Ohio's Model Curriculum | Mathematics

with Instructional Supports

Statistics and Probability

Ohio | Department of Education

Mathematics Model Curriculum

with Instructional Supports

Statistics and Probability

TABLE OF CONTENTS

Introduction

STATISTICS AND PROBABILITY MODEL CURRICULUM DRAFT

This is a **draft** document that pulls from the 2018 Algebra 2/Math 3 Course Model Curriculum and the 2018 Statistics and Probability Conceptual Category Model Curriculum documents. Due to that fact, some pieces are missing, and some course titles have been changed. Anything that is not relevant to this new course and should be deleted is shown by a ~~cross out~~ and notes that have been added to the adopted portion of the model curriculum are highlighted in red font. During the next standards revision, this document will go through the review process that includes public comment. Until that time the Department will continue to receive feedback on the document. Although the Model Curriculum is adopted by the State Board of Education and therefore cannot be change. The Instructional Supports are not, so additions and revisions can be made to those sections. Therefore, Ohio's Statistics and Probability Workgroup has added some essential information to the Instructional Supports for clarity to help guide teachers to teach this course. These sections still need activities, examples and instructional tools and resources. These pieces will be added during the next revision to the model curriculum.

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course

INTRODUCTION, CONTINUED

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Standards for Mathematical Practice, continued

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Standards for Mathematical Practice, continued

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation ${}^{(y - 2)}/_{(x - 1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Standards for Mathematical Practice, continued

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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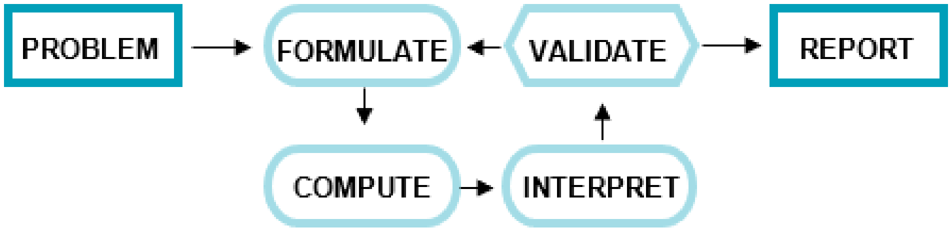
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum

with Instructional Supports

Statistics and Probability

STANDARDS	MODEL CURRICULUM (S.ID.1-4)
Statistics and Probability **INTERPRET CATEGORICAL AND QUANTITATIVE DATA** **Summarize, represent, and interpret data on a single count or measurement variable.** **S.ID.1** Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★ **S.ID.2** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★ **S.ID.3** In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★ **S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★	**Expectations for Learning** *Note: This model curriculum was only written for S.ID.4. There was no model curriculum written for S.ID.1-3 at this level.* In Algebra 1/Math 1, student build upon the GAISE Model that was introduced in middle school. They compare the center and spread of two different data sets using mean absolute deviation and standard deviation. In ~~Algebra 2/Math 3~~ Statistics and Probability**,** students extend their knowledge of mean and standard deviation to normal distributions. The learning at this level is at the developmental Level C. See pages ~~99~~ 27 and ~~102~~ 31 for more information on Level C. **ESSENTIAL UNDERSTANDINGS** • Only some bell-shaped curves are normal. • If a bell-shaped curve is normal, then 68% of the distribution is within one standard deviation of the mean; 95% is within two standard deviations of the mean; and 99.7% is within three standard deviations of the mean. This is known as the Empirical Rule. **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data.

STANDARDS	MODEL CURRICULUM (S.ID.1-4)
S.ID.1-4, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** • By using the Empirical Rule, determine whether or not a distribution is normal (or approximately normal). • Estimate area using the 68-95-99.7% rule (Empirical Rule). • Estimate the area under a normal curve (population percentages) with tables and technology. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 1, page 3~~ **THE GAISE MODEL** • GAISE Model, pages 14 – 15 ○ Focus of the standard for Algebra 2/Math 3 in the cluster is Level C, pages 61-88 **CONNECTIONS ACROSS STANDARDS** • Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6). • Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.3-6).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.ID.1-4)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- Normal curves are not addressed in Algebra 1/Math 1.
- Although the S.ID.1-3 standards appear in Algebra 1/Math 1, the level of depth and rigor will need to be increased to GAISE Level C for this course.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

In Algebra 1/Math 1, students apply concepts of statistical problem solving by using the GAISE model in the context of real-world applications. Students develop formal methods of assessing how a model fits data. In this course, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

The parenthetical lists in S.ID.2 for measures of center and spread are not exhaustive. For example, variance is typically taught along with the standard deviation.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. At Level C, students should be able to select data analysis techniques appropriate for the type of data available, produce descriptive statistical analyses, and describe in context the important characteristics of the data.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- These standards are typically included in TMM-010 Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- These standards are included in an AP Statistics course.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.ID.1-4)

Instructional Tools/Resources

This section will be published at a later time.

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (S.ID.5-6)</th></tr>
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Statistics and Probability

INTERPRET CATEGORICAL AND QUANTITATIVE DATA

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3)

b. Informally assess the fit of a function by discussing residuals. (A2, M3)

c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

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Expectations for Learning

Note: This model curriculum was only written for S.ID.6. There was no model curriculum written for S.ID.5 at this level.

For this cluster, the GAISE Model framework continues to be used: Formulating Questions; Collecting Data; Analyzing Data; and Interpreting Results. In Algebra 1/Math 1 students find the equation of a linear model, with and without technology using precise language to describe the relationship between variables. In Statistics and Probability concepts are now extended to quadratic and exponential functions. Up to this point students have only analyzed the fit of the model by looking at the closeness of the data points to a linear model. Now students are introduced to the idea of a residual, and they use them to informally assess the fit of the model.

The learning at this level is at the developmental Level C. See pages ~~99~~ 27 and ~~102~~ 31 for more information on Level C.

ESSENTIAL UNDERSTANDINGS

- A linear, quadratic, or exponential function can be used as a model for association of two quantitative variables.
- $\hat{y}$ is often used as the symbol for the predicted *y*-value for a given *x*-value.
- A residual is the difference between the actual *y*-value and the *y*-value predicted by the chosen model $(y - \hat{y})$.

