Ohio's Model Curriculum | Mathematics

with Instructional Supports

Algebra 1 Course

Ohio | Department of Education

Mathematics Model Curriculum

with Instructional Supports

Algebra 1 Course

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

Standards for Mathematical Practice—Algebra 1

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.

Students learn that patience is often required to fully understand what a problem is asking. They discern between useful and extraneous information. They expand their repertoire of expressions and functions that can be used to solve problems.

MP.2 Reason abstractly and quantitatively.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; of considering the units involved; of attending to the meaning of quantities, not just how to compute them; and of knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.

Students reason through the solving of equations, recognizing that solving an equation involves more than simply following rote rules and steps. They use language such as "If ___, then ___" when explaining their solution methods and provide justification for their reasoning.

MP.4 Model with mathematics.

Students also discover mathematics through experimentation and by examining data patterns from real-world contexts. They apply their new mathematical understanding of exponential, linear, and quadratic functions to real-world problems.

MP.5 Use appropriate tools strategically.

Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret results. They construct diagrams to solve problems.

MP.6 Attend to precision.

Students use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They make use of the definition of function when deciding if an equation can describe a function by asking, "Does every input value have exactly one output value?"

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Standards for Mathematical Practice, continued

MP.7 Look for and make use of structure.
Students develop formulas such as $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. Students see that the expression $5 + (n - 2)^2$ takes the form of 5 plus "something squared," and because "something squared" must be positive or zero, the expression can be no smaller than 5.

MP.8 Look for and express regularity in repeated reasoning.
Students see that the key feature of a line in the plane is an equal difference in outputs over equal intervals of inputs, and that the result of evaluating the expression $\frac{y_2 - y_1}{x_2 - x_1}$ for points on the line is always equal to a certain number m. Therefore, if (x, y) is a generic point on this line, the equation $m = \frac{y - y_1}{x - x_1}$ will give a general equation of that line.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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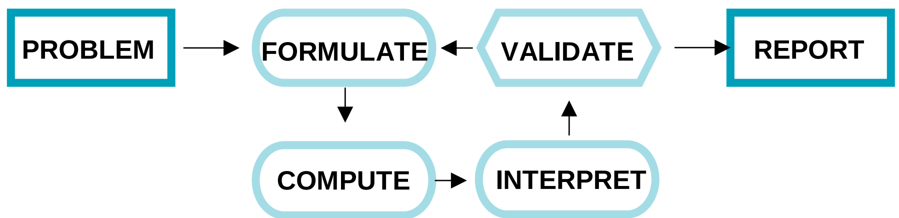
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum

with Instructional Supports

Algebra 1 Course

STANDARDS	MODEL CURRICULUM
Number and Quantity **QUANTITIES** **Reason quantitatively and use units to solve problems.** **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★ **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.★ **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★	**Expectations for Learning** In elementary grades, students use units for distance, time, money, mass, etc. In grades 6, 7, and 8, students work with rates, especially speed, as a quotient of measurements. In this cluster, students extend the use of units to more complicated applications including rates, formulas, interpretation of scale and origin in graphs, data displays, and related applications. Next, students will apply modeling within the context of the algebra concepts studied and begin to develop strategies to solve more complicated mathematical problems. **ESSENTIAL UNDERSTANDINGS** • Units are necessary when representing quantities in a modeling situation to make sense of the problem in context. • A particular quantity can be represented with units from multiple systems of measurement. • Quantities in different units of measure can be compared using equivalent units. • Derived quantities are calculated by multiplying or dividing known quantities, along with their units, e.g., 40 miles in 8 hours is 5 miles per hour. • Quantities can be converted within a system of units, e.g., feet to inches, and between two systems of units, e.g., feet to meters. • There are some contexts in which the origin of a graph or data display is essential to show, and there are other contexts in which the origin of a graph or data display where it is common to omit the origin, e.g., stock prices over time. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Determine reasonableness of results.
- Attend to the meaning of quantities.
- Consider mathematical units involved in a problem.

INSTRUCTIONAL FOCUS

- When modeling, consider the scale when choosing or deriving suitable units.
- Choose a level of accuracy appropriate for the given context.
- Convert measurements within a system of units, e.g., convert 4.6 feet to inches and feet, or between a two systems of units, e.g., convert 4.6 feet to meters.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 1, pages 3-4

CONNECTIONS ACROSS STANDARDS

- Create equations that describe numbers or relationships (A.CED.1-4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **SEEING STRUCTURE IN EXPRESSIONS** **Interpret the structure of expressions.** **A.SSE.1**. Interpret expressions that represent a quantity in terms of its context. ★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, to factor* $3x(x - 5) + 2(x - 5)$*, students should recognize that the "*$x - 5$*" is common to both expressions being added, so it simplifies to* $(3x + 2)(x - 5)$*; or see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*	**Expectations for Learning** Students build expressions in grades K-5 with arithmetic operations. As they move into the middle grades and progress through high school, students build expressions with algebraic components, beginning with linear, exponential, and quadratic expressions. In later courses, they build algebraic expressions with polynomial, rational, radical, and trigonometric expressions. In this cluster, they focus on interpreting the components of linear, exponential, and quadratic expressions and their meaning in mathematical and real-world contexts. Also, students determine when rewriting or manipulating expressions is helpful in order to reveal different insights into a mathematical or real-world context. **ESSENTIAL UNDERSTANDINGS** • An expression is a collection of terms separated by addition or subtraction. • A term is a product of a number and a variable raised to a nonnegative integer exponent. • Components of an expression or expressions within an equation may have meaning in a mathematical context, e.g., $y = mx + b$, b represents the y-intercept; $b^2 - 4ac$ in the quadratic formula indicates the number and nature of solutions to the equation. • Components of an expression may have meaning in a real-world context, e.g., in data surcharges, $60 + 0.05x$ the 60 represents the fixed costs and the 0.05 represents the cost per unit of data. • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. **MATHEMATICAL THINKING** • Attend to the meaning of quantities. • Use precise mathematical language. • Apply grade-level concepts, terms, and properties. • Look for and make use of structure. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Identify the components, such as terms, factors, or coefficients, of an expression and interpret their meaning in terms of a mathematical or real-world context.
- Explain the meaning of each part of an expression, including linear, simple exponential, and quadratic expressions, in a mathematical or real-world context.
- Analyze an expression and recognize that it can be rewritten in different ways.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 1, Number 1, page 3
- Algebra 1, Number 4, pages 9-10

