



# Ohio

## Ohio's Learning Standards | Mathematics

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## Introduction

### PROCESS

To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education.

### UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as  $(a + b)(x + y)$  and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding

$(a + b + c)(x + y)$ . Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 4 with the eight Standards for Mathematical Practice.

## Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

### 1. Make sense of problems and persevere in solving them.

In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students may use concrete objects, pictures, or drawings to help them conceptualize and solve problems, such as “Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?” or “Describe another situation where there would be 5 groups of 3 or  $5 \times 3$ .” Students may check their thinking by asking themselves, “Does this make sense?” Students listen to other students’ strategies and are able to make connections between various methods for a given problem.

### 2. Reason abstractly and quantitatively.

Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. For example: students apply their understanding of the meaning of the equal sign as “the same as” to interpret an equation with an unknown. When given  $4 \times \square = 40$ , they might think:

- 4 groups of some number is the same as 40

- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Teachers might ask, “How do you know” or “What is the relationship between the quantities?” to reinforce students’ reasoning and understanding.

### 3. Construct viable arguments and critique the reasoning of others.

Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilitates by asking questions such as “How did you get that?” and “Why is that true?” Students explain their thinking to others and respond to others’ thinking. For example, after investigating patterns on the 100s chart, students might explain why the pattern makes sense.

### 4. Model with mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

For example, students use various contexts and a variety of models (e.g., circles, squares, rectangles, fraction bars, and number lines) to represent and develop understanding of fractions. Students use models to represent both equations and story problems and can explain their thinking. They evaluate their results in the context of the situation and reflect on whether the results make sense. Students should be encouraged to answer questions, such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?”

## Standards for Mathematical Practice, continued

### 5. Use appropriate tools strategically.

Third graders consider the available tools (including drawings and estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students should be encouraged to answer questions such as, “Why was it helpful to use \_\_\_?”

### 6. Attend to precision.

As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.

### 7. Look for and make use of structure.

Students look closely to discover a pattern or structure. For instance, students use properties of operations (e.g., commutative and distributive properties) as strategies to multiply and divide. Teachers might ask, “What do you notice when \_\_\_?” or “How do you know if something is a pattern?”

### 8. Look for and express regularity in repeated reasoning.

Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of  $7 \times 8$ , they might decompose 7 into 5 and 2 and then multiply  $5 \times 8$  and  $2 \times 8$  to arrive at  $40 + 16$  or 56. In addition, third graders continually evaluate their work by asking themselves, “does this make sense? Students should be encouraged

to answer questions, such as, “what is happening in this situation?” Or “what predictions or generalizations can this pattern support?”

## CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

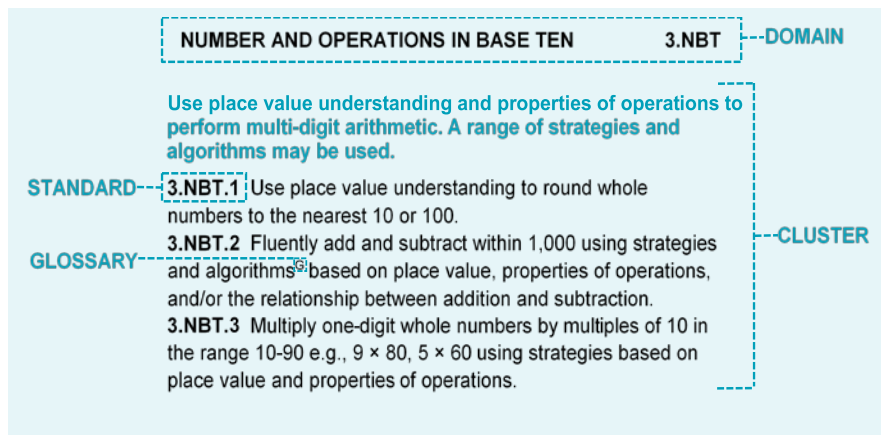
## How to Read the Grade Level Standards

**Standards** define what students should understand and be able to do.

**Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

**Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.

<sup>G</sup> shows there is a definition in the glossary for this term.



These standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, does not necessarily mean that teachers must teach topic A before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, “Students who already know ... should next come to learn ....” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Therefore, educators, researchers, and mathematicians used their collective experience and professional judgment along with state and international comparisons as a basis to make grade placements for specific topics.

