Ohio's Grade 8-Geometry Learning Progressions by Topic

Ohio | Department of Education

Learning progressions were critical in the development, review, and revision of Ohio's Learning Standards (OLS). Ohio's learning progressions were developed during Ohio's international benchmarking project and provided guidance to the writing and revision committees of Ohio's Learning Standards. Ohio believes that learning progressions are important for the understanding and coherence of mathematical topics within and across the grade levels. The Ohio Department of Education has reformatted Ohio's Learning Standards by domain and topic to show the progression of concepts and skills across the grade levels.

This document serves as a companion document to the learning standards; it does not replace them. Curriculum leaders and teachers can use this document to better understand the standards and to analyze where their curriculum fits within each student's learning progression. Professional Learning Communities can use this document in many ways such as the following:

Multi-grade groups of teachers

1. Select a domain or topic within the standards, beginning at the lowest grade of the domain or topic, and then identify the main concepts at that grade. Follow each concept progressing through the grades by identifying how the concept changes and increases in rigor and understanding for the student. Additionally, identify new concepts introduced in subsequent grades and follow them through the years.
2. Building on previous scenario, identify and analyze the connections within the learning progressions. For instance, how is ratio and proportion connected (used to develop the essential understandings) to slope or other topics in grades 6-8 and high school?

Grade-level teachers or an individual teacher

1. Use learning progressions and regular classroom formative assessments to assist in identifying where students are in the learning progression. Then develop scaffolded supports to accelerate students in an effort to bring their understandings and skills to the appropriate level or to go deeper into the content. Note that going deeper does not imply going to the next level in the progression, rather building stronger understandings of the content or making connections to other concepts or skills.
2. Make connections among the standards; between standards within a domain or topic; between standards within a cluster; and between clusters across domains. Also use the Mathematics – K-8 Critical Areas of Focus, Algebra 1 Critical Area of Focus, Geometry Critical Area of Focus, and the model curriculum to make further connections.
3. Use learning progressions to identify gaps. When gaps are identified, this document can be used to quickly scaffold previous learning.

Note: This document is best printed on legal size paper, but can also be printed on other sizes.

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<td>Understand the connections between proportional relationships, lines, and linear equations.
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
Define, evaluate, and compare functions.
8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
Use functions to model relationships between quantities.
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. For example, if a person walks $^1/_2$ mile in each $^1/_4$ hour, compute the unit rate as the complex fraction $^{(1/2)}/_{(1/4)}$ miles per hour, equivalently 2 miles per hour.
7.RP.2 Recognize and represent proportional relationships between quantities.</td>
<td>Reason quantitatively and use units to solve problems.
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Solve equations and inequalities in one variable.
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Interpret functions that arise in applications in terms of the context.
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★(A2, M3)
Construct and compare linear, quadratic, and exponential models, and solve problems.
F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★
a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
<td>Understand similarity in terms of similarity transformations.
G.SRT.1 Verify experimentally the properties of dilations[G] given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations[G] to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
Prove and apply theorems both formally and informally involving similarity using a variety of methods.
G.SRT.4 Prove and apply theorems about triangles. Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.
Define trigonometric ratios, and solve problems involving right triangles.
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8 Solve problems involving right triangles.★
a. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)
Understand and apply theorems about circles.
G.C.1 Prove that all circles are similar using transformational arguments.
G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a</td>
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<tr><th>Grades 6, 7, and 8</th><th>Algebra 1</th><th>Geometry</th></tr>
<tr><td>a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
Draw, construct, and describe geometrical figures and describe the relationships between them.
7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.
a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.
b. Represent proportional relationships within and between similar figures.
Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.
7.G.4 Work with circles.
a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle.
b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.</td><td>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
Interpret linear models.
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★</td><td>diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Find arc lengths and areas of sectors of circles.
G.C.5 Find arc lengths and areas of sectors of circles.
a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. (G, M2)
b. Derive the formula for the area of a sector, and use it to solve problems. (G, M2)
Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.
G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
Understand the relationships between lengths, areas, and volumes.
G.GMD.5 Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.
G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.
Apply geometric concepts in modeling situations.
G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★</td></tr>
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ARITHMETIC OPERATIONS ON RATIONAL NUMBERS AND POLYNOMIALS

Grades 6, 7, and 8

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p,
in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\,3/4$ inches long in the center of a door that is $27\,1/2$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ pound of chocolate equally? How many $3/4$ cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.2 Fluently divide multi-digit numbers using a standard algorithm.

