Ohio's K-8 Learning Progressions

The concept of learning progressions was critical in the development, review, and revision of Ohio's Learning Standards. Ohio's learning progressions were developed during Ohio's international benchmarking project and provided guidance to the writing and revision committees of Ohio's Learning Standards. Ohio believes that the concept of learning progressions is important for the understanding and coherence of mathematical topics within and across the grade levels. The Ohio Department of Education has reformatted Ohio's Learning Standards by domains to show the progression of concepts and skills across the grade levels.

This document serves as a companion document to the learning standards; it does not replace them. Curriculum leaders and teachers can use this document to better understand the standards and to analyze where their curriculum fits within the progression of learning for their students. The following examples are ways to use this document for professional learning communities and curriculum development.

Multi-grade groups of teachers

Example 1: Select a domain within the standards, beginning at the lowest grade of the domain, then identify the main concepts at that grade. Follow each concept progressing through the grades by identifying how the concept changes and increases in rigor and understanding for the student. Additionally, identify new concepts introduced in subsequent grades and follow them through the years.

Example 2: Building on example one, begin to identify the connections among the learning progressions. For instance, how is Measurement and Data connected (used to develop the essential understandings) to other topics in grades 6-8? How is Measurement and Data used in the service of learning other concepts and skills in kindergarten through grade 5?

Example 3: Use the learning progressions to identify where concepts and skills have moved. Some concepts and skills have moved to earlier grades, other to later grades.

Grade level or individual teachers

Example 4: In partnership with regular classroom formative assessment, teachers can use the learning progression to assist in identifying where students are in the progression. Then they can develop supports to accelerate the students to bring their understandings and skills to the appropriate level or to go deeper into the content. Note that going deeper does not imply going to the next level in the progression, rather building stronger understandings of the content or making connections to other concepts or skills.

Example 5: It is important to make connections among the standards; between standards within a domain; between standards within a cluster; and between clusters across domains. Also use the Mathematics – K-8 Critical Areas of Focus to make further connections.

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Counting and Cardinality

Kindergarten

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward within 100 beginning from any given number other than 1.
3. Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies.
   a. When counting objects, establish a one-to-one relationship by saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle; or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Compare numbers.

6. Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.
7. Compare (without using inequality symbols) two numbers between 0 and 10 when presented as written numerals.

Number and Operations in Base Ten

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
Work with numbers 11-19 to gain foundations for place value. **1.** Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects and, when appropriate, drawings or equations; understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**Extend the counting sequence.** **1.** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Understand place value.** **2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones - called a "ten;" the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones; and the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **3.** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **Use place value understanding and properties of operations to add and subtract.** **4.** Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used.	**Understand place value.** **1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: **a.** 100 can be thought of as a bundle of ten tens — called a "hundred." **b.** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.** Count forward and backward within 1,000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5. **3.** Read and write numbers to 1,000 using base-ten numerals, number names, expanded form, and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc. **4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. **Use place value understanding and properties of operations to add and subtract.** **5.** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **6.** Add up to four two-digit numbers using strategies based on place value and properties of operations.	**Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.** **1.** Use place value understanding to round whole numbers to the nearest 10 or 100. **2.** Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.** Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., 9 × 80, 5 × 60 using strategies based on place value and properties of operations.	**Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.** **1.** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division. **2.** Read and write multi-digit whole **3.** numbers using standard form, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. **4.** Use place value understanding to round multi-digit whole numbers to any place through 1,000,000. **Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.** **5.** Fluently add and subtract multi-digit whole numbers using a standard algorithm. **6.** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**Understand the place value system.** **1.** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $^1/10$ of what it represents in the place to its left. **2.** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **3.** Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (^1/10) + 9 \times (^1/100) + 2 \times (^1/1000)$. **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.** Use place value understanding to round decimals to any place, millions through hundredths. **Perform operations with multi-digit whole numbers and with decimals to hundredths.** **1.** Fluently multiply multi-digit whole numbers using a standard algorithm.

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Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
	Understand that when adding two-digit numbers, tens are added to tens; ones are added to ones; and sometimes it is necessary to compose a ten. **5.** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**7.** Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, hundreds are added or subtracted from hundreds, tens are added or subtracted from tens, ones are added or subtracted from ones; and sometimes it is necessary to compose or decompose tens or hundreds. **8.** Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900. **9.** Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.		**7.** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**2.** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **3.** Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. **a.** Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place). **b.** Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). **c.** Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers that work well with one another). For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.

Numbers and Operations - Fractions

Grade three

Develop understanding of fractions as numbers.

Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b (which may be greater than 1) on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3. Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size or on the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size.

