Ohio's Learning Standards | Mathematics

Grade 4

Ohio | Department of Education

Table of Contents

Introduction

PROCESS

To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 4 with the eight Standards for Mathematical Practice.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers. Students might use an equation strategy to solve the word problem. For example, students could solve the problem "Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?" with the equation 3 × $12 + $15 = a.

Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2. Reason abstractly and quantitatively.
Fourth graders should recognize that a number represents a specific quantity. They connect the quality to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. Students might use base 10 blocks or drawings to demonstrate 154 × 6, as 154 added six times, and develop an understanding of the distributive property. For example: 154 × 6
= (100 + 50 + 4)× 6
= (100 × 6) + (50 × 6) + (4 × 6)
= 600 + 300 + 24 = 924

3. Construct viable arguments and critique the reasoning of others.
In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. Students explain and defend their answers and solution strategies as they answer question that require an explanation. For example, "Vincent cuts 2 meters of string into 4 centimeter pieces for a craft. How many pieces of string does Vincent have? Explain your reasoning." Students ask appropriate questions and they decide if explanations make sense.

Standards for Mathematical Practice, continued

4. Model with mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense. For example, students may use money (i.e. dollars and coins) or base10 blocks to solve the following problem: Elsie buys a drink for $1.39 and a granola bar for $0.89. How much change will she receive if she pays with a $5 bill?

5. Use appropriate tools strategically.

Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper, a number line, or base 10 blocks to represent, compare, add, and subtract decimals to the hundredths. Students in fourth grade use protractors to measure angles. They use other measurement tools to understand the relative size of units within a given system and express measurements given in larger units in terms of smaller units.

6. Attend to precision.

As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. For instance, they may use graph paper or a number line to represent, compare, add, and subtract decimals to the hundredths. Students in fourth grade use protractors to measure angles. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

7. Look for and make use of structure.

In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as arrays and area models to the multiplication principal of counting. They generate number or shape patterns that follow a given rule using two-column tables.

8. Look for and express regularity in repeated reasoning.

Students in fourth grade should notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

Continued on next page

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT, CONTINED

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

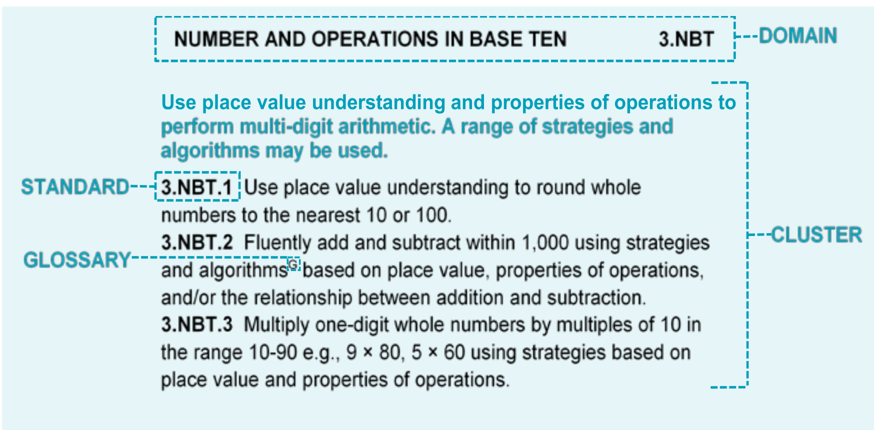
How to Read the Grade Level Standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

G shows there is a definition in the glossary for this term.



These standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, does not necessarily mean that teachers must teach topic A before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Therefore, educators, researchers, and mathematicians used their collective experience and professional judgment along with state and international comparisons as a basis to make grade placements for specific topics.

Grade 4

In Grade 4, instructional time should focus on three critical areas:

Critical Area 1: Developing an understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends as part of effectively and efficiently performing multi-digit arithmetic
Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, and area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. Students efficiently and effectively add and subtract multi-digit whole numbers.

Critical Area 2: Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal, e.g., $^{15}/_{9} = {}^{5}/_{3}$, and they develop methods such as using models for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Students solve measurement problems involving conversion of measurements and fractions.

Critical Area 3: Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, and particular angle measures
Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems.

GRADE 4 OVERVIEW

OPERATIONS AND ALGEBRAIC THINKING

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

NUMBER AND OPERATIONS IN BASE TEN

- Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.
- Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.

NUMBER AND OPERATIONS—FRACTIONS

- Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified).
- Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

MEASUREMENT AND DATA

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

GEOMETRY

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Grade 4

OPERATIONS AND ALGEBRAIC THINKING 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Table 2, page 19. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

4.OA.3 Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

NUMBER AND OPERATIONS IN BASE TEN 4.NBT

Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division.

4.NBT.2 Read and write multi-digit whole numbers using standard form, word form, and expanded form[G]. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.

4.NBT.4 Fluently[G] add and subtract multi-digit whole numbers using a standard algorithm[G].

