Ohio's Model Curriculum | Mathematics

with Instructional Supports

Math 2 Course

Ohio | Department of Education

Mathematics Model Curriculum
with Instructional Supports
Math 2 Course

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Introduction

PURPOSE OF THE MODEL CURRICULUM

Just as the standards are required by Ohio Revised Code, so is the development of the model curriculum for those standards. Throughout the development of the standards (2016-17) and the model curriculum (2017-18), the Ohio Department of Education (ODE) has involved educators from around the state at all levels, Pre-K–16. The model curriculum reflects best practices and the expertise of Ohio educators, but it is not a complete a curriculum nor is it mandated for use. The purpose of Ohio's model curriculum is to provide clarity to the standards, a foundation for aligned assessments, and guidelines to assist educators in implementing the standards.

COMPONENTS OF THE MODEL CURRICULUM

The model curriculum contains two sections: Expectations for Learning and Content Elaborations.

Expectations for Learning: This section begins with an introductory paragraph describing the cluster's position in the respective learning progression, including previous learning and future learning. Following are three subsections: Essential Understandings, Mathematical Thinking, and Instructional Focus.

- **Essential Understandings** are the important concepts students should develop. When students have internalized these conceptual understandings, application and transfer of learning results.
- **Mathematical Thinking** statements describe the mental processes and practices important to the cluster.
- **Instructional Focus** statements are key skills and procedures students should know and demonstrate.

Together these three subsections guide the choice of lessons and formative assessments and ultimately set the parameters for aligned state assessments.

Content Elaborations: This section provides further clarification of the standards, links the critical areas of focus, and connects related standards within a grade or course.

COMPONENTS OF INSTRUCTIONAL SUPPORTS

The Instructional Supports section contains the **Instructional Strategies** and **Instructional Tools/Resources** sections which are designed to be fluid and improving over time, through additional research and input from the field. The **Instructional Strategies** are descriptions of effective and promising strategies for engaging students in observation, exploration, and problem solving targeted to the concepts and skills in the cluster of standards. Descriptions of common misconceptions as well as strategies for avoiding or overcoming them and ideas for adapting instructions to meet the needs of all students are threaded throughout. The **Instruction Tools/Resources** are links to relevant research, tools, and technology. In our effort to make sure that our Instructional Supports reflect best practices, this section is under revision and will be published in 2018.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The descriptions of the mathematical practices in this document provide examples of how student performance will change and grow as students engage with and master new and more advanced mathematical ideas across the grade levels.

MP.1 Make sense of problems and persevere in solving them.
Students persevere when attempting to understand the differences between quadratic functions and the linear and exponential functions they studied previously. They create diagrams of geometric problems to help make sense of the problems.

MP.2 Reason abstractly and quantitatively.
Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; of considering the units involved; of attending to the meaning of quantities, not just how to compute them; and of knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.
Students construct proofs of geometric theorems based relationships between sine and cosine of complementary angles.

MP.4 Model with mathematics.
Students apply their mathematical understanding of quadratic functions to real-world problems. They also discover mathematics through experimentation and by examining patterns in data from real-world contexts.

MP.5 Use appropriate tools strategically.
Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the result.

MP.6 Attend to precision.
To avoid the extraneous solutions, students make use of the definition of the solution of the equation by asking, "Does this value make the equation a correct statement?"

MP.7 Look for and make use of structure.
Students develop formulas such as $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. Students see that the expression $5 + (n - 2)^2$ takes the form of 5 plus "something squared," and because "something squared" must be positive or zero, the expression can be no smaller than 5.
Continued on next page

Standards for Mathematical Practice, continued

MP.8 Look for and express regularity in repeated reasoning.

Students understand that when figures are scaled by a factor of k, the effect on their lengths, areas, and volumes remain the same such that they are multiples of k, k^2, and k^3.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing the stopping distance for a car.
- Modeling a savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

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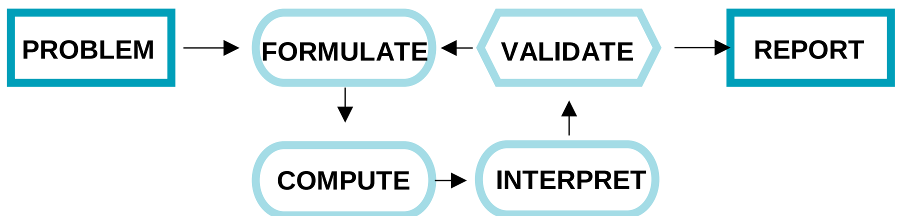
Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Mathematics Model Curriculum
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STANDARDS	MODEL CURRICULUM
Algebra **SEEING STRUCTURE IN EXPRESSIONS** **Interpret the structure of expressions.** **A.SSE.1**. Interpret expressions that represent a quantity in terms of its context. ★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	**Expectations for Learning** Students build expressions in grades K-5 with arithmetic operations. As they move into the middle grades and progress through high school, students build expressions with algebraic components, beginning with linear, exponential, and quadratic expressions. In later courses, they build algebraic expressions with polynomial, rational, radical, and trigonometric expressions. In this cluster, they focus on interpreting the components of linear, exponential, and quadratic expressions and their meaning in mathematical and real-world contexts. They also determine when rewriting or manipulating expressions is helpful in order to reveal different insights into a mathematical or real-world context. **ESSENTIAL UNDERSTANDINGS** • An expression is a collection of terms separated by addition or subtraction. • A term is a product of a number and a variable raised to a nonnegative integer exponent. • Components of an expression or expressions within an equation may have meaning in a mathematical context, e.g., $b^2 - 4ac$ in the quadratic formula indicates the number and nature of solutions to the equation. • Components of an expression may have meaning in a real-world context. • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. **MATHEMATICAL THINKING** • Attend to the meaning of quantities. • Use precise mathematical language • Apply grade-level concepts, terms, and properties. • Look for and make use of structure.