Grade Four

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$, or $5/4 = (1/4) + (1/4) + (1/4) + (1/4) + (1/4)$.*

b. Understand a multiple of a/b as a multiple of $1/b$ and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)*

Grade Five

Use equivalent fractions as a strategy to add and subtract fractions (Fractions need not be simplified.)

1. Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example*, use visual models and properties of operations to show $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. *In general, $a/b + c/d = (a/b \times d/d) + (c/d \times b/b) = (ad + bc)/bd$.*

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified.)

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a part of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing).

Grade three	Grade Four	Grade Five
Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $^3/_8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.** **5.** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $^3/_{10}$ as $^{30}/_{100}$, and add $^3/_{10} + {}^4/_{100} = {}^{34}/_{100}$.* In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade. **6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as $^{62}/_{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **7.** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	**a.** Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication. **b.** Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $^a/_b = {}^{(n\times a)}/_{(n\times b)}$ to the effect of multiplying $^a/_b$ by 1. **6.** Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **7.** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $(^1/_3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(^1/_3) \div 4 = (^1/_{12})$ because $(^1/_{12}) \times 4 = (^1/_3)$.* **b.** Interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for $4 \div (^1/_5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (^1/_5) = 20$ because $20 \times (^1/_5) = 4$.* **c.** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $^1/_2$ pound of chocolate equally? How many $^1/_3$ cup servings are in 2 cups of raisins?*

Operations and Algebraic Thinking

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
Understand addition as putting together and adding to and understand subtraction as taking apart and taking from. **1.** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds such as claps, acting out situations, verbal explanations, expressions, or equations. Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **2.** Solve addition and subtraction problems (written or oral) and add and subtract within 10 by using objects or drawings to represent the problem. **3.** Decompose numbers and record compositions for numbers less than or equal to 10 into pairs in more than one way by using objects and, when appropriate, drawings or equations.	**Represent and solve problems involving addition and subtraction.** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 1. **2.** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **Understand and apply properties of operations and the relationship between addition and subtraction.** **3.** Apply properties of operations as strategies to add and subtract. *For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative Property of Addition); to add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative Property of Addition).* Students need not use formal terms for these properties. ***4.*** Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	**Represent and solve problems involving addition and subtraction.** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 1. **Add and subtract within 20.** **2.** Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.OA.6 for a list of mental strategies. **Work with equal groups of objects to gain foundations for multiplication.** **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	**Represent and solve problems involving multiplication and division.** **1.** Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that a x b means a groups of b objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each). ***2.*** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **3.** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 2. Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **4.** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × _ = 48, 5 = _ ÷ 3, 6 × 6 = _.*	**Use the four operations with whole numbers to solve problems.** **1.** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **2.** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Glossary, Table 2. Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **3.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**Write and interpret numerical expressions.** **1.** Use parentheses in numerical expressions and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary. **2.** Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.* **Analyze patterns and relationships.** **3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0 and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or, when appropriate, an equation. Fluently add and subtract within 5.	**Add and subtract within 20.** **5.** Relate counting to addition and subtraction, e.g., by counting on 2 to add 2. **6.** Add and subtract within 20, demonstrating fluency with various strategies for addition and subtraction within 10. Strategies may include counting on; making ten, e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14; decomposing a number leading to a ten, e.g., 13 − 4 = 13 − 3 − 1 = 10 − 1 = 9; using the relationship between addition and subtraction, e.g., knowing that 8 + 4 = 12, one knows 12 − 8 = 4; and creating equivalent but easier or known sums, e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13. **Work with addition and subtraction equations.** ***7.*** Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6; 7 = 8 – 1; 5 + 2 = 2 + 5; 4 + 1 = 5 + 2.* ***8.*** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations:* *8 + _ = 11; 5 = _ - 3; 6 + 6= _*	**4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	**Understand properties of multiplication and the relationship between multiplication and division.** **5.** Apply properties of operations as strategies to multiply and divide. *For example, if 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (Commutative Property of Multiplication); 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (Associative Property of Multiplication); knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive Property).* Students need not use formal terms for these properties. **6.** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* **Multiply and divide within 100.** **7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8 or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two one-digit numbers. (continues on next page)	**Gain familiarity with factors and multiples.** **4.** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **Generate and analyze patterns.** **5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
			Solve problems involving the four operations and identify and explain patterns in arithmetic. **8.** Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected. **9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*		

