Ohio's Learning Standards | Mathematics

Data Science Foundations-DRAFT

Ohio | Department of Education

Table of Contents

Introduction to Ohio's Learning Standards for Mathematics

PROCESS

To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education.

Then in 2019, Ohio started the Strengthening Math Pathways Initiative. Two groups were formed: the Math Pathways Advisory Council and the Math Pathways Architects. The advisory council, made of representatives from education stakeholder groups, aligned systems and structures between secondary and postsecondary mathematics. The Math Pathways Architects, made up of high school and collegiate math faculty, aligned the mathematics between the two systems. One of the outcomes was four proposed Algebra 2 equivalent courses: Quantitative Reasoning, Statistics and Probability, Data Science Foundations, and Discrete Math/Computer Science. A workgroup was formed for each of these courses. This document is the result of the Data Science Foundations Workgroup with oversight from the Math Pathways Architects.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 10 with the eight Standards for Mathematical Practice.

Introduction to Ohio's Learning Standards for Computer Science

Substitute House Bill Number 170 took effect in March 2018, requiring the State Board of Education of Ohio to adopt standards and a model curriculum for grade K-12 instruction in computer science. A team of Ohio educators came together to develop and write the computer science standards and model curriculum, and the State Board adopted these in December 2018.

Ohio's Standards in Computer Science are fully aligned to Ohio's five-year strategic plan for education, Each Child, Our Future. The strategic plan acknowledges a major education policy shift around technology. A student's ability to use technology strategically is now identified as foundational and just as important as mathematics and English language arts, from which all other learning is built.

GUIDING ASSUMPTIONS

The team of Ohio educators that developed the standards and model curriculum had a clear goal – to encourage districts and educators to give all Ohio students opportunities to learn computer science. Beginning in the earliest grades and continuing through grade 12, Ohio's students will develop a foundation of computer science knowledge and gain experiences in computational thinking and problem solving to become creators and innovators of computing technology. Ohio's Computer Science Standards and Model Curriculum will give students experiences that help them discover and take part in a world continually influenced by technology and to understand the role of computing in that world.

OVERVIEW OF THE COMPUTER SCIENCE STANDARDS CONTENT

The standards will support a progression of learning in each core concept or strand to provide computer science experiences for all Ohio students. The K-8 standards integrate computer science into instruction across subject areas including mathematics, science, history, English language arts, fine arts, world language and career and technology courses. The high school computer science standards provide both foundational and advanced opportunities districts can use to design as separate courses or, when appropriate, integrate into other disciplines.

Ohio's Computer Science Standards and Model Curriculum are organized in the following strands:

- **Computing Systems** – Addresses how devices, including hardware and software, interact to accomplish tasks and how students can troubleshoot computing systems when they do not work as intended.
- **Networks and the Internet** – Addresses how networks connect to share information and resources and how students can apply cybersecurity concepts to protect information.
- **Data and Analysis** – Addresses how data can be collected and stored; analyzed and communicated; and used to make more accurate predictions.
- **Algorithmic Thinking and Programming** – Addresses program development, including the use of algorithms, variables, control structures and modules.
- **Impacts of Computing** – Addresses computing's influence on our world by examining the relationship between computing and culture, computing's impact on social interaction, and legal and ethical implications of computing.

Introduction to Ohio's Learning Standards for Computer Science, continued

Computational Thinking is a problem -solving process that students use to engage with concepts in the computer science standards. This thinking involves formulating problems in a way that can be carried out by a computer. Using computational thinking to solve a problem includes breaking down the problem into manageable parts; recognizing patterns; excluding irrelevant details to abstract or identify general principles that generate these patterns; and developing step -by -step sequences or algorithms to solve the problem and similar problems. Computational thinking can be applied with or without computers, for example, through "unplugged" activities. While computational thinking is a focus in computer science, it also is used in content areas beyond computer science.

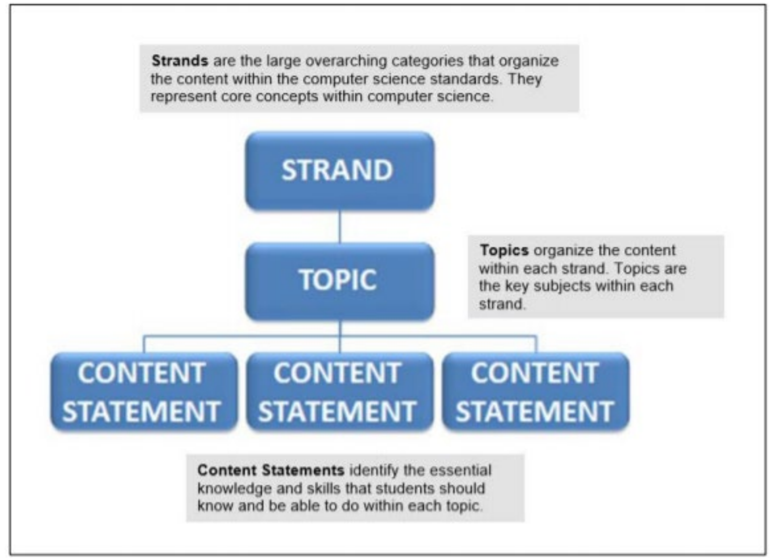
It is important that computer science not be confused with other aspects and uses of computer technology in schools, including:

- **Computer literacy** "refers to the general use of computers and programs, such as productivity software." Examples of computer literacy include performing an internet search and creating a digital presentation.
- **Educational (computer) technology** "applies computer literacy to school subjects. For example, students in an English class can use a web -based application to collaboratively create, edit and store an essay online."
- **Digital citizenship** "refers to the appropriate and responsible use of technology, such as choosing an appropriate password and keeping it secure."
- **Information technology** "often overlaps with computer science but is mainly focused on industrial applications of computer science, such as installing [and operating] software rather than creating it. Information technology professionals often have a background in computer science."

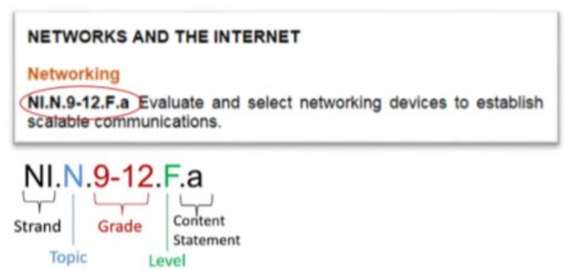
(K -12 Computer Science Framework, 2016, pp.13 -14)

OVERVIEW OF THE COMPUTER SCIENCE STANDARDS FRAMEWORK

Ohio's Computer Science Standards are organized by strands, topics and content statements.



Grades 9-12 - Content statements are organized by grade band into two levels – Foundational and Advanced. See an example of a content statement for high school and its corresponding content statement code below. This content statement addresses the topic of Networking within the Networks and the Internet strand, at the Foundational Level.



