Ohio's Model Curriculum | Mathematics

High School Conceptual Category: Algebra

Ohio | Department of Education

Mathematics Model Curriculum
High School Conceptual Category: Algebra

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018. *Note: The Instructional Supports section will be published by high school course only.*

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Standards for Mathematical Practice, continued

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Standards for Mathematical Practice, continued

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $^{(y-2)}/_{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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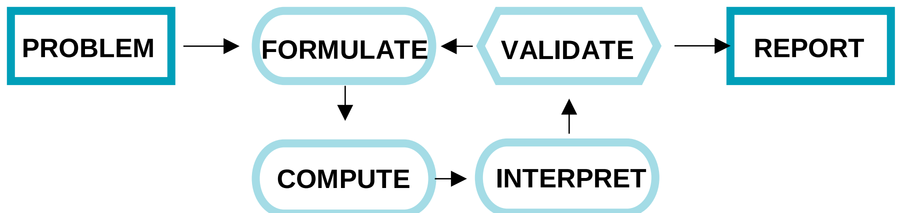
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.
Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum
High School Conceptual Category: Algebra

STANDARDS	MODEL CURRICULUM
SEEING STRUCTURE IN EXPRESSIONS **Interpret the structure of expressions.** **A.SSE.1**. Interpret expressions that represent a quantity in terms of its context. ★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	**Expectations for Learning-Algebra 1** Students build expressions in grades K-5 with arithmetic operations. As they move into the middle grades and progress through high school, students build expressions with algebraic components, beginning with linear, exponential, and quadratic expressions. In later courses, they build algebraic expressions with polynomial, rational, radical, and trigonometric expressions. In this cluster, they focus on interpreting the components of linear, exponential, and quadratic expressions and their meaning in mathematical and real-world contexts. Also, students determine when rewriting or manipulating expressions is helpful in order to reveal different insights into a mathematical or real-world context. **ESSENTIAL UNDERSTANDINGS** • An expression is a collection of terms separated by addition or subtraction. • A term is a product of a number and a variable raised to a nonnegative integer exponent. • Components of an expression or expressions within an equation may have meaning in a mathematical context, e.g., $y = mx + b$, b represents the y-intercept; $b^2 - 4ac$ in the quadratic formula indicates the number and nature of solutions to the equation. • Components of an expression may have meaning in a real-world context, e.g., in data surcharges, $60 + 0.05x$ the 60 represents the fixed costs and the 0.05 represents the cost per unit of data. • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. **MATHEMATICAL THINKING** • Attend to the meaning of quantities. • Use precise mathematical language. • Apply grade-level concepts, terms, and properties. • Look for and make use of structure.

Expectations for Learning-Algebra 1, continued

INSTRUCTIONAL FOCUS

- Identify the components, such as terms, factors, or coefficients, of an expression and interpret their meaning in terms of a mathematical or real-world context.
- Explain the meaning of each part of an expression, including linear, simple exponential, and quadratic expressions, in a mathematical or real-world context.
- Analyze an expression and recognize that it can be rewritten in different ways.

Expectations for Learning-Math 1

Students build expressions in grades K-5 with arithmetic operations. As they move into the middle grades and progress through high school, students build expressions with algebraic components, beginning with linear and exponential expression. Then in Math 2 quadratic expressions. In later courses, they build algebraic expressions with polynomial, rational, radical, and trigonometric expressions. In this cluster, they focus on interpreting the components of linear and exponential expressions and their meaning in mathematical and real-world contexts. They also determine when rewriting or manipulating expressions is helpful in order to reveal different insights into a mathematical or real-world context.

ESSENTIAL UNDERSTANDINGS

- An expression is a collection of terms separated by addition or subtraction.
- A term is a product of a number and a variable raised to a nonnegative integer exponent.
- Components of an expression or expressions within an equation may have meaning in a mathematical context, e.g., $y = mx + b$, b represents the y-intercept.
- Components of an expression may have meaning in a real-world context, e.g., in data surcharges, $60 + 0.05x$, the 60 represents the fixed costs and the 0.05 represents the cost per unit of data.
- Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context.

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	Expectations for Learning-Math 1, continued **MATHEMATICAL THINKING** • Attend to the meaning of quantities. • Use precise mathematical language. • Apply grade-level concepts, terms, and properties. • Look for and make use of structure. **INSTRUCTIONAL FOCUS** • Identify the components, such as terms, factors, or coefficients, of an expression and interpret their meaning in terms of a mathematical or real-world context. • Explain the meaning of each part of an expression, including linear and simple exponential expressions in a mathematical or real-world context. • Analyze an expression and recognize that it can be rewritten in different ways. *Continued on next page*

Expectations for Learning-Math 2

Students build expressions in grades K-5 with arithmetic operations. As they move into the middle grades and progress through high school, students build expressions with algebraic components, beginning with linear, exponential, and quadratic expressions. In later courses, they build algebraic expressions with polynomial, rational, radical, and trigonometric expressions. In this cluster, they focus on interpreting the components of linear, exponential, and quadratic expressions and their meaning in mathematical and real-world contexts. They also determine when rewriting or manipulating expressions is helpful in order to reveal different insights into a mathematical or real-world context.

ESSENTIAL UNDERSTANDINGS

- An expression is a collection of terms separated by addition or subtraction.
- A term is a product of a number and a variable raised to a nonnegative integer exponent.
- Components of an expression or expressions within an equation may have meaning in a mathematical context, e.g., $b^2 - 4ac$ in the quadratic formula indicates the number and nature of solutions to the equation.
- Components of an expression may have meaning in a real-world context.
- Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context.

MATHEMATICAL THINKING

- Attend to the meaning of quantities.
- Use precise mathematical language.
- Apply grade-level concepts, terms, and properties.
- Look for and make use of structure.

INSTRUCTIONAL FOCUS

- Identify the components, such as terms, factors, or coefficients, of an expression and interpret their meaning in terms of a mathematical or real-world context.
- Explain the meaning of each part of an expression, including linear, simple exponential, and quadratic expressions, in a mathematical or real-world context.
- Analyze an expression and recognize that it can be rewritten in different ways.

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Expectations for Learning-Algebra 2/Math 3

Students build expressions in grades K-5 with arithmetic operations. As they move into the middle grades and progress through high school, students build expressions with algebraic components, beginning with linear, exponential, and quadratic expressions. In later courses, students may build algebraic expressions with logarithmic expressions. In this cluster, they focus on interpreting the components of expressions with polynomial, rational, radical, and trigonometric expressions and their meaning in mathematical and real-world contexts. They also determine when rewriting or manipulating expressions is helpful in order to reveal different insights into a mathematical or real-world context.