CONNECTIONS ACROSS STANDARDS

- Write expressions in equivalent forms (A.SSE.3).
- Create equations in one or two variables (A.CED.1-2).
- Interpret expressions for functions in terms of the situations they model (F.LE.5).
- Interpret linear models (S.ID.7).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **SEEING STRUCTURE IN EXPRESSIONS** **Write expressions in equivalent forms to solve problems.** **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Factor a quadratic expression to reveal the zeros of the function it defines. **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.*	**Expectations for Learning** Previously, students combine like terms and recognize equivalent expressions. In this cluster, students focus on the form of an expression that is most useful for a particular purpose. Students rewrite quadratic expressions by factoring and completing the square, and they use these forms to analyze the graphs of the functions they define. Students also rewrite exponential expressions using properties of exponents using integer exponents. In Algebra 2, students use these skills to analyze higher degree polynomial functions. **ESSENTIAL UNDERSTANDINGS** • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. • The factored form of a quadratic expression reveals the zeros of the function it defines. • The vertex form of a quadratic expression reveals the vertex and the maximum or minimum value of the function it defines. • Completing the square of a quadratic expression generates the vertex form of a quadratic expression. • Understanding the properties of exponents is essential for rewriting exponential expressions. **MATHEMATICAL THINKING** • Plan a solution pathway. • Determine the appropriate form of an expression in context. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** **For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients, and limit exponential expressions to expressions with integer exponents.* • Determine the appropriate equivalent form of an expression for a given purpose. • Factor a quadratic expression so that the zeros of the function it defines can be identified. • Complete the square for a quadratic expression to identify the vertex and maximum or minimum value of the function it defines. • Rewrite exponential expressions by using properties of exponents. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 4, pages 9-10 **CONNECTIONS ACROSS STANDARDS** • Interpret key features of graphs (F.IF.4). • Interpret the structure of expressions (A.SSE.1-2). • Analyze functions using different representations (F.IF.8).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS** **Perform arithmetic operations on polynomials.** **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **a.** Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)	**Expectations for Learning** In previous courses, students develop an understanding of the properties of integers as a number system under the operations of addition, subtraction, and multiplication. They also learn to combine like terms and simplify linear expressions. In this cluster, students explore the commonalities and differences between integers and polynomials regarding the operations of addition, subtraction, and multiplication. Students will also simplify linear and quadratic expressions, or those that simplify to linear or quadratic. In Algebra 2/Math 3, students extend these ideas to include higher-degree polynomials. **ESSENTIAL UNDERSTANDINGS** • Polynomials form a system (like the integers) in which addition, subtraction, and multiplication always result in another polynomial, but sometimes division does not. **MATHEMATICAL THINKING** • Compute accurately and efficiently. • Use different properties of operations flexibly. • Recognize and apply mathematical concepts, terms, and their properties. • Draw a picture or create a model to represent mathematical thinking. **INSTRUCTIONAL FOCUS** • Add, subtract, and multiply polynomial expressions, focusing on those that simplify to linear or quadratic expressions. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 4, pages 9-10 **CONNECTIONS ACROSS STANDARDS** • Interpret the structure of expressions (A.SSE.1).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **CREATING EQUATIONS** **Create equations that describe numbers or relationships.** **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1) *Continued on next page*	**Expectations for Learning** In middle school, students create simple equations and simple inequalities and use them to solve problems. In this cluster, students extend this knowledge to write equations and inequalities for more complicated situations, focusing on linear, simple exponential, and quadratic equations. Students also rearrange formulas to highlight a particular variable. In Algebra 2, students model even more complicated situations. *Note: Simple exponential functions include integer exponents only.* **ESSENTIAL UNDERSTANDINGS** • Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts. • The graphical solution of a system of equations or inequalities is the intersection of the graphs of the equations or inequalities. • Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given. • A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations. **MATHEMATICAL THINKING** • Create a model to make sense of a problem. • Represent the concept symbolically. • Plan a solution pathway. • Determine the reasonableness of results. • Consider mathematical units and scale when graphing. *Continued on next page*

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A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★

a. Focus on formulas in which the variable of interest is linear or square. *For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle* $A = (\pi)r^2$ *to highlight radius r.* (A1)

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Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship. Focus on situations where the equations will be linear, exponential, and quadratic.
- For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales. Focus on situations where the equations will be linear, exponential, and quadratic.
- Identify the constraints implied by the scenario, and represent them with equations or inequalities.
- Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario.
- Solve formulas for a given variable.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 1, Number 1, pages 3-4
- Algebra 1, Number 4, pages 9-10

CONNECTIONS ACROSS STANDARDS

- Interpret the structure of expressions (A.SSE.1).
- Solve equations and inequalities in one variable (A.REI.3).
- Interpret parameters of linear or exponential functions (F.LE.5).
- Represent and interpret equations and inequalities (including systems) with two variables graphically (A. REI.10).
- Build a function that models a relationship between two quantities (F.BF.1).
- Interpret the slope and intercept of a linear model (S.ID.7).
- Solve systems of equations (A.REI.6).
- Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.1).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM

Algebra
REASONING WITH EQUATIONS AND INEQUALITIES
Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
	Expectations for Learning
	In previous courses, students solve simple equations using a variety of methods and investigate whether a linear equation (8.EE.7) or a system of linear equations (8.EE.8) has one solution, infinitely many solutions, or no solutions. In this cluster, students explain the process for finding a solution for any type of simple equation. Similar to proofs in Geometry, students provide reasons for the steps they follow to solve an equation. In Algebra 2/Math 3, students solve simple rational and radical equations and explain why extraneous solutions may arise.
	