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.

Algebra 1

Reason quantitatively and use units to solve problems.

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

Perform arithmetic operations on polynomials.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

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<td>Know that there are numbers that are not rational, and approximate them by rational numbers.
8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.
8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
Work with radicals and integer exponents.
8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8; and the population of the world as 7×10^9; and determine that the world population is more than 20 times larger.
8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.
Understand and apply the Pythagorean Theorem.
8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Understand and apply the Pythagorean Theorem.
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</td>
<td>Reason quantitatively and use units to solve problems.
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★</td>
<td>Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
Define trigonometric ratios, and solve problems involving right triangles.
G.SRT.8 Solve problems involving right triangles.★
a. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)</td>
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LINEAR EXPRESSIONS, EQUATIONS, AND INEQUALITIES		
Grades 6, 7, and 8	**Algebra 1**	**Geometry**
Analyze and solve linear equations and pairs of simultaneous linear equations. **8.EE.7** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. **a.** Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. **b.** Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)* **Use properties of operations to generate equivalent expressions.** **7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.2** In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by $p - 0.15p$) is equivalent to $(1 - 0.15)p$, which is equivalent to $0.85p$ or finding 85% of the original price.*	**Interpret the structure of expressions.** **A.SSE.1**. Interpret expressions that represent a quantity in terms of its context. ★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **Create equations that describe numbers or relationships.** **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **b.** Focus on applying simple quadratic expressions. (A1, M2) **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1) **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★ **a.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R, or rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r.* (A1) **Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Solve equations and inequalities in one variable.** **A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Solve systems of equations.** **A.REI.5** Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A.REI.6** Solve systems of linear equations algebraically and graphically. **a.** Limit to pairs of linear equations in two variables. (A1, M1)	**Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.** **G.GPE.5** Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

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LINEAR EXPRESSIONS, EQUATIONS, AND INEQUALITIES		
Grades 6, 7, and 8	**Algebra 1**	**Geometry**
Solve real-life and mathematical problems using numerical and algebraic expressions and equations. **7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* *Note: For instructional purposes combine Linear Expressions, Equations, and Inequalities with* Linear Functions.	**Represent and solve equations and inequalities graphically.** **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	

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Understand the connections between proportional relationships, lines, and linear equations.
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
Analyze and solve linear equations and pairs of simultaneous linear equations.
8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically.
a. Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously.
b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)
Define, evaluate, and compare functions.
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.
8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1) (2,4) and (3,9), which are not on a straight line.
Use functions to model relationships between quantities.
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the
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Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Create equations that describe numbers or relationships.
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
a. Focus on applying linear and simple exponential expressions. (A1, M1)
Understand the concept of a function, and use function notation.
F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.
Interpret functions that arise in applications in terms of the context.
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★(A2, M3)
b. Focus on linear, quadratic, and exponential functions. (A1, M2)
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
b. Focus on linear, quadratic, and exponential functions. (A1, M2)
Analyze functions using different representations.
F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★
a. Graph linear functions and indicate intercepts. (A1, M1)
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Experiment with transformations in the plane.
G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.
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Ohio's Grade 8-Geometry Learning Progressions by Topic