A Note on Rigor and Algebra 2 Equivalency

Ohio law states that students must have four units of mathematics and that one of those units should be Algebra 2/Math 3 or its equivalent. Ohio has decided to expand guidance around what it means to be *equivalent* to Algebra 2.

It has been decided that *equivalent* refers to the level of rigor and reasoning, not content. There are many branches of mathematics that are equally rigorous but have different content focuses. All equivalent courses should have the same level of rigor and reasoning that are needed to be successful in an entry-level, credit-bearing postsecondary mathematics course.

Ohio has defined rigor as the following:

> "Students use mathematical language to communicate effectively and to describe their work with clarity and precision. Students demonstrate how, when, and why their procedure works and why it is appropriate. Students can answer the question, 'How do we know?'"

This can be illustrated in the table to the right.

Currently, four courses have been determined to be equivalent to Algebra 2: Advanced Quantitative Reasoning, Statistics and Probability, Data Science Foundations and Discrete Math/Computer Science. This document explains what should be included in a Data Science Foundations course in order to be considered Algebra 2 equivalent. Like a traditional Algebra 2 course, this course should be a year-long course in order for students to earn the credit necessary for graduation.

Rigorous courses are…	vs	Rigorous courses are not…
Defined by complexity, which is a measure of the thinking, action, or knowledge that is needed to complete the task		Characterized by difficulty, which is a measure of effort required to complete a task
Measured in depth of understanding		Measured by the amount of work
Opportunities for precision in reasoning, language, definitions, and notation that are sufficient to appropriate age/course		Based on procedure alone
Determined by students' process		Measured by assigning difficult problems
Opportunities for students to make decisions in problem solving		Defined only by the resources used
Opportunities to make connections		Taught in isolation
Supportive of the transfer of knowledge to new situations		Repetitive
Driven by students developing efficient explanations of solutions and why they work, providing opportunities for thinking and reasoning about contextual problems and situations		Focused on getting an answer
Defined by what the student does with what you give them		Defined by what you give the student

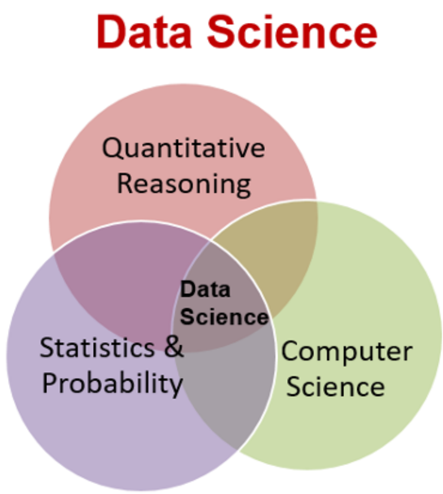
What is Data Science Foundations?

THE NEED FOR DATA SCIENCE

Contemporary mathematics education has not been keeping up with the rapid emergence of data and computing. Looking at the world's job market, many fields deal with big data sets. According to research, a vast majority of faster-growing job categories include data-centered, well-paid, exciting, and challenging jobs. To help students thrive in data-related fields of education or careers, students should be exposed not only to Statistics and Probability but also be well-equipped with a basic understanding of data science. It is evident that students who develop data fluency are better prepared for 21st century careers.

WHAT IS DATA SCIENCE?

Data science is a blend of quantitative reasoning, statistics and computer science to gain meaningful insights from data. The difference between data science and statistics is that where statistics focuses on explaining the data, data science focuses on using data to make predictions and decisions. Students will reason with and think critically about data in all forms. They will develop their understanding of data analysis, sampling, correlation/causation, bias and uncertainty, probability, modeling with data, making and evaluating data-based arguments, the power of data in society and more.



GAISE II

The Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education has updated as data types have expanded beyond being classified as quantitative and categorical which may include collections of sounds, pictures and videos. Data can also be multidimensional and display many variables simultaneously.

GAISE II highlights:

- The importance of asking questions throughout the statistical problem-solving process (formulating a statistical investigative question, collecting or considering data, analyzing data, and interpreting results), and how this process remains at the forefront of statistical reasoning for all studies involving data;
- The consideration of different data and variable types, the importance of carefully planning how to collect data or how to consider data to help answer statistical investigative questions, and the process of collecting, cleaning, interrogating, and analyzing the data;
- The inclusion of multivariate thinking throughout all Pre-K–12 educational levels;
- The role of probabilistic thinking in quantifying randomness throughout all levels;
- The recognition that modern statistical practice is intertwined with technology, and the importance of incorporating technology as feasible;
- The enhanced importance of clearly and accurately communicating statistical information; and
- The role of assessment at the school level, especially items that measure conceptual understanding and require statistical reasoning involving the statistical problem-solving process.

What is Data Science Foundations?, continued

The statistical problem-solving process is as follows:

I. Formulate Statistical Investigative Questions
II. Collect/Consider the Data
II. Analyze the Data
IV. Interpret the Results

This can be illustrated in the figure below:

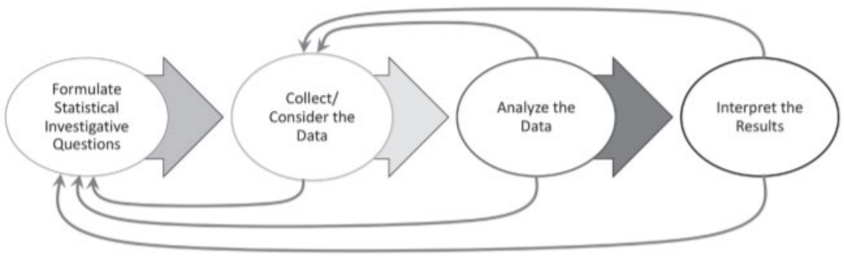


Figure : Statistical problem-solving process

OHIO'S DATA SCIENCE FOUNDATIONS COURSE

This document describes Ohio's Algebra 2 equivalent Data Science Foundations course which integrates Ohio's Math (especially the Statistics and Probability standards) and Computer Science Learning Standards. The course teaches students to reason with and think critically about data in all forms. A Data Science Foundations course may include ideas such as describing big data; usability and usefulness of data; structured vs unstructured data; data extraction techniques; data storage; privacy issues; and data mining.

Follow-on data science courses may incorporate concepts such as vectors, matrices, and trigonometry.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Standards for Mathematical Practice, continued

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Standards for Mathematical Practice, continued

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Computer Science Practices[1]

Practice 1. Fostering an Inclusive Computing Culture

Building an inclusive and diverse computing culture requires strategies for incorporating perspectives from people of different genders, ethnicities and abilities. Incorporating these perspectives involves understanding the personal, ethical, social, economic and cultural contexts in which people operate. Considering the needs of diverse users during the design process is essential to producing inclusive computational products. By the end of Grade 12, students should be able to do the following:

- Include the unique perspectives of others and reflect on one's own perspectives when designing and developing computational products.
- Address the needs of diverse end users during the design process to produce artifacts with broad accessibility and usability.
- Employ self- and peer-advocacy to address bias in interactions, product design, and development methods.