	ESSENTIAL UNDERSTANDINGS
	• Solving equations is a process of reasoning based on properties of operations and properties of equality.
	• Assuming no errors in the equation-solving process,
	  o A result that is false (e.g., 0 = 1) indicates the initial equation must have had no solutions; and
	  o A result that is always true (e.g., 0 = 0) indicates the initial equation must have been an identity.
	• Adding or subtracting the same value or expression to both sides of an equation results in an equivalent equation.
	• Multiplying or dividing both sides by the same value or expression (except by 0) results in an equivalent equation.
	• The Addition Property of Equality and Subtraction Property of Equality can be used interchangeably, since subtracting a number is the same as adding its opposite.
	• The Multiplication Property of Equality and the Division Property of Equality can be used interchangeably (except when multiplying by 0), since dividing a number is the same as multiplying the number by its inverse.
	• Squaring both sides of an equation introduces new solutions; thus, when taking the square root of both sides of an equation two possible equations must be considered.
	Continued on next page

	Expectations for learning, continued **MATHEMATICAL THINKING** • Explain mathematical reasoning. • Plan a solution pathway. **INSTRUCTIONAL FOCUS** *Note: Although, rote memorization of the names of the properties is not encouraged, it is expected for teachers to use formal language so that students gain familiarity and are able to recognize and apply the correct terminology.* • Justify the steps in solving an equation. • Solve equations in which there is one solution; equations in which there is no solution; and equations in which there are infinitely many solutions. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 1, pages 3-4 **CONNECTIONS ACROSS STANDARDS** • Solve linear equations and inequalities in one variable (A.REI.3). • Create equations that describe numbers or a relationship (A.CED.1).

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STANDARDS	MODEL CURRICULUM
Algebra **REASONING WITH EQUATIONS AND INEQUALITIES** **Solve equations and inequalities in one variable.** **A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **A.REI.4** Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (+) **c.** Derive the quadratic formula using the method of completing the square.	**Expectations for Learning** In previous courses, students solve linear equations and inequalities. In this cluster, students extend this knowledge to solve equations with numeric and letter coefficients. Students also solve quadratic equations (with real solutions) using a variety of methods. In other standards, students learn to factor quadratics; this cluster builds on that idea to solve quadratic equations with the Zero Product Property. In Algebra 2, students use these skills to solve more complicated equations. **ESSENTIAL UNDERSTANDINGS** • An appropriate solution path can be determined depending on whether the equation is linear or quadratic in the variable of interest. • Quadratic equations and expressions can be transformed into equivalent forms, leading to different solution strategies, including inspection, taking square roots, completing the square, applying the quadratic formula, or utilizing the Zero Product Property after factoring. • When the coefficients of the variable of interest are letters, the solving process is the same as when the coefficients are numbers. • The discriminant can show the nature and number of solutions a quadratic has. • (+) The quadratic formula is derived from the process of completing the square. **MATHEMATICAL THINKING** • Generalize concepts based on properties of equality. • Solve routine and straightforward problems accurately. • Plan a solution pathway. • Solve math problems using appropriate strategies. • Solve multi-step problems accurately. • (+) Use formal reasoning with symbolic representation. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

**For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*

- Recognize when an equation or inequality is linear in one variable, and plan a solution strategy.
- Solve linear equations and inequalities with coefficients represented by letters.
  - For inequalities, graph solutions sets on a number line.
- Solve compound linear inequalities in one-variable.
  - Graph solution sets on a number line.
- Recognize when an equation is quadratic in one variable, and choose an appropriate solution strategy:
  - using inspection, e.g., $(x-3)^2 = 0$ or $x^2 = -5$;
  - taking square roots, e.g., $x^2 = 8$;
  - using Zero-Product Property after factoring;
  - completing the square; or
  - applying the quadratic formula.
- Determine if a quadratic functions has one solution, two solutions, or no real solutions based on the discriminant.
- (+) Formally derive the quadratic formula using completing the square.

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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 1, pages 3-4 • Algebra 1, Number 4, pages 9-10 **CONNECTIONS ACROSS STANDARDS** • Understand solving equations as a process of reasoning (A.REI.1). • Rearrange formulas to highlight a quantity of interest (A.CED.4). • Interpret the structure of expression (A.SSE.1). • Use the structure of an expression to identify ways to rewrite it (A.SSE.2). • Write expressions in equivalent forms to solve problems (A.SSE.3). • Create equations in one variable (A.CED.1). • Graph solutions to a linear inequality on a coordinate plane. (A.REI.12)

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Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **REASONING WITH EQUATIONS AND INEQUALITIES** **Solve systems of equations.** **A.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A.REI.6** Solve systems of linear equations algebraically and graphically. **a.** Limit to pairs of linear equations in two variables. (A1, M1) **A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	**Expectations for Learning** In previous courses, students solve systems of linear equations graphically with an emphasis on the meaning of the solution. In this cluster, students solve systems of linear and quadratic equations in two variables graphically and algebraically, with a focus on the meaning of a solution to a system of equations. In Algebra 2, students solve systems of equations in three variables. Students who plan to take advanced mathematics courses (+) will represent systems of equations with matrices and use inverse matrices to solve the system. **ESSENTIAL UNDERSTANDINGS** • The graph of a linear equation is the set of ordered pairs that make the equation true. Therefore, multiplying that equation by a non-zero constant produces an equivalent equation, which has the same set of ordered pairs that make the equation true. • If a system of equations in two variables has a unique solution, then the sum of one equation and a (non-zero) multiple of the other equation also has that same solution. • The graphical solution to a system of equations in two variables is the intersection of the equations when graphed. • The solution to a system of equations in two variables is the set of ordered pairs that satisfies both equations. • A system of two linear equations can have no solutions, one solution, or infinitely many solutions. • A system of a linear equation and a quadratic equation can have no solutions, one solution, or two solutions. **MATHEMATICAL THINKING** • Determine reasonableness of results using informal reasoning. • Solve multi-step problems accurately. • Plan a solution pathway. • Use technology strategically to deepen the understanding. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

Note: For Algebra 1 – A.REI.7, the example in the standards is not appropriate, as students do not know equations of circles. Instead, use systems with an equation of a line and an equation of a parabola. In Geometry and Algebra 2, systems with an equation of a circle can be included.