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Identify the components, such as terms, factors, or coefficients, of an expression and interpret their meaning in terms of a mathematical or real-world context.
- Explain the meaning of each part of an expression, including linear, simple exponential, and quadratic expressions, in a mathematical or real-world context.
- Analyze an expression and recognize that it can be rewritten in different ways.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Math 2, Number 2, pages 4-5

CONNECTIONS ACROSS STANDARDS

- Write expressions in equivalent forms (A.SSE.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **SEEING STRUCTURE IN EXPRESSIONS** **Write expressions in equivalent forms to solve problems.** **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Factor a quadratic expression to reveal the zeros of the function it defines. **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **c.** Use the properties of exponents to transform expressions for exponential functions. *For example 8^t can be written as 2^{3t}.*	**Expectations for Learning** Previously, students rewrite exponential expressions using properties of exponents. In Math 2, students rewrite quadratic expressions by factoring and completing the square, and they use these forms to analyze the graphs of the functions they define. In Math 3, students use these skills to analyze higher degree polynomial functions. **ESSENTIAL UNDERSTANDINGS** • Expressions may potentially be rearranged or manipulated in ways to reveal different insights into the mathematical or real-world context. • The factored form of a quadratic expression reveals the zeros of the function it defines. • The vertex form of a quadratic expression reveals the vertex and the maximum or minimum value of the function it defines. • Completing the square of a quadratic expression generates the vertex form of a quadratic expression. • Understanding the properties of exponents is essential for rewriting exponential expressions. **MATHEMATICAL THINKING** • Plan a solution pathway. • Determine the appropriate form of an expression in context. **INSTRUCTIONAL FOCUS** **For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients. Also exponential expressions should be limited to expressions with integer exponents.* • Determine the appropriate equivalent form of an expression for a given purpose. • Factor a quadratic expression so that the zeros of the function it defines can be identified. • Complete the square for a quadratic expression to identify the vertex and maximum or minimum value of the function it defines. • Rewrite exponential expressions by using properties of exponents.

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Math 2, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Interpret key features of graphs (F.IF.4). • Interpret the structure of expressions (A.SSE.1-2). • Analyze functions using different representations (F.IF.8).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

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Algebra

ARITHMETIC WITH POLYNOMIAL AND RATIONAL EXPRESSIONS

Perform arithmetic operations on polynomials.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)

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Expectations for Learning

In previous courses, students develop an understanding of the properties of integers as a number system under the operations of addition, subtraction, and multiplication. They also learn to combine like terms and simplify linear expressions. In this cluster, students explore the commonalities and differences between integers and polynomials regarding the operations of addition, subtraction, and multiplication. Students will also simplify linear and quadratic expressions, or those that simplify to linear or quadratic. In Algebra 2/Math 3, students extend these ideas to include higher-degree polynomials.

ESSENTIAL UNDERSTANDINGS

- Polynomials form a system (like the integers) in which addition, subtraction, and multiplication always result in another polynomial, but sometimes division does not.

MATHEMATICAL THINKING

- Compute accurately and efficiently.
- Use different properties of operations flexibly.
- Recognize and apply mathematical concepts, terms, and their properties.
- Draw a picture or create a model to represent mathematical thinking.

INSTRUCTIONAL FOCUS

- Add, subtract, and multiply polynomial expressions, focusing on those that simplify to linear or quadratic expressions.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Math 2, Number 2, pages 4-5

CONNECTIONS ACROSS STANDARDS

- Interpret the structure of expressions (A.SSE.1).

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INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

This section is under revision and will be published in 2018.

Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
Algebra **CREATING EQUATIONS** **Create equations that describe numbers or relationships.** **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ **b.** Focus on applying simple quadratic expressions. (A1, M2) **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **b.** Focus on applying simple quadratic expressions. (A1, M2) **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★ **c.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r.* (M2)	**Expectations for Learning** In Math 1, students create linear and exponential equations and use them to solve problems. In this cluster, students extend this knowledge to include writing quadratic equations. Students also continue to rearrange formulas to highlight a particular variable. In Math 3, students model even more complicated situations. **ESSENTIAL UNDERSTANDINGS** • Regularity in repeated reasoning can be used to create equations that model mathematical or real-world contexts. • The graphical solution of a system of equations or inequalities is the intersection of the graphs of the equations or inequalities. • Solutions to an equation, inequality, or system may or may not be viable, depending on the scenario given. • A formula relating two or more variables can be solved for one of those variables (the variable of interest) as a shortcut for repeated calculations. **MATHEMATICAL THINKING** • Create a model to make sense of a problem. • Represent the concept symbolically. • Plan a solution pathway. • Determine the reasonableness of results. • Consider mathematical units and scale when graphing. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Given a mathematical or real-world context, express the relationship between quantities by writing an equation or inequality that must be true for the given relationship. Focus on situations where the equations will be linear, exponential, and quadratic. • For equations or inequalities relating two variables, graph the relationships on coordinate axes with proper labels and scales. Focus on situations where the equations will be linear, exponential, and quadratic. • Identify the constraints implied by the scenario, and represent them with equations or inequalities. • Determine the feasibility (possibility) of a solution based upon the constraints implied by the scenario. • Solve formulas for a given variable. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Math 2, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Interpret the structure of expressions (A.SSE.1). • Build a function that models an exponential or quadratic relationship between two quantities (F.BF.1).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
Algebra **REASONING WITH EQUATIONS AND INEQUALITIES** **Solve equations and inequalities in one variable.** **A.REI.4** Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. (+) **c.** Derive the quadratic formula using the method of completing the square.	**Expectations for Learning** In previous courses, students solve linear equations and inequalities, and they solve equations with numeric and letter coefficients. In this cluster, students solve quadratic equations (with real solutions) using a variety of methods. In other standards, students learn to factor quadratics; this cluster builds on that idea to solve quadratic equations with the Zero Product Property. In Math 3, students use these skills to solve more complicated equations. **ESSENTIAL UNDERSTANDINGS** • An appropriate solution path can be determined depending on whether the equation is linear or quadratic in the variable of interest. • Quadratic equations and expressions can be transformed into equivalent forms, leading to different solution strategies, including inspection, taking square roots, completing the square, applying the quadratic formula, or utilizing the Zero Product Property after factoring. • When the coefficients of the variable of interest are letters, the solving process is the same as when the coefficients are numbers. • The discriminant can show the nature and number of solutions a quadratic has. • (+) The quadratic formula is derived from the process of completing the square. **MATHEMATICAL THINKING** • Generalize concepts based on properties of equality. • Solve routine and straightforward problems accurately. • Plan a solution pathway. • Solve math problems using appropriate strategies. • Solve multi-step problems accurately. • (+) Use formal reasoning with symbolic representation. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.