LINEAR FUNCTIONS		
Grades 6, 7, and 8	Algebra 1	Geometry
rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **Investigate patterns of association in bivariate data.** **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4) **7.RP.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*. **d.** Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. **Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* *Note: For instructional purposes combine* Linear Expressions, Equations, and Inequalities *with Linear Functions.*	**F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from context. **i.** Focus on linear and exponential functions. (A1, M1) **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ **Build new functions from existing functions.** **F.BF.4** Find inverse functions. **a.** Informally determine the input of a function when the output is known. (A1, M1) **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2) (put in quad and exponents) **Interpret expressions for functions in terms of the situation they model.** **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.★ **Interpret linear models.** **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★	

Ohio’s Grade 8-Geometry Learning Progressions by Topic

EXPONENTIAL EXPRESSIONS, EQUATIONS, AND FUNCTIONS		
Grades 6, 7, and 8	**Algebra 1**	**Geometry**
Work with radicals and integer exponents. **8.EE.1** Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** **8.G.9** Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres. **Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **7.G.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **7.G.6** Solve real-world and mathematical problems involving area, volume, and surface area of two-	**Interpret the structure of expressions.** **A.SSE.1**. Interpret expressions that represent a quantity in terms of its context. ★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Represent and solve equations and inequalities graphically.** **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A.REI.11** Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations. **Write expressions in equivalent forms to solve problems.** **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **a.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.* **Create equations that describe numbers or relationships.** **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Interpret functions that arise in applications in terms of the context.** **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals*	**Experiment with transformations in the plane.** **G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch. **Explain volume formulas, and use them to solve problems.** **G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ **Understand the relationships between lengths, areas, and volumes.** **G.GMD.6** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.

EXPONENTIAL EXPRESSIONS, EQUATIONS, AND FUNCTIONS		
Grades 6, 7, and 8	**Algebra 1**	**Geometry**
and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams[G], double number line diagrams[G], or equations. **c.** Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means $^{30}/_{100}$ times the quantity; solve problems involving finding the whole, given a part and the percent. **Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	*where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★(A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **Analyze functions using different representations.** **F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★ **e.** Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **b.** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change[G] in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay.* (A2, M3) **i.** Focus on exponential functions evaluated at integer inputs. (A1, M2) **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from context. **i.** Focus on linear and exponential functions. (A1, M1) **ii.** Focus on situations that exhibit quadratic or exponential relationships. (A1, M2) **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	

Ohio's Grade 8-Geometry Learning Progressions by Topic

EXPONENTIAL EXPRESSIONS, EQUATIONS, AND FUNCTIONS		
Grades 6, 7, and 8	Algebra 1	Geometry
	F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2) **Interpret expressions for functions in terms of the situation they model.** **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.★	

Ohio's Grade 8-Geometry Learning Progressions by Topic

<table>
<tr><th colspan="3">QUADRATIC EXPRESSIONS, EQUATIONS, AND FUNCTIONS</th></tr>
<tr><th>Grades 6, 7, and 8</th><th>Algebra 1</th><th>Geometry</th></tr>
<tr><td>
Work with radicals and integer exponents.
8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.
7.G.4 Work with circles.
a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle.
b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.
7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.
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Interpret the structure of expressions.
A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
Write expressions in equivalent forms to solve problems.
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
Perform arithmetic operations on polynomials.
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)
Create equations that describe numbers or relationships.
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. ★
b. Focus on applying simple quadratic expressions. (A1, M2)
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
b. Focus on applying simple quadratic expressions. (A1, M2)
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★
a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law $V = IR$ to highlight resistance R, or rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r. (A1)
Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Solve equations and inequalities in one variable.
A.REI.4 Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
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Experiment with transformations in the plane.
G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.
Translate between the geometric description and the equation for a conic section.
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Explain volume formulas, and use them to solve problems.
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
Understand the relationships between lengths, areas, and volumes.
G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively.
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Ohio's Grade 8-Geometry Learning Progressions by Topic

QUADRATIC EXPRESSIONS, EQUATIONS, AND FUNCTIONS		
Grades 6, 7, and 8	Algebra 1	Geometry
	(+) **c.** Derive the quadratic formula using the method of completing the square. **Solve systems of equations.** **A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$. **Represent and solve equations and inequalities graphically.** **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A.REI.11** Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations. **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$. **Interpret functions that arise in applications in terms of the context.** **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★(A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **Analyze functions using different representations.** **F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★ **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	