- Substitute a solution into the original system and a manipulation of the system to show solutions are the same.
- Solve a system of linear equations in two variables algebraically using substitution, algebraically using elimination, and by graphing.
- Solve a system of a linear equation and a quadratic equation in two variables algebraically using substitution and by graphing.
- Discuss the efficiency and effectiveness of various methods of solving systems of equations.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Algebra 1, Number 2, pages 5-7
- Algebra 1, Number 4, pages 9-10

CONNECTIONS ACROSS STANDARDS

- Solve linear and quadratic equations in one variable (A.REI.3-4).
- Graph linear and quadratic models (F.IF.4, 7).
- Rearrange formulas (A.CED.4).
- Solve systems of equations and inequalities graphically (A. REI.10-12).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM

Algebra
REASONING WITH EQUATIONS AND INEQUALITIES
Represent and solve equations and inequalities graphically.
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equation *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of the equation *f(x)* = *g(x)*; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
	Expectations for Learning
	In prior courses, students graph linear relationships and identify slope and intercepts. In this cluster, students extend this knowledge to include the idea that a graph represents all of the solutions of an equation. Students use graphs and tables of equations in two variables to approximate solutions to equations in one variable. They also graph solutions to linear inequalities in two variables. In Algebra 2 or Math 3, students similarly study the relationship between the graph and the solutions of rational, radical, absolute value, polynomial, and exponential equations.
	
	ESSENTIAL UNDERSTANDINGS
	• A point of intersection of any two graphs represents a solution of the two equations that define the two graphs.
	• An equation in one variable can be rewritten as a system of two equations in two variables, by thinking of each side of the equation as a function, i.e., writing *y* = left hand side and *y* = right hand side.
	    o The approximate solution(s) to an equation in one variable is the *x*-value(s) of the intersection(s) of the graphs of the two functions.
	    o Two-variable graphical and numerical (tabular) techniques to solve an equation with one variable always work and are particularly useful when algebraic methods are not applicable, e.g., $x^2 - 3x + 2 = 2^x$.
	• A half plane represents the solutions of a linear inequality in two variables.
	• The intersection of two half planes represents the solution set to two inequalities in two variables.
	
	MATHEMATICAL THINKING
	• Use technology strategically to deepen understanding.
	• Plan a solution pathway.
	• Create a model to make sense of a problem.
	Continued on next page

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Rewrite a one-variable equation as two separate functions and use the *x*-coordinate of their intersection point to determine the solution of the original equation. • Approximate intersections of graphs of two equations using technology, tables of values, or successive approximations (focus on equations with linear, exponential, and quadratic expressions). • Graph the solution set of a linear inequality in two variables. • Graph the solution set of a system of linear inequalities in two variables. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Algebra 1, Number 2, pages 5-7 **CONNECTIONS ACROSS STANDARDS** • Solve equations in one variable (A.REI.3-4). • Create equations in two variables (A.CED.2). • Graph functions expressed symbolically (F.IF.7). • Solve systems of equations graphically (A.REI.6-7). • Analyze functions using different representations (F.IF.9).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Functions **INTERPRETING FUNCTIONS** **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$. **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.	**Expectations for Learning** In the eighth grade, students have learned a semi-formal definition of a function and know that a function pairs an input value with an output value. Eighth grade students do not use function notation nor the terms domain and range. In this cluster, students will now expand their understanding of functions to include function notation and the terms domain and range. Also, students will evaluate and interpret functions, including sequences as functions. Distinguishing between relations and functions is not a primary focus. This cluster is the foundation for all future work with functions. **ESSENTIAL UNDERSTANDINGS** • Function notation illustrates a formal connection between inputs and outputs. • Functions can be tied to real-world scenarios given by tables, graphs, equations, or verbal descriptions. • Function notation *f(x)* is shorthand for the output of *f* when the input is *x*. • Function notation, *f(x),* is a new representation for students and is articulated as "*f* of *x*", and it is not related to multiplication. • Sequences are functions whose domain is a subset of the integers, paying careful attention to how a sequence is indexed. For example, the sequence may be indexed from 0 to *n*, from 1 to $n - 1$, or something else. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to describe mathematical reasoning. • Represent a concept symbolically. • Determine reasonableness of results. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Make connections among different representations (tables, graphs, symbols, and verbal descriptions) of functions, focusing on linear, quadratic, and exponential functions. • Solve problems with functions represented in tables, graphs, symbols, and verbal descriptions. • Explain function notation in a real-world context. For example, if *f(x)* represents the height of particle at *x* seconds, then *f*(1) represents the height of the particle at 1 second. • Interpret number patterns as sequences and their graphs as discrete points. When the number pattern arises from a context, consider whether it is appropriate to "connect the dots." • Use function notation to specify sequences, both explicitly and recursively. (Subscript notation is not required.) • Relate linear functions to arithmetic sequences, and relate exponential functions to geometric sequences. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1,Number 2, pages 5-7 **CONNECTIONS ACROSS STANDARDS** • Build a function that models a relationship between two quantities (F.BF.1a, 2). • Build new functions from existing functions (F.BF.4a). • Interpret expressions for functions in terms of the situation they model (F.LE.5). • Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.2). • Represent and solve equations and inequalities graphically (A.REI.10).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Functions **INTERPRETING FUNCTIONS** **Interpret functions that arise in applications in terms of the context.** **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)	**Expectations for Learning** In eighth grade, students model linear relations between two quantities; analyze graphs to determine where they are increasing and decreasing; and determine if relations are linear or non-linear. In this cluster, students interpret additional key features of the graphs and tables of linear, quadratic, and exponential functions only. They also determine the domain of a function by looking at a graph or table. In a real-life scenario students can find the restrictions on the domain. In Algebra 2, students extend identifying and interpreting key features of functions to include periodicity. Students also have to select appropriate functions that model the data presented. Average rate of change over a specific interval will also be included in Algebra 2. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, and then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem. • The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols. • Functions can have continuous or discrete domains. • A quadratic function is symmetrical about its axis of symmetry. **MATHEMATICAL THINKING** • Connect mathematical relationships to contextual scenarios. • Attend to meaning of quantities. • Determine reasonableness of results. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

**Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*

- For linear functions represented as tables, graphs, or verbal descriptions, interpret intercepts and rates of change in the contexts of the problem.
- For exponential functions, interpret intercepts, growth/decay rates, and end behaviors in the contexts of the problems, given tables, graphs, and verbal descriptions.
- For quadratic functions, interpret intercepts; maximum or minimum; symmetry; intervals of increase or decrease; and end behavior, given tables, graphs, and verbal descriptions.
- Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required).
- Determine whether to connect points on a graph based on the context (continuous vs. discrete domain).
- Demonstrate understanding of domain in the context of a real-world problem.
- Compare the key features of quadratic functions to linear and exponential functions. For example:
    - Linear functions are either always increasing, decreasing, or constant.
    - Exponential functions are either always increasing or decreasing.
    - Quadratic functions increase to a maximum then decrease or decrease to a minimum then increase.

Continued on next page

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 2, pages 5-7 • Algebra 1, Number 5, pages 11-12 **CONNECTIONS ACROSS STANDARDS** • Create equations that describe numbers or relationships (A.CED.2a, b). • Represent and solve equations and equations and inequalities graphically (A.REI.10). • Understand the concept of a function, and use function notation (F.IF.1-3). • Graph linear functions and indicate intercepts (F.IF.7a). • Graph quadratic functions and indicate maxima and minima (F.IF.7b). • Graph simple exponential functions, indicating intercepts, and end behavior (F.IF.7e). • Interpret expressions for functions in terms of the situation they model (F.LE.5). • Analyze functions using different representations (F.IF.9b). • Interpret linear models (S.ID.7).

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Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Functions **INTERPRETING FUNCTIONS** **Analyze functions using different representations.** **F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★ **a.** Graph linear functions and indicate intercepts. (A1, M1) **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) **e.** Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) **i.** Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1) *Continued on next page*	**Expectations for Learning** In eighth grade, students graph and write linear functions, but their knowledge of key features of functions is limited to slope and *y*-intercept. They are exposed to non-linear functions and can distinguish between linear and non-linear functions. In this cluster, students graph linear, quadratic, and exponential functions given a symbolic representation and indicate intercepts and end behavior. They compare linear, quadratic, and exponential functions given various representations. In Algebra 2, students graph polynomial, square root, cube root, trigonometric, piecewise-defined, (+) rational, and (+) logarithmic functions. They identify and interpret key features (as applicable) including intercepts, end behavior, period, midline, amplitude, symmetry, asymptotes, maxima/minima, and zeros. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, and then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • The graph of a linear function shows intercepts and rate of change. • The graph of an exponential function shows the *y*-intercept and end behaviors. • The graph of a quadratic function shows intercepts and maximum or minimum. • Function families have commonalities in shapes and features of their graphs. • The factored form of a quadratic function reveals the zeros of the function (i.e., the *x*-intercepts of the graph); the vertex form of a quadratic function reveals the maximum or minimum of the function; the standard form of a quadratic function reveals the *y*-intercept of the graph. • Different representations (graphs, tables, symbols, verbal descriptions) illuminate key features of functions and can be used to compare different functions. • More generally, writing a function in different ways can reveal different features of the graph of a function.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change*[G] *in functions such as* $y = (1.02)^t$, *and* $y = (0.97)^t$ *and classify them as representing exponential growth or decay.* (A2, M3) **i.** Focus on exponential functions evaluated at integer inputs. (A1, M2) **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)	**Expectations for Learning, continued** **MATHEMATICAL THINKING** • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Analyze a mathematical model. **INSTRUCTIONAL FOCUS** **Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.* • Given symbolic representations of linear, quadratic, and exponential functions, create accurate graphs showing all key features. • Identify the key features of the graph of a quadratic function by factoring, using the quadratic formula, or completing the square • Compare and contrast linear, quadratic, and exponential functions given by graphs, tables, symbols, or verbal descriptions. • Determine the zeros of a quadratic function by factoring, using the quadratic formula, or completing the square. • Use different forms of quadratic functions (standard form, vertex form, factored form) to reveal different features. • Explore the relationship of the symbolic representation of a function and its graph by adjusting parameters. • *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 2, pages 5-7 • Algebra 1, Number 5, pages 11-12 **CONNECTIONS ACROSS STANDARDS** • Interpret functions that arise in applications in terms of the context (F.IF.4). • Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line) (A.REI.10). • Construct and compare linear, quadratics, and exponential models and solve problems (F.LE.1-2). • Interpret expressions for functions in terms of the situation they model (F.LE.5). • Solve quadratic equations in one variable (A.REI.4). • Write expressions in equivalent forms to solve problems (A.SSE.3). • Build a function that models a relationship between two quantities (F.BF.1).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Functions **BUILDING FUNCTIONS** **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from context. **i.** Focus on linear and exponential functions. (A1, M1) **ii.** Focus on situations that exhibit quadratic or exponential relationships. (A1, M2) **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	**Expectations for Learning** In the eighth grade, students create functions to model relationships between two quantities. In this cluster, students write linear, exponential, and quadratic functions symbolically given the relationship between two quantities. Relationships between quantities could be given as tables, graphs, or within a context. Students also write explicit and recursive rules for arithmetic and geometric sequences. In Algebra 2, students build functions from other functions allowing students to model more complex situations. This includes combining functions of various types using arithmetic operations or (+) composition. **ESSENTIAL UNDERSTANDINGS** • Functions can be written as explicit expressions, recursive processes, and in other ways. • An arithmetic sequence (informally, an addition pattern) has a starting term and a common difference between terms. • A geometric sequence (informally, a multiplication pattern) has a starting term and a common ratio between terms. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. • Some sequences can be defined recursively or explicitly, while others cannot be defined by a formula. • The relationships between quantities can be modeled with functions that are linear, exponential, quadratic, or none of these. **MATHEMATICAL THINKING** • Make and modify a model to represent mathematical thinking. • Discern and use a pattern or structure. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Model relationships with linear functions, which may be arithmetic sequences, using tables, graphs, symbols, and words within a context.
- Model relationships with exponential functions, which may be geometric sequences, using tables, graphs, symbols, and words within a context.
- Model relationships with quadratic functions using tables, graphs, symbols, and words within a context.
- Model relationships that are not linear, exponential, or quadratic using tables, graphs, symbols, and words within a context.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 2, pages 5-7
- Algebra 1, Number 5, pages 11-12