## Grade 3

In Grade 3, instructional time should focus on five critical areas:

### **Critical Area 1: Developing understanding of multiplication and division and strategies for multiplication and division within 100**

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

### **Critical Area 2: Developing understanding of fractions, especially unit fractions (fractions with numerator 1)**

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example,  $\frac{1}{2}$  of the paint in a small bucket could be less paint than  $\frac{1}{3}$  of the paint in a larger bucket, but  $\frac{1}{3}$  of a ribbon is longer than  $\frac{1}{5}$  of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

### **Critical Area 3: Developing understanding of the structure of rectangular arrays and of area**

Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

### **Critical Area 4: Describing and analyzing two-dimensional shapes**

Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

### **Critical Area 5: Solving multi-step problems**

Students apply previous understanding of addition and subtraction strategies and algorithms to solve multi-step problems. They reason abstractly and quantitatively by modeling problem situations with equations or graphs, assessing their processes and results, and justifying their answers through mental computation and estimation strategies. Students incorporate multiplication and division within 100 to solve multi-step problems with the four operations.

## GRADE 3 OVERVIEW

### OPERATIONS AND ALGEBRAIC THINKING

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

### NUMBER AND OPERATIONS IN BASE TEN

- Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.

### NUMBER AND OPERATIONS—FRACTIONS

- Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

### MEASUREMENT AND DATA

- Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.

### MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### GEOMETRY

- Reason with shapes and their attributes.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.



## Grade 3

### OPERATIONS AND ALGEBRAIC THINKING

### 3.OA

#### Represent and solve problems involving multiplication and division.

**3.OA.1** Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that  $a \times b$  means  $a$  groups of  $b$  objects each; however, because of the commutative property, students may also interpret  $5 \times 7$  as the total number of objects in 7 groups of 5 objects each).

**3.OA.2** Interpret whole number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .*

**3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Table 2, page 18. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

**3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations  $8 \times \square = 48$ ;  $5 = \square \div 3$ ;  $6 \times 6 = \square$ .*

#### Understand properties of multiplication and the relationship between multiplication and division.

**3.OA.5** Apply properties of operations as strategies to multiply and divide. *For example, if  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known (Commutative Property of Multiplication);  $3 \times 5 \times 2$  can be found by  $3 \times 5 = 15$ , then  $15 \times 2 = 30$ , or by  $5 \times 2 = 10$ , then  $3 \times 10 = 30$  (Associative Property of Multiplication); knowing that  $8 \times 5 = 40$  and  $8 \times 2 = 16$ , one can find  $8 \times 7$  as  $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$  (Distributive Property). Students need not use formal terms for these properties.*

**3.OA.6** Understand division as an unknown-factor problem. *For example, find  $32 \div 8$  by finding the number that makes 32 when multiplied by 8.*

#### Multiply and divide within 100.

**3.OA.7** Fluently<sup>6</sup> multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ , or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two one-digit numbers.

#### Solve problems involving the four operations, and identify and explain patterns in arithmetic.

**3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.

**OPERATIONS AND ALGEBRAIC THINKING, continued****Solve problems involving the four operations, and identify and explain patterns in arithmetic. (continued)**

**3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

**NUMBER AND OPERATIONS IN BASE TEN****3.NBT****Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.**

**3.NBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100.

**3.NBT.2** Fluently add and subtract within 1,000 using strategies and algorithms<sup>G</sup> based on place value, properties of operations, and/or the relationship between addition and subtraction.

**3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g.,  $9 \times 80$ ,  $5 \times 60$  using strategies based on place value and properties of operations.

**NUMBER AND OPERATIONS—FRACTIONS****3.NF****Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.**

**3.NF.1** Understand a fraction  $\frac{1}{b}$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $\frac{a}{b}$  as the quantity formed by  $a$  parts of size  $\frac{1}{b}$ .

**3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram<sup>G</sup>.

- Represent a fraction  $\frac{1}{b}$  on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $\frac{1}{b}$  and that the endpoint of the part based at 0 locates the number  $\frac{1}{b}$  on the number line.
- Represent a fraction  $\frac{a}{b}$  (which may be greater than 1) on a number line diagram by marking off  $a$  lengths  $\frac{1}{b}$  from 0. Recognize that the resulting interval has size  $\frac{a}{b}$  and that its endpoint locates the number  $\frac{a}{b}$  on the number line.

**3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
- Recognize and generate simple equivalent fractions, e.g.,  $\frac{1}{2} = \frac{2}{4}$ ,  $\frac{4}{6} = \frac{2}{3}$ . Explain why the fractions are equivalent, e.g., by using a visual fraction model<sup>G</sup>.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form  $3 = \frac{3}{1}$ ; recognize that  $\frac{6}{1} = 6$ ; locate  $\frac{4}{4}$  and 1 at the same point of a number line diagram.*
- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

**MEASUREMENT AND DATA****3.MD****Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.****3.MD.1** Work with time and money.

