or create a model to make sense of a problem involving similar figures.
- Identify corresponding sides and angles of similar figures.
- Compare and contrast the relationship between the angle measures and side lengths in a scale drawing and its original figure.
- Apply proportional understanding to the relationship between side lengths in similar figures.
- Investigate the relationship between the areas of similar figures.
- Identify the impact of scale on length and area.
- Compute actual lengths and areas from a scale drawing.
- Reproduce a scale drawing using a different scale.
- Draw scaled figures with proper figure labels, scale, and dimensions.

Drawing Geometric Figures

- Draw a picture or create a model of triangles or quadrilaterals with given conditions.
- Investigate whether a given set of side lengths and angle measures determines a unique triangle, creates multiple triangles, or does not create a triangle.
- Investigate quadrilaterals with a given set of side lengths, angle measures, and relationship between sides (parallel, perpendicular, neither) to observe types and properties.
- Investigate whether it is possible to construct more than one quadrilateral with the same given set of side lengths, angle measures, and relationship between sides.
- Discover the sum of the interior angles of triangles and quadrilaterals.

Slicing Three-Dimensional Figures

- Investigate the various outcomes of slicing three-dimensional figures and the resulting two-dimensional face.

Continued on next page

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 1, page 43](#)
- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 3, pages 46-47](#)
- [Ohio's K-8 Learning Progression, Geometry, page 21](#)

CONNECTIONS ACROSS STANDARDS

- Analyze proportional relationships, and use them to solve real-world and mathematical problems (7.RP. 1 – 3).
- Construct simple equations to solve for unknown dimensions in similar figures (7.EE.4.a).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>GEOMETRY Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.</p> <p>7.G.4 Work with circles.</p> <ol style="list-style-type: none"> Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. <p>7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</p> <p>7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p>	<p>Expectations for Learning</p> <p>Grade 7 is the first experience that students have with measuring radius, diameter, and circumference of circles and exploring their relationships. Students build on earlier experiences with angle measurement to solve problems that involve special angle pairs. Students will apply their understanding of area, surface area, and volume to solve real-world problems using a variety of strategies. <i>Note: The volume of a pyramid and surface area and volume of a cylinder are not addressed at this grade level.</i> This cluster could also be extended for some students to include the surface area of cylinders.</p> <p>In Grade 8, students will apply their knowledge of circles and expand their understanding of volume to cones, cylinders, and spheres. Although, students' formal understanding of rational and irrational numbers is not defined until Grade 8, at this grade, they are expected to just have an intuitive understanding of pi and its approximation. In Grade 8, students will extend their knowledge of special angle pairs to relationships when parallel lines are cut by a transversal.</p> <p>The student understanding of circles aligns with van Hiele Level 2 (Informal Deduction/Abstraction). The student understanding of special angle pairs, area, surface area, and volume aligns with van Hiele Level 1 (Analysis).</p> <p>ESSENTIAL UNDERSTANDINGS</p> <p>Circles</p> <ul style="list-style-type: none"> Points on a circle are the same distance from the center. A circle is created by connecting all the points equidistant from the center point The radius is the distance from any point on the circle to the center. The diameter is a straight line that passes through a circle and its center point where both end points lie on the circle. The diameter is twice the radius. The circumference is the distance around a circle. Pi is a constant ratio of a circle's circumference divided by its diameter. The symbol for pi is π. <p><i>Continued on next page</i></p>

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

Special Angle Pairs

- Two angles are supplementary when their angle measures have a sum of 180 degrees.
- Two angles are complementary when their angle measures have a sum of 90 degrees.
- Vertical angles are the angles opposite each other when two lines intersect; the angles are congruent.
- Vertical angles are congruent because they are both supplementary to the same angle.
- Two angles are adjacent when they have a common side and a common vertex and do not overlap.

Applications of Area, Surface Area, and Volume

- A right prism is a prism whose bases are parallel to one another and whose lateral faces are rectangles.
- Prisms can have bases other than rectangles.
- The base and top surface are the same shape and size.
- In a right prism the bases are perpendicular to the vertical sides.

MATHEMATICAL THINKING

- Explain mathematical reasoning.
- Use accurate mathematical vocabulary to describe mathematical reasoning.
- Make and modify a model to represent mathematical thinking.
- Solve real-world mathematical problems.