CONNECTIONS ACROSS STANDARDS

- Create equations that describe numbers or relationships (A.CED.2).
- Fit a linear function for a scatterplot that suggests a linear association (S.ID.6c).
- Interpret linear models (S.ID.7).
- Construct and compare linear, quadratic, and exponential models, and solve problems (F.LE.1).
- Analyze functions using different representations (F.IF.8).

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Functions

BUILDING FUNCTIONS

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)

a. Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)

F.BF.4 Find inverse functions.

a. Informally determine the input of a function when the output is known. (A1, M1)

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Expectations for Learning

In eighth grade, students learn that functions map inputs to outputs. In this cluster, students informally reverse this to find the input of a function when the output is known. In later classes, (+) some students more fully develop the concepts, procedures, and notation for inverses of functions.

In eighth grade, students attend to slope and intercepts for graphs of linear functions, without explicit attention to transformations of the graphs. In this cluster, students transform graphs of quadratic functions. Transformations of quadratic functions can be interpreted conveniently by observing the effect on the vertex and whether the parabola opens up or down. Students do not perform transformations of the form $f(kx)$. In Algebra 2, students perform all types of transformations for various function families and recognize even and odd functions.

ESSENTIAL UNDERSTANDINGS

- Sometimes the input of a function can be found when the output is given.
- Vertical and horizontal transformations of $y = x^2$ are as follows:
  - horizontal shift: $g(x) = (x - h)^2$;
  - vertical stretch/shrink: $g(x) = ax^2$ when $a > 0$;
  - vertical shift: $g(x) = x^2 + k$;
  - reflection across the x-axis: $g(x) = -x^2$; and
  - a combination of transformations: $g(x) = a(x - h)^2 + k$.

MATHEMATICAL THINKING

- Explain mathematical reasoning.

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	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** ** Limit to situations where inverse values are unique. Exclude formal notation; exclude finding the inverse algebraically; exclude switching x and y; exclude reflecting about the line $y = x$. Transformations occur in the quadratic expression rather than inside the function notation.* • Use graphs and tables to find the input value of a function when given an output, and interpret the values in context. • Transform graphs of quadratic functions and interpret the transformations geometrically. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 2, pages 5-7 • Algebra 1, Number 5, pages 11-12 **CONNECTIONS ACROSS STANDARDS** • Understand the concept of a function and use function notation (F.IF.1-2). • Analyze functions using different representations (F.IF.7a, b, and e).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
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STANDARDS	MODEL CURRICULUM
Functions **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2)	**Expectations for Learning** In eighth grade, students interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. Students also see examples of non-linear functions and learn and apply the properties of integer exponents. In Algebra 1, students compare across linear, exponential, and quadratic functions. **ESSENTIAL UNDERSTANDINGS** • Linear functions have a constant additive change. • Exponential functions have a constant multiplicative change. • Linear and exponential functions both have initial values. • To highlight the constant growth/decay rate, *r*, often expressed as a percentage, exponential functions can be written in the form, $f(n) = a(1 + r)^n$. • To highlight the growth/decay factor, *b*, exponential functions can be written in the form, $f(n) = a(b)^n$. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. • The phrase "eventually exceeds" (F.LE.3) directs the focus towards large values in the domain and the consideration of the base and *y*-intercept of the exponential function and the leading coefficient of the linear or quadratic function. • For large domain values, the growth of linear and quadratic functions is dominated by the leading term. **MATHEMATICAL THINKING** • Represent a concept symbolically. • Make and modify a model to represent mathematical thinking. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Aim toward a multifaceted understanding of additive versus multiplicative change across different representations.
- For linear functions (arithmetic sequences), focus on the constant rate of change across the tables, graphs, contexts, and the explicit and recursive representations.
- For exponential functions (geometric sequences), focus on the constant growth/decay rate (or factor) across the tables, graphs, contexts, and the explicit and recursive representations.
- Use graphs, tables, and contexts to see that as the domain value increases, the values of an exponential function will eventually exceed the corresponding values of a linear or quadratic function.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 2, pages 5-7
- Algebra 1, Number 5, pages 11-12