- Recognize when an equation or inequality is linear or quadratic in one variable, and plan a solution strategy.
- Recognize when an equation is quadratic in one variable, and choose an appropriate solution strategy:
    - using inspection, e.g., $(x-3)^2=0$ or $x^2=-5$;
    - taking square roots, e.g., $x^2=8$;
    - using Zero-Product Property after factoring;
    - completing the square; or
    - applying the quadratic formula.
- Determine if a quadratic functions has one solution, two solutions, or no real solutions based on the discriminant.
- (+) Formally derive the quadratic formula using completing the square.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS

- Math 2, Number 2, pages 4-5

CONNECTIONS ACROSS STANDARDS

- Rearrange formulas to highlight a quantity of interest (A.CED.4).
- Use the structure of an expression to identify ways to rewrite it (A.SSE.2).
- Write expressions in equivalent forms to solve problems (A.SSE.3).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Algebra **REASONING WITH EQUATIONS AND INEQUALITIES** **Solve systems of equations.** **A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$.	**Expectations for Learning** In Math 1, students solve systems of linear equations in two variables graphically and algebraically, with a focus on the meaning of a solution to a system of equations. In Math 2, students extend this knowledge to solve systems of linear and quadratic equations in two variables. In Math 3, students solve systems of equations in three variables. Students who plan to take advanced mathematics courses (+) will represent systems of equations with matrices and use inverse matrices to solve the system. **ESSENTIAL UNDERSTANDINGS** • The graphical solution to a system of equations in two variables is the intersection of the equations when graphed. • The solution to a system of equations in two variables is the set of ordered pairs that satisfies both equations. • A system of a linear equation and a quadratic equation can have no solutions, one solution, or two solutions. **MATHEMATICAL THINKING** • Determine reasonableness of results using informal reasoning. • Solve multi-step problems accurately. • Plan a solution pathway. • Use technology strategically to deepen the understanding. **INSTRUCTIONAL FOCUS** *Note: For Math 2, students should work with systems of equations in two variables that include an equation of a line and an equation of a parabola, as well as an equation of a line and an equation of a circle.* • Solve a system of a linear equation and a quadratic equation in two variables algebraically using substitution and by graphing. • Discuss the efficiency and effectiveness of various methods of solving systems of equations. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Math 2, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Solve linear and quadratic equations in one variable (A.REI.4). • Graph linear and quadratic models (F.IF.4, 7). • Rearrange formulas (A.CED.4). • Solve systems of equations and inequalities graphically (A. REI.11).

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Algebra

REASONING WITH EQUATIONS AND INEQUALITIES

Represent and solve equations and inequalities graphically.

A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equation *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of the equation *f(x)* = *g(x)*; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations

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Expectations for Learning

In previous courses, students use graphs to solve equations with linear and simple exponential expressions. In Math 2, they extend this to solving equations with quadratic expressions. Then in Algebra 2, students will similarly study the relationship between the graph and solutions of equations with rational, radical, absolute value, polynomial, exponential, and trigonometric expressions.

ESSENTIAL UNDERSTANDINGS

- A point of intersection of any two graphs represents a solution of the two equations that define the two graphs.
- An equation in one variable can be rewritten as a system of two equations in two variables, by thinking of each side of the equation as a function, i.e., writing *y* = left hand side and *y* = right hand side.
    - The approximate solution(s) to an equation in one variable is the *x*-value(s) of the intersection(s) of the graphs of the two functions.
    - Two-variable graphical and numerical (tabular) techniques to solve an equation with one variable always work and are particularly useful when algebraic methods are not applicable, e.g., $x^2 - 3x + 2 = 2^x$.

MATHEMATICAL THINKING

- Use technology strategically to deepen understanding.
- Plan a solution pathway.
- Create a model to make sense of a problem.

INSTRUCTIONAL FOCUS

- Rewrite a one-variable equation as two separate functions and use the *x*-coordinate of their intersection point to determine the solution of the original equation.
- Approximate intersections of graphs of two equations using technology, tables of values, or successive approximations (focus on equations with linear, quadratic, and exponential expressions).

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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREA OF FOCUS** • Math 2, Number 2, pages 4-5 **CONNECTIONS ACROSS STANDARDS** • Solve quadratic equations in one variable (A.REI.4). • Create equations in two variables (A.CED.2). • Graph functions expressed symbolically (F.IF.7). • Analyze functions using different representations (F.IF.9).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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STANDARDS	MODEL CURRICULUM
Functions **INTERPRETING FUNCTIONS** **Interpret functions that arise in applications in terms of the context.** **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)	**Expectations for Learning** Working with linear and exponential functions in Math 1, students interpret key features of graphs and tables. They also determine the domain of a function by looking at a graph or table. In a real-life scenario students can find the restrictions on the domain. In this cluster, students apply these concepts to quadratic functions and compare them to the prior learning of linear and exponential functions from Math 1. In Math 3, students extend identifying and interpreting key features of functions to include periodicity. Students also have to select appropriate functions that model the data presented. Average rate of change over a specific interval will also be included in Math 3. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, and then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • Key features (as listed in the standard) of a function can be illustrated graphically and interpreted in the context of the problem. • The sensible domain for a real-world context should be accurately represented in graphs, tables, and symbols. • A quadratic function is symmetrical about its axis of symmetry. • Functions can have continuous or discrete domains. • More generally, writing a function in different ways can reveal different features of the graph of a function. **MATHEMATICAL THINKING** • Connect mathematical relationships to contextual scenarios. • Attend to meaning of quantities. • Determine reasonableness of results.

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

**Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients.*

- For quadratic functions, interpret intercepts; maximum or minimum; symmetry; intervals of increase or decrease; and end behavior given tables, graphs, and verbal descriptions.
- Use written descriptions or inequalities to describe intervals on which a function is increasing/decreasing and/or positive/negative (neither interval notation nor set builder notation are required).
- Determine whether to connect points on a graph based on the context (continuous vs. discrete domain).
- Demonstrate understanding of domain in the context of a real-world problem.
- Compare the key features of quadratic functions to linear and exponential functions. For example,
  - Linear functions are either always increasing, decreasing, or constant.
  - Exponential functions are either always increasing or decreasing.
  - Quadratic functions increase to a maximum then decrease or decrease to a minimum then increase.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Math 2, Number 3, pages 6-7