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Expectations for Learning, continued**INSTRUCTIONAL FOCUS****Circles**

- Identify and define characteristics of circles: radius, diameter, circumference (perimeter), and area.
- Explore radius and diameter relationships.
- Investigate the relationship between a circles' radius or diameter and its circumference and area.
- Explain the relationship between the radius (and/or diameter) and the length measure of the circle (circumference).
- Discover an approximate value of pi.
- Know the formulas for the area and circumference of a circle.
- Apply the formulas for the area and circumference of a circle to solve real-world problems.

Special Angle Pairs

- Investigate the relationships of special angle pairs through modeling.
- Identify and define special angle pairs: supplementary, complementary, vertical, and adjacent angles.
- Use special angle pairs to write and solve equations for multi-step problems.
- Measure and find patterns among the angles of intersecting lines including cases where angles lie within polygons.

Applications of Area, Surface Area, and Volume

- Draw a picture or create a model to compose and decompose two- and three-dimensional figures to find area, surface area, and volume.
- Differentiate between linear, square, and cubic units and when to use them.
- Solve real-world problems involving area, surface area, and volume.

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Content Elaborations

- [Ohio's K-8 Critical Area of Focus Grade 7, Number 3, pages 46-47](#)
- [Ohio's K-8 Learning Progressions, Geometry, page 21](#)

CONNECTIONS ACROSS STANDARDS

- Write and solve equations to solve geometric problems (7.EE.4.a).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>STATISTICS AND PROBABILITY Use sampling to draw conclusions about a population.</p> <p>7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population.</p> <p>a. Differentiate between a sample and a population.</p> <p>b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.</p>	<p>Expectations for Learning</p> <p>In earlier grades students have been using data, both categorical and numerical, to answer simple statistical questions, but have paid little attention to how the data were selected. In 6th grade students use center and variability to describe data. In 7th grade, students will begin to investigate populations, sampling, and bias. Students move from Level A to Level B in the GAISE model by sampling more than one group. Using these concepts, students will begin to draw their own conclusions and support the conclusions of others. In high school, students will further develop their understanding using experimental designs through random sampling.</p> <p>ESSENTIAL UNDERSTANDINGS</p> <ul style="list-style-type: none"> • Statistics is the name for the science of collecting, analyzing, and interpreting data. • A population consists of everyone in a specific group and a sample is a subset from a specific group. • Results from a sample can be generalized for a much larger population. • Sampling variability exists because the sample proportion varies from sample to sample. • Bias, a systematic favoritism in the data collection process, can occur in the way the sample is selected or in the way data are collected. <p>MATHEMATICAL THINKING</p> <ul style="list-style-type: none"> • Construct valid conclusions. • Critique reasoning used to draw conclusions. • Formally explain mathematical reasoning. • Use formal and precise mathematical language. <p><i>Continued on next page</i></p>

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Differentiate between a sample and a population.
- Describe what makes a sample an accurate representation of a population.
- Describe how sample size affects inferences made about the population.
- Develop informal understanding of bias.
- Determine what factors create bias such as wording, length, timing of questions, and the choice of individuals.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 4, page 48](#)
- [Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23](#)
- [GAISE Model, pages 14 – 15](#)
 - [Focus of 7th grade is Level A – B, pages 22-59](#)

CONNECTIONS ACROSS STANDARDS

- Broaden the understanding of the framework of the GAISE Model (7.SP.2).
- Describe and analyze distributions (7.SP.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>STATISTICS AND PROBABILITY Broaden understanding of statistical problem solving. 7.SP.2 Broaden statistical reasoning by using the GAISE model:</p> <p>a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. <i>For example, “How do the heights of seventh graders compare to the heights of eighth graders?”</i> (GAISE Model, step 1)</p> <p>b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)</p> <p>c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)</p> <p>d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)</p>	<p>Expectations for Learning</p> <p>Students build on their previous work from 6th grade (6.SP.1-3) to broaden their statistical reasoning through the use of the GAISE model framework. Most of the expectations for students in 6th grade were at Level A.</p> <p>In 7th grade, students should be progressing from Level A to Level B, where the student directed process of selecting a sample and understanding its connection to the population. Students at Level B become more aware that some questions have distinct answers whereas others have answers that can vary. Whereas in Level A, the teachers posed most of the statistical questions, in Level B students begin to pose their own questions. The questions students form are no longer limited to the classroom. They begin to develop an awareness of design differences. Students at Level B start comparing two populations (two classrooms) and start informally comparing a sample to a population (compare classroom to whole school). The GAISE model framework will continue through high school.</p> <p>ESSENTIAL UNDERSTANDINGS</p> <ul style="list-style-type: none"> • Statistics is the name for the science of collecting, analyzing, and interpreting data. • The GAISE model framework is used to analyze and interpret data and has four steps: Formulate the Question; Collect Data to Answer the Question; Analyze the Data; and Interpret Results. • Data are not just numbers; they are numbers generated with respect to a particular context and situation. • There are two types of data: categorical and numerical. • Categorical data are sorted into groups and categories. • Numerical data are measurable. • A statistical question anticipates a response that varies, from one individual to the next, and this variability is described in terms of spread and overall shape. • A distribution shows all values of data and how often they occur. • A set of data has a distribution which can be described by its center, spread, and overall shape. • A measure of variation is a single number that describes the extent to which data vary in a distribution.

