Ohio's Model Curriculum | Mathematics

High School Conceptual Category: Statistics and Probability

Ohio | Department of Education

Mathematics Model Curriculum
High School Conceptual Category: Statistics and Probability

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018. *Note: The Instructional Supports section will be published by high school course only.*

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Standards for Mathematical Practice, continued

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Standards for Mathematical Practice, continued

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $^{(y-2)}/_{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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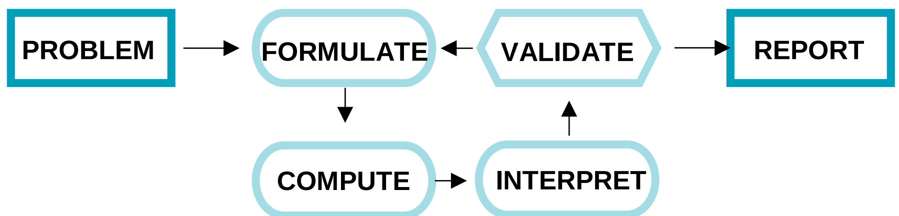
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum
High School Conceptual Category: Statistics and Probability

STANDARDS	MODEL CURRICULUM
INTERPRET CATEGORICAL AND QUANTITATIVE DATA **Summarize, represent, and interpret data on a single count or measurement variable.** **S.ID.1** Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★ **S.ID.2** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★ **S.ID.3** In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★ *Continued on next page*	**Expectations for Learning-Algebra 1/Math 1** In middle school students learn about the framework of the GAISE model of statistical problem solving. It consists of four steps: Formulating Questions; Collecting Data; Analyzing Data; and Interpreting Results. Students integrate this model whenever they use statistical reasoning. This process will continue throughout high school as students deepen their statistical reasoning skills. Middle school students create dot plots, histograms, and box plots and draw informal comparisons between two populations using graphs. They also summarize data sets using mean absolute deviation. In Algebra 1/Math 1 students should use and expand their learning to more sophisticated problems and by comparing single or multiple data sets through graphical representations. Standard deviation is a new concept for students, and it builds upon their previous understanding of mean absolute deviation (MAD). In Algebra 2/Math 3**,** students then extend their knowledge of mean and standard deviation from Algebra 1/Math 1 to normal distributions. **The GAISE Model** Students will use the GAISE Model framework for statistical problem solving in all courses. The GAISE Model should not be taught in isolation. Students are building on the framework that was developed in middle school. As students progress through the courses, the learning will move towards a greater level of precision and complexity. Students in middle school start at Level A and move towards Level B. As students progress from Level A to Levels B and C, the learning becomes less teacher-driven and more student-driven. In this cluster students are at Level B moving towards Level C, and Steps 1 and 2 continue to be emphasized with added depth on Steps 3 and 4. "Understanding the statistical concepts of GAISE model Level B enables a student to grow in appreciation that data analysis is an investigative process consisting of formulating their own questions; collecting appropriate data through various sources; analyzing data through graphs and simple summary measures; and interpreting results with an eye toward inference to a population based on a sample" (Guideline for Assessment and Instruction in Statistics Education (GAISE) Report, 2007, page 58).

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★	**Expectations for Learning-Algebra 1/Math 1, continued** **The GAISE Model, continued** **Step 1: Formulate the Question** • Students should pose their own statistical question of interest (Level C). • Students are starting to form questions that allow for generalizations of a population (Level B-C). **Step 2: Collect Data** • Students should begin to use random selection or random assignment (Level B). **Step 3: Analyze Data** • Students measure variability within a single group using MAD, IQR, and/or standard deviation (Level B). • Students compare measures of center and spread between groups using displays and values (Level B). • Students describe potential sources of error (Level B). • Students understand and use particular properties of distributions as tools of analysis moving toward using global characteristics of distributions (Level B-C). **Step 4: Interpret Results** • Students acknowledge that looking beyond the data is feasible by interpreting differences in shape, center, and spread (Level B). • Students determine if a sample is representative of a population and start to move towards generalization (Level B-C). • Students note the difference between two groups with different conditions (Level B). *Continued on next page*

Expectations for Learning-Algebra 1/Math 1, continued

ESSENTIAL UNDERSTANDINGS

- Univariate quantitative data can be represented using dot plots, box plots, and histograms.
- Mean and median are approximately equal for symmetric distributions, but tend to be different for nonsymmetric distributions.
- Standard deviation is a measure of variation from the mean (spread).
- Extreme values (outliers) have an effect on the shape, center, and spread of a distribution.
    - The median and interquartile range are appropriate measures of center and spread if the distribution is extremely skewed or has outliers.
    - The mean and standard deviation are appropriate measures of center and spread if the distribution is not skewed and has no extreme outliers.