CONNECTIONS ACROSS STANDARDS

- Create equations that describe numbers or relationships (A.CED.2b).
- Graph quadratic functions and indicate maxima and minima (F.IF.7b).
- Analyze functions using different representations (F.IF.9b).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM
Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Functions **INTERPRETING FUNCTIONS** **Analyze functions using different representations.** **F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★ **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) **i.** Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2) *Continued on next page*	**Expectations for Learning** In Math 1, students graph linear and exponential functions given a symbolic representation and indicate intercepts and end behavior. They compare linear and exponential functions given various representations. In this cluster, students graph quadratic functions and indicate key features such as maxima/minima. They compare linear, quadratic, and exponential functions given various representations. In Math 3, students graph polynomial, square root, cube root, trigonometric, piecewise-defined, (+) rational, and (+) logarithmic functions. Students identify and interpret key features (as applicable) including intercepts, end behavior, period, midline, amplitude, symmetry, asymptotes, maxima/minima, and zeros. *Note on differences between standards: In F.IF.4 and F.IF.5, the emphasis is on the context of the problem and on making connections among graphs, tables, and the context. In F.IF.7, the emphasis is on creating a graph given a symbolic representation, then identifying the key features of the graph and connecting the key features to the symbols.* **ESSENTIAL UNDERSTANDINGS** • The graph of a quadratic function shows intercepts and maximum or minimum. • The factored form of a quadratic function reveals the zeros of the function (i.e., the x-intercepts of the graph); the vertex form of a quadratic function reveals the maximum or minimum of the function; the standard form of a quadratic function reveals the y-intercept of the graph. • Different representations (graphs, tables, symbols, verbal descriptions) illuminate key features of functions and can be used to compare different functions. **MATHEMATICAL THINKING** • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Analyze a mathematical model.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change*[G] *in functions such as* $y = (1.02)^t$, *and* $y = (0.97)^t$ *and classify them as representing exponential growth or decay.* (A2, M3) **i.** Focus on exponential functions evaluated at integer inputs. (A1, M2) **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)	**Expectations for Learning, continued** **INSTRUCTIONAL FOCUS** **Remember, in this course, for exponential functions, assessments should focus on integer exponents only. For quadratic functions, students should work with expressions in which the leading coefficient can be any real number, but assessment questions should focus on expressions with leading coefficients of 1 with occasional questions using other simple leading coefficients..* • Given symbolic representations of quadratic functions, create accurate graphs showing all key features. • Identify the key features of the graph of a quadratic function by factoring, using the quadratic formula, or completing the square. • Compare and contrast linear, quadratic, and exponential functions given by graphs, tables, symbols, or verbal descriptions. • Determine the zeros of a quadratic function by factoring, using the quadratic formula, or completing the square. • Use different forms of quadratic functions (standard form, vertex form, factored form) to reveal different features. • Explore the relationship of the symbolic representation of a function and its graph by adjusting parameters. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 3, pages 6-7 **CONNECTIONS ACROSS STANDARDS** • Interpret functions that arise in applications in terms of the context (F.IF.4) • Solve quadratic equations in one variable (A.REI.4). • Write expressions in equivalent forms to solve problems (A.SSE.3). • Build a function that models a relationship between two quantities (F.BF.1).

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Functions

BUILDING FUNCTIONS

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from context.

ii. Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)

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Expectations for Learning

In Math 1, students write linear and exponential functions symbolically given the relationship between two quantities. They also write explicit and recursive rules for arithmetic and geometric sequences.

In Math 2, students focus on quadratic situations and compare to linear and exponential relationships. Students also continue to work with sequences.

In Math 3, students build functions from other functions allowing students to model more complex situations. This includes combining functions of various types using arithmetic operations or (+) composition.

ESSENTIAL UNDERSTANDINGS

- Functions (including quadratic functions) can be written as explicit expressions, recursive processes, and in other ways.
- Some sequences may be defined recursively or explicitly while others cannot be defined by a formula.
- The relationships between quantities can be modeled with functions that are linear, exponential, quadratic, or none of these.

MATHEMATICAL THINKING

- Make and modify a model to represent mathematical thinking.
- Discern and use a pattern or structure.

INSTRUCTIONAL FOCUS

- Model relationships with linear, exponential, and quadratic functions using tables, graphs, symbols, and words in context.
- Model relationships that are not linear, exponential, or quadratic using tables, graphs, symbols, and words in context.

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	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 3, pages 6-7 **CONNECTIONS ACROSS STANDARDS** • Create equations that describe numbers or relationships (A.CED.2). • Analyze functions using different representations (F.IF.8).

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STANDARDS	MODEL CURRICULUM
Functions **BUILDING FUNCTIONS** **Build new functions from existing functions.** **F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) **a.** Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)	**Expectations for Learning** In eighth grade and Math 1, students attend to slope and intercepts for graphs of linear functions, without explicit attention to transformations of the graphs. In this cluster, students transform graphs of quadratic functions. Transformations of quadratic functions can be interpreted conveniently by observing the effect on the vertex and whether the parabola opens up or down. Students do not perform transformations of the form *f(kx).* In Math 3, students perform all types of transformations for various function families and recognize even and odd functions. **ESSENTIAL UNDERSTANDINGS** • Vertical and horizontal transformations of $y = x^2$ are as follows: o horizontal shift: $g(x) = (x - h)^2$; o vertical stretch/shrink: $g(x) = ax^2$ when $a > 0$; o vertical shift: $g(x) = x^2 + k$; o reflection across the x-axis: $g(x)= -x^2$; and o a combination of transformations: $g(x) = a(x - h)^2 + k$. **MATHEMATICAL THINKING** • Explain mathematical reasoning. **INSTRUCTIONAL FOCUS** **Transformations occur in the quadratic expression rather than inside the function notation.* • Transform graphs of quadratic functions, and interpret the transformations geometrically. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 3, pages 6-7 **CONNECTIONS ACROSS STANDARDS** • Analyze functions using different representations (F.IF.7b).