Geometry – Grades K-5

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). **1.** Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. **2.** Correctly name shapes regardless of their orientations or overall size. **3.** Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid"). **Describe, compare, create, and compose shapes.** **4.** Describe and compare two- or three-dimensional shapes, in different sizes and orientations, using informal language to describe their commonalities, differences, parts, and other attributes. **5.** Model shapes in the world by building shapes from components (such as sticks and clay balls) and drawing shapes. **6.** Combine simple shapes to form larger shapes.	**Reason with shapes and their attributes.** **1.** Distinguish between defining attributes, e.g., triangles are closed and three-sided, versus non-defining attributes, e.g., color, orientation, overall size; build and draw shapes that possess defining attributes. **2.** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism." **3.** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths,* and *quarters,* and use the phrases *half of, fourth of,* and *quarter of.* Describe the whole as two of or four of the shares in real-world contexts. Understand for these examples that decomposing into more equal shares creates smaller shares.	**Reason with shapes and their attributes.** **1.** Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders. **2.** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **3.** Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds,* or fourths and quarters, and use the phrases *half of, third of,* or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.	**Reason with shapes and their attributes.** **1.** Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles). **2.** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as $^1/_4$ of the area of the shape.*	**Draw and identify lines and angles and classify shapes by properties of their lines and angles.** **1.** Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **2.** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.	**Graph points on the coordinate plane to solve real-world and mathematical problems.** **1.** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate. **2.** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation. **Classify two-dimensional figures into categories based on their properties.** **3.** Identify and describe commonalities and differences of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles). **4.** Identify and describe commonalities and differences of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids, and rhombuses.

Measurement and Data

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
Identify, describe, and compare measurable attributes. **1.** Identify and describe measurable attributes (length, weight, and height) of a single object using vocabulary terms such as long/short, heavy/light, or tall/short. **2.** Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute and describe the difference. *For example, directly compare the heights of two children, and describe one child as taller/shorter.* **Classify objects and count the number of objects in each category.** **3.** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. The number of objects in each category should be less than or equal to ten. Counting and sorting coins should be limited to pennies.	**Measure lengths indirectly and by iterating length units.** **1.** Order three objects by length; compare the lengths of two objects indirectly by using a third object. **2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **Work with time and money.** **3.** Work with time and money. **a.** Tell and write time in hours and half-hours using analog and digital clocks. **b.** Identify pennies and dimes by name and value. (continues on next page)	**Measure and estimate lengths in standard units.** **1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **3.** Estimate lengths using units of inches, feet, centimeters, and meters. **4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit. **Relate addition and subtraction to length.** **5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same whole number units, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	**Solve problems involving money and measurement and estimation of intervals of time, liquid volumes, and masses of objects.** **1.** Work with time and money. **a.** Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. **b.** Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbol appropriately (not including decimal notation). **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Glossary, Table 2. **Represent and interpret data.** **3.** Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. *For example,* create *a bar graph in which each square in the bar graph might represent 5 pets,* then determine how many more/less in two given categories.	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** **1.** Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. *For example,* express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300, ... **2.** Solve real-world problems involving money, time, and metric measurement. **a.** Using models, add and subtract money and express the answer in decimal notation. **b.** Using number line diagrams, clocks, or other models, add and subtract intervals of time in hours and minutes. **c.** Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects. **3.** Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.	**Convert like measurement units within a given measurement system.** **1.** Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems. **Represent and interpret data.** **2.** Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions 1/2, 1/4, 1/8, or decimals. **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **3.** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b.** A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
	Represent and interpret data. **4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **6.** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. **Work with time and money.** **7.** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **8.** Solve problems with money. **a.** Identify nickels and quarters by name and value. **b.** Find the value of a collection of quarters, dimes, nickels, and pennies. **c.** Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbols appropriately (not including decimal notation).	**4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **5.** Recognize area as an attribute of plane figures and understand concepts of area measurement. **a.** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. **b.** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **7.** Relate area to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning. **c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and $b + c$ is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model). **d.** Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	**Represent and interpret data.** **4.** Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade. **Geometric measurement: understand concepts of angle and measure angles.** **5.** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement. **a.** Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. **b.** Understand angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **6.** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **7.** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	**4.** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative Property of Multiplication. **b.** Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real- world problems.

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
		Represent and interpret data. **9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plot, where the horizontal scale is marked off in whole-number units. **10.** Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph. See Glossary, Table 1.	**Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **8.** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.		