ESSENTIAL UNDERSTANDINGS

- An expression is a collection of terms separated by addition or subtraction.
- A term is a product of a number and a variable raised to a nonnegative integer exponent.
- Components of an expression or expressions within an equation may have meaning in a mathematical context, e.g., $b^2 - 4ac$ in the quadratic formula indicates the number and nature of solutions to the equation.
- Components of an expression may have meaning in a real-world context, e.g., $5{,}000(1.07)^t$ can be used to model compound interest formula with a 7% interest rate and \$5,000 principal.
- Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context.

MATHEMATICAL THINKING

- Attend to the meaning of quantities.
- Use precise mathematical language.
- Apply grade-level concepts, terms, and properties.
- Look for and make use of structure.

INSTRUCTIONAL FOCUS

- Identify the components, such as terms, factors, or coefficients, of an expression and interpret their meaning in terms of a mathematical or real-world context.
- Explain the meaning of each part of an expression, including linear, exponential, quadratic, polynomial, rational, radical, and trigonometric expressions, in a mathematical or real-world context.
- Analyze an expression and recognize that it can be rewritten in different ways.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 1, Number 1, page 3
- Algebra 1, Number 4, pages 9-10
- Math 1, Number 1, page 3
- Math 2, Number 2, pages 4-5
- Algebra 2/Math 3, Number 2, pages 4-5

CONNECTIONS ACROSS STANDARDS

Algebra 1

- Write expressions in equivalent forms (A.SSE.3).
- Create equations in one or two variables (A.CED.1-2).
- Interpret expressions for functions in terms of the situations they model (F.LE.5).
- Interpret linear models (S.ID.7).

Math 1

- Create equations in one or two variables (A.CED.1-2).
- Interpret expressions for functions in terms of the situations they model (F.LE.5).
- Interpret linear models (S.ID.7).

Math 2

- Write expressions in equivalent forms (A.SSE.3).

Algebra 2/Math 3

- Write expressions in equivalent forms (A.SSE.4).
- Rewrite rational expressions in different forms (A.APR.6).
- Create equations in one or two variables (A.CED.1, A.CED.2).
- Combine standard function types using arithmetic operations (F.BF.1b).

STANDARDS	MODEL CURRICULUM
SEEING STRUCTURE IN EXPRESSIONS **Write expressions in equivalent forms to solve problems.** **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Factor a quadratic expression to reveal the zeros of the function it defines. (A1, M2) **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (A1, M2) **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.* (+) **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★	**Expectations for Learning-Algebra 1** Previously, students combine like terms and recognize equivalent expressions. In this cluster, students focus on the form of an expression that is most useful for a particular purpose. Students rewrite quadratic expressions by factoring and completing the square, and they use these forms to analyze the graphs of the functions they define. Students also rewrite exponential expressions using properties of exponents using integer exponents. In Algebra 2, students use these skills to analyze higher degree polynomial functions. **ESSENTIAL UNDERSTANDINGS** • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. • The factored form of a quadratic expression reveals the zeros of the function it defines. • The vertex form of a quadratic expression reveals the vertex and the maximum or minimum value of the function it defines. • Completing the square of a quadratic expression generates the vertex form of a quadratic expression. • Understanding the properties of exponents is essential for rewriting exponential expressions. **MATHEMATICAL THINKING** • Plan a solution pathway. • Determine the appropriate form of an expression in context. *Continued on next page*

Expectations for Learning, Algebra 1

INSTRUCTIONAL FOCUS

**For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients. Also exponential expressions should be limited to expressions with integer exponents.*

- Determine the appropriate equivalent form of an expression for a given purpose.
- Factor a quadratic expression so that the zeros of the function it defines can be identified.
- Complete the square for a quadratic expression to identify the vertex and maximum or minimum value of the function it defines.
- Rewrite exponential expressions by using properties of exponents.

Expectations for Learning-Math 1

In middle school, students explore the properties of exponents informally using patterns. In Math 1, students are expected to formally know the properties of exponents and rewrite exponential expressions with integer exponents using properties of exponents. In Math 3, students expand their skills and knowledge to situations involving rational exponents.

ESSENTIAL UNDERSTANDINGS

- Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context.
- Understanding the properties of exponents is essential for rewriting exponential expressions.

MATHEMATICAL THINKING

- Plan a solution pathway.
- Determine the appropriate form of an expression in context.

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	Expectations for Learning-Math 1, continued **INSTRUCTIONAL FOCUS** **Limit exponential expression to expression with integer exponents.* • Determine the appropriate equivalent form of an expression for a given purpose. • Rewrite exponential expressions by using properties of exponents. *Continued on next page*

	Expectations for Learning-Math 2 Previously, students rewrite exponential expressions using properties of exponents. In Math 2, students rewrite quadratic expressions by factoring and completing the square, and they use these forms to analyze the graphs of the functions they define. In Math 3, students use these skills to analyze higher degree polynomial functions. **ESSENTIAL UNDERSTANDINGS** • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. • The factored form of a quadratic expression reveals the zeros of the function it defines. • The vertex form of a quadratic expression reveals the vertex and the maximum or minimum value of the function it defines. • Completing the square of a quadratic expression generates the vertex form of a quadratic expression. • Understanding the properties of exponents is essential for rewriting exponential expressions. **MATHEMATICAL THINKING** • Plan a solution pathway. • Determine the appropriate form of an expression in context. **INSTRUCTIONAL FOCUS** **For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients. Also exponential expressions should be limited to expressions with integer exponents.* • Determine the appropriate equivalent form of an expression for a given purpose. • Factor a quadratic expression so that the zeros of the function it defines can be identified. • Complete the square for a quadratic expression to identify the vertex and maximum or minimum value of the function it defines. • Rewrite exponential expressions by using properties of exponents.

Expectations for Learning-Algebra 2/Math 3

In previous courses, students write arithmetic and geometric sequences both recursively and with a formula, connecting them with linear and exponential functions, respectively. In this cluster, (+) students derive the formula for the sum of finite geometric series.

Note: The standard A.SSE.4 is not required for all students, but is intended for students who are pursing advanced mathematics coursework.

ESSENTIAL UNDERSTANDINGS

- Understanding the properties of exponents is essential for rewriting exponential expressions.
- (+) Geometric series can model real-world phenomena, such as determining mortgage payments.