Practice 2: Collaborating Around Computing

Collaborative computing is the process of performing a computational task by working in pairs and on teams. Because it involves asking for the contributions and feedback of others, effective collaboration can lead to better outcomes than working independently. Collaboration requires individuals to navigate and incorporate diverse perspectives, conflicting ideas, disparate skills, and distinct personalities. Students should use collaborative tools to effectively work together and to create complex artifacts. By the end of Grade 12, students should be able to do the following:

- Cultivate working relationships with individuals possessing diverse perspectives, skills and personalities.
- Create team norms, expectations and equitable workloads to increase efficiency and effectiveness.
- Solicit and incorporate feedback from, and provide constructive feedback to, team members and other stakeholders.
- Evaluate and select technological tools that can be used to collaborate on a project.

Practice 3: Recognizing and Defining Computational Problems

The ability to recognize appropriate and worthwhile opportunities to apply computation is a skill that develops over time and is central to computing. Solving a problem with a computational approach requires defining the problem, breaking it down into parts, and evaluating each part to determine whether a computational solution is appropriate. By the end of Grade 12, students should be able to do the following:

- Identify complex, interdisciplinary, real-world problems that can be solved computationally.
- Decompose complex real-world problems into manageable subproblems that could integrate existing solutions or procedures. Compose complex real-world problems into manageable subproblems that could integrate existing solutions or procedures.
- Evaluate whether it is appropriate and feasible to solve a problem computationally.

[1] The "Computer Science Practices" section has been modified from chapter two of The K-12 Computer Science Framework Statements by Grade Band, (K–12 Computer Science Framework. (2016). Retrieved from https://k12cs.org/framework-statements-by-grade-band/This work is licensed under Creative Commons (CC BY-NC-SA 4.0).

Computer Science Practices, continued

Practice 4: Developing and Using Abstractions

Abstractions are formed by identifying patterns and extracting common features from specific examples to create generalizations. Using generalized solutions and parts of solutions designed for broad reuse simplifies the development process by managing complexity. By the end of Grade 12, students should be able to do the following:

- Extract common features from a set of interrelated processes or complex phenomena.
- Evaluate existing technological functionalities and incorporate them into new designs.
- Create modules and develop points of interaction that can apply to multiple situations and reduce complexity.
- Model phenomena and processes and simulate systems to understand and evaluate potential outcomes.

Practice 5: Creating Computational Artifacts

The process of developing computational artifacts embraces both creative expression and the exploration of ideas to create prototypes and solve computational problems. Students create artifacts that are personally relevant or beneficial to their community and beyond. Computational artifacts can be created by combining and modifying existing artifacts or by developing new artifacts. Examples of computational artifacts include programs, simulations, visualizations, digital animations, robotic systems, and apps. By the end of Grade 12, students should be able to do the following:

- Plan the development of a computational artifact using an iterative process that includes reflection on and modification of the plan, taking into account key features, time and resource constraints, and user expectations.
- Create a computational artifact for practical intent, personal expression, or to address a societal issue.
- Modify an existing artifact to improve or customize it.

Practice 6: Testing and Refining Computational Artifacts

Testing and refinement is the deliberate and iterative process of improving a computational artifact. This process includes debugging (identifying and fixing errors) and comparing actual outcomes to intended outcomes. Students also respond to the changing needs and expectations of end users and improve the performance, reliability, usability, and accessibility of artifacts. By the end of Grade 12, students should be able to do the following:

- Systematically test computational artifacts by considering all scenarios and using test cases.
- Identify and fix errors using a systematic process.
- Evaluate and refine a computational artifact multiple times to enhance its performance, reliability, usability, and accessibility.

Practice 7: Communicating About Computing

Communication involves personal expression and exchanging ideas with others. In computer science, students communicate with diverse audiences about the use and effects of computation and the appropriateness of computational choices. Students write clear comments, document their work, and communicate their ideas through multiple forms of media. Clear communication includes using precise language and carefully considering possible audiences. By the end of Grade 12, students should be able to do the following:

- Select, organize, and interpret large data sets from multiple sources to support a claim.
- Describe, justify, and document computational processes and solutions using appropriate terminology consistent with the intended audience and purpose.
- Articulate ideas responsibly by observing intellectual property rights and giving appropriate attribution.

Equity and Computer Science[1]

COMPUTER SCIENCE FOR ALL

To help realize the vision of computer science for all students, equity must be at the forefront of the state's efforts to implement the computer science standards. Equity is more than whether classes are available. It includes how those classes are taught, how students are recruited and how the classroom culture supports diverse learners and promotes retention. When equity exists, schools expect academic success for every student and makes that success accessible to every student. The result of such equity is a classroom of diverse students based on factors such as race, gender, disability, socioeconomic status and English language proficiency. All these students have high expectations and feel empowered to learn.

Computer science faces intense challenges related to access, opportunity and culture.

- The 2015 National Assessment of Educational Progress (NAEP) survey showed that only 44 percent of 12th grade students attend high schools that offer any computer science courses (Change the Equation, 2016). This data showed that students with the least access are Black, Latino and Native American; from lower income backgrounds; and from rural areas.

Even when computer science courses are available, there are wide gaps in participation and the level of instruction.

- For the 2015 Advanced Placement (AP) Computer Science exam, only 21.9 percent of students were female, 3.9 percent were Black or African American, 9 percent were Hispanic or Latino and 0.4 percent were American Indian (College Board, 2016). The potential impact of these gaps in participation is illustrated in the statistic that females who take high school AP Computer Science are 10 times more likely to major in computer science in college than students who do not take this course. Similarly, African American students are seven times more likely, and Hispanic students 8.5 times more likely, to major in computer science when they have taken high school AP Computer Science (Morgan & Klaric, 2007).
- Especially in schools with large numbers of African American and Latino students, computer classes too commonly offer only basic, rudimentary user skills rather than engaging students in the problem-solving and computational thinking practices that form the foundation of computer science (Margolis et al., 2012). The lack of representation in computer science after K–12 reflects the lack of access and participation in grades K–12. In 2015, only 24.7 percent of workers employed in computer and mathematical occupations were female. Only 8.6 percent were Black or African American, and only 6.8 percent were Hispanic or Latino (Bureau of Labor Statistics, 2015).

Equity and Computer Science, continued

EFFORTS TO INCREASE EQUITY

Even when schools have made computer science courses available to students, inequity can be perpetuated at the classroom level. Educators can work to ensure equity through changes in curriculum, instruction and classroom culture.