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Broaden understanding of the GAISE model

Step 1 – Formulate the Question:

- Begin to pose student generated statistical questions with variability that go beyond the classroom.
- Recognize the distinction between a population, census, and a sample.

Step 2 - Collect Data to Answer the Question:

- Design a collection method to answer a statistical question.
- Conduct sample surveys of two or more groups or comparative experiments.

Step 3 - Analyze the Data:

- Use properties of distributions (center, spread, shape) as tools of analysis.
- Determine variability (spread) within a group.
- Compare individual to individual, individual to group, group to group.
- Summarize the numerical data sets in relation to the context using graphical displays:
 - histograms
 - interquartile range (IQR) and mean absolute deviation (MAD)
 - boxplots and five number summaries: lower extreme (min), upper extreme (max), median (Q2), lower quartile (Q1), and upper quartile (Q3)
- Show distribution as all values of data and how often they occur.

Step 4 - Interpret Results:

- Draw conclusions and make generalizations from the analysis of the data between two groups.
 - Describe differences between two or more groups using center, spread, and shape.
- Acknowledge whether a sample may or may not be representative of a larger population.

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Expectations for Learning, continued

MATHEMATICAL THINKING

- Make sense of and create statistical problems.
- Formally explain mathematical reasoning.
- Use formal and precise mathematical language.
- Analyze student created questions.
- Pay attention to and make sense of quantities.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 4, page 48](#)
- [Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23](#)
- [GAISE Model pages 14 – 15](#)
 - [Focus of 7th grade is Levels A – B, pages 22-59](#)

CONNECTIONS ACROSS STANDARDS

- Extend the GAISE Model to address questions involving two populations versus one population (7.SP.1).
- Describe and analyze distributions (7.SP.3).
- Data may also be generated using a chance process (7.SP.5-8).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>STATISTICS AND PROBABILITY Summarize and describe distributions representing one population and draw informal comparisons between two populations.</p> <p>7.SP.3 Describe and analyze distributions.</p> <p>a. Summarize quantitative data sets in relation to their context by using mean absolute deviation^G (MAD), interpreting mean as a balance point.</p> <p>b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot^G (line plot), the separation between the two distributions of heights is noticeable.</i></p>	<p>Expectations for Learning</p> <p>In 6th grade, students analyzed distributions (center, spread, and shape) in a single population. In 7th grade, they will begin to draw comparisons between two populations. For the first time, students will interpret mean as a balance point and use the mean absolute deviation (MAD) as the measure of variability from the mean. It is imperative students develop the conceptual understanding of the MAD. Students will take their conceptual understanding of MAD in 7th grade and apply it to standard deviation in high school.</p> <p>ESSENTIAL UNDERSTANDINGS</p> <ul style="list-style-type: none"> • Descriptive statistics may include measures of center and spread. • There is variability between groups. • Data can be represented in different ways to persuade people. • The important purpose of a measure of center is not the value itself, but the interpretation it provides for the variation of the data. • The sum of the distances from each data point below the mean to the mean equals the sum of the distance from each data point above the mean to the mean. • Mean absolute deviation (MAD) is one way to measure the extent to which a distribution is stretched or squeezed. • The mean absolute deviation (MAD) is the average distance that each data value is from the mean. <p>MATHEMATICAL THINKING</p> <ul style="list-style-type: none"> • Formally explain mathematical reasoning. • Use formal and precise mathematical language. • Pay attention to and make sense of quantities. • Solve real-world problems accurately. • Determine the reasonableness of results. • Analyze visual models. <p><i>Continued on next page</i></p>

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Interpret mean as a balance point.
- Explore, explain, and calculate the mean absolute deviation (MAD).
- Summarize data using MAD within a context.
- Summarize and describe distributions representing one population.
- Informally compare distributions representing two populations using MAD, histograms, dot plots, and/or boxplots.