CONNECTIONS ACROSS STANDARDS

- Build a function that models a relationship between two quantities (F.BF.1a, 2).
- Interpret functions that arise in applications in terms of the context (F.IF.4b, 5b).
- Analyze functions using different representations (F.IF.7a, b, and e).
- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6c).
- Interpret linear models (S.ID.7).
- Interpret the structure of expressions (A.SSE.1).
- Interpret the parameters in a linear or exponential function in terms of a context (F.LE.5).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Functions **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** **Interpret expressions for functions in terms of the situation they model.** **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.★	**Expectations for Learning** This standard does not present new expectations for student learning. Rather, it emphasizes important habits to complement F.LE.1-3. In this cluster, students connect their understanding of the defining characteristics of linear functions (initial value and rate of change) to the defining characteristics of exponential functions (initial value and growth rate/growth factor) and by interpreting them in the context of a real-world problem. **ESSENTIAL UNDERSTANDINGS** • Linear functions have a constant additive change. • Exponential functions have a constant multiplicative change. • Linear and exponential functions both have initial values. • To highlight the constant growth/decay rate, r, often expressed as a percentage, exponential functions can be written in the form, $f(n) = a(1 + r)^n$. • To highlight the growth/decay factor, b, exponential functions can be written in the form, $f(n) = a(b)^n$. • An arithmetic sequence is a linear function, and a geometric sequence is an exponential function. **MATHEMATICAL THINKING** • Connect mathematical relationships to contextual scenarios. • Use accurate mathematical vocabulary to describe mathematical reasoning. • Attend to meaning of quantities. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. **INSTRUCTIONAL FOCUS** • For linear functions (arithmetic sequences), focus on the constant rate of change across the tables, graphs, contexts, and the explicit and recursive representations. • For exponential functions (geometric sequences), focus on the constant growth/decay rate (or factor) across the tables, graphs, contexts, and the explicit and recursive representations.

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 2, pages 5-7 **CONNECTIONS ACROSS STANDARDS** • Build a function that models a relationship between two quantities (F.BF.1a, 2). • Interpret functions that arise in applications in terms of the context (F.IF.4-5). • Analyze functions using different representations (F.IF.7a, e). • Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.6c). • Interpret linear models (S.ID.7). • Interpret the structure of expressions (A.SSE.1).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
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STANDARDS	MODEL CURRICULUM
Statistics and Probability **INTERPRETING CATEGORICAL AND QUANTITATIVE DATA** **Summarize, represent, and interpret data on a single count or measurement variable.** **S.ID.1** Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★ **S.ID.2** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★ **S.ID.3** In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★	**Expectations for Learning** In middle school students learn about the framework of the GAISE model of statistical problem solving. It consists of four steps: Formulating Questions; Collecting Data; Analyzing Data; and Interpreting Results. Students integrate this model whenever they use statistical reasoning. This process will continue throughout high school as students deepen their statistical reasoning skills. Middle school students create dot plots, histograms, and box plots and draw informal comparisons between two populations using graphs. They also summarize data sets using mean absolute deviation. In Algebra 1/Math 1 students should use and expand their learning to more sophisticated problems and by comparing single or multiple data sets through graphical representations. Standard deviation is a new concept for students, and it builds upon their previous understanding of mean absolute deviation (MAD). In Algebra 2/Math 3**,** students then extend their knowledge of mean and standard deviation from Algebra 1/Math 1 to normal distributions. **The GAISE Model** Students will use the GAISE Model framework for statistical problem solving in all courses. The GAISE Model should not be taught in isolation. Students are building on the framework that was developed in middle school. As students progress through the courses, the learning will move towards a greater level of precision and complexity. Students in middle school start at Level A and move towards Level B. As students progress from Level A to Levels B and C, the learning becomes less teacher-driven and more student-driven. In this cluster students are at Level B moving towards Level C, and Steps 1 and 2 continue to be emphasized with added depth on Steps 3 and 4. "Understanding the statistical concepts of GAISE model Level B enables a student to grow in appreciation that data analysis is an investigative process consisting of formulating their own questions; collecting appropriate data through various sources; analyzing data through graphs and simple summary measures; and interpreting results with an eye toward inference to a population based on a sample" (Guideline for Assessment and Instruction in Statistics Education (GAISE) Report, 2007, page 58). *Continued on next page*

Expectations for Learning, continued

The GAISE Model, continued

Step 1: Formulate the Question
- Students should pose their own statistical question of interest (Level C).
- Students are starting to form questions that allow for generalizations of a population (Level B-C).

Step 2: Collect Data
- Students should begin to use random selection or random assignment (Level B).

Step 3: Analyze Data
- Students measure variability within a single group using MAD, IQR, and/or standard deviation (Level B).
- Students compare measures of center and spread between groups using displays and values (Level B).
- Students describe potential sources of error (Level B).
- Students understand and use particular properties of distributions as tools of analysis moving toward using global characteristics of distributions (Level B-C).

Step 4: Interpret Results
- Students acknowledge that looking beyond the data is feasible by interpreting differences in shape, center, and spread (Level B).
- Students determine if a sample is representative of a population and start to move towards generalization (Level B-C).
- Students note the difference between two groups with different conditions (Level B).

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Expectations for Learning, continued

ESSENTIAL UNDERSTANDINGS

- Univariate quantitative data can be represented using dot plots, box plots, and histograms.
- Mean and median are approximately equal for symmetric distributions, but tend to be different for nonsymmetric distributions.
- Standard deviation is a measure of variation from the mean (spread).
- Extreme values (outliers) have an effect on the shape, center, and spread of a distribution.
    - The median and interquartile range are appropriate measures of center and spread if the distribution is extremely skewed or has outliers.
    - The mean and standard deviation are appropriate measures of center and spread if the distribution is not skewed and has no extreme outliers.

MATHEMATICAL THINKING

- Use accurate and precise mathematical vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve real-world and statistical problems.
- Use appropriate tools to display and analyze data.

Continued on next page

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Compare the mean to the median of the same data set and relate them to the shape of the distribution (symmetric, skewed).
- Develop the formula for and a conceptual understanding of standard deviation by building on the conceptual understanding and formula of mean absolute deviation.
- Compare two or more distributions based upon their means and standard deviations.
- Explain how outliers affect the mean, the median, and standard deviation.
- Given two or more data sets or graphs, do the following:
    - Compare the shape (symmetric, skewed, uniform).
    - Compare the spread (greater than, less than, equal).
    - Compare the centers (mean, median).
- Interpret the mean, standard deviation, outliers, as well as differences and similarities between two or more sets of data within a context.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Algebra 1, Number 3, page 8

THE GAISE MODEL

- GAISE Model, pages 14 – 15
    - Focus of the cluster for Algebra 1/Math 1 is Level B, pages 37-60

CONNECTIONS ACROSS STANDARDS

- Summarize, represent, and interpret data on two categorical and quantitative variables (S.ID.5-6).

