Advanced Quantitative Reasoning Critical Areas of Focus-DRAFT

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.

- What are the concepts?
- What are the procedures and skills?
- What are the key mathematical practices?
- What are the relationships students are to make?
- What further information is needed? For example, what does prove mean?
- What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards *within* each course's Critical Areas of Focus.

- What are the relationships among the conceptual categories, domains, clusters and standards?
- Why is each relationship important?
- What are the differences?
- How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections *across* the Critical Areas of Focus within a course. This information will help create a sequence of units for the course.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.

- What understandings does this learning build upon?
- What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.

- What is taught now but not in Ohio's Learning Standards?
- What content is essentially the same? Identify the differences.
- What will be new content for this grade?

Advanced Quantitative Reasoning

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CRITICAL AREA OF FOCUS #1

Communication and Analysis

Within this critical area students develop conclusions based on quantitative information and critical thinking. They recognize, make, and evaluate underlying assumptions in estimation, modeling, and data analysis. Students then organize and present thoughts and processes using mathematical and statistical evidence. They communicate clear and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation. Students demonstrate numerical reasoning orally and in writing coherent statements and paragraphs.

In the context of real-world applications, students make and investigate mathematical conjectures. They are able to defend their conjectures and respectfully question conjectures made by their classmates. This leads to the development of mathematical arguments and informal proofs, which are ways of expressing particular kinds of reasoning and justification. Explanations (oral and written) include mathematical arguments and rationales, not just procedural descriptions or summaries. Listening to others' explanations gives students opportunities to develop their own understandings. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language. Additionally, conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. **This critical area of focus cross cuts all the other critical areas of focus.**

Standards for Mathematical Practices

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.

This critical area cross cuts all the rest of the standards, so all standards in this document also fall under this crucial area!!!

Algebra – Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #2

Modeling with Number and Quantity

Numeracy is the ability to read, understand and work with quantitative information in order to purposefully use and analyze mathematics in a wide range of situations including in one's personal life, as a citizen and in one's career. Numeracy involves much more than arithmetic skills and straightforward procedural competencies. It involves the ability to be strategic in how and when to use mathematics, understand and interpret mathematical representations, construct and destruct numbers, estimate quantities appropriately, and make sound decisions. Students develop and use the concepts of numeracy to investigate and explain quantitative relationships and solve problems in a variety of real-world contexts. Mathematical concepts involving proportional reasoning such as ratios, rates of change, unit conversions and dimensional analysis are explored. In this course numeracy is applied to a variety of topics including aspects of financial literacy such as budgets and loans. (The teaching of numeracy is intended to both deepen and broaden understanding achieved in previous grades keeping the development and use of all but the most basic algebraic procedures to a minimum. Problems requiring rote use of arithmetic or algebraic procedures should be deemphasized, except when these procedures are essential to gaining deep conceptual understanding.) Modeling in this course should be grounded in concepts outlined in the GAIMME framework[G].

Standards for Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**

Number and Quantity – Quantities

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Note: Standards N.Q.4-7 are included in this course to be considered for next standards revision.

Reason quantitatively about numerical data including covariation to solve real-world problems.

N.Q.4 *Reason, model, and communicate about numerical data building upon previous knowledge of fractions, decimals, percents, scientific notation and estimation and using that knowledge flexibly in a variety of circumstances and a range of number values.★*

N.Q.5 *Reason, model, and communicate about proportions including the following:★*

a. *Distinguishing between proportional and non-proportional situations.*

b. *Using dimensional analysis.*

c. *Analyzing and comparing growth and decay using absolute and relative change.*

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #2, CONTINUED

Modeling with Number and Quantity

Number and Quantity – Quantities

Note: Standards N.Q.4-7 are included in this course to be considered for next standards revision.

Reason quantitatively about numerical data including covariation to solve real-world problems.

N.Q.6 *Use models to solve and communicate about contextual financial questions such as budgets, credit card debt, installment savings, amortization schedules, mortgage and other loan scenarios.★*

N.Q.7 *Identify and explain personal and societal consequences of financial decisions and other scenarios.★*

Algebra – Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

Geometry – Modeling with Geometry

Apply geometric concepts in modeling situations.

G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #3

Modeling with Algebra and Functions

The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of the course. The narrative discussion and diagram (page 19) of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context. Students make decisions by analyzing mathematical models, including situations in which the student must recognize and/or make assumptions. Students synthesize and generalize what they have learned about a variety of function families. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. Students model functions in a variety of real-world topics including aspects of financial literacy. Modeling in this course should be grounded in concepts outlined in the GAIMME framework[G].

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
5. Use appropriate tools strategically.
8. Look for and express regularity in repeated reasoning.