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STANDARDS	MODEL CURRICULUM
Functions **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2)	**Expectations for Learning** In eighth grade, students interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. Students also see examples of non-linear functions and learn and apply the properties of integer exponents. In Math 1, students compare linear and exponential functions. In Math 2, students compare across linear, exponential, and quadratic functions. **ESSENTIAL UNDERSTANDINGS** • The phrase "eventually exceeds" (F.LE.3) directs the focus towards large values in the domain and consideration of the base and *y*-intercept of the exponential function and the leading coefficient of the linear or quadratic function. • For large domain values, the growth of linear and quadratic functions is dominated by the leading term. **MATHEMATICAL THINKING** • Represent a concept symbolically. • Make and modify a model to represent mathematical thinking. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. **INSTRUCTIONAL FOCUS** • Use graphs, tables, and contexts to see that as the domain value increases, the values of an exponential function will eventually exceed the corresponding values of a linear or quadratic function. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 3, pages 6-7 **CONNECTIONS ACROSS STANDARDS** • Build a function that models a relationship between two quantities (F.BF.1aii). • Interpret functions that arise in applications in terms of the context (F.IF.4b). • Analyze functions using different representations (F.IF.7b, 9b).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
Geometry **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** **Understand similarity in terms of similarity transformations.** **G.SRT.1** Verify experimentally the properties of dilations[G] given by a center and a scale factor: **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations[G] to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	**Expectations for Learning** The standards in this cluster make more precise the informal notion of "same shape, different size." In middle school, students represent proportional relationships within and between similar figures; create scale drawings; describe the effect of dilations on two-dimensional figures; and understand similarity transformations as a sequence of basic rigid motions and dilations. In this cluster, students verify the properties (given center and scale factor) of dilations and use those properties to establish the AA criterion for triangles. They also explore the relationships among corresponding parts of similar figures. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • A dilation requires a center and a scale factor. • A similarity transformation often requires a sequence of basic rigid motions, in addition to a dilation. • A scale factor is a ratio corresponding lengths between figures. • A similarity transformation with a scale factor of 1 is a special case, which is a congruence transformation. • While the definition of similarity applies to polygons, it also applies to non-polygonal shapes, e.g. circles, parabolas, etc. • The AA criterion is equivalent to the AAA criterion because the angle sum in a triangle is 180 degrees. • The AA criterion and the AAA criterion apply only to triangles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Use formal reasoning with symbolic representation. • Determine reasonableness of results. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Make connections between terms and properties. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Use basic rigid motions and dilations to map similar figures onto one another. • Given a figure, carry out a similarity transformation, and then verify its properties. • Know the precise definitions of dilation and similarity. • Identify center and scale factor of a dilation. • Determine the scale factor. • Establish that triangles with two pairs of corresponding congruent angles are similar. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 4, page 8 **CONNECTIONS ACROSS STANDARDS** • Prove theorems involving similarity (G.SRT.4-5). • Apply right triangles trigonometry (G.SRT.6-8). • Apply the understanding that all circles are similar (G.C.1). • Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6). • Use coordinates to prove simple geometric theorems algebraically (G.GPE.6). • Apply geometric concepts in modeling situations (G.MG.1-3).

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Instructional Strategies *This section is under revision and will be published in 2018.* **Instructional Tools/Resources** *This section is under revision and will be published in 2018.*

STANDARDS	MODEL CURRICULUM
Geometry **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** **Prove and apply theorems both formally and informally involving similarity using a variety of methods.** **G.SRT.4** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.	**Expectations for Learning** In middle school, students draw, construct, and describe geometric figures; use informal arguments to establish facts about similar triangles; and explain a proof of the Pythagorean Theorem and its converse. In this cluster, students prove theorems and solve problems involving similarity of triangles. They will also solve problems by applying these theorems to geometric figures that can be decomposed into triangles. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstraction) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • The altitude to the hypotenuse divides a right triangle into two triangles that are similar to the original triangle. • A line parallel to the side of a triangle makes similar triangles and divides the other two side lengths proportionally. • Two right triangles are similar if they have another congruent angle. • Polygons can be divided into congruent and/or similar triangles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Use formal reasoning with symbolic representation. • Make conjectures. • Plan a solution pathway. • Justify relationships in geometric figures. • Determine reasonableness of results. • Create a drawing and add components as appropriate. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Form conjectures and construct a valid argument for why the conjecture is true or not true, both formally and informally.
- Recognize when polygons are divided into congruent and/or similar triangles.
- Justify relationships in geometric figures that can be decomposed into triangles.
- Solve problems using triangle congruence and similarity.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Math 2, Number 4, page 8

CONNECTIONS ACROSS STANDARDS

- Understand similarity (G.SRT.1-3).
- Define trigonometric ratios, and solve problems involving right triangles (G.SRT.6-8).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.5-6).
- Use coordinate geometry (G.GPE.4).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.6).
- Find arc lengths and areas of sectors of circles (G.C.5).
- Apply geometric concepts in modeling situations (G.MG.2-3).
- Write equations in one variable, and use them to solve problems (A.CED.1).
- Solve quadratic equations in one variable (A.REI.4).

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STANDARDS	MODEL CURRICULUM
Geometry **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** **Define trigonometric ratios, and solve problems involving right triangles.** **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.8** Solve problems involving right triangles.★ **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)	**Expectations for Learning** In middle school, students draw, construct, and describe geometric figures; use informal arguments to establish facts about similar triangles; and explain a proof of the Pythagorean Theorem and its converse. In this cluster, students use similarity to define trigonometric ratios and then solve problems using right triangles (excluding inverse trigonometric functions). The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Because right triangles with the same acute angle are similar, within-figure ratios are equal. Three of these possible ratios are named sine, cosine, and tangent. • The sine of an acute angle is equal to the cosine of its complement and vice versa. • Given an angle and a side length of a right triangle, the triangle can be solved, which means finding the missing sides and angles. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Discern and use a pattern or structure. • Plan a solution pathway. • Justify relationships in geometric figures. • Determine reasonableness of results. • Create a drawing and add components as appropriate. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. • Connect mathematical relationships to real-world encounters. *Continued on next page*

Expectations for Learning, continued

INSTRUCTIONAL FOCUS

- Define trigonometric ratios for acute angles (sine, cosine, tangent).
- Explain and apply the relationship between sine and cosine of complementary angles.
- Solve problems involving right triangles (excluding inverses of trigonometric functions).
- Use the Pythagorean Theorem to explore exact trigonometric ratios for 30, 45, and 60 degree angles (fluency not required).
- Use triangle similarity criteria to define trigonometric ratios.
- Given the sine, cosine, or tangent of an angle, find other trigonometric ratios in the triangle.
- Solve mathematical and real-world problems given a side and an angle (or the sine, cosine, or tangent of an angle) of a right triangle.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Math 2, Number 4, page 8

CONNECTIONS ACROSS STANDARDS

- Understand similarity (G.SRT.1-3).
- Prove and apply theorems involving similarity (G.SRT.4-5).
- Apply geometric concepts in modeling situations (G.MG.1-3).
- Use coordinates to prove simple geometric theorems algebraically (G.GPE.4).
- Write equations in one variable, and use them to solve problems (A.CED.1).
- Solve quadratic equations in one variable (A.REI.4).

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Geometry

CIRCLES

Understand and apply theorems about circles.

G.C.1 Prove that all circles are similar using transformational arguments.

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Expectations for Learning

In middle school, students have worked with measurements of circles such as circumference and area. In Math 1, students solve problems using the relationships among the arcs and angles created by radii, chords, and tangents. They construct inscribed and circumscribed circles of a triangle. In this course, they extend their understanding of similarity to circles.

The student understanding of this cluster aligns begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction).

ESSENTIAL UNDERSTANDINGS

- All circles are similar as follows: translate one circle so that its center maps onto the center of the other, and then dilate about the common center by the ratio of the radii.

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary.
- Make connections between concepts, terms, and properties.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Solve mathematical and real-world problems accurately.
- Determine reasonableness of results.
- Consider mathematical units involved in a problem.
- Make sound decisions about using tools.