MATHEMATICAL THINKING

- Use accurate and precise mathematical vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve real-world and statistical problems.
- Use appropriate tools to display and analyze data.

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Expectations for Learning-Algebra 1/Math 1, continued

INSTRUCTIONAL FOCUS

- Compare the mean to the median of the same data set and relate them to the shape of the distribution (symmetric, skewed).
- Develop the formula for and a conceptual understanding of standard deviation by building on the conceptual understanding and formula of mean absolute deviation.
- Compare two or more distributions based upon their means and standard deviations.
- Explain how outliers affect the mean, the median, and standard deviation.
- Given two or more data sets or graphs, do the following:
  - Compare the shape (symmetric, skewed, uniform).
  - Compare the spread (greater than, less than, equal).
  - Compare the centers (mean, median).
- Interpret the mean, standard deviation, outliers, as well as differences and similarities between two or more sets of data within a context.

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Expectations for Learning-Algebra 2/Math 3

In Algebra 1/Math 1, student build upon the GAISE Model that was introduced in middle school. They compare the center and spread of two different data sets using mean absolute deviation and standard deviation. In Algebra 2/Math 3**,** students extend their knowledge of mean and standard deviation to normal distributions.

The learning at this level is at the developmental Level C. See pages 24 and 26 for more information on Level C.

ESSENTIAL UNDERSTANDINGS

- Only some bell shaped curves are normal.
- If a bell-shaped curve is normal, then 68% of the distribution is within one standard deviation of the mean; 95% is within two standard deviations of the mean; and 99.7% is within three standard deviations of the mean. This is known as the Empirical Rule.

MATHEMATICAL THINKING

- Use accurate and precise mathematical vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve real-world and statistical problems.
- Use appropriate tools to display and analyze data.

INSTRUCTIONAL FOCUS

- By using the Empirical Rule, determine whether or not a distribution is normal (or approximately normal).
- Estimate area using the 68-95-99.7% rule (Empirical Rule).
- Estimate the area under a normal curve (population percentages) with tables and technology.

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Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 3, page 8
- Math 1, Number 4, page 8
- Algebra 2/Math 3, Number 1, page 3

THE GAISE MODEL

- GAISE Model, pages 14 – 15
    - Focus of the cluster for Algebra 1/Math 1 is Level B, pages 37-60
    - Focus of the standard for Algebra 2/Math 3 in the cluster is Level C, pages 61-88

CONNECTIONS ACROSS STANDARDS

Algebra 1/Math 1

- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.5-6).

Algebra 2/Math 3

- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6).
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.3-6).

STANDARDS	MODEL CURRICULUM
INTERPRET CATEGORICAL AND QUANTITATIVE DATA **Summarize, represent, and interpret data on two categorical and quantitative variables.** **S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★ **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3) **b.** Informally assess the fit of a function by discussing residuals. (A2, M3) **c.** Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)	**Expectations for Learning-Algebra 1/Math 1** For this cluster, the GAISE Model framework continues to be used: Formulating Questions; Collecting Data; Analyzing Data; and Interpreting Results. In the middle grades, students visually approximate a linear model and informally judge its goodness of fit. In Algebra 1/Math 1, students extend this knowledge to find the equation of a linear model, with and without technology. They will also use more precise language to describe the relationship between variables. In Algebra 2/Math 3, concepts extend to quadratic and exponential functions as well as working with residuals. The learning at this level is at the developmental Level B. See pages 10-11 for more information on Level B. **ESSENTIAL UNDERSTANDINGS** *Note: Students should be able to talk sensibly about the meanings of joint, marginal, and conditional frequencies within a context but should not be held responsible for precise usage of this vocabulary.* • Row totals and column totals constitute the marginal frequencies. • Individual table entries represent joint frequencies. • A relative frequency is found by dividing the frequency count by the total number of observations for a whole set or subset. o A marginal relative frequency is calculated by dividing the row (or column) total by the table total. o A joint relative frequency is calculated by dividing the table entry by the table total. o A conditional relative frequency is calculated by restricting to one row or one column of the table. • Relative frequencies are useful in considering association between two categorical variables. • A linear function can be used as a model for a linear association of two quantitative variables. *Continued on next page*

Expectations for Learning-Algebra 1/Math 1, continued

MATHEMATICAL THINKING

- Use accurate and precise mathematical vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve real-world and statistical problems.
- Use appropriate tools to display and analyze data.
- Accurately make computations using data.
- Determine reasonableness of predictions.