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

NUMBER AND OPERATIONS IN BASE TEN, continued

Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000. (continued)

4.NBT.6 Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

NUMBER AND OPERATIONS—FRACTIONS 4.NF

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.1 Explain why a fraction $^{a}/_{b}$ is equivalent to a fraction $^{(n \times a)}/_{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $^{1}/_{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified).

4.NF.3 Understand a fraction $^{a}/_{b}$ with $a > 1$ as a sum of fractions $^{1}/_{b}$.

- **a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- **b.** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model[G]. *Examples:* $^{3}/_{8} = {}^{1}/_{8} + {}^{1}/_{8} + {}^{1}/_{8}$; $^{3}/_{8} = {}^{1}/_{8} + {}^{2}/_{8}$; $2\,{}^{1}/_{8} = 1 + 1 + {}^{1}/_{8} = {}^{8}/_{8} + {}^{8}/_{8} + {}^{1}/_{8}$.
- **c.** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- **d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

NUMBER AND OPERATIONS—FRACTIONS, continued

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified). (continued)

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

- **a.** Understand a fraction $^a/_b$ as a multiple of $^1/_b$. *For example, use a visual fraction model to represent* $^5/_4$ *as the product* $5 \times (^1/_4)$*, recording the conclusion by the equation* $^5/_4 = 5 \times (^1/_4)$ *or* $^5/_4 = (^1/_4) + (^1/_4) + (^1/_4) + (^1/_4) + (^1/_4)$.
- **b.** Understand a multiple of $^a/_b$ as a multiple of $^1/_b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times (^2/_5)$ *as* $6 \times (^1/_5)$*, recognizing this product as* $^6/_5$*. (In general,* $n \times (^a/_b) = {}^{(n \times a)}/_b$*.)*
- **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat* $^3/_8$ *of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express* $^3/_{10}$ *as* $^{30}/_{100}$*, and add* $^3/_{10} + {}^4/_{100} = {}^{34}/_{100}$*.* In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* $^{62}/_{100}$*; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

MEASUREMENT AND DATA **4.MD**

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. *For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...*

MEASUREMENT AND DATA, continued **4.MD**

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (continued)

4.MD.2 Solve real-world problems involving money, time, and metric measurement.

- **a.** Using models, add and subtract money and express the answer in decimal notation.
- **b.** Using number line diagrams[G], clocks, or other models, add and subtract intervals of time in hours and minutes.
- **c.** Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.

4.MD.3 Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. *For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.*

Represent and interpret data.

4.MD.4 Display and interpret data in graphs (picture graphs, bar graphs, and line plots[G]) to solve problems using numbers and operations for this grade.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

- **a.** Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $^{1}/_{360}$ of a circle is called a "one-degree angle," and can be used to measure angles.
- **b.** Understand an angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6 Measure angles in whole number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

GEOMETRY **4.G**

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 − 5 = 9 is a subtraction within 20, and 55 − 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

Algorithm. *See also:* computation algorithm.

Associative property of addition. See Table 3, page 20.

Associative property of multiplication. See Table 3, page 20.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1] *See also:* first quartile and third quartile.

Commutative property. See Table 3, page 20.

Complex fraction. A fraction $^{A}/_{B}$ where *A* and/or *B* are fractions (*B* nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Dot plot. *See also:* line plot.

---

[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/ standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fluency. The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

Fluently. *See also:* fluency.

Fraction. A number expressible in the form $^a/_b$ where *a* is a whole number and *b* is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3, page 20.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or −*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 − 6 = 9. *See also:* first quartile, third quartile.

Justify: To provide a convincing argument for the truth of a statement to a particular audience.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

---

[3] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

[4] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $^3/_4$ and $^4/_3$ are multiplicative inverses of one another because $^3/_4 \times {}^4/_3 = {}^4/_3 \times {}^3/_4 = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $^5/_{50}$ = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of equality. See Table 4, page 96.

Properties of inequality. See Table 5, page 97.

Properties of operations. See Table 3, page 20.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a nonzero denominator.

Rational number. A number expressible in the form $^a/_b$ or $-^a/_b$ for some fraction $^a/_b$. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Standard Algorithm. See computational algorithm.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Trapezoid. 1. A trapezoid is a quadrilateral with at least one pair of parallel sides. (inclusive definition) 2. A trapezoid is a quadrilateral with exactly one pair of parallel sides. (exclusive definition) *Districts may choose either definition to use for instruction. Ohio's State Tests' items will be written so that either definition will be acceptable.*

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Verify: To check the truth or correctness of a statement in specific cases.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

[5] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Table 1. Common Addition and Subtraction Situations.

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN[1]**
PUT TOGETHER/ TAKE APART[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	DIFFERENCE UNKNOWN	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
COMPARE[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Key: ☐ Extends across Grades 3-5 to include grade level appropriate numbers

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean "makes" or "results in" but always does mean "is the same number as."

[2] Either addend can be unknown, so there are three variations of these problem situations. *Both Addends Unknown* is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the *Bigger Unknown* or *Smaller Unknown* situations, one version directs the correct operation (the version using more for the *Bigger Unknown* and using less for the *Smaller Unknown*). The other versions are more difficult.