MATHEMATICAL THINKING

- Use accurate and precise mathematical vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve real-world and statistical problems.
- Use appropriate tools to display and analyze data.
- Accurately make computations using data.
- Determine reasonableness of predictions.

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STANDARDS	MODEL CURRICULUM (S.ID.5-6)
S.ID.5-6, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** **Quantitative Data** • Reason about the context and the data to judge whether a linear, quadratic, or exponential model (or none of these) is appropriate. • Fit quadratic and exponential models to data using technology. • Use the chosen model to make contextual conclusions. • Discuss residual values to assess the appropriateness of a linear model. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 1, page 3~~ **THE GAISE MODEL** • GAISE Model, pages 14 – 15 ○ Focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88 **CONNECTIONS ACROSS STANDARDS** • Create equations that describe numbers or relationships (A.CED.2). • Build a function that models a relationship between two quantities (F.BF.1). • Interpret linear models (S.ID.9).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.ID.5-6)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- It revisits many Algebra 1/Math 1 linear regression topics.
- It builds on Algebra 1/Math 1/Math 2 exponential function and quadratic function topics by matching these functions to data, assessing fit and analyzing residuals.
- It revisits the probability topics from Geometry/Math 2 that will be the foundation of inference for proportions.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

In this course, we make use of quadratic and exponential functions by matching these to real-world data, assessing fit, and analyzing residuals.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. At Level C, statistical investigative questions expand from summative and comparative situations to include questions about associations and relationships among multiple variables, including predictions. Once an appropriate plan for collecting data to answer a statistical investigative question has been implemented and the resulting data are in hand, the next step usually is to summarize the data using tools such as graphical displays and numerical summaries.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This cluster includes quadratic and exponential models, which are not a part of TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This cluster includes quadratic and exponential models, which are not a part of AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (S.ID.7-9)
Statistics and Probability **INTERPRET CATEGORICAL AND QUANTITATIVE DATA** **Interpret linear models.** **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★ **S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.★ **S.ID.9** Distinguish between correlation and causation.★	**Expectations for Learning** *Note: This model curriculum was only written for S.ID.9. There was no model curriculum written for S.ID.7-8 at this level.* In Algebra 1/Math 1, students interpret the slope and intercept of a linear model. They also work with the correlation coefficient. In ~~Algebra 2/Math 3~~ Statistics and Probability, students are now introduced to and explore the distinction between correlation and causation. The learning of standard S.ID.9 is at developmental Level C. See pages ~~99~~ 27 and ~~102~~ 31 for more information on Level C. **ESSENTIAL UNDERSTANDINGS** • There are three main methods of data production in statistics: surveys of samples to estimate population parameters; randomized experiments to compare treatments and to show cause; and observational studies to indicate possible associations among variables. Students should understand the distinctions among these three and decide if appropriate inferences have been drawn. • Causation is a cause and effect relationship between two variables. • Correlation (i.e., strong correlation) does not imply causation. • Causation cannot be established after the research has been completed; it can only be established through well-designed experiments (not observational studies and not surveys). *Continued on next page* **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data. • Determine reasonableness of predictions.

STANDARDS	MODEL CURRICULUM (S.ID.7-9)
S.ID.7-9, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** • Compare and contrast situations when changes in one variable cause changes in another. • Compare and contrast when the direction of causation is not clear. • Compare and contrast when changes in both variables are caused by something else. • Use surveys of samples to estimate population parameters. • Use randomized experiments to compare treatments and to show cause. • Use observational studies to indicate possible associations among variables. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 1, page 3~~ **THE GAISE MODEL** • GAISE Model, pages 14 – 15 ○ The focus of S.ID.9 is at Level C for Algebra 2/Math 3, pages 61-85 **CONNECTIONS ACROSS STANDARDS** • Summarize, represent, and interpret data in two categories and quantitative variables (S.ID.6). • Interpret the structure of functions (F.IF.4). • Build a function that models a relationship between two quantities (F.BF.1).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.ID.7-9)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- In this course we introduce the fact that correlation is necessary, but not sufficient, to show causation.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

In this course, students explore bivariate relationships more closely by exploring issues of measurement error and incomplete data, and by re-expressing data. Students also analyze data more formally using not only a visual inspection of a scatterplot and the correlation coefficient (r), but also the coefficient of determination (r^2), the residual plot, and least squares regression as a method of finding the line of best fit.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. Level C introduces a more formal way of exploring linear relationships between two quantitative variables by interpreting correlation coefficient and a least-squares regression line and interpreting results. This leads to inferences of causation and inferences from a sample to a population. Students should describe in context the important characteristics of the data.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- A TMM 010–Introductory Statistics course has more focus on extrapolation and emphasizes the predictive characteristics of bivariate relationships.
- Hypothesis testing for the slope is also included in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- An AP Statistics course has more focus on extrapolation and emphasizes the predictive characteristics of bivariate relationships.
- Hypothesis testing for the slope is also included in an AP Statistics course.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.ID.7-9)

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (S.ID.10-13 DRAFT STANDARDS)
Statistics and Probability **INTERPRET CATEGORICAL AND QUANTITATIVE DATA** **Recognize and interpret the normal distribution model.** **S.ID.10** Understand discrete and continuous distributions and distinguish between them. Compare to uniform continuous, uniform discrete, binomial, and normal distributions.★ **a.** Calculate probabilities for various distributions. **b.** Use one kind of distribution to approximate the other. **S.ID.11** Visually compare a data distribution to the standard normal distribution. Recognize that normal distributions can be used to represent some population distributions. Understand that a normal distribution is determined by its mean and standard deviation. ★ **S.ID.12** Calculate and use probability from a normal distribution. ★ **a.** Determine proportions and percentiles from a normal distribution. Understand that the 50th percentile is a measure-of-center, the median. **b.** Compare measures of relative position in data sets: z-scores and percentiles. **S.ID.13** Use the standard normal distribution to approximate binomial distributions.★	*Note: There is no model curriculum written for these standards, as these are draft standards to be submitted for review in the next standards revision.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.ID.10-13)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2??