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STANDARDS	MODEL CURRICULUM

Statistics and Probability
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA
Summarize, represent, and interpret data on two categorical and quantitative variables.
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★
S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★
c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)
	Expectations for Learning
	For this cluster, the GAISE Model framework continues to be used: Formulating Questions; Collecting Data; Analyzing Data; and Interpreting Results. In the middle grades, students visually approximate a linear model and informally judge its goodness of fit. In Algebra 1/Math 1, students extend this knowledge to find the equation of a linear model, with and without technology. They will also use more precise language to describe the relationship between variables. In Algebra 2/Math 3, concepts extend to quadratic and exponential functions as well as working with residuals.
	
	The learning at this level is at the developmental Level B. See pages 62-63 for more information on Level B.
	
	ESSENTIAL UNDERSTANDINGS
	Note: Students should be able to talk sensibly about the meanings of joint, marginal, and conditional frequencies within a context but should not be held responsible for precise usage of this vocabulary.
	• Row totals and column totals constitute the marginal frequencies.
	• Individual table entries represent joint frequencies.
	• A relative frequency is found by dividing the frequency count by the total number of observations for a whole set or subset.
	    o A marginal relative frequency is calculated by dividing the row (or column) total by the table total.
	    o A joint relative frequency is calculated by dividing the table entry by the table total.
	    o A conditional relative frequency is calculated by restricting to one row or one column of the table.
	• Relative frequencies are useful in considering association between two categorical variables.
	• A linear function can be used as a model for a linear association of two quantitative variables.
	Continued on next page

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use accurate and precise mathematical vocabulary.
- Construct formal and informal arguments to verify claims and justify conclusions.
- Solve real-world and statistical problems.
- Use appropriate tools to display and analyze data.
- Accurately make computations using data.
- Determine reasonableness of predictions.

INSTRUCTIONAL FOCUS

Categorical Data

- Calculate and interpret, within a context, joint, marginal, and conditional relative frequencies.
- Recognize possible relationships (trends) in the context of the data by using percentages from two-way frequency tables.

Quantitative Data

- Describe, within a context, how variables are related in a linear relationship using scatter plots.
- Calculate and interpret, within a context, the slope and *y*-intercept of a linear model, given a set of data or graph, with or without technology.

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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 3, page 8 **THE GAISE MODEL** • GAISE Model, pages 14 – 15 o Focus of this cluster for Algebra 1/Math 1 is Level B moving toward Level C, pages 37-60 **CONNECTIONS ACROSS STANDARDS** • Interpret linear models (S.ID.7-8). • Build a function that models a relationship between two quantities (F.BF.1). • Distinguish between situations that can be modeled with linear and exponential functions (F.LE.1). • Create equations that describe numbers or relationships (A.CED.2).

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STANDARDS	MODEL CURRICULUM

Statistics and Probability
INTERPRETING CATEGORICAL AND QUANTITATIVE DATA
Interpret linear models.
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★
	Expectations for Learning
	In middle school, students interpret the slope and *y*-intercept of a linear model. In Algebra 1/Math 1, students build on this knowledge with more sophisticated problems. Since scales may vary, students require a deeper conceptual understanding of slope. They also need to recognize when the *y*-intercept is not always meaningful in the context of the data. This leads to the computation and interpretation of the correlation coefficient and its interpretation. In Algebra 2/Math 3, students are introduced to and explore the distinction between correlation and causation.
	
	The learning of standard S.ID.7 is at the developmental Level B. The learning of standard S.ID.8 is at developmental Level C. See pages 62-63 for more information on Level B, and see Algebra 2 Model Curriculum for more information on Level C.
	
	ESSENTIAL UNDERSTANDINGS
	• In a linear model, the slope represents the change in the predicted value for every one unit of increase in the independent (*x*) variable.
	• When appropriate, the *y*-intercept represents the predicted value of the dependent variable when $x = 0$.
	• In a linear model, the *y*-intercept may not always be appropriate for the context.
	• The correlation coefficient (*r*) is a measure of the strength of a linear association in the data. Correlation coefficients are between –1 and 1, inclusive.
	    o If *r* is close to 0, then there is a weak correlation.
	    o If *r* is close to 1, then there is a strong correlation with a positive slope.
	    o If r is close to –1, then there is a strong correlation with a negative slope.
	Continued on next page

	Expectations for Learning, continued **MATHEMATICAL THINKING** • Use accurate and precise mathematical vocabulary. • Construct formal and informal arguments to verify claims and justify conclusions. • Solve real-world and statistical problems. • Use appropriate tools to display and analyze data. • Determine reasonableness of predictions. **INSTRUCTIONAL FOCUS** • Given a linear model, interpret the slope and the *y*-intercept within a context. • Compute, with technology, and interpret correlation coefficient (*r*). **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Algebra 1, Number 3, page 8 **THE GAISE MODEL** • GAISE Model, pages 14 – 15 o The focus of S.ID.7 is at Level B for Algebra 1/Math 1, pages 37-60 o The focus of S.ID.8 is at Level C for Algebra 1/Math 1, pages 61-85 **CONNECTIONS ACROSS STANDARDS** • Interpret the structure of functions (F.IF.4). • Construct linear models, and solve problems (F.LE.1-2). • Interpret expressions for functions in terms of context (F.LE.5). • Build a function that models a relationship between two quantities (F.BF.1). • Summarize, represent, and interpret data in two categories and quantitative variables (S.ID.6).

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Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

This section is under revision and will be published in 2018.

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