Ohio | Department of Education

Ohio's 6-8 Learning Progressions by Topic

Learning progressions were critical in the development, review, and revision of Ohio's Learning Standards (OLS). Ohio's learning progressions were developed during Ohio's international benchmarking project and provided guidance to the writing and revision committees of Ohio's Learning Standards. Ohio believes that learning progressions are important for the understanding and coherence of mathematical topics within and across the grade levels. The Ohio Department of Education has reformatted Ohio's Learning Standards by domain and topic to show the progression of concepts and skills across the grade levels.

This document serves as a companion document to the learning standards; it does not replace them. Curriculum leaders and teachers can use this document to better understand the standards and to analyze where their curriculum fits within each student's learning progression. Professional Learning Communities can use this document in many ways such as the following:

Multi-grade groups of teachers

1. Select a domain or topic within the standards, beginning at the lowest grade of the domain or topic, then identify the main concepts at that grade. Follow each concept progressing through the grades by identifying how the concept changes and increases in rigor and understanding for the student. Additionally, identify new concepts introduced in subsequent grades and follow them through the years. *Note: Topics are in further developed in Grade 8-Geometry Learning Progressions document.*
2. Building on previous scenario, identify and analyze the connections among the topics within learning progressions. For instance, how is ratio and proportion connected (used to develop the essential understandings) to slope or other topics in grades 6-8 and high school.

Grade-level teachers or an individual teacher

1. Use learning progressions and regular classroom formative assessments to assist in identifying where students are in the learning progression. Then develop scaffolded supports to accelerate students in an effort to bring their understandings and skills to the appropriate level or to go deeper into the content. Note that going deeper does not imply going to the next level in the progression, rather building stronger understandings of the content or making connections to other concepts or skills.
2. Make connections among the standards; between standards within a domain or topic; between standards within a cluster; and between clusters across domains. Use Mathematics – K-8 Critical Areas of Focus, model curriculum, and learning progressions to build further connections among the standards; between standards within a domain or topic; between standards within a cluster; and between clusters across domains.
3. Use learning progressions to identify gaps. When gaps are identified, this document can be used to quickly scaffold previous learning.

Note: This document is best printed on legal size paper, but can also be printed on other sizes. Please note, only Grade 5 standards related to a topic are included in this document. The Grades 6-8 model curricula are linked in each grade-level header.

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

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K-5	6	7	8	Algebra 1	Geometry
Measurement & Data	Perimeter, Area, Surface Area, & Volume				Perimeter, Area, Surface Area, & Volume
	Univariate Data			Univariate Data	
			Bivariate Data	Bivariate Data	
		Probability			Probability

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

RATIOS AND PROPORTIONAL RELATIONSHIPS, RATE OF CHANGE, AND SLOPE			
Grade Five	**Grade Six**	**Grade Seven**	**Grade Eight**
Analyze patterns and relationships. **5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified).** **5.NF.3** Interpret a fraction as division of the numerator by the denominator ($^{a}/_{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $^{3}/_{4}$ as the result of dividing 3 by 4, noting that $^{3}/_{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $^{3}/_{4}$. If 9 people want to share a 50 pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.5** Interpret multiplication as scaling (resizing). **a.** Compare the size of a product to the size of one factor on the basis of the size of the other	**Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.2** Understand the concept of a unit rate $^{a}/_{b}$ associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $^{3}/_{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **a.** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. **b.** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **c.** Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means $^{30}/_{100}$ times the quantity; solve problems involving finding the whole, given a part and the percent. **d.** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	**Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $^{1}/_{2}$ mile in each $^{1}/_{4}$ hour, compute the unit rate as the complex fraction $^{(1/2)}/_{(1/4)}$ miles per hour, equivalently 2 miles per hour.* **7.RP.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* **d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate. **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.	**Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **Define, evaluate, and compare functions.** **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. **Use functions to model relationships between quantities.** **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it