- This content is not included in the Algebra 1/Math 1 or Geometry/Math 2 courses.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

Students explore normal distributions and use normal probabilities to make predictions and model other distributions. This particular bell-shaped curve is often used to model data that are unimodal and roughly symmetric.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. The normal distribution is fundamental to formal inference processes, so this cluster prepares students to move to Level C with respect to those processes.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is the foundational understanding of recognizing and interpreting normal distributions.
- TMM 010–Introductory Statistics will use other distributions such as the x^2 and t-distributions.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is the foundational understanding of recognizing and interpreting normal distributions.
- The AP Statistics class will use other distributions such as the x^2 and t-distributions.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (S.IC.1-2)
Statistics and Probability **MAKING INFERENCES AND JUSTIFYING CONCLUSIONS** **Understand and evaluate random processes underlying statistical experiments.** **S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★ **S.IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★	**Expectations for Learning** This is students' first exposure to the necessity of randomness when sampling to make an inference about a population which is consistent with the GAISE Model (Step 4, Level C). In middle school, students are exposed to simulations (observed) and compare them to predicted (expected) outcomes based on probability. In this cluster, students decide if results are consistent with a given model by using simulations or computing probabilities. Understanding the statistical concepts of GAISE model Level C enables a student to build on the foundations developed in Levels A and B. Although the previous levels are revisited, students are now expected to take prior learning to a deeper statistical nature. They are expected to draw on basic concepts from earlier work; extend the concept to cover a wider scope of investigatory issues; and develop a deeper understanding of inferential reasoning and its connection to probability. Students should be able to provide a more sophisticated interpretation that integrates the context and objectives of a study, and they should also be able to see limitations based on data. **ESSENTIAL UNDERSTANDINGS** • Random sampling guarantees that the sample chosen is representative of the population which ensures that the statistical conclusions will be valid. • A random sample must be generated through a chance selection process. • A statistic is generated from sample data to estimate the corresponding parameter for the entire population. • A population parameter is a measure of some characteristic in the population such as the population proportion or the population mean. • Experimental results are not a perfect match for theoretical models by the nature of variability. **MATHEMATICAL THINKING** • Make connections between samples and their populations. • Determine whether results are reasonable. • Use precise mathematical language. *Continued on next page*

STANDARDS	MODEL CURRICULUM (S.IC.1-2)
S.IC.1-2, continued	**Expectations for Learning** **INSTRUCTIONAL FOCUS** • Explain and interpret, within a context, the results of a data-generating process. • Explain, within a context, what a point on a dot plot from a simulation represents. • Decide if an outcome is unusual in a specified distribution. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 1, page 3~~ **THE GAISE MODEL** • GAISE Model, pages 14 – 15 ○ The focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88 **CONNECTIONS ACROSS STANDARDS** • Make inferences and justify conclusions from sample surveys, experiments, and observational studies (S.IC.4, 6). • ~~(+)~~ Develop probability models and expected values (S.MD.3-6).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.IC.1-2)

Instructional Strategies

HOW IS THIS DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- This content is not included in the Algebra 1/Math 1 or Geometry/Math 2 courses.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

In this cluster, students should understand that there are a variety of ways to sample from a population. Understanding the differences between the methods and knowing when to use which is essential.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. Students should be able to understand that a sample can be used to answer statistical investigative questions about a population. They should further be able to recognize the limitations and scope of the data collected by describing the group or population from which the data are collected.

In statistics, randomness is incorporated into the sample selection procedure to provide a method that is "fair" (unbiased) and to improve the chances of selecting a representative sample.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is included in the TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is included in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

RANDOMNESS

Randomness is discussed informally in Algebra 1/Math 1. This cluster asks students to identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn. Specifically, utilizing random selection allows us to infer our results to the population while random assignment of experimental units in a well-designed experiment allows us to establish causal relationships. Random assignment is discussed further in the next cluster.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.IC.1-2)

RANDOMNESS AND THE GAISE II CONNECTION

The idea of representative samples is introduced at GAISE Level B and Level C. The idea that random samples are extremely likely to be representative is an essential idea. Representativeness allows generalization to the population from which the random sample was drawn.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (S.IC.3-6)
Statistics and Probability **MAKING INFERENCES AND JUSTIFYING CONCLUSIONS** **Make inferences and justify conclusions from sample surveys, experiments, and observational studies.** **S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★ **S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★ **S.IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.★ **S.IC.6** Evaluate reports based on data.★	**Expectations for Learning** Previously, students have been informally introduced to data collection methods and bias. In this cluster, the concept of randomization is introduced in data collection methods. Students are also introduced to the concept of margin of error, and they begin to formalize the concept of statistical significance. **The GAISE Model** The GAISE Model is a framework for all statistical problem solving and should not be taught in isolation. For this cluster, the focus is on Steps 2, 3, and 4 at Level C. Students are building on the framework developed in earlier grades. ~~Algebra 2/Math 3~~ Statistics and Probability students use more in-depth reasoning and a greater level of precision and complexity. **Step 1: Formulate the Question** • Students should be fluent in posing their own statistical question of interest. • Students should form questions to allow generalizations be made about a population **Step 2: Collect Data** • Students should purposefully design for differences through random selection or random assignment. • Students design samples through selection. • Students design experiments through randomization. **Step 3: Analyze Data** • Students understand and use global characteristics of distributions in analysis. • Students compare group to group using displays and measures of variability. • Students describe and quantify sampling error. **Step 4: Interpret Variability** • Students are able to look beyond the data in some contexts. • Students are able to generalize from a sample to population. • Students are aware of the effects of randomization on the results of experiments. • Students understand and distinguish between observational studies and experiments. *Continued on next page*