Content Elaborations

- [Ohio's K-8 Critical Area of Focus, Grade 7, Number 4, page 48](#)
- [Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22-23](#)
- [GAISE Model, pages 14 – 15](#)
 - [Focus of 7th grade is Level A – B, pages 22-59](#)

CONNECTIONS ACROSS STANDARDS

- A sample allows results to be generalized to a much larger set of data, the population from which the sample was selected (7.SP.1).
- Broaden understanding of the framework of GAISE Model (7.SP.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

STANDARDS	MODEL CURRICULUM
<p>STATISTICS AND PROBABILITY Investigate chance processes and develop, use, and evaluate probability models.</p> <p>7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.</p> <p>7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. <i>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</i></p> <p>7.SP.7 Develop a probability model⁶ and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p><i>Continued on next page</i></p>	<p>Expectations for Learning</p> <p>Grade 7 introduces the formal study of probability. Students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions. It is imperative students use simulations to model concepts of probability as a foundation for further work in high school.</p> <p>ESSENTIAL UNDERSTANDINGS</p> <ul style="list-style-type: none"> • Probability is the study of the chance (likelihood) that a particular event will occur. • The theoretical probability of an event describes how often the event will occur in an infinite number of repetitions of a chance process. It also is the long run ratio of the number of times the event occurs divided by the number of times that the chance process is repeated. • Probability is a number between 0 and 1 that has no units. <ul style="list-style-type: none"> ○ Near 1 is most likely; near 0 is least likely; and $\frac{1}{2}$ is neither likely nor unlikely. • An outcome is the number of times an event happens. • A probability model provides a probability for each possible non-overlapping outcome for a chance process. <ul style="list-style-type: none"> ○ A sample space is the collection of all possible individual outcomes. ○ An event is an outcome or set of outcomes in an experiment; it is a subset of the sample space. <ul style="list-style-type: none"> ▪ A simple event has one outcome. ▪ A compound event has more than one outcome. ○ The total probability of all such outcomes is 1. • Frequency (absolute frequency) is a quantity that has no units represented by real number greater than or equal to zero. It is the number of items occurring in a given set; it is a count. <i>Note: Frequency has a different meaning in statistics than is used in common usage, mathematics, and physics.</i> • Relative (observed or experimental) frequency is the ratio of times an event occurs to the number of occasions which it might occur in the same period. e.g., if a coin is flipped 1,000 times and get heads 498 times, the relative frequency is $\frac{498}{1000}$ or 0.498.

- a. Develop a uniform probability model^G by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
- b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

- 7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.
- a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space^G for which the compound event occurs.
- b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., “rolling double sixes,” identify the outcomes in the sample space which compose the event.

Continued on next page

Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS, CONTINUED

- The relative frequency of an event is based on the outcomes of collected data.
 - A simulation is the use of a probability model to imitate a real situation.
 - The simulation is supposed to give similar results to the real situation and predicts what should occur.
 - In the long-run (increased trials), the relative frequency approaches theoretical probability of the event. *Note: Middle school students may use experimental probability interchangeable with relative frequency.*
- In cases when it is difficult or impossible to compute the theoretical probability, the relative frequency can be used to estimate the theoretical probability of the event by running experiments and counting the number of outcomes.

MATHEMATICAL THINKING

- Use precise mathematical vocabulary to describe mathematical reasoning.
- Develop and use probability models.
- Make and analyze conjectures.
- Use technology strategically to deepen understanding.

Continued on next page

- c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Identify a question to explore using probability.
- Design a probability model (uniform and non-uniform) based on observed frequencies to answer the question.
 - Collect data (through simulations or experiments).
 - Organize data (lists, tables, tree diagrams, etc.) into an appropriate sample space.
 - Define events as subsets of the sample space.
 - Assign a probability to an event as a ratio (number of times the event occurs to the total number of trials).
 - Discuss the likelihood of an event as a number (fractions, decimals, percents).
- Analyze results and explain possible discrepancies between observed (experimental) and theoretical outcomes.
- Use probability from a repeated chance process to predict the likelihood of a long-run event.