OHIO’S 6-8 LEARNING PROGRESSIONS BY TOPIC

RATIOS AND PROPORTIONAL RELATIONSHIPS, RATE OF CHANGE, AND SLOPE			
Grade Five	**Grade Six**	**Grade Seven**	**Grade Eight**
factor, without performing the indicated multiplication. **b.** Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence ${}^{a}/_{b} = {}^{(n\times a)}/_{(n\times b)}$ to the effect of multiplying ${}^{a}/_{b}$ by 1. **5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	**Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time. **Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	**a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. **b.** Represent proportional relationships within and between similar figures. **Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.	models, and in terms of its graph or a table of values. **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

ARITHMETIC OPERATIONS ON RATIONAL NUMBERS			
Grade Five	**Grade Six**	**Grade Seven**	**Algebra 1**
Perform operations with multi-digit whole numbers and with decimals to hundredths. **5.NBT.5** Fluently[G] multiply multi-digit whole numbers using a standard algorithm[G]. **5.NBT.6** Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NBT.7** Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. **a.** Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place). **b.** Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). **c.** Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). *For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.* **Use equivalent fractions as a strategy to add and subtract fractions. (Fractions need not be simplified).** **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show* $^{2}/_{3} + {}^{5}/_{4} = {}^{8}/_{12} + {}^{15}/_{12} = {}^{23}/_{12}$. *In general,* $^{a}/_{b} + {}^{c}/_{d} = ({}^{a}/_{b} \times {}^{d}/_{d}) + ({}^{c}/_{d} \times {}^{b}/_{b}) = {}^{(ad + bc)}/_{bd}$. **5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $({}^{2}/_{3}) \div ({}^{3}/_{4})$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $({}^{2}/_{3}) \div ({}^{3}/_{4}) = {}^{8}/_{9}$ *because* $^{3}/_{4}$ *of* $^{8}/_{9}$ *is* $^{2}/_{3}$. *(In general,* $({}^{a}/_{b}) \div ({}^{c}/_{d}) = {}^{ad}/_{bc}$.*) How much chocolate will each person get if 3 people share* $^{1}/_{2}$ *pound of chocolate equally? How many* $^{3}/_{4}$ *cup servings are in* $^{2}/_{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $^{3}/_{4}$ *mi and area* $^{1}/_{2}$ *square mi?* **Compute fluently with multi-digit numbers and find common factors and multiples.** **6.NS.2** Fluently divide multi-digit numbers using a standard algorithm. **6.NS.3** Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **b.** Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. **c.** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **d.** Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are	**Reason quantitatively and use units to solve problems.** **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.★ **Perform arithmetic operations on polynomials.** **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **a.** Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2) **Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. *Note: Algebra 1 is included here, because there is a gap in alignment between Grade 7 and high school.*

ARITHMETIC OPERATIONS ON RATIONAL NUMBERS			
Grade Five	**Grade Six**	**Grade Seven**	**Algebra 1**
denominators, e.g., by using visual fraction models[G] or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* ${}^{2}/_{5} + {}^{1}/_{2} = {}^{3}/_{7}$*, by observing that* ${}^{3}/_{7} < {}^{1}/_{2}$*.* **Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified).** **5.NF.3** Interpret a fraction as division of the numerator by the denominator (${}^{a}/_{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret* ${}^{3}/_{4}$ *as the result of dividing 3 by 4, noting that* ${}^{3}/_{4}$ *multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size* ${}^{3}/_{4}$*. If 9 people want to share a 50 pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. **a.** Interpret the product $({}^{a}/_{b}) \times q$ as a parts of a partition of q into b equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show* $({}^{2}/_{3}) \times 4 = {}^{8}/_{3}$*, and create a story context for this equation. Do the same with* $({}^{2}/_{3}) \times ({}^{4}/_{5}) = {}^{8}/_{15}$*. (In general,* $({}^{a}/_{b}) \times ({}^{c}/_{d}) = {}^{ac}/_{bd}$*.)* **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF.5** Interpret multiplication as scaling (resizing). **a.** Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **b.** Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case);		integers, then $-({}^{p}/_{q}) = {}^{(-p)}/_{q} = {}^{p}/_{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts. **c.** Apply properties of operations as strategies to multiply and divide rational numbers. **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional ${}^{1}/_{10}$ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar $9\,{}^{3}/_{4}$ inches long in the center of a door that is $27\,{}^{1}/_{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.	

