Ohio's Model Curriculum | Mathematics

High School Conceptual Category: Number and Quantity

Ohio | Department of Education

Mathematics Model Curriculum
High School Conceptual Category: Number and Quantity

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018. *Note: The Instructional Supports section will be published by high school course only.*

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Standards for Mathematical Practice, continued

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Standards for Mathematical Practice, continued

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $^{(y-2)}/_{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x − 1)(x + 1)$, $(x − 1)(x^2 + x + 1)$, and $(x − 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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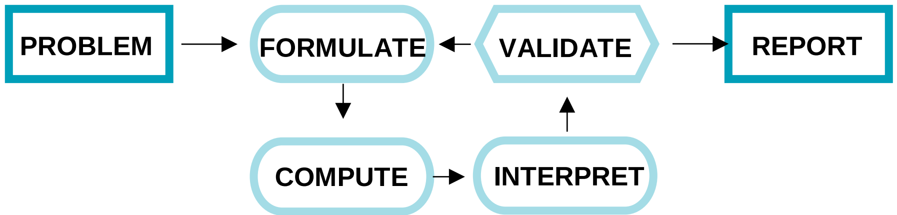
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum
High School Conceptual Category: Number and Quantity

STANDARDS	MODEL CURRICULUM
THE REAL NUMBER SYSTEM **Extend the properties of exponents to rational exponents.** **N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	**Expectations for Learning-Algebra 2/Math 3** In previous grades, students learn and apply the properties of integer exponents to generate equivalent numerical expressions. Now students will discover that those properties hold for rational exponents as well. In later courses, they may use this understanding to explore exponential functions with continuous domains. **ESSENTIAL UNDERSTANDINGS** • Radicals can be expressed in terms of rational exponents. • The properties of integer exponents hold for rational exponents. • The "properties of radicals" can be derived from the definitions that relate radicals to rational exponents and the properties of exponents. **MATHEMATICAL THINKING** • Use properties of exponents flexibly. • Discern and use a pattern or a structure. • Generalize concepts based on patterns. • Use formal reasoning with symbolic representation. **INSTRUCTIONAL FOCUS** • Rewrite expressions between radical form and rational exponent form. • Explain the definition of a radical using properties of rational exponents. • Use the properties of radicals to rewrite a radical in an equivalent form. • Add, subtract, multiply, and divide radicals. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/Math 3** • Solve simple rational and radical equations in one variable (A.REI.2). • Analyze functions using different representations (F.IF.7-8). • Use the properties of exponents to transform expressions for exponential functions (A.SSE.3). • Build a function that models a relationship between two quantities (F.BF.1, 4). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.4).

STANDARDS	MODEL CURRICULUM
THE REAL NUMBER SYSTEM **Use properties of rational and irrational numbers.** **N.RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**Expectations for Learning-Algebra 2/Math 3** In previous courses, students perform arithmetic operations with rational and irrational numbers. Now they generalize the results of operations on rational and irrational numbers. The emphasis is on developing mathematical reasoning habits that can be used in numerous future situations. **ESSENTIAL UNDERSTANDINGS** • The sum or product of two rational numbers is rational. • The sum of a rational number and an irrational number is irrational. • The product of a nonzero rational number and an irrational number is irrational. **MATHEMATICAL THINKING** • Use informal reasoning. • Make and analyze mathematical conjectures. • Explain mathematical reasoning. **INSTRUCTIONAL FOCUS** • Explain why the following statements are always true and provide examples to illustrate the following: o The sum or product of two rational numbers is rational. o The sum of a rational number and an irrational number is irrational. o The product of a nonzero rational number and an irrational number is irrational. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/Math 3** • Extend the properties of exponents to rational exponents (N.RN.1-2). • Perform arithmetic operations on polynomials (A.APR.1).