INSTRUCTIONAL FOCUS

Categorical Data

- Calculate and interpret, within a context, joint, marginal, and conditional relative frequencies.
- Recognize possible relationships (trends) in the context of the data by using percentages from two-way frequency tables.

Quantitative Data

- Describe, within a context, how variables are related in a linear relationship using scatter plots.
- Calculate and interpret, within a context, the slope and *y*-intercept of a linear model, given a set of data or graph, with or without technology.

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	## Expectations for Learning-Algebra 2/Math 3 For this cluster, the GAISE Model framework continues to be used: Formulating Questions; Collecting Data; Analyzing Data; and Interpreting Results. In Algebra 1/Math 1 students find the equation of a linear model, with and without technology using precise language to describe the relationship between variables. In Algebra 2/Math 3, concepts are now extended to quadratic and exponential functions. Up to this point students have only analyzed the fit of the model by looking at the closeness of the data points to a linear model. Now students are introduced to the idea of a residual, and they use them to informally assess the fit of the model. The learning at this level is at the developmental Level C. See pages 24 and 26 for more information on Level C. **ESSENTIAL UNDERSTANDINGS** • A linear, quadratic, or exponential function can be used as a model for association of two quantitative variables. • $\hat{y}$ is often used as the symbol for the predicted *y*-value for a given *x*-value. • A residual is the difference between the actual *y*-value and the *y*-value predicted by the chosen model ($y - \hat{y}$). **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data. • Accurately make computations using data. • Determine reasonableness of predictions. **INSTRUCTIONAL FOCUS** **Quantitative Data** • Reason about the context and the data to judge whether a linear, quadratic, or exponential model (or none of these) is appropriate. • Fit quadratic and exponential models to data using technology. • Use the chosen model to make contextual conclusions. • Discuss residual values to assess the appropriateness of a linear model.

	Content Elaborations
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	• Algebra 1, Number 3, page 8
	• Math 1, Number 4, page 8
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	THE GAISE MODEL
	• GAISE Model, pages 14 – 15
	  o Focus of this cluster for Algebra 1/Math 1 is Level B moving toward Level C, pages 37-60
	  o Focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88
	
	CONNECTIONS ACROSS STANDARDS
	Algebra 1/Math 1
	• Interpret linear models (S.ID.7-8).
	• Build a function that models a relationship between two quantities (F.BF.1).
	• Distinguish between situations that can be modeled with linear and exponential functions (F.LE.1).
	• Create equations that describe numbers or relationships (A.CED.2).
	
	Algebra 2/Math 3
	• Create equations that describe numbers or relationships (A.CED.2).
	• Build a function that models a relationship between two quantities (F.BF.1).
	• Interpret linear models (S.ID.9).

STANDARDS	MODEL CURRICULUM

INTERPRET CATEGORICAL AND QUANTITATIVE DATA
Interpret linear models.
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★
S.ID.9 Distinguish between correlation and causation.★
	Expectations for Learning-Algebra 1/Math 1
	In middle school, students interpret the slope and *y*-intercept of a linear model. In Algebra 1/Math 1, students build on this knowledge with more sophisticated problems. Since scales may vary, students require a deeper conceptual understanding of slope. They also need to recognize when the *y*-intercept is not always meaningful in the context of the data. This leads to the computation and interpretation of the correlation coefficient and its interpretation. In Algebra 2/Math 3, students are introduced to and explore the distinction between correlation and causation.
	
	The learning of standard S.ID.7 is at the developmental Level B. The learning of standard S.ID.8 is at developmental Level C. See pages 10-11 for more information on Level B, and see pages 24 and 26 for more information on Level C.
	