MATHEMATICAL THINKING

- Discern and use a pattern or a structure.
- Solve real-world problems accurately.
- Use different properties of operations flexibly.
- (+) Generalize concepts based on patterns.

INSTRUCTIONAL FOCUS

- Rewrite exponential expressions by using properties of exponents.
- (+) Derive the formula for the sum of a finite geometric series.
- (+) Use the formula for the sum of a finite geometric series to solve problems in real-world contexts.

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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 4, pages 9-10 • Math 1, Number 1, page 3 • Math 2, Number 2, pages 4-5 • Algebra 2/Math 3, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** **Algebra** • Interpret key features of graphs (F.IF.4). • Interpret the structure of expressions (A.SSE.1-2). • Analyze functions using different representations (F.IF.8). **Math 1** • Interpret key features of graphs (F.IF.4). • Interpret the structure of expressions (A.SSE.1). • Analyze functions using different representations (F.IF.8). **Math 2** • Interpret key features of graphs (F.IF.4). • Interpret the structure of expressions (A.SSE.1-2). • Analyze functions using different representations (F.IF.8). **Algebra 2/Math 3** • Rewrite expressions using rational exponent properties (N.RN.1-2). • Factors are used to find zeros of polynomials (A.APR.2-3). • Use properties of exponents to interpret expressions for exponential functions (F.IF.8b). • Graph exponential functions (F.IF.7f). • Exponential equations can be written as logarithmic equations (F.LE.4).

STANDARDS	MODEL CURRICULUM
ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS **Perform arithmetic operations on polynomials.** **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **a.** Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) **b.** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)	**Expectations for Learning-Algebra 1/Math 2** In previous courses, students develop an understanding of the properties of integers as a number system under the operations of addition, subtraction, and multiplication. They also learn to combine like terms and simplify linear expressions. In this cluster, students explore the commonalities and differences between integers and polynomials regarding the operations of addition, subtraction, and multiplication. Students will also simplify linear and quadratic expressions, or those that simplify to linear or quadratic. In Algebra 2/Math 3, students extend these ideas to include higher-degree polynomials. **ESSENTIAL UNDERSTANDINGS** • Polynomials form a system (like the integers) in which addition, subtraction, and multiplication always result in another polynomial, but sometimes division does not. **MATHEMATICAL THINKING** • Compute accurately and efficiently. • Use different properties of operations flexibly. • Recognize and apply mathematical concepts, terms, and their properties. • Draw a picture or create a model to represent mathematical thinking. **INSTRUCTIONAL FOCUS** • Add, subtract, and multiply polynomial expressions, focusing on those that simplify to linear or quadratic expressions. *Continued on next page*

Expectations for Learning-Algebra 2/Math 3

In previous courses, students simplify linear and quadratic expressions using the operations of addition, subtraction, and multiplication. In this cluster, extend these ideas to include polynomial expressions of higher degree.

ESSENTIAL UNDERSTANDINGS

- Polynomials form a system (like the integers) in which addition, subtraction, and multiplication always result in another polynomial, but sometimes division does not.

MATHEMATICAL THINKING

- Compute accurately and efficiently.
- Use different properties of operations flexibly.
- Recognize and apply mathematical concepts, terms, and their properties.

INSTRUCTIONAL FOCUS

- Add, subtract, and multiply polynomial expressions, focusing on those of degree greater than two.

Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 4, pages 9-10 • Math 2, Number 2, pages 4-5 • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 1** • Interpret the structure of expressions (A.SSE.1). **Math 2** • Interpret the structure of expressions (A.SSE.1). **Algebra 2/Math 3** • Use properties of exponents (N.RN.2). • Interpret the structure of expressions (A.SSE.1). • Use polynomial operations to examine the relationship between zeros and factors of polynomials (A.APR.2-5).

STANDARDS	MODEL CURRICULUM
ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS **Understand the relationship between zeros and factors of polynomials.** **A.APR.2** Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$. In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. **A.APR.3** Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Expectations for Learning-Algebra 2/Math 3** In previous courses, students learn to factor quadratic expressions and divide integers using long division. In this cluster, students apply these concepts to understand and use the Remainder Theorem for polynomials. Students will also identify zeros of polynomial functions and use the zeros, along with concepts of end behavior and *y*-intercepts from F.IF.4 and F.IF.7, to sketch graphs of the functions. **ESSENTIAL UNDERSTANDINGS** • The division of polynomials is analogous to the division of integers, yielding quotients and remainders. • If a polynomial $p(x)$ is divided by $(x - a)$, the remainder is the constant $p(a)$. • $p(a) = 0$, if and only if $(x - a)$ is a factor of $p(x)$. • The real solutions of the polynomial equation $p(x) = 0$ are the zeros of the function $p(x)$, and are the x-intercepts of the graph $y = p(x)$. • The zeros of a polynomial function can help to construct a rough sketch of its graph. • An nth degree polynomial function has at most n zeros. **MATHEMATICAL THINKING** • Solve multi-step mathematical problems accurately. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. *Continued on next page*

Expectations for Learning, Algebra 2/Math 3, continued

INSTRUCTIONAL FOCUS

- Divide polynomials using long division. Using synthetic division is discouraged until fluency in long division is achieved and the relationship between the two methods is understood.
- Rewrite a function as a sum of the remainder and the product of the quotient and divisor (If $p(x)$ is divided by $(x - a)$, then $p(x) = (x - a)q(x) + r$).
- Explain why when a polynomial $p(x)$ is divided by a linear polynomial $(x - a)$, the remainder, r, is equal to $p(a)$. Note the special case: When $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.
- Factor a polynomial expression to identify the zeros of the function it defines.
- Using the zeros, make a rough sketch of the graph of a polynomial function, with consideration of end behavior, the y-intercept, and perhaps other points.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 2/Math 3, Number 2, pages 4-6

CONNECTIONS ACROSS STANDARDS

Algebra 2/Math 3

- Interpret the structure of expressions (A.SSE.1b).
- Perform arithmetic operations on polynomials (A.APR.1).
- Analyze functions using different representations (F.IF.7).