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
<tr><td>
QUANTITIES
Reason quantitatively and use units to solve problems.
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★
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Expectations for Learning-Algebra 1/Math 1
In elementary grades, students use units for distance, time, money, mass, etc. In grades 6, 7, and 8, students work with rates, especially speed, as a quotient of measurements. In this cluster, students extend the use of units to more complicated applications including rates, formulas, interpretation of scale and origin in graphs, data displays, and related applications. Next, students will apply modeling within the context of the algebra concepts studied and begin to develop strategies to solve more complicated mathematical problems.
ESSENTIAL UNDERSTANDINGS

Units are necessary when representing quantities in a modeling situation to make sense of the problem in context.
A particular quantity can be represented with units from multiple systems of measurement.
Quantities in different units of measure can be compared using equivalent units.
Derived quantities are calculated by multiplying or dividing known quantities, along with their units, e.g., 40 miles in 8 hours is 5 miles per hour.
Quantities can be converted within a system of units, e.g., feet to inches, and between two systems of units, e.g., feet to meters.
There are some contexts in which the origin of a graph or data display is essential to show, and there are other contexts in which the origin of a graph or data display where it is common to omit the origin, e.g., stock prices over time.

MATHEMATICAL THINKING

Determine reasonableness of results.
Attend to the meaning of quantities.
Consider mathematical units involved in a problem.

INSTRUCTIONAL FOCUS

When modeling, consider the scale when choosing or deriving suitable units.
Choose a level of accuracy appropriate for the given context.
Convert measurements within a system of units, e.g., convert 4.6 feet to inches and feet, or between a two systems of units, e.g., convert 4.6 feet to meters.

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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 1, pages 3-4 • Math 1, Number 1, page 3 **CONNECTIONS ACROSS STANDARDS** **Algebra 1/Math 1** • Create equations that describe numbers or relationships (A.CED.1-4).

STANDARDS	MODEL CURRICULUM
THE COMPLEX NUMBER SYSTEM **Perform arithmetic operations with complex numbers.** **N.CN.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (+) **N.CN.3** Find the conjugate of a complex number; use conjugates to find magnitudes and quotients of complex numbers.	**Expectations for Learning-Algebra 2/Math 3** In previous courses, students perform operations with polynomial and solve quadratic equations with real solutions. The expansion to the set of complex numbers allows students to find complex solution of the quadratic equations. Now in Algebra 2/Math 3 they extend this knowledge to recognize and rewrite complex numbers and expressions, using properties of polynomial operations. **ESSENTIAL UNDERSTANDINGS** • Complex numbers exist as a solution to a quadratic equation when taking the square root of a negative number. • Any number can be written as a complex number with a real and an imaginary component. • Complex numbers can be rewritten using polynomial operations (addition, subtraction, multiplication) and properties of exponents. • Associative, commutative, and distributive properties apply to rewriting complex numbers. **MATHEMATICAL THINKING** • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Generalize concepts based on patterns (repeated reasoning). • Use different properties of operations flexibly. **INSTRUCTIONAL FOCUS** • Write a complex number in the form *a* + *bi*, identifying the real and imaginary components. • Perform operations with complex numbers (addition, subtraction, multiplication). • Rewrite powers of imaginary numbers utilizing properties of exponents and the definition: $i^2 = -1$. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/Math 3** • Solve quadratic equations with real coefficients that have complex solutions (N.CN.7). • Rewrite expressions involving radicals and rational exponents using the properties of exponents (N.RN.2). • Perform arithmetic operations on polynomials (A.APR.1). • (+) Extend polynomial identities to the complex numbers (N.CN.8).