STANDARDS	MODEL CURRICULUM (S.IC.3-6)
S.IC.3-6, continued	**Expectations for Learning, continued** **ESSENTIAL UNDERSTANDINGS** • Surveys, observational studies, and experiments are different methods for data collection and each have their own advantages. • Surveys and observational studies involve a researcher collecting information about a sample without imposing a treatment on subjects. • A researcher should utilize a chance process to assign treatment groups in experiments. • Causality can be established with well-designed experiments; surveys and observational studies cannot determine causality. • The decision process based on sample data does not guarantee a correct answer to the underlying statistical question. • The characteristics of distributions of sample statistics are simulation models for random sampling; it is predictable only if the sampling is random. • The interval (observed statistic $\pm$ margin of error) should include the plausible values for the true population parameter. • The results from an experiment are statistically significant if the differences between treatment groups are unlikely to have occurred by chance alone. **MATHEMATICAL THINKING** • Make sense of the structure of distributions. • Use precise vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve statistical problems in real-world context. • Determine whether results are reasonable. *Continued on next page*

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (S.IC.3-6)</th></tr>
<tr><td>S.IC.3-6, continued</td><td>

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Identify and choose when to use surveys, experiments, and observational studies.
- Explain the role of randomization as it relates to bias in surveys, experiments, and observational studies.
- Use a sample mean or sample proportion to estimate a population mean or population proportion.
- Explain how sampling variability relates to margin of error in the context of estimating population mean or proportion.
- Use simulation to estimate margin of error in the context of estimating population mean or population proportion.
- Develop an understanding of the definition of *statistical significance* as compared to everyday language *significance*; determine if the results of an experiment are statistically significant; draw conclusions in context based on a report.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- ~~Algebra 2/Math 3, Number 1, page 3~~

THE GAISE MODEL

- GAISE Model, pages 14 – 15
  - The focus of this cluster for Algebra 2/Math 3 is Level C, pages 61-88

CONNECTIONS ACROSS STANDARDS

- Understand and evaluate random processes (S.IC.1-2).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.IC.3-6)

Instructional Strategies

HOW IS THIS DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- This content is not included in the Algebra 1/Math 1 or Geometry/Math 2 courses.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

In this cluster, the concept of randomization is introduced in data collection methods. Students are also introduced to the concept of margin of error, and they begin to formalize the concept of statistical significance.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. Regarding causality and prediction, students use primary or secondary data to pose statistical investigative questions for surveys, observational studies, and experiments. Students distinguish between surveys, observational studies, and experiments and understand what constitutes good practice in designing them. They develop simulations and generate summary statistics and data visualization to determine approximate sampling distributions and compute *p*-values from those distributions. Students understand what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation and understand how to interpret simulated *p*-values appropriately. They also interpret the margin of error associated with an estimate of a population characteristic.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

In this course students will be introduced to the idea of hypothesis testing but will not do formal inference as they would in TMM 010–Introductory Statistics. This course will also limit the discussion to inference of one sample and not delve into two samples.

Some topics commonly included in TMM 010–Introductory Statistics and not this course may include but are not limited to the following:

- Inference for two proportions and two means
- Type I error
- Confidence level interpretation (confidence *interval* interpretation is included)
- *Z*-test for means

Continued on next page

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.IC.3-6)

The following topics are touched upon in this course but will go to greater depth in TMM 010–Introductory Statistics:

- Formal inference for proportions and means
- The Central Limit Theorem

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

In this course students will be introduced to the idea of hypothesis testing but will not do formal inference as they would in AP Statistics. This course will also limit the discussion to inference of one sample and not delve into two samples.

Topics that commonly occur in an AP Statistics course that are **not included** in this course are the following:

- Inference for two proportions and two means
- Verifying conditions for inference
- Type I, type II errors and power
- Confidence level interpretation (confidence *interval* interpretation is included)
- *Z*-test for means

The following topics are touched upon in this course but will go into greater depth in AP Statistics:

- Formal inference for proportions and means
- The Central Limit Theorem

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

RANDOMNESS

Randomness is discussed informally in Algebra 1/Math 1. This cluster asks students to identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn. Specifically, utilizing random selection allows us to infer our results to the population while random assignment of experimental units in a well-designed experiment allows us to establish causal relationships.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.IC.3-6)

RANDOMNESS AND THE GAISE II CONNECTION

As discussed in the previous cluster, the idea of representative samples is introduced at GAISE Level B and Level C. The idea that random samples are extremely likely to be representative is an essential idea. Representativeness allows generalization to the population from which the random sample was drawn. Also, Level C introduces the importance of random assignment in experiments, so that the only likely difference between control and treatment groups is the treatment, and that the effect of other confounding variables is equally distributed across the groups being compared. A well-designed experiment that uses randomness is the only way to determine cause-and-effect. The idea of random variability between samples of the same size is used to compute margin of error when estimating a population parameter as well as to make decisions when testing a hypothesis.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (S.CP.1-5)
Statistics and Probability **CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY** **Understand independence and conditional probability, and use them to interpret data.** **S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★ **S.CP.2** Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★ **S.CP.3** Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ *Continued on next page*	*Note: There is no model curriculum written at this level for this cluster.*

STANDARDS	MODEL CURRICULUM (S.CP.1-5)
S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★ **S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★	

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.CP.1-5)

Instructional Strategies

HOW IS THIS DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- The standards in this cluster are explored in greater depth in this course with a focus on conditional probability and independence.
- This cluster develops the foundation for additional work with probability distributions.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

This cluster focuses on the concept of independence between two categorical variables. It further focuses on the understanding of independence, including symbolic notation, formulas, and visual representations, such as two-way frequency tables, Venn diagrams, and tree diagrams. Applications of independence and conditional probability are critical components in this course.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. Probability is a measure of the chance that something will happen. It is a measure of certainty or uncertainty. At Level B, students recognize probability as a long-run relative frequency. At Level C, students apply and solve problems using the concept of independence and conditional probability.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is included in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This is content included in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.CP.1-5)
Instructional Tools/Resources *This section will be published at a later time.*

STANDARDS	MODEL CURRICULUM (S.CP.6-9)
Statistics and Probability **CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY** **Use the rules of probability to compute probabilities of compound events in a uniform probability model.** **S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ **S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★ ~~(+)~~ **S.CP.8** Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★ ~~(+)~~ **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.★	*Note: There is no model curriculum written at this level for this cluster.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.CP.6-9)

Instructional Strategies

HOW IS THIS DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- The standards in this cluster are explored in greater depth in this course, with a focus on conditional probability.
- This cluster explores and applies rules of probability, including the addition and multiplication rules, to compute probabilities of compound events in a uniform probability model.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

This cluster focuses on using formal rules of probability. It further focuses on understanding the rules of probability, emphasizing symbolic notation and formulas in the context of real-world problems. A brief introduction to permutations and combinations is included.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

In previous grades, students were exposed to GAISE Levels A and B. It is the intention of this course for students to achieve proficiency at GAISE Level C. Probability is a measure of the chance that something will happen. It is a measure of certainty or uncertainty. At Level B, students recognize probability as a long-run relative frequency. At Level C, students apply and solve real-world problems using the formal rules of probability.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is included in TMM 010–Introductory Statistics with the exception of permutations and combinations.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is included in AP Statistics, with the exception of permutations and combinations.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.CP.6-9)

Instructional Tools/Resources

This section will be published at a later time.