- Educators can reach *students with disabilities* using learning accommodations, curricular modifications and established techniques for differentiated instruction. For example, the Quorum programming language accommodates students with visual impairments by enabling the programming language to be read by computer screen readers (Quorum, 2019). Recent research shows ways to use Universal Design for Learning (UDL) to develop and refine introductory computer science experiences for a wider range of learners (Hansen et al., 2016). Educators also can apply instructional strategies used in other content areas to support struggling learners and students with disabilities. For example, if verbal prompting helps in math instruction, it will likely help in computer science instruction (Snodgrass, Israel, & Reese, 2016).
- A variety of approaches make programming more accessible to *young learners and beginners.* Visual, block-based programming languages allow students to program without the obstacle of syntax errors (errors in typing commands) found in traditional text-based languages. Programming environments on tablets have made programming even more accessible to younger children by reducing the number of available commands and the amount of reading required to navigate the options (Strawhacker&Bers,2014).
- To address a *lack of computer and internet access*, educators can help students learn many computer science topics, such as algorithmic thinking, searching, sorting and logic through “unplugged” activities.
- To reach *females and underrepresented minorities*, teachers can use strategies to work against issues such as the threat of stereotyping or bias. For example, stereotype threats can be mitigated by altering the wording of test questions to be gender-neutral and using examples that are equally relevant to both males and females (Kumar, 2012). It also is important for students to have diverse role models in the field so they can imagine themselves as a computer scientist. Role models also help dispel stereotypes of how computer scientists look and act (Goode, 2008).

Below, are other practices that teachers can adopt and adapt to change classroom culture and broaden participation in computer science:

- Connect computer science to concepts that motivate children, like fairness and social justice (Denner et al., 2015).
- Practice culturally relevant pedagogy to tie computer science to students’ experiences, culture and interests (Margolis et al., 2014).
- Designing projects and instruction to be socially relevant and meaningful for diverse students helps them “build personal relationships with CS concepts and applications -- an important process for discovering the relevance of CS for their own lives.” (Margolis et. al, 2012, p. 76)
- Reflect on beliefs and actions to address stereotypes among students and teachers (Margolis et al., 2014).

EQUITY AND THE COMPUTER SCIENCE STANDARDS

The computer science standards reflect the writing team’s considerations on equity. The standards describe concepts and skills all students can benefit from, regardless of whether they go on to postsecondary computer-science education or a career in computer science.

Equity and Computer Science, continued

Equity is woven into the computer science concepts and skills across grade levels. This is especially apparent in the core concept or strand involving *Impacts of Computing.* Here, students examine the social implications of the digital world, including their impacts on equity and access to computing. Specific content statements address equity directly. For example, in grade 3, students identify diverse user needs and "how computing devices have built-in features to increase accessibility to all users." In grade 7, students "evaluate various technologies to identify issues of bias and accessibility." Students in grade 8 build on prior learning to work against existing inequities; they propose guidelines "to positively impact bias and accessibility in the design of future technologies."

As students design computational products, they engage in computer science practices that also directly involve consideration of equity, inclusion and diversity. Students foster inclusion as they develop products that "include the unique perspectives of others" and "address the needs of diverse end users." Students encourage diversity through working in teams "with individuals possessing diverse perspectives." Involving students in such practices stresses the need to practice equity when doing computer science. Through such practices, students can see the benefit of, for example, considering the products they develop from the perspectives of a diverse group of end-users, such as those with visual impairments and English language learners.

---

[1] The "Equity and Computer Science" section has been modified from chapter two of The K-12 Computer Science Framework, "Equity in Computer Science Education." (K–12 Computer Science Framework. (2016). Retrieved from http://www.k12cs.org.) This work is licensed under Creative Commons (CC BY-NC-SA 4.0).

Mathematical Content Standards for High School

PROCESS

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> *(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).*

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students. However, standards with a (+) symbol will not appear on Ohio's State Tests.

The high school standards are listed in conceptual categories:

- Modeling
- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including Calculus.

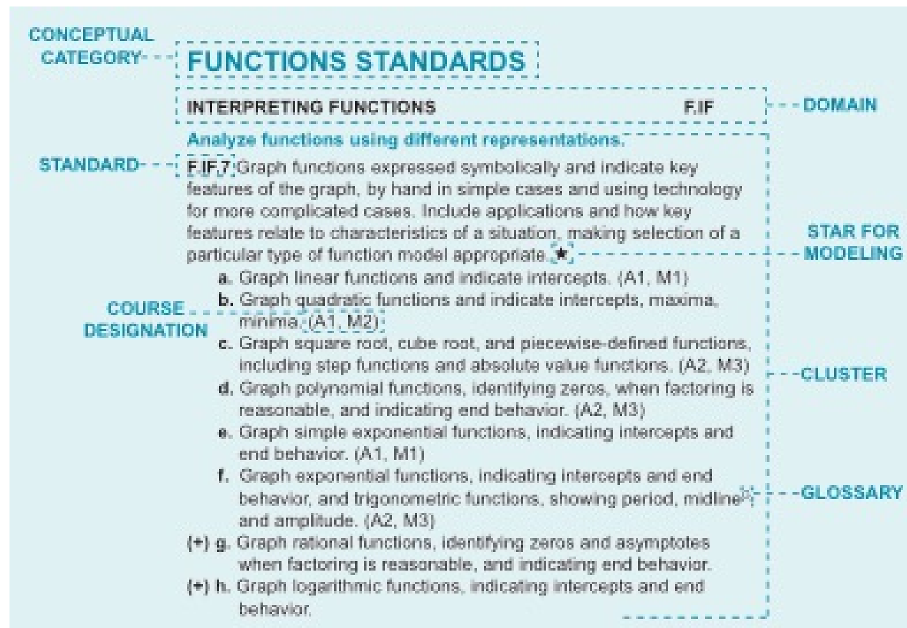
Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Proofs in high school mathematics should not be limited to geometry. Mathematically proficient high school students employ multiple proof methods, including algebraic derivations, proofs using coordinates, and proofs based on geometric transformations, including symmetries. These proofs are supported by the use of diagrams and dynamic software and are written in multiple formats including not just two-column proofs but also proofs in paragraph form, including mathematical symbols. In statistics, rather than using mathematical proofs, arguments are made based on empirical evidence within a properly designed statistical investigation.