INSTRUCTIONAL FOCUS

- Use transformational arguments to prove that all circles are similar.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Math 2, Number 5, page 9

CONNECTIONS ACROSS STANDARDS

- Understand similarity in terms of similarity transformations (G.SRT.2).
- Find arc length and areas of sectors of circles (G.C.5).

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STANDARDS	MODEL CURRICULUM
Geometry **CIRCLES** **Find arc lengths and areas of sectors of circles.** **G.C.5** Find arc lengths and areas of sectors of circles. **a.** Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. **b.** Derive the formula for the area of a sector, and use it to solve problems.	**Expectations for Learning** In middle school, students are limited to working with measurements of circles such as circumference and area. This cluster spans Geometry/ Math 2 and Algebra 2/ Math 3. In Geometry/Math 2, students are using part-to-whole proportional reasoning to find arc lengths and sector areas, in which the arc or central angle is measured in degrees. In Algebra 2/ Math 3, students derive and use formulas relating degree and radian measure. The student understanding of this cluster aligns with van Hiele Level 2 (Informal Deduction/Abstraction). *Note: Since in Math 2 students focus on quadratics with leading coefficients of 1 with occasional uses of other simple coefficients, geometry standards should only apply to equations where the squared terms have a coefficient of 1 or occasionally other simple leading coefficients.* **ESSENTIAL UNDERSTANDINGS** • A central angle that turns through n one-degree angles is said to have an angle measure of n degrees. • The measure of an arc is equal to the measure of the corresponding central angle and is expressed in degrees, while the length of an arc is expressed in units of linear measure. • The arc length is a part of the circumference of a circle. • The ratio of the central angle to 360 degrees is equal to the ratio of the length of the arc to the circumference of the circle. • The sector area is a part of the area of a circle. • The ratio of the central angle to 360 degrees is equal to the ratio of the area of the sector to the area of the circle. • Because all circles are similar, if the radius of the circle is scaled by k, the corresponding arc length is multiplied by k and the sector area is multiplied by k^2. *Continuation on next page*

	Expectations for Learning, continued **MATHEMATICAL THINKING** • Consider mathematical units involved in a problem. • Make connections between concepts and terms. • Generalize concepts based on patterns. • Use proportional reasoning (part to whole). • Draw a picture to make sense of a problem. • Solve real-world and mathematical problems accurately. • Plan a solution pathway. • Attend to the meaning of quantities. **INSTRUCTIONAL FOCUS** • Develop understanding of the formulas for arc length and area of a sector through derivation. • Solve problems using arc lengths and areas of sectors of circles. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 5, page 9 **CONNECTIONS ACROSS STANDARDS** • Understand and apply theorems about circles (G.C.1). • Explain volume formulas, and use them to solve problems (G.GMD.1). • Understand similarity in terms of similarity transformations (G.SRT.2). • Create equations that describe numbers or relationships (A.CED.1, 2, 4). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6).

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STANDARDS	MODEL CURRICULUM
Geometry **EXPRESSIONS GEOMETRIC PROPERTIES WITH EQUATIONS** **Translate between the geometric description and the equation for a conic section.** **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	**Expectations for Learning** In middle school, students use the Pythagorean Theorem to find distances between points within the coordinate system. In the high school, students complete the square to solve quadratic equations. In this cluster, students derive the equation of a circle using the Pythagorean Theorem. They also complete the square to find the center and radius of a circle. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). **ESSENTIAL UNDERSTANDINGS** • The equation of a circle relates a fixed center, a fixed radius, and a set of variable points, which are the points on the circle. • Just as the distance formula is an application of the Pythagorean Theorem so is the equation of a circle. **MATHEMATICAL THINKING** • Use precise mathematical language. • Discern and use a pattern or structure. • Make connections between concepts, terms, and properties within the grade level and with previous grade levels. • Justify relationships in geometric figures. • Use technology strategically to deepen understanding. • Solve routine and straightforward problems accurately. **INSTRUCTIONAL FOCUS** • Use the Pythagorean Theorem to derive the equation of a circle. • Given the equation of a circle that is not in standard form, find the center and radius of the circle by completing the square. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 5, page 9 **CONNECTIONS ACROSS STANDARDS** • Solve equations and inequalities in one variable (A.REI.4). • Prove that all circles are similar (G.C.1). • Prove theorems about triangles (G.SRT.4). • Write equations in two variables, and use them to solve problems (A.CED.2). • Solve quadratic equations in one variable (A.REI.4). • Solve systems of linear and quadratic equations (A.REI.7).

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STANDARDS	MODEL CURRICULUM
Geometry **EXPRESSIONS GEOMETRIC PROPERTIES WITH EQUATIONS** **Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.** **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2) **G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	**Expectations for Learning** In middle school, students find the distance between two points in a coordinate system; work with linear functions; solve linear equations; and apply the Pythagorean Theorem in the coordinate system. In addition, they used square root symbols to represent solutions to equations, and they evaluate square roots of rational numbers. In Math 1, students use the coordinate system to justify slope criteria for parallel and perpendicular lines and compute perimeters and areas of geometric figures. In this course, these strategies are used for proof of geometric relationships with respect to properties of figures and partitioning line segments. The student understanding of this cluster begins at van Hiele Level 2 (Informal Deduction/Abstractions) and moves to Level 3 (Deduction). *Note: In Math 2, revisit G.CO.10-11 with respect to coordinate and transformational proofs. Since G.GPE.4 is no longer placed in Math 1, students may not have had exposure to these types of proofs for that content.* **ESSENTIAL UNDERSTANDINGS** • Coordinate proof is a method that uses algebraic techniques to prove geometric theorems and properties. • Properties of geometric figures, especially special quadrilaterals, can be proven on a coordinate plane using lengths of segments, slopes of lines, and equations of lines. • Coordinate proof can be used to prove that figures are congruent or similar. • Partitioning a line segment into a given ratio is an application of similar triangles. *Continued on next page*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use accurate mathematical vocabulary to represent geometric relationships.
- Make connections between terms and formulas.
- Recognize, apply, and justify mathematical concepts, terms, and their properties.
- Compute using strategies or models.
- Use formal reasoning with symbolic representation.
- Determine reasonableness of results.
- Solve multi-step problems accurately.
- Discern and use a pattern or structure.

INSTRUCTIONAL FOCUS

- Know and use the distance formula.
- Use coordinates to prove simple geometric theorems algebraically.
- Verify geometric relationships algebraically.
- Partition a line segment given a ratio.