	ESSENTIAL UNDERSTANDINGS
	• In a linear model, the slope represents the change in the predicted value for every one unit of increase in the independent (*x*) variable.
	• When appropriate, the *y*-intercept represents the predicted value of the dependent variable when $x = 0$.
	• In a linear model, the *y*-intercept may not always be appropriate for the context.
	• The correlation coefficient (*r*) is a measure of the strength of a linear association in the data. Correlation coefficients are between –1 and 1, inclusive.
	    o If *r* is close to 0, then there is a weak correlation.
	    o If *r* is close to 1, then there is a strong correlation with a positive slope.
	    o If r is close to –1, then there is a strong correlation with a negative slope.
	Continued on next page

Expectations for Learning-Algebra 1/Math 1,continued

MATHEMATICAL THINKING

- Use accurate and precise mathematical vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve real-world and statistical problems.
- Use appropriate tools to display and analyze data.
- Determine reasonableness of predictions.

INSTRUCTIONAL FOCUS

- Given a linear model, interpret the slope and the y-intercept within a context.
- Compute, with technology, and interpret correlation coefficient (r).

Expectations for Learning-Algebra 2/Math 3

In Algebra 1/Math 1, students interpret the slope and intercept of a linear model. They also work with the correlation coefficient. In Algebra 2/Math 3, students are now introduced to and explore the distinction between correlation and causation.

The learning of standard S.ID.9 is at developmental Level C. See pages 24 and 26 for more information on Level C.

ESSENTIAL UNDERSTANDINGS

- There are three main methods of data production in statistics: surveys of samples to estimate population parameters; randomized experiments to compare treatments and to show cause; and observational studies to indicate possible associations among variables. Students should understand the distinctions among these three and decide if appropriate inferences have been drawn.
- Causation is a cause and effect relationship between two variables.
- Correlation (i.e., strong correlation) does not imply causation.
- Causation cannot be established after the research has been completed; it can only be established through well-designed experiments (not observational studies and not surveys).

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	Expectations for Learning-Algebra 2/Math 3, continued **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data. • Determine reasonableness of predictions. **INSTRUCTIONAL FOCUS** • Compare and contrast situations when changes in one variable cause changes in another. • Compare and contrast when the direction of causation is not clear. • Compare and contrast when changes in both variables are caused by something else. • Use surveys of samples to estimate population parameters. • Use randomized experiments to compare treatments and to show cause. • Use observational studies to indicate possible associations among variables. *Continued on next page*

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

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THE GAISE MODEL

- GAISE Model, pages 14 – 15
    - The focus of S.ID.7 is at Level B for Algebra 1/Math 1, pages 37-60
    - The focus of S.ID.8 is at Level C for Algebra 1/Math 1, pages 61-85
    - The focus of S.ID.9 is at Level C for Algebra 2/Math 3, pages 61-85

CONNECTIONS ACROSS STANDARDS

Algebra 1/Math 1

- Interpret the structure of functions (F.IF.4).
- Construct linear models, and solve problems (F.LE.1-2).
- Interpret expressions for functions in terms of context (F.LE.5).
- Build a function that models a relationship between two quantities (F.BF.1).
- Summarize, represent, and interpret data in two categories and quantitative variables (S.ID.6).

Algebra 2/Math 3

- Summarize, represent, and interpret data in two categories and quantitative variables (S.ID.6).
- Interpret the structure of functions (F.IF.4).
- Build a function that models a relationship between two quantities (F.BF.1).

STANDARDS	MODEL CURRICULUM
MAKING INFERENCES AND JUSTIFYING CONCLUSIONS **Understand and evaluate random processes underlying statistical experiments.** **S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★ **S.IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★	**Expectations for Learning-Algebra 2/Math 3** This is students' first exposure to the necessity of randomness when sampling to make an inference about a population which is consistent with the GAISE Model (Step 4, Level C). In middle school, students are exposed to simulations (observed) and compare them to predicted (expected) outcomes based on probability. In this cluster, students decide if results are consistent with a given model by using simulations or computing probabilities. Understanding the statistical concepts of GAISE model Level C enables a student to build on the foundations developed in Levels A and B. Although the previous levels are revisited, students are now expected to take prior learning to a deeper statistical nature. They are expected to draw on basic concepts from earlier work; extend the concept to cover a wider scope of investigatory issues; and develop a deeper understanding of inferential reasoning and its connection to probability. Students should be able to provide a more sophisticated interpretation that integrates the context and objectives of a study, and they should also be able to see limitations based on data. **ESSENTIAL UNDERSTANDINGS** • Random sampling guarantees that the sample chosen is representative of the population which ensures that the statistical conclusions will be valid. • A random sample must be generated through a chance selection process. • A statistic is generated from sample data to estimate the corresponding parameter for the entire population. • A population parameter is a measure of some characteristic in the population such as the population proportion or the population mean. • Experimental results are not a perfect match for theoretical models by the nature of variability. **MATHEMATICAL THINKING** • Make connections between samples and their populations. • Determine whether results are reasonable. • Use precise mathematical language. *Continued on next page*