How to Read the High School Content Standards

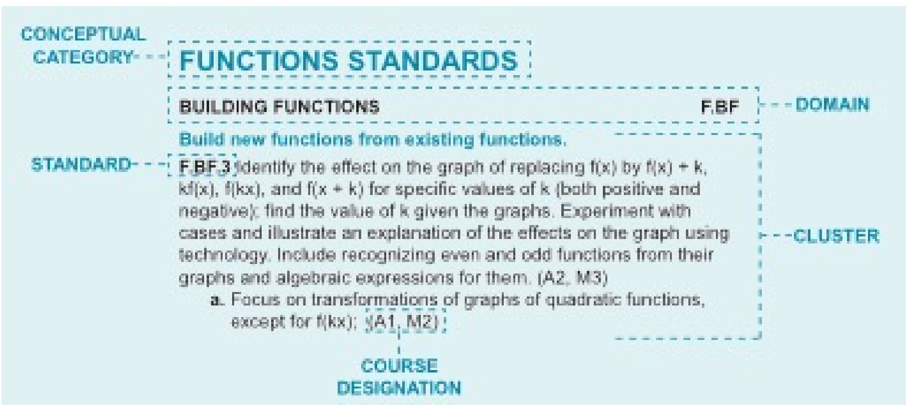
- **Conceptual Categories** are areas of mathematics that cross through various course boundaries.
- **Standards** define what students should understand and be able to do.
- **Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
- **Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.
- G shows there is a definition in the glossary for this term.
- (★) indicates that modeling should be incorporated into the standard. (See the Conceptual Category of Modeling pages 60-61)
- (+) indicates that it is a standard for students who are planning on taking advanced courses. Standards with a (+) sign will not appear on Ohio's State Tests.

Some standards have course designations such as (A1, M1) or (A2, M3) listed after an **a., b.,** or **c.** These designations help teachers know where to focus their instruction within the standard. In the example below the beginning section of the standard is the stem. The stem shows what the teacher should be doing for all courses. (Notice in the example below that modeling (★) should also be incorporated.) Looking at the course designations, an Algebra 1 teacher should be focusing his or her instruction on **a.** which focuses on linear functions; **b.** which focuses on quadratic functions; and **e.** which focuses on simple exponential functions. An Algebra 1 teacher can ignore **c., d.,** and **f,** as the focuses of these types of functions will come in later courses. However, a teacher may choose to touch on these types of functions to extend a topic if he or she wishes.



How to Read the High School Content Standards, continued

Notice that in the standard below, the stem has a course designation. This shows that the full extent of the stem is intended for an Algebra 2 or Math 3 course. However, **a.** shows that Algebra 1 and Math 2 students are responsible for a modified version of the stem that focuses on transformations of quadratics functions and excludes the *f(kx)* transformation. However, again a teacher may choose to touch on different types of functions besides quadratics to extend a topic if he or she wishes.



Critical Areas of Focus

CRITICAL AREA OF FOCUS #1

Communication and Analysis

Students develop conclusions based on quantitative information and critical thinking. They recognize, make, and evaluate underlying assumptions in estimation, modeling, and data analysis, and then organize and present thoughts and processes using mathematical and statistical evidence. They communicate clear and complete information so their audience can better understand the contextual and quantitative information in a given situation or context. Students demonstrate numerical reasoning orally and in writing as they craft coherent statements and paragraphs.

In the context of real-world applications, students make and investigate mathematical conjectures. They defend and question their conjectures and those of their classmates, precursors to formal proof. Oral and written explanations move beyond procedural descriptions or summaries to include mathematical arguments and rationales. As students listen to others' explanations, they develop their own understandings. **This critical area of focus cross cuts all the other critical areas of focus.**

CRITICAL AREA OF FOCUS #2

Data and Visualizations

Students engage in fundamental notions of data analysis as they gather and analyze real-world data. Using descriptive statistics, students gain experience managing large, real-world datasets—a practice too often overlooked in traditional mathematics courses at the secondary level. An emphasis is placed on creating and interpreting visualizations of real-world processes using computational tools.

CRITICAL AREA OF FOCUS #3

Distributions, Probability, and Simulations

Students generate data using simulations—often of their own design. Then they use numerical summaries to describe the resulting distributions. They additionally use simulations for informal inference using generated probabilities—an approach not typically encountered in traditional courses. This approach scaffolds student computational thinking by allowing students to observe and create models in the process of applying fundamental programming structures.

CRITICAL AREA OF FOCUS #4

Data Collection Methods: Traditional and Modern

Students extend their knowledge of data collection as they investigate the effects that various methods have on the interpretation of patterns they discover. Moreover, they explore sampling error and bias, and discuss approaches to mitigating their impact on the analysis process and outcomes. This course also introduces students to modern data collection methods, such as participatory sensing[G], using mobile devices, and web services to systematically explore interesting aspects of their worlds ranging from health to culture.

CRITICAL AREA OF FOCUS #5

Predictions and Models

Students create and apply mathematical and statistical models to predict future observations and investigate how data scientists measure the success of these predictions. This is done by identifying, analyzing, and implementing efficient steps in the process of regression analysis. Learners design and develop models based on real-world data, evaluating the usefulness of the models in the context of the problem or situation. Students engage in ongoing reflection during the modeling and testing phases as they further develop and strengthen their metacognitive skills.

CRITICAL AREA OF FOCUS #6

Data and Society

As our world becomes increasingly interconnected with technology, nearly every facet of modern life is impacted by data. Individuals and communities influence computing through their behaviors and cultural and social interactions. Likewise, computing influences new cultural practices. Informed and responsible citizens should understand the social implications of the digital world, including equity and access to computing (K-12 Computer Science Framework (K12CSF), p. 92). Students enrolled in this course learn how data impacts equity, privacy, security, legal, and ethical issues.

Data Science Foundations Course Overview

MATHEMATICS

NUMBER AND QUANTITY

QUANTITIES

- Reason quantitatively and use units to solve problems.

VECTOR AND MATRIX QUANTITIES

- Perform operations on matrices, and use matrices in applications.

ALGEBRA

CREATING EQUATIONS

- Create equations that describe numbers or relationships.

REASONING WITH EQUATIONS AND INEQUALITIES

- Represent and solve equations and inequalities graphically.

FUNCTIONS

INTERPRETING FUNCTIONS

- Understand the concept of a function, and use function notation.
- Interpret functions that arise in applications in terms of the context.

BUILDING FUNCTIONS

- Build a function that models a relationship between two quantities.

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

- Construct and compare linear, quadratic, and exponential models, and solve problems.
- Interpret expressions for functions in terms of the situation they model.

GEOMETRY

MODELING IN GEOMETRY

- Apply geometric concepts in modeling situations.

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving the,
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STATISTICS AND PROBABILITY

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

- Understand independence and conditional probability, and use them to interpret data.