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Math 2 Number 4, page 8
- Math 2 Number 5, page 9

CONNECTIONS ACROSS STANDARDS

- Understand and apply theorems about circles (G.C.1, 5).
- Prove theorems involving similarity (G.SRT.4-5).
- Understand the relationships between lengths, areas, and volumes in similar figures (G.GMD.6).
- Create equations that describe numbers or relationships (A.CED.4).
- Represent and solve equations graphically (A.REI.4).
- Apply geometric concepts in modeling situations (G.MG.3).

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STANDARDS	MODEL CURRICULUM
Geometry **GEOMETRIC MEASUREMENT AND DIMENSION** **Explain volume formulas, and use them to solve problems.** **G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	**Expectations for Learning** In middle school, students use established circumference, area, and volume formulas for two- and three-dimensional figures. Instead of using area and volume formulas rotely, students in this cluster give informal justifications for these formulas and use them to solve problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • A three-dimensional solid can be viewed as a stack of layers. • If all of the layers of a three-dimensional solid have the same area, then the volume is the area of the base times the height. • The volume remains unchanged when layers parallel to the base in a three-dimensional solid are shifted. • A cone's volume is $\frac{1}{3}$ of the volume of a cylinder if their base areas are equal and their heights are congruent. • A pyramid's volume is $\frac{1}{3}$ of the volume of a prism if their base areas are equal and their heights are congruent. • Volume, like area, is additive, so to find the volume of a composite figure, cut the figure into pieces of known volume, and add or subtract as appropriate. • The cross sections of a cylinder are circles of equal area. • The cross sections of a prism are congruent to the base, so therefore the areas are equal. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Make and modify a model to represent mathematical thinking. • Attend to meaning of quantities. • Consider mathematical units involved in a problem. • Solve real-world and mathematical problems accurately. • Determine reasonableness of results. • Use informal reasoning.

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Explain and justify the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. • Apply volume formulas in real-world and mathematical problems. • (+) Informally apply Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 6, page 10 **CONNECTIONS ACROSS STANDARDS** • Understand and apply theorems about circles (G.C.5). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). • Apply geometric concepts in modeling situations (G.MG.1-3). • Create equations that describe numbers or relationships (A.CED.2, 4).

INSTRUCTIONAL SUPPORTS FOR THE MODEL CURRICULUM

Instructional Strategies

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Instructional Tools/Resources

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STANDARDS	MODEL CURRICULUM
Geometry **GEOMETRIC MEASUREMENT AND DIMENSION** **Visualize relationships between two-dimensional and three-dimensional objects.** **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**Expectations for Learning** In middle school, students identify cross-sections as a result of slicing right rectangular prisms and pyramids. In this cluster, which supports the previous cluster, students extend the identification of cross-sections to include other three-dimensional solids. Students will also identify three-dimensional objects created when a two-dimensional object is rotated about a line. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Two-dimensional figures can be used to understand three-dimensional solids. • A three-dimensional figure can be created by rotating a two-dimensional figure about a line. **MATHEMATICAL THINKING** • Draw a picture or create a model to make sense of a problem. • Use technology strategically to deepen understanding. • Make connections between concepts, terms, and properties. **INSTRUCTIONAL FOCUS** • Identify two-dimensional cross-sections of three-dimensional objects. • Identify three-dimensional objects formed by rotations of two-dimensional objects. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 6, page 10 **CONNECTIONS ACROSS STANDARDS** • Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3). • Understand the relationships between lengths, areas, and volumes (G.GMD.5-6). • Apply geometric concepts in modeling situations (G.MG.1-3).

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STANDARDS	MODEL CURRICULUM
Geometry **GEOMETRIC MEASUREMENT AND DIMENSION** **Understand the relationships between lengths, areas, and volumes.** **G.GMD.5** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. **G.GMD.6** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.	**Expectations for Learning** In middle school, students solve problems involving two-dimensional similar figures and calculate the volumes of three-dimensional figures. In this cluster, students extend their knowledge of similarity to explore and understand how changes to length or angle measure in one figure will result in similar or non-similar figures. Students will also understand the effect that a scale factor has on the length, area, and volume of similar figures and use this relationship to solve problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Changes to the lengths and/or angle measures of a figure result in similar and non-similar figures. • When changes to a figure result in similar figures with a scale factor of k, the lengths of the resulting figures are a multiple of k. • When changes to a figure result in similar figures with a scale factor of k, the areas of the resulting figures are a multiple of k^2. • When changes to a figure result in similar figures with a scale factor of k, the volume of the resulting figures are a multiple of k^3. **MATHEMATICAL THINKING** • Use precise mathematical language. • Draw a picture or create a model to make sense of a problem. • Determine reasonableness of results. • Solve multi-step problems accurately. • Plan a solution pathway. • Solve mathematical and real-world problems accurately. • Consider mathematical units involved in a problem. • Attend to the meaning of quantities. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Generalize concepts based on patterns. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Classify objects as similar or non-similar when the lengths or angles of figures are changed. • Explain the types of changes to a figure that result in similar and non-similar figures. • Use geometry and algebra to explain how length, area, and volume are affected when scaling is applied. • Solve problems involving length, area, and volume of figures under scaling. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 6, page 10 **CONNECTIONS ACROSS STANDARDS** • Explain volume formulas, and use them to solve problems (G.GMD.1, (+) 2, 3). • Understand similarity in terms of similarity transformations (G.SRT.1-2). • Apply geometric concepts in modeling situations (G.MG.2-3). • Create equations that describe numbers or relationships (A.CED.2, 4). • Solve quadratic equations in one variable (A.REI.4). • Build a function that models a relationship between two quantities (F.BF.1a).

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STANDARDS	MODEL CURRICULUM
Geometry **MODELING WITH GEOMETRY** **Apply geometric concepts in modeling situations.** **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★ **G.MG.2** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★ **G.MG.3** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★	**Expectations for Learning** In middle school, students work with nets, area, and volume; use appropriate tools to represent situations; and solve real-life and mathematical problems. In this cluster, students make sense of the world around them by using geometric models and their properties to solve more sophisticated problems. The student understanding of this cluster begins at van Hiele Level 1 (Analysis) and moves to Level 2 (Informal Deduction/Abstraction). **ESSENTIAL UNDERSTANDINGS** • Composite figures can be analyzed by approximating them with traditional two- and three-dimensional figures. • Many real-life scenarios are related to length, area, and volume. **MATHEMATICAL THINKING** • Use accurate mathematical vocabulary to represent geometric relationships. • Make connections between terms and properties. • Recognize, apply, and justify mathematical concepts, terms, and their properties. • Use formal reasoning with symbolic representation. • Determine reasonableness of results. • Use proportional reasoning. • Plan a solution pathway. • Connect mathematical relationships to real-world encounters. • Draw a picture or create a model to represent a problem. **INSTRUCTIONAL FOCUS** • Use geometric shapes, their measures, and their properties to describe objects. • Identify useful quantities for modeling situations. • Apply concepts of density based on area and volume. • Solve design problems geometrically. *Continued on next page*

	Content Elaborations **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 6, page 10 **CONNECTIONS ACROSS STANDARDS** • Solve problems involving right triangles (G.SRT.8). • Use volume formulas to solve problems (G.GMD.3-4, 6). • Model a relationship given a verbal description (F.IF.4). • Show key features of a function (F.IF.7). • Write a function that describes a relationship between two quantities (F.BF.1). • Interpret parts of an expression (A.SSE.1). • Write expressions in equivalent forms (A.SSE.3). • Create equations to describe relationships (A.CED.1-2, 4).