STANDARDS	MODEL CURRICULUM
ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS **Use polynomial identities to solve problems.** **A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* (+) **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example by using coefficients determined for by Pascal's Triangle.* The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.	**Expectations for Learning-Algebra 2/Math 3** In previous courses, students examine writing an algebraic expression in different but equivalent forms. In this cluster, students prove polynomial identities based on numerical relationships. Students pursuing advanced mathematics courses (+) apply the Binomial Theorem and prove it by induction or a combinatorial argument. *Note: A.APR.5 is not required for all students, but is intended for students who are pursuing advanced mathematics.* **ESSENTIAL UNDERSTANDINGS** • When two expressions are equivalent, an equation relating the two is called an identity because it is true for all values of the variables. • To prove an algebraic identity means to show, using the properties of operations, that the equation is always true, for any values of the variables. • Recognize patterns in numerical relationships and be able to express these patterns as algebraic identities. • (+) Pascal's Triangle can be used to generate coefficients in the Binomial Theorem. **MATHEMATICAL THINKING** • Discern and use a pattern or structure. • Generalize concepts based on patterns. • Explain mathematical reasoning. • Use informal reasoning. **INSTRUCTIONAL FOCUS** • Produce algebraic proofs for various polynomial identities. • Represent numerical relationships using identities. • (+) Apply the Binomial Theorem. *Continued on next page*

	Content Elaborations
	OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/Math 3** • Interpret the structure of expressions (A.SSE.2). • Perform arithmetic operations on polynomials (A.APR.1). • Create equations that describe numbers or relationships (A.CED.1-2).

STANDARDS	MODEL CURRICULUM
ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS **Rewrite rational expressions.** **A.APR.6** Rewrite simple rational expressions[G] in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (+) **A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	**Expectations for Learning-Algebra 2/Math 3** In previous courses, students learn properties of and operations with rational numbers. They also learn to factor quadratic expressions. In A.APR.2, students use polynomial division to think about zeros of a polynomial function. In this cluster, students use polynomial division to rewrite rational expressions, which are quotients of polynomial expressions, in other forms. Students pursuing advanced mathematics courses (+) will add, subtract, multiply, and divide rational expressions. **ESSENTIAL UNDERSTANDINGS** • The result of polynomial division can be written as an expression showing both the quotient and the remainder, in the same way that the improper fraction can be written in the form of a mixed number, for example, 14 divided by 3, or $\frac{14}{3}$, results in the quotient 4 remainder 2 or $4 + \frac{2}{3}$. • The properties of operations on rational numbers hold for rational expressions. In other words, rational expressions are fractions, and the arithmetic is the same. • (+) Rational expressions are closed under addition, subtraction, multiplication and division, excluding division by the zero polynomial. **MATHEMATICAL THINKING** • Make sound decisions about using tools. • Use technology strategically to deepen understanding. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. **INSTRUCTIONAL FOCUS** • Rewrite simple rational expressions (denominators with linear polynomials and numerators with at most quadratic polynomials) as equivalent expressions using inspection (e.g., for monomial denominators) or long division. • (+) Add, subtract, multiply, and divide (with nonzero denominators) rational expressions. *Continued on next page*

	Content Elaborations
	OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/Math 3** • Graph rational functions (F.IF.7g). • Build new functions from existing functions (F.BF.3). • Know and apply the Remainder Theorem (A.APR.2).

STANDARDS	MODEL CURRICULUM
CREATING EQUATIONS **Create equations that describe numbers or relationships.** **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **c.** Extend to include more complicated function situations with the option to solve with technology. (A2, M3) **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **c.** Extend to include more complicated function situations with the option to graph with technology. (A2, M3) *Continued on next page*	**Expectations for Learning-Algebra 1** In middle school, students create simple equations and simple inequalities and use them to solve problems. In this cluster, students extend this knowledge to write equations and inequalities for more complicated situations, focusing on linear, simple exponential, and quadratic equations. Students also rearrange formulas to highlight a particular variable. In Algebra 2, students model even more complicated situations. *Note: Simple exponential functions include integer exponents only.* **ESSENTIAL UNDERSTANDINGS** • Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts. • The graphical solution of a system of equations or inequalities is the intersection of the equations or inequalities. • Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given. • A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations. **MATHEMATICAL THINKING** • Create a model to make sense of a problem. • Represent the concept symbolically. • Plan a solution pathway. • Determine the reasonableness of results. • Consider mathematical units and scale when graphing. *Continued on next page*

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1) **a.** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3) **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★ **a.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R*, or rearrange the formula for the area of a circle* $A = (\pi)r^2$ *to highlight radius* r*.* (A1) **b.** Focus on formulas in which the variable of interest is linear. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R*.* (M1) **c.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange the formula for the area of a circle* $A = (\pi)r^2$ *to highlight radius* r*.* (M2) *Continued on next page*	**Expectations for Learning-Algebra 1, continued** **INSTRUCTIONAL FOCUS** • Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship. Focus on situations where the equations will be linear, exponential, and quadratic. • For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales. Focus on situations where the equations will be linear, exponential, and quadratic. • Identify the constraints implied by the scenario, and represent them with equations or inequalities. • Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario. • Solve formulas for a given variable. **Expectations for Learning-Math 1** In middle school, students create simple equations and simple inequalities and use them to solve problems. In this cluster, students extend this knowledge to write equations and inequalities for more complicated situations, focusing on linear and simple exponential equations. Students also rearrange formulas to highlight a particular variable. In Math 2, students model situations that include quadratic equations. *Note: Simple exponential functions include integer exponents only.* **ESSENTIAL UNDERSTANDINGS** • Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts. • The graphical solution of a system of equations or inequalities is the intersection of the equations or inequalities. • Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given. • A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations. *Continued on next page*

d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)	**Expectations for Learning-Math 1, continued** **MATHEMATICAL THINKING** • Create a model to make sense of a problem. • Represent the concept symbolically. • Plan a solution pathway. • Determine the reasonableness of results. • Consider mathematical units and scale when graphing. **INSTRUCTIONAL FOCUS** • Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship. Focus on situations where the equations will be linear and exponential. • For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales. Focus on situations where the equations will be linear and exponential. • Identify the constraints implied by the scenario, and represent them with equations or inequalities. • Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario. • Solve formulas for a given variable. *Continued on next page*

	Expectations for Learning-Math 2 In Math 1, students create linear and exponential equations and use them to solve problems. In this cluster, students extend this knowledge to include writing quadratic equations. Students also continue to rearrange formulas to highlight a particular variable. In Math 3, students model even more complicated situations. **ESSENTIAL UNDERSTANDINGS** • Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts. • The graphical solution of a system of equations or inequalities is the intersection of the equations or inequalities. • Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given. • A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations. **MATHEMATICAL THINKING** • Create a model to make sense of a problem. • Represent the concept symbolically. • Plan a solution pathway. • Determine the reasonableness of results. • Consider mathematical units and scale when graphing. *Continued on next page*

Expectations for Learning-Math 2, continued

INSTRUCTIONAL FOCUS

- Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship. Focus on situations where the equations will be linear, exponential, and quadratic.
- For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales. Focus on situations where the equations will be linear, exponential, and quadratic.
- Identify the constraints implied by the scenario, and represent them with equations or inequalities.
- Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario.
- Solve formulas for a given variable.