USING PROBABILITY TO MAKE DECISIONS

- Use probability to evaluate outcomes of decisions

Data Science Foundations Course Overview, continued

COMPUTER SCIENCE

COMPUTER SCIENCE GRADES 9-12 FOUNDATIONAL LEVEL

DATA AND ANALYSIS

- Data Collection and Storage
- Visualization and Communication
- Inference and Modeling

ALGORITHMIC THINKING AND PROGRAMMING

- Algorithms
- Variables and Data Representation
- Modularity

IMPACTS OF COMPUTING

- Culture
- Social Interaction
- Safety, Law and Ethics

COMPUTER SCIENCE PRACTICES

1. Fostering an Inclusive Computing Culture
2. Collaborating Around Computing
3. Recognizing and Defining Computational Problems
4. Developing and Using Abstractions
5. Creating Computational Artifacts
6. Testing and Refining Computational Artifacts
7. Communicating About Computing

COMPUTER SCIENCE GRADES 9-12 ADVANCED LEVEL

DATA AND ANALYSIS

- Data Collection and Storage
- Visualization and Communication
- Inference and Modeling

ALGORITHMIC THINKING AND PROGRAMMING

- Algorithms
- Control Structures
- Modularity

IMPACTS OF COMPUTING

- Culture

High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

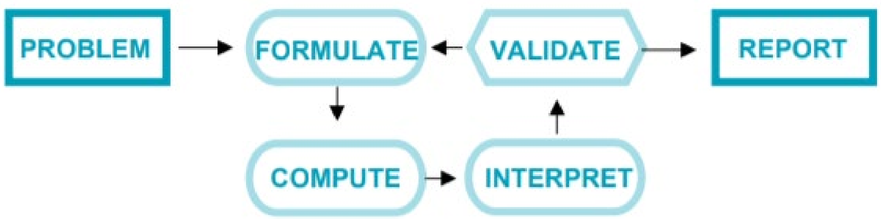
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

High School—Modeling, continued

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

MODELING STANDARDS

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

High School—Number and Quantity

NUMBERS AND NUMBER SYSTEMS

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, “number” means “counting number”: 1, 2, 3... Soon after that, 0 is used to represent “none” and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system— integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole number exponents leads to new and productive notation. For example, properties of whole number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)\,3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

QUANTITIES

In real-world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Standards

QUANTITIES **N.Q**

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

VECTOR AND MATRIX QUANTITIES **N.VM**

Perform operations on matrices, and use matrices in applications.

(+) **N.VM.6** Use matrices[G] to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

High School—Algebra

EXPRESSIONS

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

EQUATIONS AND INEQUALITIES

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

High School—Algebra, continued

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = (\frac{(b_1+b_2)}{2})h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

CONNECTIONS WITH FUNCTIONS AND MODELING

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Standards

CREATING EQUATIONS A.CED

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1)

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★

REASONING WITH EQUATIONS AND INEQUALITIES A.REI

Represent and solve equations and inequalities graphically.

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = {}^{100}/_{v}$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph, e.g., the trace of a seismograph; by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

CONNECTIONS TO EXPRESSIONS, EQUATIONS, MODELING, AND COORDINATES.
Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Standards

INTERPRETING FUNCTIONS F.IF

Understand the concept of a function, and use function notation.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **c.** Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)

BUILDING FUNCTIONS F.BF

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from context.
- **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3)
- **c.** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS F.LE

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes— as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of “same shape” and “scale factor” developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non- right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

High School—Geometry, continued

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

CONNECTIONS TO EQUATIONS

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry Standards

MODELING WITH GEOMETRY **G.MG**

Apply geometric concepts in modeling situations.

G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

High School—Statistics and Probability

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

CONNECTIONS TO FUNCTIONS AND MODELING

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Standards

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA S.ID

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

Summarize, represent, and interpret data on two categorical and quantitative variables, continued.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

- **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3)
- **b.** Informally assess the fit of a function by discussing residuals. (A2, M3)
- **c.** Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S.ID.9 Distinguish between correlation and causation.★

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS S.IC

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*★

Statistics and Probability Standards, continued

MAKING INFERENCES AND JUSTIFYING CONCLUSIONS, continued

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★

S.IC.6 Evaluate reports based on data.★

CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY S.CP

Understand independence and conditional probability, and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

USING PROBABILITY TO MAKE DECISIONS S.MD

Use probability to evaluate outcomes of decisions.

S.MD.7 Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

Computer Science Grades 9 - 12—Foundational Level

DATA AND ANALYSIS

Data Collection and Storage

DA.DCS.9-12.F.a Analyze patterns in a real-world data store through hypothesis, testing and use of data tools to gain insight and knowledge.
DA.DCS.9-12.F.b Investigate data storage systems to compare and contrast how data is stored and accessed.

Visualization and Communication

DA.VC.9-12.F.a Analyze the benefits and limitations of data visualization or multisensory artifacts and tools to communicate which is most appropriate to solve a real-world problem.

Inference and Modeling

DA.IM.9-12.F.a Evaluate a model by creating a hypothesis, testing it and refining it to discover connections and trends in the data.

ALGORITHMIC THINKING AND PROGRAMMING

Algorithms

ATP.A.9-12.F.a Define and use appropriate problem solving strategies and visual artifacts to create and refine a solution to a real-world problem.
ATP.A.9-12.F.b Define and implement an algorithm by decomposing problem requirements from a problem statement to solve a problem.

Variables and Data Representation

ATP.VDR.9-12.F.a Identify types of variables and data and utilize them to create a computer program that stores data in appropriate ways.

Control Structures

ATP.CS.9-12.F.a Define control structures and Boolean logic and use them to solve real-world scenarios.
ATP.CS.9-12.F.b Use appropriate syntax to create and use a method.
ATP.CS.9-12.F.c Use data scoping to isolate data.

Modularity

ATP.M.9-12.F.a Break down a solution into procedures using systematic analysis and design.
Equivalent to: ATP.A.9-12.F.b Define and implement an algorithm by decomposing problem requirements from a problem statement to solve a problem.
ATP.M.9-12.F.b Create computational artifacts by systematically organizing, manipulating and/or processing data.
Addressed in:
ATP.VDR.9-12.A.a Utilize different data storage structures to store larger and more complex data than variables can contain.
ATP.VDR.9-12.A.b Identify the appropriate data structures or variables to use to design a solution to a complex problem

IMPACTS OF COMPUTING

Culture

IC.Cu.9-12.F.b Explore other professions to understand how computing has and will impact them positively and negatively.

Social Interactions

IC.SI.9-12.F.b Analyze the collection and generation of data through automated processes to explain the privacy concerns that are not always evident to users.

Safety, Law and Ethics

IC.SLE.9-12.F.c Analyze the collection and generation of data through automated processes to explain the legal concerns that are not always evident to users.

Computer Science Grades 9 - 12—Advanced Level

DATA AND ANALYSIS

Data Collection and Storage

DA.DCS.9-12.A.a Create multidimensional data collections that can be utilized through various methods to solve complex data problems.
DA.DCS.9-12.A.b Investigate data storage and collection tools to analyze tradeoffs and limitations.