Ohio's Grade 8-Geometry Learning Progressions by Topic

QUADRATIC EXPRESSIONS, EQUATIONS, AND FUNCTIONS		
Grades 6, 7, and 8	Algebra 1	Geometry
	a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) **i.** Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1) **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from context. **ii.** Focus on situations that exhibit quadratic or exponential relationships. (A1, M2) **Build new functions from existing functions.** **F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) **a.** Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)	

Ohio's Grade 8-Geometry Learning Progressions by Topic

LINES, ANGLES, AND TWO-DIMENSIONAL SHAPES INCLUDING RIGHT TRIANGLES AND THE PYTHAGOREAN THEOREM		
Grades 6, 7, and 8	**Algebra 1**	**Geometry**
Draw, construct, and describe geometrical figures and describe the relationships between them. **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions. **Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. **7.G.5.** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **Understand congruence and similarity using physical models, transparencies, or geometry software.** **8.G.1** Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **a.** Lines are taken to lines, and line segments are taken to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines. **8.G.2** Understand that a two-dimensional figure is congruent[G] to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.) **8.G.3** Describe the effect of dilations[G], translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections,	**Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**Prove geometric theorems both formally and informally using a variety of methods.** **G.CO.9** Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **G.CO.10** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **G.CO.11** Prove and apply theorems about parallelograms. *Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **Make geometric constructions.** **G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. **Classify and analyze geometric figures.** **G.CO.14** Classify two-dimensional figures in a hierarchy based on properties. **Understand similarity in terms of similarity transformations.** **G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Prove and apply theorems both formally and informally involving similarity using a variety of methods.** **G.SRT.4** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles. **Define trigonometric ratios, and solve problems involving right triangles.** **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. **G.SRT.8** Solve problems involving right triangles.★ **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)

Ohio's Grade 8-Geometry Learning Progressions by Topic

LINES, ANGLES, AND TWO-DIMENSIONAL SHAPES INCLUDING RIGHT TRIANGLES AND THE PYTHAGOREAN THEOREM		
Grades 6, 7, and 8	**Algebra 1**	**Geometry**
translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.) **8.G.5.** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*		**Understand and apply theorems about circles.** **G.C.1** Prove that all circles are similar using transformational arguments. **G.C.2** Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G.C.3** Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle. (+) **G.C.4** Construct a tangent line from a point outside a given circle to the circle. **Translate between the geometric description and the equation for a conic section.** **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.** **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2)

Ohio's Grade 8-Geometry Learning Progressions by Topic

PERIMETER, AREA, SURFACE AREA, AND VOLUME		
Grades 6, 7, and 8	**Algebra 1**	**Geometry**
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. **8.G.9.** Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres. **Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. **Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.3.** Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4.** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. **7.G.6.** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.1.** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. **6.G.2.** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.3.** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same	**Reason quantitatively and use units to solve problems.** **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★ **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.★ **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★ **Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**Find arc lengths and areas of sectors of circles.** **G.C.5** Find arc lengths and areas of sectors of circles. **a.** Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. (G, M2) **b.** Derive the formula for the area of a sector, and use it to solve problems. (G, M2) **Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.** **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2) **G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ **Explain volume formulas, and use them to solve problems.** **G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* **G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ **Visualize relationships between two-dimensional and three-dimensional objects.** **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Understand the relationships between lengths, areas, and volumes.** **G.GMD.5** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures. **G.GMD.6** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2, and k^3, respectively. **Apply geometric concepts in modeling situations.** **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★ **G.MG.2** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★ **G.MG.3** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

Ohio's Grade 8-Geometry Learning Progressions by Topic

PERIMETER, AREA, SURFACE AREA, AND VOLUME		
Grades 6, 7, and 8	Algebra 1	Geometry
second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.4.** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.		