Table 2. Common Multiplication and Division Situations[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 X 6 = ?	3 X ? = 18, AND 18 ÷ 3 = ?	? X 6 = 18, AND 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? **Measurement example.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? **Measurement example.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? **Measurement example.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? **Measurement example.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? **Measurement example.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Key: ☐ Grades 3 (introduce), 4-5 (extend) ▩ Grades Grades 4 (introduce), 5 (extend)

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. Properties of Operations.

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system and the complex number sytem.

ASSOCIATIVE PROPERTY OF ADDITION	$(a+b)+c=a+(b+c)$
COMMUTATIVE PROPERTY OF ADDITION	$a+b=b+a$
ADDITIVE IDENTITY PROPERTY OF 0	$a+0=0+a=a$
EXISTENCE OF ADDITIVE INVERSES	For ever a there exists $-a$ so that $a+(-a)=(-a)+a=0$
ASSOCIATIVE PROPERTY OF MULTIPLICATION	$(a\times b)\times c=a\times(b\times c)$
COMMUTATIVE PROPERTY OF MULTIPLICATION	$a\times b=b\times a$
MULTIPLICATIVE IDENTITY PROPERTY OF 1	$a\times 1=1\times a=a$
EXISTENCE OF MULTIPLICATIVE INVERSES	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION	$a\times(b+c)=a\times b+a\times c$

Acknowledgements

ADVISORY COMMITTEE MEMBERS

Aaron Altose
The Ohio Mathematics Association of Two-Year Colleges

Jeremy Beardmore
Ohio Educational Service Center Association

Jessica Burchett
Ohio Teachers of English to Speakers of Other Languages

Jeanne Cerniglia
Ohio Education Association

Margie Coleman
Cochair

Jason Feldner
Ohio Association for Career and Technical Education

Brad Findell
Ohio Higher Education

Gregory D. Foley
Ohio Mathematics and Science Coalition

Margaret (Peggy) Kasten
Cochair

Courtney Koestler
Ohio Mathematics Education Leadership Council

Scott Mitter
Ohio Math and Science Supervisors

Tabatha Nadolny
Ohio Federation of Teachers

Eydie Schilling
Ohio Association for Supervision and Curriculum Development

Kim Yoak
Ohio Council of Teachers of Mathematics

WORKING GROUP MEMBERS

Darry Andrews
Higher Education, Ohio State University, C

Bridgette Beeler
Teacher, Perrysburg Exempted Local, NW

Melissa Bennett
Teacher, Minford Local, SE

Dawn Bittner
Teacher, Cincinnati Public Schools, SW

Katherine Bunsey
Teacher, Lakewood City, NE

Hoyun Cho
Higher Education, Capital University, C

Viki Cooper
Curriculum Specialist/Coordinator, Pickerington Local, C

Ali Fleming
Teacher, Bexley City, C

Linda Gillum
Teacher, Springboro City Schools, SW

Gary Herman
Curriculum Specialist/Coordinator, Putnam County ESC, NW

William Husen
Higher Education, Ohio State University, C

Kristen Kelly
Curriculum Specialist/Coordinator, Cleveland Metropolitan School District, NE

Endora Kight Neal
Curriculum Specialist/Coordinator, Cleveland Metropolitan School District, NE

Julie Kujawa
Teacher, Oregon City, NW

Sharilyn Leonard
Teacher, Oak Hill Union Local Schools, SE

Michael Lipnos
Curriculum Specialist/Coordinator, Aurora City, NE

Dawn Machacek
Teacher, Toledo Public Schools, NW

Janet McGuire
Teacher, Gallia County Schools, SE

Jill Madonia
Curriculum Specialist/Coordinator, Akron Public Schools, NE

Cindy McKinstry
Teacher, East Palestine City, NE

Cindy Miller
Curriculum Specialist/Coordinator, Maysville Local, SE

Anita O'Mellan
Higher Education, Youngstown State University, NE

Sherryl Proctor
Teacher, Vantage Career Center, NW

Diane Reisdorff
Teacher, Westlake City, NE

Susan Rice
Teacher, Mount Vernon City, C

Tess Rivero
Teacher, Bellbrook-Sugarcreek Schools, SW

Benjamin Shaw
Curriculum Specialist/Coordinator, Mahoning County ESC, NE

Julia Shew
Higher Education, Columbus State Community College, C

Tiffany Sibert
Teacher, Lima Shawnee Local, NW

Jennifer Statzer
Principal, Greenville City, SW

Karma Vince
Teacher, Sylvania City, NW

Jennifer Walls
Teacher, Akron Public Schools, NE

Gaynell Wamer
Teacher, Toledo City, NW

Victoria Warner
Teacher, Greenville City, SW

Mary Webb
Teacher, North College Hill, SW

Barb Weidus
Curriculum Specialist/Coordinator, New Richmond Exempted Village, SW

Sandra Wilder
Teacher, Akron Public Schools, NE

Tong Yu
Teacher,Cincinnati Public Schools, SW