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (S.MD.1-4)</th></tr>
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Statistics and Probability

USING PROBABILITY TO MAKE DECISIONS

Calculate expected values, and use them to solve problems.

~~(+)~~ **S.MD.1** Define a random variable[G] for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution[G] using the same graphical displays as for data distributions.★

~~(+)~~ **S.MD.2** Calculate the expected value[G] of a random variable; interpret it as the mean of the probability distribution.★

~~(+)~~ **S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★

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Expectations for Learning

In middle school, students begin exploring the differences between expected values and observed values and create data displays based upon counts. Students now are asked to develop and define random variables; to graph the associated probability distribution; and to compute expected values in anticipation of using the expected value to make decisions.

ESSENTIAL UNDERSTANDINGS

- Random variables are numeric representations of outcomes resulting from a
- chance process.
- The mean of a probability distribution is the expected value of the random variable.
- A probability distribution is the list of all possible outcomes and their respective probabilities.

MATHEMATICAL THINKING

- Attend to the meaning of quantities in the context.
- Create a model to make sense of a problem.
- Make and modify a model to represent mathematical thinking.

INSTRUCTIONAL FOCUS

- Recognize random variables in everyday settings.
- Distinguish between algebraic variables and random variables.
- Assign the values of the random variable.
- Compute the probabilities for the values of the random variable.
- Compute expected values.
- Compute, explain, and interpret the expected value of a random variable within
- a context.
- Create and interpret graphical and tabular displays of probability distributions.

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STANDARDS	MODEL CURRICULUM (S.MD.1-4)
~~(+)~~ **S.MD.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* ★	**Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** ~~*These standards are not assigned to any particular course.*~~ **CONNECTIONS ACROSS STANDARDS** • (+) Know and apply Binomial Theorem (A.APR.5). • (+) Use probability to evaluate outcomes of decisions (S.MD.5-7). • Understand and evaluate random processes using simulation (S.IC.2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.MD.1-4)

Instructional Strategies

HOW IS THIS DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- This cluster was not included in the Algebra 1/Math 1 or Geometry/Math 2 courses.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

This cluster introduces the probability distribution of a random variable, which is a foundational concept for understanding inferential statistics. The focus is placed on solving real-world problems with expected value and interpreting expected value as the mean of the probability distribution.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

This cluster is implicit in GAISE but is not explored in detail.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- The language of expected value is not included in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is included in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (S.MD.5-7)
Statistics and Probability **USING PROBABILITY TO MAKE DECISIONS** **Use probability to evaluate outcomes of decisions.** ~~(+)~~ **S.MD.5** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★ **a.** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **b.** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* ~~(+)~~ **S.MD.6** Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★ ~~(+)~~ **S.MD.7** Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★	**The Expectations for Learning** Students are using probability, random variables, and expected values to make decisions and judgments regarding the fairness of games and strategies for playing games. **ESSENTIAL UNDERSTANDINGS** • A game is "fair" when the expected net winnings are zero. • Expected values are the mean of a large number of trials. **MATHEMATICAL THINKING** • Determine reasonableness of decisions. **INSTRUCTIONAL FOCUS** • Evaluate the decisions of others based on probability models. • Solve real-world problems using probability models and expected values. Use probabilities to make decisions. • Assign payoff values for games of chance and interpret them in the context of the problem. • Decide if a game is "fair". • Use random number tables or random number generators to make selections. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** ~~*These standards are not assigned to any particular course.*~~ **CONNECTIONS ACROSS STANDARDS** • ~~(+)~~ Calculate expected values, and use them to make decisions (S.MD.1-4). • Use rules of probability to compute probabilities (S.CP.6-9).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (S.MD.5-7)

Instructional Strategies

HOW IS THIS DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- This cluster was not included in the Algebra 1/Math 1 or Geometry/Math 2 courses.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

This cluster builds on the work in S.MD.1-4. The focus is placed on evaluating the outcomes of decisions using expected value and probability concepts.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

This cluster is implicit in GAISE but is not explored in detail.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- The language of expected value is not included in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is included in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (N.Q.1-3)
Number and Quantity **QUANTITIES** **Reason quantitatively and use units to solve problems.** **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★ **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.★ **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★	*Note: There is no model curriculum written at this level for this cluster.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (N.Q.1-3)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- In this course, implications of accuracy and precision are applied to the practice of gathering and representing data. In particular, it is important to attend to problems with measurement error and misleading scales in data distributions.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

This course emphasizes precision in the practice of gathering and representing data. In particular, students attend to accuracy and precision of measurement when collecting data, select appropriate scales when creating visual representations of data, and analyze the appropriateness of these practices in existing statistical sources. Additionally, students extend the use of unit conversion when transforming elements of a data set to create comparable data and when computing the transformed statistics that will result from those conversions.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

This cluster is implicit in GAISE but is not explored in detail.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is not explicitly included in TMM 010–Introductory Statistics.
- Clear use of appropriate units on graphs and in results is fundamental to proper statistical communication.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is not explicitly included in AP Statistics.
- Clear use of appropriate units on graphs and in results is fundamental to proper statistical communication.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (N.Q.1-3)
Instructional Tools/Resources *This section will be published at a later time.*