- a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock.
- b. Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the \$ and ¢ symbol appropriately (not including decimal notation).

**3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Table 2, page 18.

**Represent and interpret data.**

**3.MD.3** Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. *For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.*

**3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot<sup>Ⓒ</sup>, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**

**3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement.

- a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
- b. A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units.

**3.MD.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

**3.MD.7** Relate area to the operations of multiplication and addition.

- a. Find the area of a rectangle with whole number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems, and represent whole number products as rectangular areas in mathematical reasoning.
- c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths  $a$  and  $b + c$  is the sum of  $a \times b$  and  $a \times c$  (represent the distributive property with visual models including an area model).
- d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

**MEASUREMENT AND DATA, continued**

**Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

**3.MD.8** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

**GEOMETRY****3.G**

**Reason with shapes and their attributes.**

**3.G.1** Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).

**3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as  $\frac{1}{4}$  of the area of the shape.*

## Glossary

### Addition and subtraction within 5, 10, 20, 100, or 1000.

Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively.

Example:  $8 + 2 = 10$  is an addition within 10,  $14 - 5 = 9$  is a subtraction within 20, and  $55 - 18 = 37$  is a subtraction within 100.

**Additive inverses.** Two numbers whose sum is 0 are additive inverses of one another. Example:  $\frac{3}{4}$  and  $-\frac{3}{4}$  are additive inverses of one another because  $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$ .

**Algorithm.** See also: computation algorithm.

**Associative property of addition.** See Table 3, page 96.

**Associative property of multiplication.** See Table 3, page 96.

**Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

**Box plot.** A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.<sup>1</sup> See also: first quartile and third quartile.

**Commutative property.** See Table 3, page 96.

**Complex fraction.** A fraction  $\frac{A}{B}$  where  $A$  and/or  $B$  are fractions ( $B$  nonzero).

**Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

**Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

**Congruent.** Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

**Counting on.** A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

**Dilation.** A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

**Dot plot.** See also: line plot.

<sup>1</sup> Adapted from Wisconsin Department of Public Instruction, <http://dpi.wi.gov/standards/mathglos.html>, accessed March 2, 2010.

<sup>2</sup> Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

<sup>3</sup> Adapted from Wisconsin Department of Public Instruction, *op. cit.*

<sup>4</sup> Adapted from Wisconsin Department of Public Instruction, *op. cit.*

**Expanded form.** A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example,  $643 = 600 + 40 + 3$ .

**Expected value.** For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

**First quartile.** For a data set with median  $M$ , the first quartile is the median of the data values less than  $M$ . Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

**Fluency.** The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

**Fluently.** *See also:* fluency.

**Fraction.** A number expressible in the form  $\frac{a}{b}$  where  $a$  is a whole number and  $b$  is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

**Identity property of 0.** *See* Table 3, page 96.

**Independently combined probability models.** Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

**Integer.** A number expressible in the form  $a$  or  $-a$  for some whole number  $a$ .

**Interquartile Range.** A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the interquartile range is  $15 - 6 = 9$ . *See also:* first quartile, third quartile.

**Justify:** To provide a convincing argument for the truth of a statement to a particular audience.

**Line plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.<sup>3</sup>

**Mean.** A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set  $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the mean is 21.

**Mean absolute deviation.** A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set  $\{2, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$ , the mean absolute deviation is 20.

**Median.** A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

**Midline.** In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

**Multiplication and division within 100.**

Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example:  $72 \div 8 = 9$ .

**Multiplicative inverses.** Two numbers whose product is 1 are multiplicative inverses of one another. Example:  $\frac{3}{4}$  and  $\frac{4}{3}$  are multiplicative inverses of one another because  $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$ .

**Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

**Percent rate of change.** A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by  $\frac{5}{50} = 10\%$  per year.

**Probability distribution.** The set of possible values of a random variable with a probability assigned to each.

**Properties of equality.** See Table 4, page 96.

**Properties of inequality.** See Table 5, page 97.

**Properties of operations.** See Table 3, page 96.

**Probability.** A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

**Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The

set of all outcomes is called the sample space, and their probabilities sum to 1. See *also*: uniform probability model.

**Prove:** To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

**Random variable.** An assignment of a numerical value to each outcome in a sample space.

**Rational expression.** A quotient of two polynomials with a nonzero denominator.

<sup>5</sup> Adapted from Wisconsin Department of Public Instruction, *op. cit.*

**Rational number.** A number expressible in the form  $\frac{a}{b}$  or  $-\frac{a}{b}$  for some fraction  $\frac{a}{b}$ . The rational numbers include the integers.