Content Elaborations

- [Ohio's K-8 Critical Areas of Focus, Grade 7, Number 5, page 49](#)
- [Ohio's K-8 Learning Progressions, Statistics and Probability, pages 22 – 23](#)
- [Glossary – see Probability Model and Uniform Probability Model](#)

CONNECTIONS ACROSS STANDARDS

- Extend the idea of a population to the context of probability (7.SP.1).
- Use proportional relationships to describe probability (7.RP.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM**Instructional Strategies**

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Instructional Tools/Resources

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Acknowledgments

Wendy Arnold

Teacher, Lake Local Schools, NW

Robert Batty

Curriculum Specialist/Coordinator,
Darke County ESC, SW

Dawn Bittner (WG)

Teacher,
Cincinnati Public Schools, SW

Annette Blue

Teacher, Hudson City Schools, NE

Adam Brown

Teacher, Mathews Local Schools, NE

Jeanne Cerniglia (AC)

Teacher,
Southeast Local Schools, NE

Hoyun Cho (WG)

Higher Education,
Capital University, C

Beth Drum

Teacher,
Caldwell Exempted Village, SE

Matthew Dunn

Teacher, Waverly City Schools, SE

Lisa Floyd-Jefferson

Curriculum Specialist/Coordinator,
Reynoldsburg City Schools, C

Brittany Hammonds

Teacher,
Greenfield Exempted Village, SW

Michael Hernandez

Teacher,
Manchester Local Schools, NE

Michele Heron

Higher Education,
Kent State University, NE

Steve Hiner

Curriculum Specialist/Coordinator,
Columbus City Schools, C

Michael Huler

Curriculum Specialist/Coordinator,
Westerville City Schools, C

Melissa Jacobs

Curriculum Specialist/Coordinator,
Toledo Public Schools, NW

Niki Jelil

Teacher, Hamilton City Schools, SW

Kamlesh Jindal

Retired Educator, SW

Carole Katz

Curriculum Specialist/Coordinator,
Beachwood City Schools, NE

Heather Kidd

Curriculum Specialist/Coordinator,
Northwest Local Schools, SW

Melissa Kincaid

Consultant,
Hamilton County ESC, SW

Kathy Koepp

Curriculum Specialist/Coordinator,
Elyria City Schools, NE

Rita Leskovec

Teacher, Cleveland Metropolitan
School District, NE

Nikki Littleton

Curriculum Specialist/Coordinator,
Warren City Schools, NE

Denise Lutz

Administrator, South-Western City
School District, C

Alicia Mailhot

Teacher,
Valley View Local Schools, SW

Cindy McKinstry (WG)

Teacher, East Palestine Schools, NE

Amanda Mickey

Teacher, Chillicothe City Schools, C

Cindy Miller (WG)

Curriculum Specialist/Coordinator,
Maysville Local Schools, SE

Wendi Moorman

Curriculum Specialist/Coordinator,
Mercer County ESC, NW

Jerry Moreno

Higher Education,
John Carroll University, NE

Acknowledgements, continued

Kristan Northcutt

Teacher, Dayton City Schools, SW

Melissa Novak

*Teacher,
Strongsville City Schools, NE*

Robin Phillips

*Teacher,
Oak Hill Union Local Schools, SE*

Rebecca Prelog

Teacher, Kent City Schools, NE

Christina Sherman

*Consultant,
Hamilton County ESC, SW*

Jenni Stadtmiller

*Curriculum Specialist/Coordinator,
Hamilton County ESC, SW*

Joyce Tatro Manes

Teacher, Adena Local Schools, C

Kelli Tebbe

*Curriculum Specialist/Coordinator,
Auglaize County ESC, NW*

Betsy Thomas

Teacher, Lebanon City Schools, SW

Stephanie Wagoner

*Curriculum Specialist/Coordinator,
Brown County ESC, SW*

Ruthie Ware

*Curriculum Specialist/Coordinator,
Muskingum Valley ESC, SE*

Jessica Wills

Teacher, Hamilton Local Schools, C

Heather Wukelich

*Curriculum Specialist/Coordinator,
Austintown Local Schools, NE*

*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.