STANDARDS	MODEL CURRICULUM (A.APR.5)
Algebra **ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS** **Use polynomial identities to solve problems.** (+) **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, by using coefficients determined for by Pascal's Triangle.* The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.	**Expectations for Learning** *Note: In this cluster, only standard A.APR.5 was assigned to this course. The crossed-out lines below show those statements that relate to standard A.APR.4 and so are not applicable to this course.* In previous courses, students examine writing an algebraic expression in different but equivalent forms. In this cluster, students prove polynomial identities based on numerical relationships. Students pursuing advanced mathematics courses (+) apply the Binomial Theorem and prove it by induction or a combinatorial argument. *Note: A.APR.5 is not required for all students, but is intended for students who are pursuing advanced mathematics.* **ESSENTIAL UNDERSTANDINGS** • ~~When two expressions are equivalent, an equation relating the two is called an identity because it is true for all values of the variables.~~ • ~~To prove an algebraic identity means to show, using the properties of operations, that the equation is always true, for any values of the variables.~~ • ~~Recognize patterns in numerical relationships and be able to express these patterns as algebraic identities.~~ • (+) Pascal's Triangle can be used to generate coefficients in the Binomial Theorem. **MATHEMATICAL THINKING** • Discern and use a pattern or structure. • Generalize concepts based on patterns. • Explain mathematical reasoning. • Use informal reasoning. **INSTRUCTIONAL FOCUS** • ~~Produce algebraic proofs for various polynomial identities.~~ • ~~Represent numerical relationships using identities.~~ • (+) Apply the Binomial Theorem. *Continued on next page*

STANDARDS	MODEL CURRICULUM (A.APR.5)
A.APR.5, continued	**Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • ~~Algebra 2/Math 3, Number 2, pages 4-6~~ **CONNECTIONS ACROSS STANDARDS** • ~~Interpret the structure of expressions (A.SSE.2).~~ • ~~Perform arithmetic operations on polynomials (A.APR.1).~~ • Create equations that describe numbers or relationships (A.CED.1-2).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (A.APR.5)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- This content is not part of the Algebra 1/Math 1 or Geometry/Math 2 courses.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

Pascal's triangle can be used to determine combinations, so the Binomial Theorem can be explored as an application of combinations. When using a formula to compute binomial probabilities, Pascal's triangle gives the number of combinations, $nCr.$ This cluster further supports the work of S.CP.9.

Common variations of the combinations formula include the following:

- $nCr\ p^r(1-p)^{n-r}$
- $\binom{n}{k}\ p^k(1-p)^{n-k}$

HOW DOES THE CLUSTER CONNECT TO GAISE II?

This cluster is not included in GAISE.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- Combinations, nCr, are used in the formula for computing binomial probabilities in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- Combinations, nCr, are used in the formula for computing binomial probabilities in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (A.CED.1-3)
Algebra **CREATING EQUATIONS** **Create equations that describe numbers or relationships.** **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1)	**Expectations for Learning** *Note: In this cluster, only standards A.CED.1-3 were assigned to this course. The crossed-out lines below show those statements that relate to standard A.CED.4 and so are not applicable to this course.* In previous courses, students create equations and inequalities, focusing on linear, exponential, and quadratic equations. They also rearrange formulas to highlight a particular variable. In this course, students extend these skills to more complicated situations. **ESSENTIAL UNDERSTANDINGS** • Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts. • The graphical solution of a system of equations or inequalities is the intersection of the equations or inequalities. • Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given. • ~~A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations.~~ **MATHEMATICAL THINKING** • Create a model to make sense of a problem. • Represent the concept symbolically. • Plan a solution pathway. • Determine the reasonableness of results. • Consider mathematical units and scale when graphing. **INSTRUCTIONAL FOCUS** • Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship. • For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales. • Identify the constraints implied by the scenario, and represent them with equations or inequalities. • Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario. • ~~Solve formulas for a given variable.~~

STANDARDS	MODEL CURRICULUM (A.CED.1-3)
A.CED.1-3, continued	**Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • ~~Algebra 2/Math 3, Number 4, pages 8-10~~ **CONNECTIONS ACROSS STANDARDS** • Model and interpret the relationship between two quantities (F.IF.4). • ~~Rewrite expressions involving radicals and rational exponents (N.RN.2).~~ • ~~Use the structure of an expression to identify ways to rewrite it (A.SSE.2).~~ • Write a function that describes a relationship between two quantities (F.BF.1b,~~c~~). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (A.CED.1-3)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- In previous courses, students focused on linear, exponential and quadratic equations. In this course, students extend their work with linear, exponential and quadratic equations to include power and logarithmic functions, which are useful when finding appropriate bivariate data transformations.
- This cluster further supports the work of S.IC.4 and S.ID.6.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

In this course, students create equations to represent bivariate data both by inspection and by using regression with technology. For example, students create equations to determine a least squares regression line when given means, standard deviations, and the correlation coefficient. Additionally, students create equations to determine the sample size necessary to achieve a desired margin of error.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

Algebra skills, such as those in this cluster, are implicit in GAISE but are not explored in detail.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This cluster is not explicitly included in TMM 010–Introductory Statistics.
- Algebraic skills are utilized to deepen understanding and as methods for problem solving throughout TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This cluster is not explicitly included in AP Statistics.
- Algebraic skills are utilized to deepen understanding and as methods for problem solving throughout AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

<table>
<tr><th>STANDARDS</th><th>MODEL CURRICULUM (F.IF.4-5)</th></tr>
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Functions

INTERPRETING FUNCTIONS

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

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Expectations for Learning

Note: In this cluster, only standards F.IF.4-5 were assigned to this course. The crossed-out lines below show those statements that relate to standard F.IF.6 and so are not applicable to this course.

Working with linear, quadratic, and exponential functions in Algebra1/Math 2, students interpreted key features of graphs and tables. Students determined the domain and understand its limitations by looking at a graph, table, and a given real-life scenario.

In this cluster, students extend key features to include periodicity, polynomials of degree greater than 2, and others given by graphs, tables, symbols, or verbal descriptions. Students select appropriate functions that model the situation presented. Average rate of change over a specific interval is included in this cluster.

In future courses, some students will extend their understanding of average rate of change to instantaneous rate of change.

Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem, and making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, then identifying the key features of the graph and connecting the key features to the symbols.