ARITHMETIC OPERATIONS ON RATIONAL NUMBERS			
Grade Five	**Grade Six**	**Grade Seven**	**Algebra 1**
explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $^{a}/_{b} = {}^{(n\times a)}/_{(n\times b)}$ to the effect of multiplying $^{a}/_{b}$ by 1. **5.NF.6** Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(^{1}/_{3}) \div 4$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $(^{1}/_{3}) \div 4 = (^{1}/_{12})$ *because* $(^{1}/_{12}) \times 4 = (^{1}/_{3})$*.* **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div (^{1}/_{5})$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div (^{1}/_{5}) = 20$ *because* $20 \times (^{1}/_{5}) = 4$*.* **c.** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share* $^{1}/_{2}$ *pound of chocolate equally? How many* $^{1}/_{3}$ *cup servings are in 2 cups of raisins?*			

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

THE REAL NUMBER SYSTEM INCLUDING SQUARE ROOTS, EXPONENTS, AND SCIENTIFIC NOTATION			
Grade Five	**Grade Six**	**Grade Seven**	**Grade Eight**
Understand the place value system. **5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $^1/_{10}$ of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10. **5.NBT.3** Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form[G], e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × ($^1/_{10}$) + 9 × ($^1/_{100}$) + 2 × ($^1/_{1000}$). **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.	**Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., − (−3) = 3, and that 0 is its own opposite. **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **c.** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.7** Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right.* **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write −3°C > −7°C to express the fact that −3°C is warmer than −7 °C.* **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**Know that there are numbers that are not rational, and approximate them by rational numbers.** **8.NS.1** Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. **8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **Work with radicals and integer exponents.** **8.EE.1** Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = {}^1/_{3^3} = {}^1/_{27}$. **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **8.EE.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8; and the population of the world as 7×10^9; and determine that the world population is more than 20 times larger. **8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading.

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OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

THE REAL NUMBER SYSTEM INCLUDING SQUARE ROOTS, EXPONENTS, AND SCIENTIFIC NOTATION			
Grade Five	**Grade Six**	**Grade Seven**	**Grade Eight**
Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate. **5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars.* **d.** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars.* **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **Apply and extend previous understandings of arithmetic to algebraic expressions.** **6.EE.1** Write and evaluate numerical expressions involving whole number exponents. **Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.		Interpret scientific notation that has been generated by technology. **Understand and apply the Pythagorean Theorem.** **8.G.6** Analyze and justify an informal proof of the Pythagorean Theorem and its converse. **8.G.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Understand and apply the Pythagorean Theorem. **8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

LINEAR EXPRESSIONS, EQUATIONS, AND INEQUALITIES			
Grade Five	**Grade Six**	**Grade Seven**	**Grade Eight**
Write and interpret numerical expressions. **5.OA.1** Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary. **5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.*	**Compute fluently with multi-digit numbers and find common factors and multiples.** **6.NS.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* **Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.7** Understand ordering and absolute value of rational numbers. **a.** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right. **b.** Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write −3°C > −7°C to express the fact that −3°C is warmer than −7 °C. **c.** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars. **d.** Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars. Apply and extend previous understandings of arithmetic to algebraic expressions. **Apply and extend previous understandings of arithmetic to algebraic expressions.** **6.EE.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.2** Write, read, and evaluate expressions in which letters stand for numbers. **a.** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 − y.* **b.** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* **c.** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas*	**Use properties of operations to generate equivalent expressions.** **7.EE.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.2** In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p − 0.15p) is equivalent to (1 − 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**Analyze and solve linear equations and pairs of simultaneous linear equations.** **8.EE.7** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. **a.** Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. **b.** Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For*