Expectations for Learning-Algebra 2/Math 3

In previous courses, students create equations and inequalities, focusing on linear, exponential, and quadratic equations. They also rearrange formulas to highlight a particular variable. In this course, students extend these skills to more complicated situations.

ESSENTIAL UNDERSTANDINGS

- Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts.
- The graphical solution of a system of equations or inequalities is the intersection of the equations or inequalities.
- Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given.
- A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations.

Continued on next page

Expectations for Learning-Algebra 2/Math 3, continued

MATHEMATICAL THINKING

- Create a model to make sense of a problem.
- Represent the concept symbolically.
- Plan a solution pathway.
- Determine the reasonableness of results.
- Consider mathematical units and scale when graphing.

INSTRUCTIONAL FOCUS

- Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship.
- For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales.
- Identify the constraints implied by the scenario, and represent them with equations or inequalities.
- Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario.
- Solve formulas for a given variable.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 1, Number 1, pages 3-4
- Algebra 1, Number 4, pages 9-10
- Math 1, Number 1, page 3
- Math 2, Number 2, pages 4-5
- Algebra 2/Math 3, Number 4, pages 8-10

Continued on next page

Content Elaborations, continued

CONNECTIONS ACROSS STANDARDS

Algebra 1

- Interpret the structure of expressions (A.SSE.1).
- Solve equations and inequalities in one variable (A.REI.3).
- Interpret parameters of linear or exponential functions (F.LE.5).
- Represent and interpret equations and inequalities (including systems) with two variables graphically (A. REI.10).
- Build a function that models a relationship between two quantities (F.BF.1).
- Interpret the slope and intercept of a linear model (S.ID.7).
- Solve systems of equations (A.REI.6).
- Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.1).

Math 1

- Interpret the structure of expressions (A.SSE.1).
- Solve equations and inequalities in one variable (A.REI.3).
- Interpret parameters of linear or exponential functions (F.LE.5).
- Represent and interpret equations and systems of inequalities with two variables graphically (A. REI.10).
- Build a function that models a linear or exponential relationship between two quantities (F.BF.1).
- Interpret the slope and intercept of a linear model (S.ID.7).
- Solve systems of equations (A.REI.6).
- Construct and compare linear and exponential models, and solve problems (F.LE.1).

Math 2

- Interpret the structure of expressions (A.SSE.1).
- Build a function that models an exponential or quadratic relationship between two quantities (F.BF.1).

Continued on next page

	Content Elaborations, continued
	CONNECTIONS ACROSS STANDARDS, CONTINUED **Algebra 2/Math 3** • Model and interpret the relationship between two quantities (F.IF.4). • Rewrite expressions involving radicals and rational exponents (N.RN.2). • Use the structure of an expression to identify ways to rewrite it (A.SSE.2). • Write a function that describes a relationship between two quantities (F.BF.1b,c). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.4).

STANDARDS	MODEL CURRICULUM

REASONING WITH EQUATIONS AND INEQUALITIES
Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
	Expectations for Learning-Algebra 1/Math 1
	In previous courses, students solve simple equations using a variety of methods and investigate whether a linear equation (8.EE.7) or a system of linear equations (8.EE.8) has one solution, infinitely many solutions, or no solutions. In this cluster, students explain the process for finding a solution for any type of simple equation. Similar to proofs in Geometry/Math 2, students provide reasons for the steps they follow to solve an equation. In Algebra 2/Math 3, students solve simple rational and radical equations and explain why extraneous solutions may arise.
	
	ESSENTIAL UNDERSTANDINGS
	• Solving equations is a process of reasoning based on properties of operations and properties of equality.
	• Assuming no errors in the equation-solving process,
	   o A result that is false (e.g., 0 = 1) indicates the initial equation must have had no solutions; and
	   o A result that is always true (e.g., 0 = 0) indicates the initial equation must have been an identity.
	• Adding or subtracting the same value or expression to both sides of an equation results in an equivalent equation.
	• Multiplying or dividing both sides by the same value or expression (except by 0) results in an equivalent equation.
	• The Addition Property of Equality and Subtraction Property of Equality can be used interchangeably, since subtracting a number is the same as adding its opposite.
	• The Multiplication Property of Equality and the Division Property of Equality can be used interchangeably (except when multiplying by 0), since dividing a number is the same as multiplying the number by its inverse.
	
	MATHEMATICAL THINKING
	• Explain mathematical reasoning.
	• Plan a solution pathway.
	Continued on next page

Expectations for Learning-Algebra 1/Math 1, continued

INSTRUCTIONAL FOCUS

Note: Although, rote memorization of the names of the properties is not encouraged, it is expected for teachers to use formal language so that students gain familiarity and are able to recognize and apply the correct terminology.

- Justify the steps in solving an equation.
- Solve equations in which there is one solution; equations in which there is no solution; and equations in which there are infinitely many solutions.

Expectations for Learning-Algebra 2/Math 3

In Algebra 1/ Math 1, students solve simple equations and justify the steps in their solution process. In this cluster, students solve simple rational and radical equations using a variety a methods and explain why extraneous solutions may arise.

ESSENTIAL UNDERSTANDINGS

- To solve a rational equation, a helpful strategy is to multiply both sides of the equation by a common denominator of the rational expressions.
    - This strategy may introduce extraneous solutions; thus, potential solutions must be checked in the original equation.
- To solve an equation with square root expressions, a helpful strategy is to square both sides after a square root expression is isolated on one side of the equation if possible.
    - This strategy may introduce extraneous solutions; thus, potential solutions must be checked in the original equation.

MATHEMATICAL THINKING

- Solve math equations accurately.
- Determine reasonableness of results.
- Use technology strategically to deepen understanding.

Continued on next page

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Solve simple rational and radical equations in one variable. • Explain why and where in the solution process the extraneous solution arose. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 1, pages 3-4 • Math 1, Number 3, page 7 • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 1/Math 1** • Solve linear equations and inequalities in one variable (A.REI.3). • Create equations that describe numbers or a relationship (A.CED.1). **Algebra 2/Math 3** • Rewrite expressions involving radicals and rational exponents (N.RN.2). • Rewrite simple rational expressions (A.APR.6). • Interpret functions that arise in applications in terms of the context (F.IF.4). • Graph rational functions (F.IF.7g). • Build new functions from existing functions (F.BF.3).