	Expectations for Learning-Algebra 2/Math 3 continued **INSTRUCTIONAL FOCUS** • Explain and interpret, within a context, the results of a data-generating process. • Explain, within a context, what a point on a dot plot from a simulation represents. • Decide if an outcome is unusual in a specified distribution. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 2/Math 3, Number 1, page 3 **THE GAISE MODEL** • GAISE Model, pages 14 – 15 o The focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88 **CONNECTIONS ACROSS STANDARDS** **Algebra 2/Math 3** • Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.4, 6). • (+) Develop probability models and expected values (S.MD.3-6).

STANDARDS	MODEL CURRICULUM
MAKING INFERENCES AND JUSTIFYING CONCLUSIONS **Make inferences and justify conclusions from sample surveys, experiments, and observational studies.** **S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★ **S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★ **S.IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.★ **S.IC.6** Evaluate reports based on data.★	**Expectations for Learning-Algebra 2/Math 3** Previously, students have been informally introduced to data collection methods and bias. In this cluster, the concept of randomization is introduced in data collection methods. Students are also introduced to the concept of margin of error, and they begin to formalize the concept of statistical significance. **The GAISE Model** The GAISE Model is a framework for all statistical problem solving and should not be taught in isolation. For this cluster, the focus is on Steps 2, 3, and 4 at Level C. Students are building on the framework developed in earlier grades. Algebra 2/Math 3 students use more in-depth reasoning and a greater level of precision and complexity. **Step 1: Formulate the Question** • Students should be fluent in posing their own statistical question of interest. • Students should form questions to allow generalizations be made about a population **Step 2: Collect Data** • Students should purposefully design for differences through random selection or random assignment. • Students design samples through selection. • Students design experiments through randomization. **Step 3: Analyze Data** • Students understand and use global characteristics of distributions in analysis. • Students compare group to group using displays and measures of variability. • Students describe and quantify sampling error. **Step 4: Interpret Variability** • Students are able to look beyond the data in some contexts. • Students are able to generalize from a sample to population. • Students are aware of the effects of randomization on the results of experiments. • Students understand and distinguish between observational studies and experiments. *Continued on next page*

Expectations for Learning, Algebra 2/Math 3, continued

ESSENTIAL UNDERSTANDINGS

- Surveys, observational studies, and experiments are different methods for data collection and each have their own advantages.
- Surveys and observational studies involve a researcher collecting information about a sample without imposing a treatment on subjects.
- A researcher should utilize a chance process to assign treatment groups in experiments.
- Causality can be established with well-designed experiments; surveys and observational studies cannot determine causality.
- The decision process based on sample data does not guarantee a correct answer to the underlying statistical question.
- The characteristics of distributions of sample statistics are simulation models for random sampling; it is predictable only if the sampling is random.
- The interval (observed statistic $\pm$ margin of error) should include the plausible values for the true population parameter.
- The results from an experiment are statistically significant if the differences between treatment groups are unlikely to have occurred by chance alone.

MATHEMATICAL THINKING

- Make sense of the structure of distributions.
- Use precise vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve statistical problems in real-world context.
- Determine whether results are reasonable.

Continued on next page

Expectations for Learning, Algebra 2/Math 3, continued

INSTRUCTIONAL FOCUS

- Identify and choose when to use surveys, experiments, and observational studies.
- Explain the role of randomization as it relates to bias in surveys, experiments, and observational studies.
- Use a sample mean or sample proportion to estimate a population mean or population proportion.
- Explain how sampling variability relates to margin of error in the context of estimating population mean or proportion.
- Use simulation to estimate margin of error in the context of estimating population mean or population proportion.
- Develop an understanding of the definition of *statistical significance* as compared to everyday language *significance*; determine if the results of an experiment are statistically significant; draw conclusions in context based on a report.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 2/Math 3, Number 1, page 3

THE GAISE MODEL

- GAISE Model, pages 14 – 15
    - The focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88

CONNECTIONS ACROSS STANDARDS

Algebra 2/Math 3

- Understand and evaluate random processes (S.IC.1-2).

