Algebra 2 and Mathematics 3
Critical Areas of Focus

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. These descriptions have been used to develop critical areas for each of the courses in both the Traditional and Integrated pathways. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.
What are the concepts?
What are the procedures and skills?
What are the key mathematical practices?
What are the relationships students are to make?
What further information is needed? For example, what does prove mean?
What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards <u>within</u> each course's Critical Areas of Focus.
What are the relationships among the conceptual categories, domains, clusters and standards?
Why is each relationship important?
What are the differences?
How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections <u>across</u> the Critical Areas of Focus within a course. This information will help create a sequence of units for the course. For example, within Critical Area of Focus #2, work with polynomials should precede work with rational expressions.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.
What understandings does this learning build upon?
What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.
What is taught now but not in Ohio's Learning Standards?
What content is essentially the same? Identify the differences.
What will be new content for this grade?

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CRITICAL AREA OF FOCUS #1

Inferences and Conclusions from Data

Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn. To model the relationships between variables, students fit a linear function to data and analyze residuals to assess the appropriateness of fit. Building on prior experiences associated with the notion of a correlation coefficient, students learn how to distinguish a statistical relationship from a cause-and-effect relationship.

Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★

- **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3)
- **b.** Informally assess the fit of a function by discussing residuals. (A2, M3)

Interpret linear models.

S.ID.9 Distinguish between correlation and causation. ★

Statistics and Probability – Making Inferences and Justifying Conclusions

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant. ★

S.IC.6 Evaluate reports based on data. ★

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CRITICAL AREA OF FOCUS #2

Polynomials, Rational and Radical Relationships

Students build on their previous work with rational numbers and integer exponents to develop understanding of rational exponents. They apply this new understanding of number to seeing the structure in exponential expressions. Students develop the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. This learning culminates with applying the Fundamental Theorem of Algebra.

A central theme of the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students realize that rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, they learn that rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. In addition, students will extend their experience with solving systems of two linear equations in two variables to solving systems of three linear equations in three variables algebraically.

Number and Quantity – The Real Number System

Extend the properties of exponents to rational exponents.

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Number and Quantity – The Complex Number System

Perform arithmetic operations with complex numbers.

N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

(+) **N.CN.8** Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*

(+) **N.CN.9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

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CRITICAL AREA OF FOCUS #2, CONTINUED

Polynomials, Rational and Radical Relationships

Algebra – Seeing Structure in Expressions

Interpret the structure of expressions.

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

- **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.*

(+) **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* ★

Algebra – Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **b.** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)

Understand the relationship between zeros and factors of polynomials.

A.APR.2 Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$. In particular, $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

A.APR.3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

(+) **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, by using coefficients determined for by Pascal's Triangle.* The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions.

A.APR.6 Rewrite simple rational expressions in different forms; write ${}^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

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Critical Area of Focus #2, continued

Polynomials, Rational and Radical Relationships

Algebra – Arithmetic with Polynomials and Rational Expressions

Rewrite rational expressions.

(+) **A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Algebra - Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve systems of equations.

A.REI.6 Solve systems of linear equations algebraically and graphically.

- **b.** Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)

Represent and solve equations and inequalities graphically.

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

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CRITICAL AREA OF FOCUS #3

Trigonometry of General Triangles and Trigonometric Functions

Building on students' previous work with functions, trigonometric ratios, and circles in Geometry or Mathematics 2, students now use the coordinate plane and the unit circle to extend trigonometry to general angles and to model periodic phenomena. This leads to the conclusion that trigonometry is applied beyond the right triangle— that is, at least to obtuse angles. Concurrently, students develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They also develop an understanding that the Laws of Sines and Cosines can be used to find missing measures of general (not necessarily right) triangles. This allows students to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Geometry – Similarity, Right Triangles, and Trigonometry

Define trigonometric ratios, and solve problems involving right triangles.

G.SRT.8 Solve problems involving right triangles. ★

(+) **b.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★ (A2, M3)

Apply trigonometry to general triangles.

(+) **G.SRT.9** Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

(+) **G.SRT.10** Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.

(+) **G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.

Geometry – Circles

Find arc lengths and areas of sectors of circles.

G.C.6 Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)

Functions – Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

Prove and apply trigonometric identities.

F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

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CRITICAL AREA OF FOCUS #4

Modeling with Functions

Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★

- **c.** Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

- **c.** Extend to include more complicated function situations with the option to graph with technology. (A2, M3)

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★ (A1, M1)

- **a.** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★

- d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ (A2, M3)

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CRITICAL AREA OF FOCUS #4, CONTINUED

Modeling with Functions

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **c.** Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

- **c.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3)
- **d.** Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3)
- **f.** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline[G], and amplitude. (A2, M3)
- (+) **g.** Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (A2, M3)
- (+) **h.** Graph logarithmic functions, indicating intercepts and end behavior. (A2, M3)

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)
- **b.** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, *and* $y = (0.97)^t$ *and classify them as representing exponential growth or decay.* (A2, M3)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

- **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3)

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CRITICAL AREA OF FOCUS #4, CONTINUED

Modeling with Functions

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)

F.BF.4 Find inverse functions.

(+) **b.** Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)

(+) **c**. Verify by composition that one function is the inverse of another. (A2, M3)

(+) **d.** Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★