ESSENTIAL UNDERSTANDINGS

- Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem.
- The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols.
- The context of a situation can suggest the shape of the graph and can suggest the function family that models relationships identified in the situation.
- ~~Linear functions have a constant rate of change.~~
- ~~Since non-linear functions do NOT have a constant rate of change, average rate of change is used the describe how the rate of change varies from interval to interval.~~
- ~~The average rate of change of a function over an interval is finding the slope of a linear function that goes through the same two points.~~

Continued on next page

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STANDARDS	MODEL CURRICULUM (F.IF.4-5)
F.IF.4-5, continued	**Expectations for Learning, continued** **MATHEMATICAL THINKING** • Connect mathematical relationships to contextual scenarios. • Attend to meaning of quantities. • Determine reasonableness of results. **INSTRUCTIONAL FOCUS** • Use key features (including periodicity) of graphs, tables, and contexts to select an appropriate function family for modeling purposes. • Utilize graphs to compare and understand behaviors and features of various types of functions. • Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required). • Demonstrate understanding of domain in the context of a real-world problem. • ~~Given a function graphically, estimate the average rate of change over a specified interval.~~ • ~~Given a function symbolically, compute the average rate of change over a specified interval.~~ • ~~Given a function represented as a table, compute the average rate of change over a specified interval.~~ **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 4, pages 8-10~~ *Continued on next page*

STANDARDS	MODEL CURRICULUM (F.IF.4-5)
	CONNECTIONS ACROSS STANDARDS • Create equations that describe numbers or relationships (A.CED.2c). • ~~Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions (F.IF.7c).~~ • ~~Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior (F.IF.7d).~~ • ~~Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude (F.IF.7f).~~ • ~~(+) Graph rational functions, identifying zeros and asymptotes, when factoring is reasonable, and indicating end behavior (F.IF.7g).~~ • ~~Analyze functions using different representations (F.IF.9).~~ • Model periodic phenomena with trigonometric functions (F.TF.5). • Fit a function to the data; use functions fitted to data to solve problems in the context of the data (S.ID.6a).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (F.IF.4-5)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- In previous courses, students modeled bivariate data and interpreted key features. In this course, students will perform similar analysis on probability distributions.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

Students continue to model and interpret key features of bivariate data and identify the function family for any particular data set. Students understand that the domain of a function is limited by the domain of the data to avoid extrapolation. Additionally, students interpret key features and parameters of probability distributions, such as a normal distribution. Key features include the following: mean, standard deviation, points of inflection, and symmetry. This cluster further supports the work of S.ID.7.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

The ability to recognize elements of functions, such as those in this cluster, are implicit in GAISE but are not explored in detail.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is included in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is included in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (F.BF.1)
Functions **BUILDING FUNCTIONS** **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*(A2, M3)	**Expectations for Learning** In prior math courses, students write linear, quadratic, and exponential functions. They also write explicit and recursive rules for arithmetic and geometric sequences. In this cluster, students build functions from other functions to model more complex situations. This includes combining functions of various types using arithmetic operations or (+) function composition. **ESSENTIAL UNDERSTANDINGS** • Known functions can be combined using arithmetic operations or (+) function composition, yielding new functions. • Addition or subtraction by a constant function can be interpreted as a vertical shift. • Multiplication or division by a non-zero constant function can be interpreted as a vertical stretch/shrink. **MATHEMATICAL THINKING** • Make and modify a model to represent mathematical thinking. • Discern and use a pattern or structure. **INSTRUCTIONAL FOCUS** • Demonstrate the effects of arithmetic operations on known functions: Choose an *x*-value; find the two *y*-values; do the operation; and then repeat with several other x-values. • For addition and subtraction of functions, compare the heights of the functions to their sum or difference. • Select appropriate functions and arithmetic operations on those functions to model situations. • (+) Compose functions algebraically and for the purpose of modeling.

STANDARDS	MODEL CURRICULUM (F.BF.1)
F.BF.1, continued	**Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 4, pages 9-10~~ **CONNECTIONS ACROSS STANDARDS** • Create equations that describe numbers or relationships (A.CED.2). • Fit a function to the data; use functions fitted to data to solve problems in the context of the data (S.ID.6a). • Interpret linear models (S.ID.7). • ~~Analyze functions using different representations (F.IF.8).~~ • ~~Perform arithmetic operations on polynomials (A.APR.1b).~~

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (F.BF.1)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- In prior math courses, students write functions that describe a relationship between two quantities. That work is continued in this course when students work with linear regression.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

This is a supporting standard that is also a part of Algebra 2 composition of functions that helps students build functions that match observed patterns in data that comprise more than one parent function.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

This content is not directly addressed in GAISE. It is a supportive standard that helps students build a function that matches observed patterns in data.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is not directly addressed in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is not directly addressed in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (F.LE.4)
Functions **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★	**Expectations for Learning** Students study exponential functions with integer domains in Algebra 1 or in Math 1 and Math 2. Earlier in this course, students develop an understanding of rational exponents in order to talk about exponential functions with a domain that is the real numbers. Based on these understandings, students in ~~Algebra 2/Math 3~~ Statistics and Probability focus on logarithms as numbers that are solutions to exponential equations. Logarithms as functions and the laws of logarithms are recommended for higher-level courses. **ESSENTIAL UNDERSTANDINGS** • Logarithms are exponents; they are numerical solutions to exponential equations, such as $10^x = 30$. **MATHEMATICAL THINKING** • Represent a concept symbolically. • Make and modify a model to represent mathematical thinking. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. **INSTRUCTIONAL FOCUS** • Solve exponential equations with an unknown exponent by translating between exponential and logarithmic forms, with the support of tables and graphs, interpreting the solution in context. *Continued on next page*

STANDARDS	MODEL CURRICULUM (F.LE.4)
F.LE.4, continued	**Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 4, pages 8-10~~ **CONNECTIONS ACROSS STANDARDS** • Interpret functions that arise in applications in terms of the context (F.IF.4). • ~~Analyze functions using different representations (F.IF.9).~~ • ~~Interpret the structure of expressions (A.SSE.3c).~~ • ~~Build new functions from existing functions (F.BF.3).~~ • ~~Extend the properties of exponents to rational exponents (N.RN.1-2).~~ • ~~Use properties of rational and irrational numbers (N.NR.3).~~