**Rectilinear figure.** A polygon all angles of which are right angles.

**Rigid motion.** A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

**Repeating decimal.** The decimal form of a rational number. *See also:* terminating decimal.

**Sample space.** In a probability model for a random process, a list of the individual outcomes that are to be considered.

**Scatter plot.** A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.<sup>5</sup>

**Similarity transformation.** A rigid motion followed by a dilation.

**Standard Algorithm.** See computational algorithm.

**Tape diagram.** A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

**Terminating decimal.** A decimal is called terminating if its repeating digit is 0.

**Third quartile.** For a data set with median  $M$ , the third quartile is the median of the data values greater than  $M$ . Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

**Transitivity principle for indirect measurement.** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

**Trapezoid.** 1. A trapezoid is a quadrilateral with at least one pair of parallel sides. (inclusive definition)  
2. A trapezoid is a quadrilateral with exactly one pair of parallel sides. (exclusive definition)

*Districts may choose either*

*definition to use for instruction. Ohio's State Tests' items will be written so that either definition will be acceptable.*

**Uniform probability model.** A probability model which assigns equal probability to all outcomes. *See also:* probability model.

**Vector.** A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

**Verify:** To check the truth or correctness of a statement in specific cases.

**Visual fraction model.** A tape diagram, number line diagram, or area model.

**Whole numbers.** The numbers 0, 1, 2, 3, ....



**Table 1. Common Addition and Subtraction Situations.**

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
<b>ADD TO</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>TAKE FROM</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN <sup>1</sup>
<b>PUT TOGETHER/ TAKE APART<sup>2</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
<b>COMPARE<sup>3</sup></b>	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

Key:  Extends across Grades 3-5 to include grade level appropriate numbers

<sup>1</sup> These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean "makes" or "results in" but always does mean "is the same number as."

<sup>2</sup> Either addend can be unknown, so there are three variations of these problem situations. *Both Addends Unknown* is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

<sup>3</sup> For the *Bigger Unknown* or *Smaller Unknown* situations, one version directs the correct operation (the version using more for the *Bigger Unknown* and using less for the *Smaller Unknown*). The other versions are more difficult.

Table 2. Common Multiplication and Division Situations<sup>1</sup>

	UNKNOWN PRODUCT $3 \times 6 = ?$	GROUP SIZE UNKNOWN (“HOW MANY IN EACH GROUP?” DIVISION) $3 \times ? = 18$ , AND $18 \div 3 = ?$	NUMBER OF GROUPS UNKNOWN (“HOW MANY GROUPS?” DIVISION) $? \times 6 = 18$ , AND $18 \div 6 = ?$
<b>EQUAL GROUPS</b>	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><b>Measurement example.</b> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><b>Measurement example.</b> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><b>Measurement example.</b> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
<b>ARRAYS<sup>2</sup>, AREA<sup>3</sup></b>	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><b>Area example.</b> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><b>Area example.</b> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><b>Area example.</b> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
<b>COMPARE</b>	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><b>Measurement example.</b> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><b>Measurement example.</b> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><b>Measurement example.</b> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
<b>GENERAL</b>	$a \times b = ?$	$a \times ? = p$ , and $p \div a = ?$	$? \times b = p$ , and $p \div b = ?$

Key:  Grades 3 (introduce), 4-5 (extend) Grades 4 (introduce), 5 (extend)

<sup>1</sup> The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

<sup>2</sup> The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>3</sup> Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

### Table 3. Properties of Operations.

Here  $a$ ,  $b$  and  $c$  stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system and the complex number system.

<b>ASSOCIATIVE PROPERTY OF ADDITION</b>	$(a + b) + c = a + (b + c)$
<b>COMMUTATIVE PROPERTY OF ADDITION</b>	$a + b = b + a$
<b>ADDITIVE IDENTITY PROPERTY OF 0</b>	$a + 0 = 0 + a = a$
<b>EXISTENCE OF ADDITIVE INVERSES</b>	For every $a$ there exists $-a$ so that $a + (-a) = (-a) + a = 0$
<b>ASSOCIATIVE PROPERTY OF MULTIPLICATION</b>	$(a \times b) \times c = a \times (b \times c)$
<b>COMMUTATIVE PROPERTY OF MULTIPLICATION</b>	$a \times b = b \times a$
<b>MULTIPLICATIVE IDENTITY PROPERTY OF 1</b>	$a \times 1 = 1 \times a = a$
<b>EXISTENCE OF MULTIPLICATIVE INVERSES</b>	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
<b>DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION</b>	$a \times (b + c) = a \times b + a \times c$

\* Students need not use formal terms for these properties.

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