Algebra – Seeing Structure in Expressions

Interpret the structure of expressions.

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems.

(+) **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. *For example, calculate mortgage payments.*★

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.*★
c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
c. Extend to include more complicated function situations with the option to graph with technology. (A2, M3)

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #3, CONTINUED

Modeling with Algebra and Functions

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1)
a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★
d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

Algebra – Reasoning with Equations and Inequalities

Solve equations and inequalities in one variable.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

A.REI.6 Solve systems of linear equations algebraically and graphically.

Represent and solve equations and inequalities graphically.

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions

Understand the concept of a function, and use function notation.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #3, CONTINUED

Modeling with Algebra and Functions

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ (A2, M3)

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3)

(+) **c.** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #3, CONTINUED

Modeling with Algebra and Functions

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

(+) F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.★ (A1, M2)

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #4

Modeling with Statistics and Probability

Students build upon their previous statistical experiences and extend their experience to new situations and deepen their understanding of the statistical process. They use graphical representations and knowledge of the context to make judgments about the appropriateness of statistical models. They develop formal means of assessing how a model fits data[G] such as the use of regression techniques to describe approximately linear relationships between quantities. Students see how the visual displays and summary statistics relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn. To model the relationships between variables, students fit a linear function to data and analyze residuals to assess the appropriateness of fit. Building on prior experiences associated with the notion of a correlation coefficient, students learn how to distinguish a statistical relationship from a cause-and-effect relationship.

Statistics and Probability in Ohio's Learning Standards is grounded in GAISE model framework[G]. Within the context of real-world applications, students investigate, represent, make decisions, and draw conclusions about data. Students formulate statistical investigative questions anticipating variability. They acknowledge variability when they collect and consider data. Students will use distributions in accounting for variability when they analyze data. Students interpret the results by looking beyond the data.

Standards for Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
5. **Use appropriate tools strategically.**
7. **Look for and make use of structure.**

Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets.★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #4, CONTINUED

Modeling with Statistics and Probability

Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (A2, M3)

b. Informally assess the fit of a function by discussing residuals. (A2, M3)

c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S.ID.9 Distinguish between correlation and causation.★

Statistics and Probability – Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.★

S.IC.6 Evaluate reports based on data.★

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #4, CONTINUED

Modeling with Statistics and Probability

Statistics and Probability –Conditional Probability and the Rules of Probability

Understand independence and conditional probability, and use them to interpret data.

S.CP.3 Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★

S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★

Statistics and Probability – Using Probability to Make Decisions

Calculate expected values, and use them to solve problems.

S.MD.1 Define a random variable[G] for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution[G] using the same graphical displays as for data distributions.★

S.MD.2 Calculate the expected value[G] of a random variable; interpret it as the mean of the probability distribution.★

S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★

S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*★

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #4, CONTINUED

Modeling with Statistics and Probability

Statistics and Probability – Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions.

S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★

a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

S.MD.6 Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★

S.MD.7 Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #5

Modeling with Geometry

Students take their prior experience in geometry and apply that learning to real-world problems which could include concepts of congruence, similarity, symmetry, Pythagorean Theorem, circles and trigonometry. They utilize geometric and visual models to represent and understand given scenarios. Geometric methods can be applied to solve real-world design and modeling problems. Of all the subjects students learn in geometry, trigonometry may have the greatest application in college and career. Students in high school should see authentic applications of trigonometry to many different contexts. Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena as well as physical phenomena. Geometric modeling can be used in Fermi[G] problems, problems which ask for rough estimates of quantities.

Students' experience with two-dimensional and three-dimensional objects may be extended. Students build upon their understanding of area and volume; how changes in dimensions result in similar and non-similar shapes; and how scaling changes lengths, areas and volumes. Additionally, students may apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections; the result of rotating a two-dimensional object about a line; and the functions represented by the cross-section.

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

Geometry – Congruence

Experiment with transformations in the plane.

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.
a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes.
b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Advanced Quantitative Reasoning Critical Areas of Focus

CRITICAL AREA OF FOCUS #5, CONTINUED

Modeling with Geometry

Geometry – Similarity, Right Triangles, and Trigonometry

Prove and apply theorems both formally and informally involving similarity using a variety of methods.

G.SRT.4 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

Define trigonometric ratios, and solve problems involving right triangles.

G.SRT.8 Solve problems involving right triangles.★

Geometry – Circles

Find arc lengths and areas of sectors of circles.

G.C.5 Find arc lengths and areas of sectors of circles.

Geometry – Geometric Measurement and Dimension

Understand the relationships between lengths, areas, and volumes.

G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.

Geometry – Modeling with Geometry

Apply geometric concepts in modeling situations.

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★

G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★

G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

Functions – Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★