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (F.LE.4)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- Logarithms are not introduced in Algebra 1/Math 1 or Geometry/Math 2.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

For this course, logarithms are introduced as a tool to re-express or rescale non-linear data using technology. Algebraic manipulation of logarithmic expressions allows students to convert equations between the linear and exponential/logarithmic forms. This skill underlies the linearization of data using logarithms. A primary purpose for inclusion of this content is to prepare students to understand data and graphs that use log scales on one or more axes.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

There is no explicit connection to GAISE.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- It is not a prominent feature in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- In AP Statistics, students are expected to linearize data using logarithmic and other methods.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (F.LE.5)
Functions **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** **Interpret expressions for functions in terms of the situation they model.** **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.★	*Note: There is no model curriculum written at this level for this cluster.*

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (F.LE.5)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- Interpreting parameters in terms of context is not a primary focus in the Algebra 1/Math 1 course, whereas it is fundamental in Statistics.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

This standard further supports the work of SID.7. Rate of change is always meaningful, however the initial amount may or may not have meaning. The use of nondeterministic language (e.g. on average, is predicted, is expected) is of paramount importance. For exponential functions, students interpret the initial amount and the growth/decay rate (or factor).

HOW DOES THE CLUSTER CONNECT TO GAISE II?

Attention to context is fundamental to the formulation of investigative questions and the interpretation of data. These are GAISE level A & B concepts.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

The use of nondeterministic language (e.g. on average, is predicted, is expected) to interpret the parameters in a linear function in terms of context is addressed. However, interpreting exponential functions is not covered in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

The use of nondeterministic language (e.g. on average, is predicted, is expected) to interpret the parameters in a linear function in terms of context is addressed. However, interpreting exponential functions is not covered in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

STANDARDS	MODEL CURRICULUM (F.TF.5)
Functions **TRIGONOMETRIC FUNCTIONS** **Model periodic phenomena with trigonometric functions.** **F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	**Expectations for Learning** Geometry/Math 2, students learn trigonometric ratios of right triangles. They study the right-triangle definitions of sine and cosine and apply their understanding to solve problems involving right triangles. In ~~Algebra 2/Math 3~~ Statistics and Probability, students are introduced to the concept of radians and extend their understanding of right triangle trigonometry to circular trigonometry. In this cluster, students apply their understanding of circular trigonometry to model periodic phenomena such as tides, electrical current, height of Ferris wheels, temperature, etc. The language in the standard states that students will "choose trig functions", but additionally, students must also choose the parameters for a trigonometric function that models real-world phenomena. In future courses, students will graph the six trigonometric functions by hand and/or using technology and will also identify periods and phase shifts by analyzing graphs and equations. **ESSENTIAL UNDERSTANDINGS** • Many real-world phenomena, including sound waves; oscillation on a spring; the motion of a pendulum; and phases of the moon, are cyclical and can be approximated by trigonometric functions. • The period is the horizontal length of one cycle, and it can be interpreted in terms of a horizontal stretch. • The equation of the midline is the average of the maximum and minimum values of the function, and it is a horizontal axis about which the graph of the function oscillates. • The amplitude is the distance between the midline and the maximum or minimum values of the function, and it can be interpreted in terms of a vertical stretch. **MATHEMATICAL THINKING** • Connect mathematical relationships and contexts. • Make and modify a model to represent mathematical thinking. *Continued on next page*

STANDARDS	MODEL CURRICULUM (F.TF.5)
F.TF.5, continued	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** • Given a real-world phenomenon, select an appropriate trigonometric function that models that phenomenon, understanding that sine and cosine functions will be used most frequently because of their tendency to model common real-world phenomena. • Given a real-world phenomenon, identify the parameters (midline, amplitude, period) that describe that phenomenon. • When phenomena are modeled with a sine ($f(x)$ = A sin(Bx + C) + D) or cosine ($f(x)$ = A cos(Bx + C) + D) function, students will choose the parameters A, B, C, and D that best model the scenario. • Interpret amplitude and midline as vertical stretches and shifts of the graphs. Include some attention to period and frequency as horizontal stretches. Reserve phase shifts as horizontal shifts for advanced courses. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • ~~Algebra 2/Math 3, Number 3, page 7~~ **CONNECTIONS ACROSS STANDARDS** • ~~Extend the domain of trigonometric functions using the unit circle (F.TF.1-2, (+) 3-4).~~ • ~~Analyze functions using different representations (F.IF.7f).~~ • ~~Build new functions from existing functions (F.BF.3, (+) 4b-d).~~ • ~~(+) Apply trigonometry to general triangles (G.SRT.8b, 9, 10, 11)~~

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM (F.TF.5)

Instructional Strategies

HOW IS THIS CLUSTER DIFFERENT FROM ALGEBRA 1/GEOMETRY OR MATH 1/MATH 2?

- Periodic functions are not introduced in Algebra 1/Math 1 or Geometry/Math 2.

DESCRIPTION OF CLUSTER AT ALGEBRA 2 EQUIVALENCY (A2E) LEVEL

In this course, use of sinusoidal regression with technology can explain data that shows a periodic pattern. This standard is not intended to be a complete lesson about unit circle trigonometry or trigonometric graphs.

HOW DOES THE CLUSTER CONNECT TO GAISE II?

This content is not directly addressed in GAISE.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education can be found on the American Statistical Association's website.

HOW IS THIS CLUSTER DIFFERENT FROM A CCP TMM 010–INTRODUCTORY STATISTICS COURSE?

- This content is not included in TMM 010–Introductory Statistics.

TMM010–Introductory Statistics Learning Outcomes can be found on the Ohio Department of Higher Education's webpage.

HOW IS THIS CLUSTER DIFFERENT FROM ADVANCED PLACEMENT (AP) STATISTICS?

- This content is not included in AP Statistics.

The Advanced Placement (AP) Statistics Course Overview can be found on College Board's webpage.

Examples and activities will be published at a later time.

Instructional Tools/Resources

This section will be published at a later time.

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