STANDARDS	MODEL CURRICULUM
REASONING WITH EQUATIONS AND INEQUALITIES **Solve equations and inequalities in one variable.** **A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **A.REI.4** Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (+) **c.** Derive the quadratic formula using the method of completing the square.	**Expectations for Learning-Algebra 1** In previous courses, students solve linear equations and inequalities. In this cluster, students extend this knowledge to solve equations with numeric and letter coefficients. Students also solve quadratic equations (with real solutions) using a variety of methods. In other standards, students learn to factor quadratics; this cluster builds on that idea to solve quadratic equations with the Zero Product Property. In Algebra 2, students use these skills to solve more complicated equations. **ESSENTIAL UNDERSTANDINGS** • An appropriate solution path can be determined depending on whether the equation is linear or quadratic in the variable of interest. • Quadratic equations and expressions can be transformed into equivalent forms, leading to different solution strategies, including inspection, taking square roots, completing the square, applying the quadratic formula, or utilizing the Zero Product Property after factoring. • When the coefficients of the variable of interest are letters, the solving process is the same as when the coefficients are numbers. • The discriminant can show the nature and number of solutions a quadratic has. • (+) The quadratic formula is derived from the process of completing the square. **MATHEMATICAL THINKING** • Generalize concepts based on properties of equality. • Solve routine and straightforward problems accurately. • Plan a solution pathway. • Solve math problems using appropriate strategies. • Solve multi-step problems accurately. • (+) Use formal reasoning with symbolic representation. *Continued on next page*

Expectations for Learning-Algebra 1, continued

INSTRUCTIONAL FOCUS

**For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*

- Recognize when an equation or inequality is linear in one variable, and plan a solution strategy.
- Solve linear equations and inequalities with coefficients represented by letters.
  - For inequalities, graph solutions sets on a number line.
- Solve compound linear inequalities in one-variable.
  - Graph solution sets on a number line.
- Recognize when an equation is quadratic in one variable, and choose an appropriate solution strategy:
  - using inspection, e.g., $(x-3)^2 = 0$ or $x^2 = -5$;
  - taking square roots, e.g., $x^2 = 8$;
  - using Zero-Product Property after factoring;
  - completing the square; or
  - applying the quadratic formula.
- Determine if a quadratic functions has one solution, two solutions, or no real solutions based on the discriminant.
- (+) Formally derive the quadratic formula using completing the square.

Continued on next page

	Expectations for Learning-Math 1
	In previous courses, students solve linear equations and inequalities. In this cluster, students extend this knowledge to solve equations with numeric and letter coefficients. In Math 2, students solve quadratic equations (with real solutions) using a variety of methods. In Math 3, students use these skills to solve more complicated equations.
	
	ESSENTIAL UNDERSTANDINGS
	• An appropriate solution path can be determined when the equation is linear in the variable of interest.
	• When the coefficients of the variable of interest are letters, the solving process is the same as when the coefficients are numbers.
	
	MATHEMATICAL THINKING
	• Generalize concepts based on properties of equality.
	• Solve routine and straightforward problems accurately.
	• Plan a solution pathway.
	• Solve math problems using appropriate strategies.
	• Solve multi-step problems accurately.
	• (+) Use formal reasoning with symbolic representation.
	
	INSTRUCTIONAL FOCUS
	• Recognize when an equation or inequality is linear in one variable, and plan a solution strategy.
	• Solve linear equations and inequalities with coefficients represented by letters.
	    o For inequalities, graph solutions sets on a number line.
	• Solve compound linear inequalities in one-variable.
	    o Graph solution sets on a number line.
	Continued on next page

	## Expectations for Learning-Math 2 In previous courses, students solve linear equations and inequalities, and they solve equations with numeric and letter coefficients. In this cluster, students solve quadratic equations (with real solutions) using a variety of methods. In other standards, students learn to factor quadratics; this cluster builds on that idea to solve quadratic equations with the Zero Product Property. In Math 3, students use these skills to solve more complicated equations. **ESSENTIAL UNDERSTANDINGS** • An appropriate solution path can be determined depending on whether the equation is linear or quadratic in the variable of interest. • Quadratic equations and expressions can be transformed into equivalent forms, leading to different solution strategies, including inspection, taking square roots, completing the square, applying the quadratic formula, or utilizing the Zero Product Property after factoring. • When the coefficients of the variable of interest are letters, the solving process is the same as when the coefficients are numbers. • The discriminant can show the nature and number of solutions a quadratic has. • (+) The quadratic formula is derived from the process of completing the square. **MATHEMATICAL THINKING** • Generalize concepts based on properties of equality. • Solve routine and straightforward problems accurately. • Plan a solution pathway. • Solve math problems using appropriate strategies. • Solve multi-step problems accurately. • (+) Use formal reasoning with symbolic representation. *Continued on next page*

Expectations for Learning-Math 2, continued

INSTRUCTIONAL FOCUS

******For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*

- Recognize when an equation or inequality is linear or quadratic in one variable, and plan a solution strategy.
- Recognize when an equation is quadratic in one variable, and choose an appropriate solution strategy:
  - using inspection, e.g., $(x-3)^2 = 0$ or $x^2 = -5$;
  - taking square roots, e.g., $x^2 = 8$;
  - using Zero-Product Property after factoring;
  - completing the square; or
  - applying the quadratic formula.
- Determine if a quadratic functions has one solution, two solutions, or no real solutions based on the discriminant.
- (+) Formally derive the quadratic formula using completing the square.

Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 1, pages 3-4 • Algebra 1, Number 4, pages 9-10 • Math 1, Number 3, page 7 • Math 2, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** **Algebra 1** • Understand solving equations as a process of reasoning (A.REI.1). • Rearrange formulas to highlight a quantity of interest (A.CED.4). • Interpret the structure of expression (A.SSE.1). • Use the structure of an expression to identify ways to rewrite it (A.SSE.2). • Write expressions in equivalent forms to solve problems (A.SSE.3). • Create equations in one variable (A.CED.1). • Graph the solution of an inequality in two variables (A.REI.12). **Math 1** • Understand solving equations as a process of reasoning (A.REI.1). • Rearrange formulas to highlight a quantity of interest (A.CED.4). • Interpret the structure of expression (A.SSE.1). • Create equations in one variable (A.CED.1). • Graph the solution of an inequality in two variables (A.REI.12). **Math 2** • Rearrange formulas to highlight a quantity of interest (A.CED.4). • Use the structure of an expression to identify ways to rewrite it (A.SSE.2). • Write expressions in equivalent forms to solve problems (A.SSE.3).