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STANDARDS	MODEL CURRICULUM
Statistics and Probability **CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY** **Understand independence and conditional probability, and use them to interpret data.** **S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★ **S.CP.2** Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★ **S.CP.3** Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★ *Continued on next page*	**Expectations for Learning** In middle school, students develop basic probability skills including probability as relative frequencies; probabilities of compound events; the development a uniform/non-uniform probability model; and the use of tree diagrams. Also students are introduced to two-way frequency tables in middle school. However, students' only prior exposure to the concept of independence was in S.ID.5 (Math 1). This cluster focuses on the concept of independence between two categorical variables. It also focuses on the understanding of independence rather than symbolic notation and formulas. Fluency with independence is expected by the end of Math 2. **ESSENTIAL UNDERSTANDINGS** • Approximations for the true probability of an event can be found by looking at the long-run relative frequency. • The sample space of a probability experiment can be modeled with a Venn diagram. • The union of an event and its complement represent the entire sample space. • The intersection of an event and its complement represent the empty set. • Conditional probability is the probability of event A occurring given that event B has occurred. It is denoted by $A\|B$ and is read "A given B." • Two events occurring in succession are said to be independent if the outcome of one event has no effect on the outcome of the other, e.g., a coin tossed twice. Otherwise, the events are dependent, e.g., two cards are drawn in succession from a standard deck of cards. • The intersection of two sets A and B is the set of elements that are common to both set A and set B. It is denoted by $A \cap B$ and is read "A intersection B" as well as "A and B." • The union of two sets A and B is the set of elements, which are in A or in B or in both. It is denoted by $A \cup B$ and is read "A union B" as well as "A or B." • If A and B are events that have no outcomes in common $(A \cap B \neq 0)$, they are said to be mutually exclusive. Mutually exclusive events cannot occur together. *Continued on next page*

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★*

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.★*

Expectations for Learning, continued

MATHEMATICAL THINKING

- Use appropriate vocabulary.
- Attend to precision.

INSTRUCTIONAL FOCUS

- Recognize and explain for two successive events, whether the outcome of the first event affects the outcome of the second event.
- Recognize and justify conceptually whether two events are independent.
- Make connections between conditional probability and independence. Recognize sample space subsets in everyday contexts.
- Identify an event and its complement.
- Identify which components of the sample space represent the union and intersection of two events.
- Explain what a conditional probability means within a context.
- Distinguish between a conditional probability (A given B) and the probability of an intersection (A and B).
- Use a two way frequency table to determine the following:
  - conditional probabilities;
  - probabilities of the sample space subsets;
  - event independence by comparing joint probabilities (P(A and B)) and the product of the separate probabilities (P(A) × P(B)); and
  - event independence by comparing the conditional probability (P(A given B)) and the probability P(A).

Content Elaborations

OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS

- Math 2, Number 1, page 3

CONNECTIONS ACROSS STANDARDS

- This will lead into the cluster (S.CP.6-9) which includes the calculations of conditional probabilities, and the use of probability formulas and set notation with probability.

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STANDARDS	MODEL CURRICULUM
Statistics and Probability **CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY** **Use the rules of probability to compute probabilities of compound events in a uniform probability model.** **S.CP.6** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★ **S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★ (+) **S.CP.8** Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★ (+) **S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems.★	**Expectations for Learning** In middle school, students develop basic probability skills including probability as relative frequencies; probabilities of compound events; development of a uniform/non-uniform probability model; and the use of tree diagrams. Also they are introduced to two-way frequency tables in middle school. Now in Math 2 this cluster formalizes the concepts of conditional probability and independence in S.CP.1-5. The focus of this cluster is developing the Addition Rule and the (+) Multiplication Rule in everyday contexts. Although permutations and combinations are part of Math 2 for students who pursue advanced mathematics, these concepts would also be appropriate in a fourth year course. Exploration of the Fundamental Counting Principle and factorials (!) may also be addressed in a fourth year course. **ESSENTIAL UNDERSTANDINGS** • Compound probabilities model real-world scenarios and must be interpreted within a context. • The conditional probability of A *given* B is the fraction of B's outcomes that also belong to A. This can be expressed by $P(A\|B) = P(A \cap B)/P(B)$. • The addition rule is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ and can also be expressed as $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. • (+) The Multiplication Rule is $P(A \text{ and } B) = P(A) * P(B\|A) = P(B) * P(A\|B)$. • (+) Permutations and combinations are strategies for counting the outcomes of a sample space. **MATHEMATICAL THINKING** • Use precise mathematical language. • Look for and make use of structure. • Compute accurately and efficiently. *Continued on next page*

	Expectations for Learning, continued **INSTRUCTIONAL FOCUS** • Recognize and justify mathematically whether two events are independent. • Generalize probability rules using patterns. • Compute probabilities accurately and efficiently. • (+) Recognize and apply counting methods to compute probabilities. • Compute the conditional probability of A given B. • Interpret and explain conditional probability within a context. • Apply the Addition Rule. • Interpret and explain the Addition Rule within a context. • (+) Apply the Multiplication Rule. • (+) Interpret and explain the Multiplication Rule within a context. • (+) Know and explain the difference between a permutation and a combination. • (+) Calculate probabilities using permutations and combinations. • (+) Interpret and explain probabilities using permutations and combinations within a context. **Content Elaborations** **OHIO'S HIGH SCHOOL CRITICAL AREAS OF FOCUS** • Math 2, Number 1, page 3 **CONNECTIONS ACROSS STANDARDS** • Understand independence and conditional probability (S.CP.1-5). • Interpret and use the structure of expressions (A.SSE.1-3).

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*(WG) refers to a member of the Working Group and (AC) refers to a member of the Advisory Committee in the Standards Revision Process.