STANDARDS	MODEL CURRICULUM
CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY **Understand independence and conditional probability, and use them to interpret data.** **S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★ **S.CP.2** Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★ **S.CP.3** Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ *Continued on next page*	**Expectations for Learning-Geometry/Math 2** In middle school, students develop basic probability skills including probability as relative frequencies; probabilities of compound events; the development a uniform/non-uniform probability model; and the use of tree diagrams. Also students are introduced to two-way frequency tables in middle school. However, students' only prior exposure to the concept of independence was in S.ID.5 (Algebra 1/Math 1). This cluster focuses on the concept of independence between two categorical variables. It also focuses on the understanding of independence rather than symbolic notation and formulas. Fluency with independence is expected by the end of Geometry/Math 2. **ESSENTIAL UNDERSTANDINGS** • Approximations for the true probability of an event can be found by looking at the long-run relative frequency. • The sample space of a probability experiment can be modeled with a Venn diagram. • The union of an event and its complement represent the entire sample space. • The intersection of an event and its complement represent the empty set. • Conditional probability is the probability of event A occurring given that event B has occurred. It is denoted by $A\|B$ and is read "A given B." • Two events occurring in succession are said to be independent if the outcome of one event has no effect on the outcome of the other, e.g., a coin tossed twice. Otherwise, the events are dependent, e.g., two cards are drawn in succession from a standard deck of cards. • The intersection of two sets A and B is the set of elements that are common to both set A and set B. It is denoted by $A \cap B$ and is read "A intersection B" as well as "A and B." • The union of two sets A and B is the set of elements, which are in A or in B or in both. It is denoted by $A \cup B$ and is read "A union B" as well as "A or B." • If A and B are events that have no outcomes in common $(A \cap B \neq 0)$, they are said to be mutually exclusive. Mutually exclusive events cannot occur together. **MATHEMATICAL THINKING** • Use appropriate vocabulary. • Attend to precision.

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★*
S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.★*
	Expectations for Learning-Geometry/Math 2, continued
	INSTRUCTIONAL FOCUS
	• Recognize and explain for two successive events, whether the outcome of the first event affects the outcome of the second event.
	• Recognize and justify conceptually whether two events are independent.
	• Make connections between conditional probability and independence. Recognize sample space subsets in everyday contexts.
	• Identify an event and its complement.
	• Identify which components of the sample space represent the union and intersection of two events.
	• Explain what a conditional probability means within a context.
	• Distinguish between a conditional probability (A given B) and the probability of an intersection (A and B).
	• Use a two way frequency table to determine the following:
	  o conditional probabilities;
	  o probabilities of the sample space subsets;
	  o event independence by comparing joint probabilities (P(A and B)) and the product of the separate probabilities (P(A) × P(B)); and
	  o event independence by comparing the conditional probability (P(A given B)) and the probability P(A).
	
	Content Elaborations
	OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS
	• Geometry, Number 1, page 3
	• Math 2, Number 1, page 3
	
	CONNECTIONS ACROSS STANDARDS
	Geometry/Math 2
	• This will lead into the cluster (S.CP.6-9) which includes the calculations of conditional probabilities, and the use of probability formulas and set notation with probability.

STANDARDS	MODEL CURRICULUM
CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY **Use the rules of probability to compute probabilities of compound events in a uniform probability model.** **S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ **S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★ (+) **S.CP.8** Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★ (+) **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.★	**Expectations for Learning-Geometry/Math 2** In middle school, students develop basic probability skills including probability as relative frequencies; probabilities of compound events; development of a uniform/non-uniform probability model; and the use of tree diagrams. Also they are introduced to two-way frequency tables in middle school. Now in Geometry/Math 2 this cluster formalizes the concepts of conditional probability and independence in S.CP.1-5. The focus of this cluster is developing the Addition Rule and the (+) Multiplication Rule in everyday contexts. Although permutations and combinations are part of Geometry/Math 2 for students who pursue advanced mathematics, these concepts would also be appropriate in a fourth year course. Exploration of the Fundamental Counting Principle and factorials (!) may also be addressed in a fourth year course. **ESSENTIAL UNDERSTANDINGS** • Compound probabilities model real-world scenarios and must be interpreted within a context. • The conditional probability of A *given* B is the fraction of B's outcomes that also belong to A. This can be expressed by $P(A \vert B) = P(A \cap B)/P(B)$. • The addition rule is $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ and can also be expressed as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. • (+) The Multiplication Rule is $P(A\ and\ B) = P(A) * P(B \vert A) = P(B) * P(A \vert B)$. • (+) Permutations and combinations are strategies for counting the outcomes of a sample space. **MATHEMATICAL THINKING** • Use precise mathematical language. • Look for and make use of structure. • Compute accurately and efficiently. *Continued on next page*