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
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REASONING WITH EQUATIONS AND INEQUALITIES

Solve systems of equations.

A.REI.5 Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.6 Solve systems of linear equations algebraically and graphically.

a. Limit to pairs of linear equations in two variables. (A1, M1)

b. Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)

A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

(+) **A.REI.8** Represent a system of linear equations as a single matrix equation in a vector variable.

(+) **A.REI.9** Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

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Expectations for Learning-Algebra 1

In previous courses, students solve systems of linear equations graphically with an emphasis on the meaning of the solution. In this cluster, students solve systems of linear and quadratic equations in two variables graphically and algebraically, with a focus on the meaning of a solution to a system of equations. In Algebra 2, students solve systems of equations in three variables. Students who plan to take advanced mathematics courses (+) will represent systems of equations with matrices and use inverse matrices to solve the system.

ESSENTIAL UNDERSTANDINGS

- The graph of a linear equation is the set of ordered pairs that make the equation true. Therefore, multiplying that equation by a non-zero constant produces an equivalent equation, which has the same set of ordered pairs that make the equation true.
- If a system of equations in two variables has a unique solution, then the sum of one equation and a (non-zero) multiple of the other equation also has that same solution.
- The graphical solution to a system of equations in two variables is the intersection of the equations when graphed.
- The solution to a system of equations in two variables is the set of ordered pairs that satisfies both equations.
- A system of two linear equations can have no solutions, one solution, or infinitely many solutions.
- A system of a linear equation and a quadratic equation can have no solutions, one solution, or two solutions.

MATHEMATICAL THINKING

- Determine reasonableness of results using informal reasoning.
- Solve multi-step problems accurately.
- Plan a solution pathway.
- Use technology strategically to deepen the understanding.

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	Expectations for Learning-Algebra 1, continued **INSTRUCTIONAL FOCUS** *Note: For Algebra 1–A.REI.7, the example in the standards is not appropriate, as students do not know equations of circles. Instead, use systems with an equation of a line and an equation of a parabola. In Geometry and Algebra 2, systems with an equation of a circle can be included.* • Substitute a solution into the original system and a manipulation of the system to show solutions are the same. • Solve a system of linear equations in two variables algebraically using substitution, algebraically using elimination, and by graphing. • Solve a system of a linear equation and a quadratic equation in two variables algebraically using substitution and by graphing. • Discuss the efficiency and effectiveness of various methods of solving systems of equations. *Continued on next page*

Expectations for Learning-Math 1

In previous courses, students solve systems of linear equations graphically with an emphasis on the meaning of the solution. In this cluster, students solve systems of linear equations in two variables graphically and algebraically, with a focus on the meaning of a solution to a system of equations. In Math 2, students extend this knowledge to solve systems of linear and quadratic equations in two variables.

ESSENTIAL UNDERSTANDINGS

- The graph of a linear equation is the set of ordered pairs that make the equation true. Therefore, multiplying that equation by a non-zero constant produces an equivalent equation, which has the same set of ordered pairs that make the equation true.
- If a system of equations in two variables has a unique solution, then the sum of one equation and a (non-zero) multiple of the other equation also has that same solution.
- The graphical solution to a system of equations in two variables is the intersection of the equations when graphed.
- The solution to a system of equations in two variables is the set of ordered pairs that satisfies both equations.
- A system of two linear equations can have no solutions, one solution, or infinitely many solutions.

MATHEMATICAL THINKING

- Determine reasonableness of results using informal reasoning.
- Solve multi-step problems accurately.
- Plan a solution pathway.
- Use technology strategically to deepen the understanding.

INSTRUCTIONAL FOCUS

- Substitute a solution into the original system and a manipulation of the system to show solutions are the same.
- Solve a system of linear equations in two variables algebraically using substitution, algebraically using elimination, and by graphing.
- Discuss the efficiency and effectiveness of various methods of solving systems of equations.

Expectations for Learning-Math 2

In Math 1, students solve systems of linear equations in two variables graphically and algebraically, with a focus on the meaning of a solution to a system of equations. In Math 2, students extend this knowledge to solve systems of linear and quadratic equations in two variables. In Math 3, students solve systems of equations in three variables. Students who plan to take advanced mathematics courses (+) will represent systems of equations with matrices and use inverse matrices to solve the system.

ESSENTIAL UNDERSTANDINGS

- The graphical solution to a system of equations in two variables is the intersection of the equations when graphed.
- The solution to a system of equations in two variables is the set of ordered pairs that satisfies both equations.
- A system of a linear equation and a quadratic equation can have no solution, one solution, or two solutions.

MATHEMATICAL THINKING

- Determine reasonableness of results using informal reasoning.
- Solve multi-step problems accurately.
- Plan a solution pathway.
- Use technology strategically to deepen the understanding.

INSTRUCTIONAL FOCUS

Note: For Math 2, students should work with systems of equations in two variables that include an equation of a line and an equation of a parabola, as well as an equation of a line and an equation of a circle.

- Solve a system of a linear equation and a quadratic equation in two variables algebraically using substitution and by graphing.
- Discuss the efficiency and effectiveness of various methods of solving systems of equations.

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Expectations for Learning-Algebra 2/Math 3

In previous courses, students solve systems of equations in two variables algebraically and graphically. In this cluster, students solve systems of equations in three variables algebraically. For students planning to take advanced mathematics courses, topics for extension include matrices and vectors.

ESSENTIAL UNDERSTANDINGS

- A system of linear equations in three variables can have no solutions, one solution, or infinitely many solutions.
- The solution to a system of linear equations in three variables is the set of values that satisfies all three equations.

MATHEMATICAL THINKING

- Use informal reasoning with symbolic representation.
- Solve multi-step problems accurately.
- Plan a solution pathway.

INSTRUCTIONAL FOCUS

- Solve a system of linear equations in three variables algebraically.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 1, Number 2, pages 5-7
- Algebra 1, Number 4, pages 9-10
- Math 1, Number 3, page 7
- Math 2, Number 2, pages 4-5
- Algebra 2/Math 3, Number 2, pages 5-6

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	Content Elaborations, continued
	CONNECTIONS ACROSS STANDARDS **Algebra 1** • Solve linear and quadratic equations in one variable (A.REI.3-4). • Graph linear and quadratic models (F.IF.4, 7). • Rearrange formulas (A.CED.4). • Solve systems of equations and inequalities graphically (A. REI.10-12). **Math 1** • Solve linear equations in one variable (A.REI.3). • Graph linear models (F.IF.4, 7). • Rearrange formulas (A.CED.4). • Solve systems of equations and inequalities graphically (A. REI.10-12). **Math 2** • Solve linear and quadratic equations in one variable (A.REI.4). • Graph linear and quadratic models (F.IF.4, 7). • Rearrange formulas (A.CED.4). • Solve systems of equations and inequalities graphically (A. REI.11). **Algebra 2/Math 3** • Rearrange formulas (A.CED.4). **Plus Standards** • (+) Represent and model with vector quantities (N.VM.1-3). • (+) Perform operations on matrices and use matrices in applications (N.VM.6-10).