Ratios and Proportional Relationships

Grade Six	Grade Seven
Understand ratio concepts and use ratio reasoning to solve problems. **1.** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **2.** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **c.** Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means $30/100$ times the quantity; solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. Previous Learning from Grade 5 Number and Operations in Base Ten, Number and Operations - Fractions	**Analyze proportional relationships and use them to solve real-world and mathematical problems.** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $(1/2)/(1/4)$ miles per hour, equivalently 2 miles per hour.* **2.** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* **d.** Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. **3.** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System

Grade Six	Grade Seven	Grade Eight
Apply and extend previous understandings of multiplication and division to divide fractions by fractions. **1.** Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ pound of chocolate equally? How many $3/4$ cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?* **Compute fluently with multi-digit numbers and find common factors and multiples.** **2.** Fluently divide multi-digit numbers using a standard algorithm. **3.** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. **4.** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* **Apply and extend previous understandings of numbers to the system of rational numbers.** **5.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **7.** Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3°C > -7°C$ to express the fact that $-3°C$ is warmer than $-7°C$.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $\lvert -30 \rvert = 30$ to describe the size of the debt in dollars.* **d.** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* **8.** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. Previous Learning from Grade 5 Number and Operations in Base Ten, Number and Operations - Fractions	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **1.** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **b.** Understand $p + q$ as the number located a distance $\lvert q \rvert$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. **d.** Apply properties of operations as strategies to add and subtract rational numbers. **2.** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. **c.** Apply properties of operations as strategies to multiply and divide rational numbers. **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **3.** Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. (continues on the next page)	**Know that there are numbers that are not rational and approximate them by rational numbers.** **1.** Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. **2.** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations

Grade Six

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.

2. Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 − y.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$*

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

4. Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

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Grade Seven

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. For example, a discount of 15% (represented by p − 0.15p) is equivalent to (1 − 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example, if a woman making $25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 ¾ inches long in the center of a door that is 27 $1/2$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

4. Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.*

Grade Eight

Work with radicals and integer exponents.

1. Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

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Grade Six	Grade Seven	Grade Eight
Reason about and solve one-variable equations and inequalities. **5.** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **7.** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **8.** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **Represent and analyze quantitative relationships between dependent and independent variables.** **9.** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times and write the equation $d = 65t$ to represent the relationship between distance and time.* Previous Learning from Grade 5 Operations and Algebraic Thinking		**Analyze and solve linear equations and pairs of simultaneous linear equations.** **7.** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.** Analyze and solve pairs of simultaneous linear equations graphically. **a.** Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. **b.** Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)*

Functions

Grade Eight

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1) (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Previous learning from Grade 5 Operations and Algebraic Thinking

Geometry - Grades 6-8

Grade Six	Grade Seven	Grade Eight
Solve real-world and mathematical problems involving area, surface area, and volume. **1.** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. **2.** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell w h$ and $V = B h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **3.** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **4.** Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. Previous Learning in Grade 5 Measurement and Data, Geometry	**Draw, construct, and describe geometrical figures and describe the relationships between them.** **1.** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. **b.** Represent proportional relationships within and between similar figures. **2.** Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions. **3.** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume. **4.** Work with circles. **5.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **6.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. **7.** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **8.** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**Understand congruence and similarity using physical models, transparencies, or geometry software.** **1.** Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **a.** Lines are taken to lines, and line segments are taken to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines. **2.** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.) **3.** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **4.** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.) **5.** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* **Understand and apply the Pythagorean Theorem.** **6.** Analyze and justify an informal proof of the Pythagorean Theorem and its converse. **7.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **8.** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** **9.** Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.

Statistics and Probability

Grade Six	Grade Seven	Grade Eight
Develop understanding of statistical problem solving. **1.** Develop statistical reasoning by using the GAISE model: **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4) **2.** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **3.** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **Summarize and describe distributions.** **4.** Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3) **5.** Summarize numerical data sets in relation to their context. **a.** Report the number of observations. **b.** Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describing shape and the presence of clusters, gaps, peaks, and outliers in a distribution. **d.** Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.	**Use sampling to draw conclusions about a population.** **1.** Understand that statistics can be used to gain information about a population by examining a sample of the population. **a.** Differentiate between a sample and a population. **b.** Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias. **Broaden understanding of statistical problem solving.** **2.** Broaden statistical reasoning by using the GAISE model: **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4) **Summarize and describe distributions representing one population and draw informal comparisons between two populations.** **3.** Describe and analyze distributions. **a.** Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point. **b.** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable. **Investigate chance processes and develop, use, and evaluate probability models.** **4.** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **5.** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.	**Investigate patterns of association in bivariate data.** **1.** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4) **2.** Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4) **3.** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4) **4.** Understand that patterns of association can also be seen in **5.** bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. **6.** Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Grade Six	Grade Seven	Grade Eight
	6. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **a.** Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? **7.** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes", identify the outcomes in the sample space which compose the event. **c.** Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?	