Ohio's Grade 8-Geometry Learning Progressions by Topic

TRANSFORMATIONS FOCUSING ON CONGRUENCE AND SIMILARITY		
Grades 6, 7, and 8	Algebra 1	Geometry
Understand congruence and similarity using physical models, transparencies, or geometry software. **8.G.1** Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **a.** Lines are taken to lines, and line segments are taken to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines. **8.G.2** Understand that a two-dimensional figure is congruent[G] to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.) **8.G.3** Describe the effect of dilations[G], translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)	**Write expressions in equivalent forms to solve problems.** **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ **Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **Build new functions from existing functions.** **F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) **a.** Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)	**Experiment with transformations in the plane.** **G.CO.1** Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length. **G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch. **G.CO.3** Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. **b.** Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes. **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motions.** **G.CO.6** Use geometric descriptions of rigid motions[G] to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent[G]. **G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **Understand similarity in terms of similarity transformations.** **G.SRT.1** Verify experimentally the properties of dilations[G] given by a center and a scale factor: **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations[G] to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Visualize relationships between two-dimensional and three-dimensional objects.** **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Ohio's Grade 8-Geometry Learning Progressions by Topic

<table>
<tr><th colspan="3">UNIVARIATE DATA</th></tr>
<tr><th>Grade 6</th><th>Grade 7</th><th>Algebra 1</th></tr>
<tr>
<td>Develop understanding of statistical problem solving.
6.SP.1 Develop statistical reasoning by using the GAISE model:
a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages. (GAISE Model, step 1)
b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)
c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)
d. Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Summarize and describe distributions.
6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3)
6.SP.5 Summarize numerical data sets in relation to their context.
a. Report the number of observations.
b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describing shape and the presence of clusters, gaps, peaks, and outliers in a distribution.
d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.</td>
<td>Use sampling to draw conclusions about a population.
7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population.
a. Differentiate between a sample and a population.
b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.
Broaden understanding of statistical problem solving.
7.SP.2 Broaden statistical reasoning by using the GAISE model:
a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1)
b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)
c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)
d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)
Summarize and describe distributions representing one population and draw informal comparisons between two populations.
7.SP.3 Describe and analyze distributions.
a. Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point.
b. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable.</td>
<td>Summarize, represent, and interpret data on a single count or measurement variable.
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.★
S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★
S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★</td>
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</table>

Ohio's Grade 8-Geometry Learning Progressions by Topic

BIVARIATE DATA		
Grades 6, 7, and 8	Algebra 1	Geometry
Investigate patterns of association in bivariate data. **8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4) **8.SP.2** Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4) **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4) **8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	**Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f(x)*. **Summarize, represent, and interpret data on two categorical and quantitative variables.** **S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of thedata (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★ **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ **c.** Fit a linear function for a scatterplot that suggests a linear association. (A1, M1) **Interpret linear models.** **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★ **S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.★	**Understand independence and conditional probability, and use them to interpret data.** **S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★*

Ohio's Grade 8-Geometry Learning Progressions by Topic

<table>
<tr><th colspan="3">PROBABILITY</th></tr>
<tr><th>Grades 6, 7, and 8</th><th>Algebra 1</th><th>Geometry</th></tr>
<tr>
<td>Investigate patterns of association in bivariate data.
8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?
Investigate chance processes and develop, use, and evaluate probability models.
7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $^1/_2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, e.g., “rolling double sixes”, identify the outcomes in the sample space which compose the event.
c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</td>
<td>Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Summarize, represent, and interpret data on two categorical and quantitative variables.
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★</td>
<td>Understand independence and conditional probability, and use them to interpret data.
S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★
S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★
S.CP.3 Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★
S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★
S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.★
Use the rules of probability to compute probabilities of compound events in a uniform probability model.
S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★
S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★
(+) S.CP.8 Apply the general Multiplication Rule in a uniform probability model[G],
P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★ (G, M2)
(+) S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.★ (G, M2)</td>
</tr>
</table>