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<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
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REASONING WITH EQUATIONS AND INEQUALITIES

Represent and solve equations and inequalities graphically.

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

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Expectations for Learning-Algebra 1/Math 1

In prior courses, students graph linear relationships and identify slope and intercepts. In this cluster, students extend this knowledge to include the idea that a graph represents all of the solutions of an equation. Students use graphs and tables of equations in two variables to approximate solutions to equations in one variable. They also graph solutions to linear inequalities in two variables. In Algebra 2/Math 3, students similarly study the relationship between the graph and the solutions of rational, radical, absolute value, polynomial, and exponential equations.

Note: Math 1 students focus on linear and simple exponential equations and inequalities. Math 2 students will focus on quadratic equations.

ESSENTIAL UNDERSTANDINGS

- A point of intersection of any two graphs represents a solution of the two equations that define the two graphs.
- An equation in one variable can be rewritten as a system of two equations in two variables, by thinking of each side of the equation as a function, i.e., writing *y* = left hand side and *y* = right hand side.
  - o The approximate solution(s) to an equation in one variable is the *x*-value(s) of the intersection(s) of the graphs of the two functions.
  - o Two-variable graphical and numerical (tabular) techniques to solve an equation with one variable always work and are particularly useful when algebraic methods are not applicable, e.g., $3x + 4 = 2^x$(M1) or $x^2 - 3x + 2 = 2^x$(A1).
- A half plane represents the solutions of a linear inequality in two variables.
- The intersection of two half planes represents the solution set to two inequalities in two variables.

MATHEMATICAL THINKING

- Use technology strategically to deepen understanding.
- Plan a solution pathway.
- Create a model to make sense of a problem.

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Expectations for Learning-Algebra 1/Math 1, continued

INSTRUCTIONAL FOCUS

- Rewrite a one-variable equation as two separate functions and use the *x*-coordinate of their intersection point to determine the solution of the original equation.
- Approximate intersections of graphs of two equations using technology, tables of values, or successive approximations (focus on equations with linear, quadratic (A1 only), and exponential expressions).
- Graph the solution set of a linear inequality in two variables.
- Graph the solution set of a system of linear inequalities in two variables.

Expectations for Learning-Math 2

In previous courses, students use graphs to solve equations with linear and simple exponential expressions. In Math 2, they extend this to solving equations with quadratic expressions. Then in Algebra 2, students will similarly study the relationship between the graph and solutions of equations with rational, radical, absolute value, polynomial, exponential, and trigonometric expressions.

ESSENTIAL UNDERSTANDINGS

- A point of intersection of any two graphs represents a solution of the two equations that define the two graphs.
- An equation in one variable can be rewritten as a system of two equations in two variables, by thinking of each side of the equation as a function, i.e., writing *y* = left hand side and *y* = right hand side.
    - o The approximate solution(s) to an equation in one variable is the *x*-value(s) of the intersection(s) of the graphs of the two functions.
    - o Two-variable graphical and numerical (tabular) techniques to solve an equation with one variable always work and are particularly useful when algebraic methods are not applicable, e.g., $3x + 4 = 2^x$(M1) or $x^2 - 3x + 2 = 2^x$(A1).

MATHEMATICAL THINKING

- Use technology strategically to deepen understanding.
- Plan a solution pathway.
- Create a model to make sense of a problem.

Expectations for Learning, Math 2, continued

INSTRUCTIONAL FOCUS

- Rewrite a one-variable equation as two separate functions and use the *x*-coordinate of their intersection point to determine the solution of the original equation.
- Approximate intersections of graphs of two equations using technology, tables of values, or successive approximations (focus on equations with linear, quadratic, and exponential expressions).

Expectations for Learning-Algebra 2/Math 3

In previous courses, students use graphs to solve equations with linear, quadratic, and simple exponential expressions. In this cluster, students will similarly study the relationship between the graph and solutions of equations with rational, radical, absolute value, polynomial, exponential, and trigonometric expressions.

ESSENTIAL UNDERSTANDINGS

- An equation in one variable can be rewritten as a system of two equations in two variables, by thinking of each side of the equation as a function, i.e., writing *y* = left hand side and *y* = right hand side.
  - The approximate solution(s) to an equation in one variable is the *x*-value(s) of the intersection(s) of the graphs of the two functions.
  - Two-variable graphical and numerical (tabular) techniques to solve an equation with one variable always work and are particularly useful when algebraic methods are not applicable, e.g., $\sin x = x^2$.

MATHEMATICAL THINKING

- Use technology strategically to deepen understanding.
- Plan a solution pathway.
- Create a model to make sense of a problem.

Continued on next page

Expectations for Learning-Algebra 2/Math 3, continued

INSTRUCTIONAL FOCUS

- Rewrite a one-variable equation as two separate functions and use the *x*-coordinate of their intersection point to determine the solution of the original equation.
- Approximate intersections of graphs of two equations using technology, tables of values, or successive approximations.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

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- Math 1, Number 2, pages 4-6
- Math 2, Number 2, pages 4-5
- Algebra 2/Math 3, Number 2, pages 4-6

CONNECTIONS ACROSS STANDARDS

Algebra 1/Math 1

- Solve equations in one variable (A.REI.3-4).
- Create equations in two variables (A.CED.2).
- Graph functions expressed symbolically (F.IF.7).
- Solve systems of equations graphically (A.REI.6-7).
- Analyze functions using different representations (F.IF.9).

Math 2

- Solve quadratic equations in one variable (A.REI.4).
- Create equations in two variables (A.CED.2).
- Graph functions expressed symbolically (F.IF.7).
- Analyze functions using different representations (F.IF.9).

Algebra 2/Math 3

- Solve simple rational and radical equations (A.REI.2).
- Construct a rough graph of a polynomial function (A.APR.3).
- Graph functions expressed symbolically (F.IF.7).
- Evaluate exponential functions using technology (F.LE.4).

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