Expectations for Learning-Geometry/Math 2, continued

INSTRUCTIONAL FOCUS

- Recognize and justify mathematically whether two events are independent.
- Generalize probability rules using patterns.
- Compute probabilities accurately and efficiently.
- (+) Recognize and apply counting methods to compute probabilities.
- Compute the conditional probability of A given B.
- Interpret and explain conditional probability within a context.
- Apply the Addition Rule.
- Interpret and explain the Addition Rule within a context.
- (+) Apply the Multiplication Rule.
- (+) Interpret and explain the Multiplication Rule within a context.
- (+) Know and explain the difference between a permutation and a combination.
- (+) Calculate probabilities using permutations and combinations.
- (+) Interpret and explain probabilities using permutations and combinations within a context.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Geometry, Number 1, page 3
- Math 2, Number 1, page 3

CONNECTIONS ACROSS STANDARDS

Geometry

- Understand independence and conditional probability (S.CP.1-5).

Math 2

- Understand independence and conditional probability (S.CP.1-5).
- Interpret and use the structure of expressions (A.SSE.1-3).

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<tr><th>STANDARDS</th><th>MODEL CURRICULUM</th></tr>
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USING PROBABILITY TO MAKE DECISIONS

Calculate expected values, and use them to solve problems.

(+) **S.MD.1** Define a random variable[G] for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution[G] using the same graphical displays as for data distributions.★

(+) **S.MD.2** Calculate the expected value[G] of a random variable; interpret it as the mean of the probability distribution.★

(+) **S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★

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Expectations for Learning

In middle school, students begin exploring the differences between expected values and observed values and create data displays based upon counts. Students now are asked to develop and define random variables; to graph the associated probability distribution; and to compute expected values in anticipation of using the expected value to make decisions.

ESSENTIAL UNDERSTANDINGS

- Random variables are numeric representations of outcomes resulting from a chance process.
- The mean of a probability distribution is the expected value of the random variable.
- A probability distribution is the list of all possible outcomes and their respective probabilities.

MATHEMATICAL THINKING

- Attend to the meaning of quantities in the context.
- Create a model to make sense of a problem.
- Make and modify a model to represent mathematical thinking.

INSTRUCTIONAL FOCUS

- Recognize random variables in everyday settings.
- Distinguish between algebraic variables and random variables.
- Assign the values of the random variable.
- Compute the probabilities for the values of the random variable.
- Compute expected values.
- Compute, explain, and interpret the expected value of a random variable within a context.
- Create and interpret graphical and tabular displays of probability distributions.

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(+) **S.MD.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* ★	**Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** *These standards are not assigned to any particular course.* **CONNECTIONS ACROSS STANDARDS** • (+) Know and apply Binomial Theorem (A.APR.5). • (+) Use probability to evaluate outcomes of decisions (S.MD.5-7). • Understand and evaluate random processes using simulation (S.IC.2).

STANDARDS	MODEL CURRICULUM
USING PROBABILITY TO MAKE DECISIONS **Use probability to evaluate outcomes of decisions.** (+) **S.MD.5** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★ **a.** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* (+) **S.MD.6** Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★ (+) **S.MD.7** Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★	**Expectations for Learning** Students are using probability, random variables, and expected values to make decisions and judgments regarding the fairness of games and strategies for playing games. **ESSENTIAL UNDERSTANDINGS** • A game is "fair" when the expected net winnings are zero. • Expected values are the mean of a large number of trials. **MATHEMATICAL THINKING** • Determine reasonableness of decisions. **INSTRUCTIONAL FOCUS** • Evaluate the decisions of others based on probability models. • Solve real-world problems using probability models and expected values. Use probabilities to make decisions. • Assign payoff values for games of chance and interpret them in the context of the problem. • Decide if a game is "fair". • Use random number tables or random number generators to make selections. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** *These standards are not assigned to any particular course.* **CONNECTIONS ACROSS STANDARDS** • (+) Calculate expected values, and use them to make decisions (S.MD.1-4). • Use rules of probability to compute probabilities (S.CP.6-9).

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*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.