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

LINEAR EXPRESSIONS, EQUATIONS, AND INEQUALITIES			
Grade Five	**Grade Six**	**Grade Seven**	**Grade Eight**
	$V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$ **6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **6.EE.4** Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* **Reason about and solve one-variable equations and inequalities.** **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.7.** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*		*example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)*

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

LINEAR FUNCTIONS		
Grades Five through Seven	**Grade Eight**	**Algebra 1**
Analyze proportional relationships and use them to solve real-world and mathematical problems. **7.RP.2** Recognize and represent proportional relationships between quantities. **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* **d.** Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. **Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the	**Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*. **Analyze and solve linear equations and pairs of simultaneous linear equations.** **8.EE.8** Analyze and solve pairs of simultaneous linear equations graphically. **a.** Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. **b.** Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* **c.** Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)* *(continued on next page)*	**Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Create equations that describe numbers or relationships.** **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **a.** Focus on applying linear and simple exponential expressions. (A1, M1) **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$. **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.* **Interpret functions that arise in applications in terms of the context.** **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **Analyze functions using different representations.** **F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★ **a.** Graph linear functions and indicate intercepts. (A1, M1)

OHIO’S 6-8 LEARNING PROGRESSIONS BY TOPIC

LINEAR FUNCTIONS		
Grades Five through Seven	**Grade Eight**	**Algebra 1**
equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* **Analyze patterns and relationships.** **5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**Define, evaluate, and compare functions.** **8.F.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8. **8.F.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **8.F.3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1) (2,4) and (3,9), which are not on a straight line.* **Use functions to model relationships between quantities.** **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **Investigate patterns of association in bivariate data.** **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4)	**F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) **b.** Focus on linear, quadratic, and exponential functions. (A1, M2) **Build a function that models a relationship between two quantities.** **F.BF.1** Write a function that describes a relationship between two quantities.★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from context. **i.** Focus on linear and exponential functions. (A1, M1) **F.BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ **Build new functions from existing functions.** **F.BF.4** Find inverse functions. **a.** Informally determine the input of a function when the output is known. (A1, M1) **Construct and compare linear, quadratic, and exponential models, and solve problems.** **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2) **Interpret expressions for functions in terms of the situation they model.** **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.★ **Interpret linear models.** **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★ *Note: Algebra 1is included here, because there are not many standards in the middle school progression surrounding this topic.*

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

LINES, ANGLES, AND TWO-DIMENSIONAL SHAPES INCLUDING RIGHT TRIANGLES AND THE PYTAGOREAN THEOREM		
Grade Five	**Grade Seven**	**Grade Eight**
Classify two-dimensional figures into categories based on their properties. **5.G.3** Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles). **5.G.4** Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids[G], and rhombuses.	**Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions. **Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**Understand congruence and similarity using physical models, transparencies, or geometry software.** **8.G.1** Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **a.** Lines are taken to lines, and line segments are taken to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines. **8.G.2** Understand that a two-dimensional figure is congruent[G] to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.) **8.G.3** Describe the effect of dilations[G], translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.) **8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

PERIMETER, AREA, AND VOLUME			
Grade Five	**Grade Six**	**Grade Seven**	**Grade Eight**
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.5** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole number products as volumes, e.g., to represent the Associative Property of Multiplication. **b.** Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems. **Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate. **5.G.2** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. *Note: Fraction Operations also lead to Perimeter, Area, and Volume in Grade Six.*	**Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.1** Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. **6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.1** Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. **Draw, construct, and describe geometrical figures and describe the relationships between them.** **7.G.3** Describe the two-dimensional figures that result from slicing three dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. **7.G.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.** **8.G.9** Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