STANDARDS	MODEL CURRICULUM
THE COMPLEX NUMBER SYSTEM **Represent complex numbers and their operations on the complex plane.** (+) **N.CN.4** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. (+) **N.CN.5** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + i\sqrt{3})^3 = 8$ *because* $(-1 + i\sqrt{3})$ *has magnitude 2 and argument 120°.* (+) **N.CN.6** Calculate the distance between numbers in the complex plane as the magnitude of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	*This cluster goes beyond the graduation requirements for all students in Ohio with respect to curriculum and assessment. The inclusion and implementation of these standards and their inclusion in fourth year courses will vary be district and/or course.*

STANDARDS	MODEL CURRICULUM
THE COMPLEX NUMBER SYSTEM **Use complex numbers in polynomial identities and equations.** **N.CN.7** Solve quadratic equations with real coefficients that have complex solutions. (+) **N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* (+) **N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	**Expectations for Learning-Algebra 2/Math 3** Previously, students solved quadratic equations with real solutions. Now they will be extending this knowledge to solve quadratic equations with complex solutions. Advanced students will extend polynomial identities to the complex numbers and will know and apply the Fundamental Theorem of Algebra. *Note: N.CN.8 and N.CN.9 are not required for all students but are intended for students who are pursuing advanced mathematics.* **ESSENTIAL UNDERSTANDINGS** • Complex numbers exist as a solution to a quadratic equation when taking the square root of a negative number. • (+) Polynomial identities extend to complex numbers. • (+) The Fundamental Theorem of Algebra holds for quadratic polynomials. **MATHEMATICAL THINKING** • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • (+) Use properties of operations flexibly. **INSTRUCTIONAL FOCUS** • Solve a quadratic equation with a complex solution. • (+) Perform operations with complex numbers (addition, subtraction, multiplication). • (+) Extend polynomial identities to the complex numbers. • (+) Know and apply the Fundamental Theorem of Algebra. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 2, pages 4-6 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/Math 3** • Perform arithmetic operations with complex numbers (N.CN.1-2). • Perform arithmetic operations on polynomials (A.APR.1). • (+) Understand the relationship between zeros and factors of polynomials (A.APR.2-3). • (+) Using polynomial identities to solve problems (A.APR.4).

STANDARDS	MODEL CURRICULUM
VECTOR AND MATRIX QUANTITIES **Represent and model with vector quantities.** (+) **N.VM.1** Recognize vector[G] quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes, e.g., $\boldsymbol{v}$, $\|\boldsymbol{v}\|$, $\|\|\boldsymbol{v}\|\|$, $\vec{v}$. (+) **N.VM.2** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. (+) **N.VM.3** Solve problems involving velocity and other quantities that can be represented by vectors.	*This cluster goes beyond the graduation requirements for all students in Ohio with respect to curriculum and assessment. The inclusion and implementation of these standards and their inclusion in fourth year courses will vary be district and/or course.*

STANDARDS	MODEL CURRICULUM
VECTOR AND MATRIX QUANTITIES **Perform operations on vectors.** (+) **N.VM.4** Add and subtract vectors. **a.** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **b.** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **c.** Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. (+) **N.VM.5** Multiply a vector by a scalar. **a.** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c\ (v_x, v_y) = (cv_x, cv_y)$. **b.** Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|c\boldsymbol{v}\| = \|c\|\ \boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	*This cluster goes beyond the graduation requirements for all students in Ohio with respect to curriculum and assessment. The inclusion and implementation of these standards and their inclusion in fourth year courses will vary be district and/or course.*

STANDARDS	MODEL CURRICULUM
VECTOR AND MATRIX QUANTITIES **Perform operations on matrices, and use matrices in applications.** (+) **N.VM.6** Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (+) **N.VM.7** Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. (+) **N.VM.8** Add, subtract, and multiply matrices of appropriate dimensions. (+) **N.VM.9** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (+) **N.VM.10** Understand that the zero and identity matrices play a role in matrix addition and multiplication analogous to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (+) **N.VM.11** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. (+) **N.VM.12** Work with 2 × 2 matrices as transformations of the plane, and	*This cluster goes beyond the graduation requirements for all students in Ohio with respect to curriculum and assessment. The inclusion and implementation of these standards and their inclusion in fourth year courses will vary be district and/or course.*

interpret the absolute value of the determinant in terms of area.	

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*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.