Visualization and Communication

DA.VC.9-12.A.a Create visualization or multisensory artifacts to communicate insights and knowledge gained from complex data analysis that answers real-world questions.

Inference and Modeling

DA.IM.9-12.A.a Create a model that simulates a complex system and uses extracted data to hypothesize, test and refine the model to discover connections or trends.

ALGORITHMIC THINKING AND PROGRAMMING

Algorithms

ATP.A.9-12.A.a Define and explain recursive algorithms to understand how and when to apply them.
ATP.A.9-12.A.b Use recursion to effectively solve problems.
ATP.A.9-12.A.d Use sorting and searching to analyze and organize data.

Variables and Data Representation

ATP.VDR.9-12.A.a Utilize different data storage structures to store larger and more complex data than variables can contain.
ATP.VDR.9-12.A.b Identify the appropriate data structures or variables to use to design a solution to a complex problem.

Control Structures

ATP.CS.9-12.A.a Write programs that use library methods and control structures and methods to solve a problem.

Modularity

ATP.M.9-12.A.a Construct solutions to problems using student created components (e.g., procedures, modules, objects).
Equivalent to: ATP.CS.9-12.F.b Use appropriate syntax to create and use a method.
ATP.M.9-12.A.b Design or redesign a solution to a large-scale computational problem by identifying generalizable patterns.
Equivalent to: ATP.PD.9-12.A.a Fully implement the most appropriate software methodology to complete a team programming project.
ATP.M.9-12.A.c Create programming solutions by reusing existing code (e.g., libraries, Application Programming Interface (APIs), code repositories).
Equivalent to: ATP.CS.9-12.A.a Write programs that use library functions, methods and control structures to solve a problem.

IMPACTS OF COMPUTING

Culture

IC.Cu.9-12.A.b Analyze the equity, access and influence of the distribution of computing resources to see their global impact.

Glossary

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]
See also: first quartile and third quartile.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2
See also: median, third quartile, interquartile range.

GAISE Model *See also:* Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 − 6 = 9.
See also: first quartile, third quartile.

Justify: To provide a convincing argument for the truth of a statement to a particular audience.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Matrix (plural 'matrices') A collection of numbers, symbols, expressions, or images arranged in a grid (rows and columns) to form a rectangular array. In mathematics it is used to represent transformations or objects. In computer science it may be used to represent a group of related data.

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

---

1. Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

2 Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

3. Adapted from Wisconsin Department of Public Instruction, *op. cit.*

4. Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Glossary, continued

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Participatory Sensing. An approach to data collection and interpretation in which individuals, acting alone or in groups, use their personal mobile devices and web services to systematically explore interesting aspects of their worlds, ranging from health to culture.

Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education. It is an updated report endorsed by the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM) to enhance the statistics standards. Like the GAISE I, it provides a framework of recommendations for developing students' foundational skills in statistical reasoning in three levels across the school years, described as levels A, B, and C. GAISE I and GAISE II can be found at https://www.amstat.org/asa/education/Guidelines-for-Assessment-and-Instruction-in-Statistics-Education-Reports.aspx.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[4]

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Verify: To check the truth or correctness of a statement in specific cases.

Computer Science References

Bureau of Labor Statistics. (2015). Labor force statistics from the Current Population Survey [Data file]. Retrieved from http://www.bls.gov/cps

Change the Equation. (2016, August 9). New data: Bridging the computer science access gap [Blog post]. Retrieved from http://changetheequation.org/blog/new-data-bridging-computer-science-access-gap-0 (no longer available)

College Board. (2016). AP program participation and performance data 2015 [Data file]. Retrieved from https://research.collegeboard.org/programs/ap/data/archived/ap-2015

Denner, J., Martinez, J., Thiry, H., & Adams, J. (2015). Computer science and fairness: Integrating a social justice perspective into an after school program. Science Education and Civic Engagement: An International Journal, 6(2): 41–54.

Goode, J. (2008, March). Increasing diversity in K–12 computer science: Strategies from the field. ACM SIGCSE Bulletin, 40(1), 362–366.

Hansen, A., Hansen, E., Dwyer, H., Harlow, D., & Franklin, D. (2016). Differentiating for diversity: Using universal design for learning in computer science education. Proceedings of the 47th ACM Technical Symposium on Computing Science Education (pp. 376–381).

K–12 Computer Science Framework. (2016). Retrieved from http://www.k12cs.org

Kumar, A. N. (2012, July). A study of stereotype threat in computer science. Proceedings of the 17th ACM Annual Conference on Innovation and Technology in Computer Science Education (pp. 273–278).

Margolis, J., Ryoo, J., Sandoval, C., Lee, C., Goode, J., & Chapman, G. (2012). Beyond access: Broadening participation in high school computer science. ACM Inroads, 3(4), 72–78.

Margolis, J., Goode, J., Chapman, G., & Ryoo, J. J. (2014). That classroom ‘magic.’ Communications of the ACM, 57(7), 31–33.

Morgan, R., & Klaric, J. (2007). AP students in college: An analysis of five-year academic careers. Research report no. 2007-4. Retrieved from https://eric.ed.gov/?id=ED561034

Computer Science References, continued

Quorum [Computer software]. (2019). Retrieved from https://quorumlanguage.com/

Snodgrass, M. R., Israel, M., & Reese, G. (2016). Instructional supports for students with disabilities in K–5 computing: Findings from a cross-case analysis. Computers & Education, 100, 1–17.

Strawhacker, A. L., & Bers, M. U. (2014, August). ScratchJr: Computer programming in early childhood education as a pathway to academic readiness and success. Poster presented at DR K–12 PI Meeting, Washington, DC.

Acknowledgements

MATH STANDARDS ADVISORY COMMITTEE MEMBERS

Aaron Altose
The Ohio Mathematics Association of Two-Year Colleges

Jeremy Beardmore
Ohio Educational Service Center Association

Jessica Burchett
Ohio Teachers of English to Speakers of Other Languages

Jeanne Cerniglia
Ohio Education Association

Margie Coleman
Cochair

Jason Feldner
Ohio Association for Career and Technical Education

Brad Findell
Ohio Higher Education

Gregory D. Foley
Ohio Mathematics and Science Coalition

Margaret (Peggy) Kasten
Cochair

Courtney Koestler
Ohio Mathematics Education Leadership Council

Scott Mitter
Ohio Math and Science Supervisors

Tabatha Nadolny
Ohio Federation of Teachers

Eydie Schilling
Ohio Association for Supervision and Curriculum Development

Kim Yoak
Ohio Council of Teachers of Mathematics

MATH STANDARDS WORKING GROUP MEMBERS

Ali Fleming
Teacher, Bexley City, C

Gary Herman
Curriculum Specialist/Coordinator, Putnam County ESC, NW

William Husen
Higher Education, Ohio State University, C

Kristen Kelly
Curriculum Specialist/Coordinator, Cleveland Metropolitan School District, NE