TRANSFORMATIONS FOCUSING ON CONGRUENCE AND SIMILARITY		
Grade Seven	**Grade Eight**	**Geometry**
Draw, construct, and describe geometrical figures and describe the relationships between them. **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions. **Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.** **7.G.4** Work with circles. **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems. **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	**Understand congruence and similarity using physical models, transparencies, or geometry software.** **8.G.1** Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **a.** Lines are taken to lines, and line segments are taken to line segments of the same length. **b.** Angles are taken to angles of the same measure. **c.** Parallel lines are taken to parallel lines. **8.G.2** Understand that a two-dimensional figure is congruent[G] to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.) **8.G.3** Describe the effect of dilations[G], translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)	**Experiment with transformations in the plane.** **G.CO.1** Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length. **G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch. **G.CO.3** Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. **b.** Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes. **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motions.** **G.CO.6** Use geometric descriptions of rigid motions[G] to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent[G]. **G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **Understand similarity in terms of similarity transformations.** **G.SRT.1** Verify experimentally the properties of dilations[G] given by a center and a scale factor: **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations[G] to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

TRANSFORMATIONS FOCUSING ON CONGRUENCE AND SIMILARITY		
Grade Seven	**Grade Eight**	**Geometry**
		Visualize relationships between two-dimensional and three-dimensional objects. **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. *Note: Geometry is included here, because there are not many standards in the middle school progression surrounding this topic.*

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

UNIVARIATE DATA			
Grade Five	**Grade Six**	**Grade Seven**	**Algebra 1**
Represent and interpret data. **5.MD.2** Display and interpret data in graphs (picture graphs, bar graphs, and line plots[G]) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions $^1/_2$, $^1/_4$, $^1/_8$, or decimals.	**Develop understanding of statistical problem solving.** **6.SP.1** Develop statistical reasoning by using the GAISE model: **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4) **6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **Summarize and describe distributions.** **6.SP.4** Display numerical data in plots on a number line, including dot plots (line plots), histograms, and box plots. (GAISE Model, step 3) **6.SP.5** Summarize numerical data sets in relation to their context. **a.** Report the number of observations. **b.** Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. **c.** Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range) as well as informally describing shape and the presence of clusters, gaps, peaks, and outliers in a distribution. **d.** Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.	**Use sampling to draw conclusions about a population.** **7.SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population. **a.** Differentiate between a sample and a population. **b.** Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias. **Broaden understanding of statistical problem solving.** **7.SP.2** Broaden statistical reasoning by using the GAISE model: **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?"* (GAISE Model, step 1) **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) **d.** Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4) **Summarize and describe distributions representing one population and draw informal comparisons between two populations.** **7.SP.3** Describe and analyze distributions. **a.** Summarize quantitative data sets in relation to their context by using mean absolute deviation (MAD), interpreting mean as a balance point. **b.** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot (line plot), the separation between the two distributions of heights is noticeable.*	**Summarize, represent, and interpret data on a single count or measurement variable.** **S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.★ **S.ID.2** In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★ **S.ID.3** In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★ *Note: Algebra 1 is included here, because there is a gap in alignment between Grade 7 and high school.*

BIVARIATE DATA		
Grade Eight	**Grade Eight**	**Algebra 1**
Pair with Grade 8 Linear Functions topics.	**Investigate patterns of association in bivariate data.** **8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4) **8.SP.2** Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4) **8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4) **8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	**Understand solving equations as a process of reasoning and explain the reasoning.** **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Understand the concept of a function, and use function notation.** **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. **Summarize, represent, and interpret data on two categorical and quantitative variables.** **S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of thedata (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★ **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ **c.** Fit a linear function for a scatterplot that suggests a linear association. (A1, M1) **Interpret linear models.** **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★ **S.ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.★ *Note: Algebra 1 is included here, because there are not many standards in the middle school progression surrounding this topic.*

OHIO'S 6-8 LEARNING PROGRESSIONS BY TOPIC

PROBABILITY		
Grade Seven	**Grade Seven**	**Grade Eight**
Pair with Grade 7 Ratio and Proportional Relationships topics.	**Investigate chance processes and develop, use, and evaluate probability models.** **7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $^1/_2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **7.SP.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **a.** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. **a.** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **b.** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes", identify the outcomes in the sample space which compose the event. **c.** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	**Investigate patterns of association in bivariate data.** **8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* *Note: 8.SP.4 connects to S.ID.5 and build toward probability in S.CP.4* *Note: See Geometry Conditional Probability Standards (S.CP) for connections with high school probability.*