Endora Kight Neal
Curriculum Specialist/Coordinator, Cleveland Metropolitan School District, NE

Dawn Machacek
Teacher, Toledo Public Schools, NW

Sherryl Proctor
Teacher, Vantage Career Center, NW

Tess Rivero
Teacher, Bellbrook-Sugarcreek Schools, SW

Jennifer Walls
Teacher, Akron Public Schools, NE

Gaynell Wamer
Teacher, Toledo City, NW

Sandra Wilder
Teacher, Akron Public Schools, NE

Acknowledgements, continued

COMPUTER SCIENCE ADVISORY COMMITTEE MEMBERS

Chris Anderson
LexisNexis

Kevin Boehner
Ohio Chamber of Commerce

Grady Burrows
BioEnterprise

Lisa Chambers
Tech Corps

Chelsey Cook-Kohn
John Marshall School of IT

Tad Douce
TRECA Education

Chris Ferruso
NFIB/Ohio

Kelly Gaier-Evans
Battelle Education

Gary Greenberg CET
Connect/ThinkTV

Stephanie Hoeppner
Computer Science Teachers Association (CSTA)

Diane Kahle
Ohio Council for Teachers of Mathematics

Dan McMullen
Greater Cleveland Partnership/OHTec

Kenneth Moore
Wright State University

Gloria Rogiers
Columbus State Community College

Tim Sisson
Cleveland Metropolitan

Nigamanth Sridhar
Cleveland State University

COMPUTER SCIENCE WRITING TEAM MEMBERS

Roberta Banfield
Cleveland Catholic Diocese

Tishina Brown-Harris
Eastern Local

Leah Burke
Westlake City

Dave Burkhart
New Lexington City

Laura Ciszweski
Auburn City

Shanti Coaston
East Cleveland City

Martin Chenzira
Pickerington Local

Patrick Connor
Cleveland Catholic Diocese

Jenifer Conard
Warren County Career Center

Pamela Cook
Notre Dame College

Sheryl Edwards
Cleveland Metropolitan

Martin Fish
Mason City

Jessica Frenchik

Strongsville City

John Fulton
Tech Elevator, Inc

Judith Gawelek
Westerville City

Melissa Goodall
Miami Valley Career Tech

Lashawna Grimes
Emmanuel Christian Academy

Briana Guevara
Cleveland Metropolitan

Acknowledgements, continued

COMPUTER SCIENCE WRITING TEAM MEMBERS, CONTINUED

Julie Hagaman
Tri-County North School District

Gary Herman
Putnam County ESC

Marvin "Mel" Hoffert
Sycamore Community

Jennifer Humphrey
Cleveland Metropolitan

Diane Kahle
Upper Arlington City

Christopher Kaminski
Fairview Park City

Barbara Kenney
West Clermont Local

Bonnie Kovacic
Perry Local

Carlo Miraldi
Great Oaks Career Campuses

Valerie Moga
Hudson City

Shellu Mumaw
Ottoville Local

Pat Murakami
Dayton Regional STEM School

Lisa Murray
Vermilion Local

Lynne Pachnowski
University of Akron

Erin Pekar
Maple Heights City

Krystina Pratt
New Albany Plain Local

Karen Plaster
University of Akron

Leslie Slaven
Lakota Local

Renee Snyder
Teaching & Learning Collaborative

Darrin Spondike
Auburn City

Rebecca Stanek
Stow-Munroe Falls City

James Stanton
Maumee City

Deborah Wiley
North Canton City Schools/
Kent State Stark University

Jonathon Wilson
New Richmond Exempted Village

Robert Wyant
Austintown Local

OHIO HIGH SCHOOL MATH PATHWAYS ADVISORY COUNCIL

Trina Barrell
Buckeye Association of School Administrators (BASA)

Michael Broadwater
Ohio Association of Secondary School Administrators (OASSA)

Mark Cortez
Higher education admissions

Kevin Duff
Ohio Excels

Shawn Grime
Ohio School Counselor Association (OSCA)

Kelly Hogan
Ohio Association of Community Colleges (OACC)

Aaron Johnson
Ohio Association for Career and Technical Education (Ohio ACTE)

Tom Kaczmarek
Ohio Education Association (OEA)

Peggy Kasten
Ohio Mathematics and Science Coalition (OMSC)

Todd Martin
The Ohio 8 Coalition

Ricardo Moena
Ohio Mathematics Initiative (OMI)

Carrie Rice
Ohio Federation of Teachers (OFT)

Brad Ritchey
Ohio School Boards Association (OSBA)/ Buckeye Association of School Administrators (BASA)

Barbara Varley
Ohio Parent Teacher Association (Ohio PTA)

Heather Wukelich
Ohio Council of Teachers of Mathematics (OCTM)

Acknowledgements, continued

OHIO HIGH SCHOOL MATH PATHWAYS ARCHITECTS

Deborah Ackley
Teacher, Toledo Public Schools, NW

Dave Burkhart
Teacher, New Lexington City Schools, SE

Doug Darbro
Higher Education, Shawnee State University, SE

Brad Findell
Higher Education, The Ohio State University, C

Derek Gulling
Teacher, Edison Local Schools (Jefferson County), NE

Christina Hamman
Curriculum Specialist/Coordinator, Medina City Schools, NE

Gary Herman
Curriculum Specialist/Coordinator, Putnam County ESC, NW

Ruth Hopkins
Teacher, Symmes Valley Local Schools, SE

Endora Kight Neal
Higher Education, Cuyahoga Community College and Curriculum Specialist/Coordinator, Cleveland Metropolitan School District, NE

Scott Mitter
Teacher, Kettering City Schools, SE

Jennifer Montgomery
Teacher, Wayne Local Schools, SW

Rachael Newell
Teacher, Perrysburg Exempted Village, NW

Stephanie Stafford
Higher Education, Cincinnati State Technical and Community College, SW

Lee Wayand
Higher Education, Columbus State Community College, C

Richelle Zbinden
Teacher, Miami Valley Career Technology Center, SW

OHIO HIGH SCHOOL DATA SCIENCE FOUNDATIONS WORKGROUP

Lauren Burke
Women in Analytics

Doug Darbo
Higher Education, Shawnee State University, SE

Brett Doudican
Curriculum Specialist/Coordinator, Greene County Career Center, SW

Todd Edwards
Higher Education, Miami University, SW

Paul Hansford
Higher Education, Sinclair Community College, SW

Sarah Luthy
Teacher, North Central Local Schools, NW

Scott Meech
Teacher, Green Local Schools (Wayne), NE

Robert Redinger
Teacher, Tallmadge City Schools, NE

Sandra Wilder
Curriculum Specialist/Coordinator, Akron Public Schools, NE