Ohio's Learning Standards | Mathematics

Grade 7

Ohio | Department of Education

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Introduction

PROCESS

To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 4 with the eight Standards for Mathematical Practice.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". When students compare arithmetic and algebraic solutions to the same problem, they identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
In grade 7, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3. Construct viable arguments and critique the reasoning of others.
In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. For example, as students notice when geometric conditions determine a unique triangle, more than one triangle, or no triangle, they have an opportunity to construct viable arguments and critique the reasoning of others. Students should be encouraged to answer questions such as these: "How did you get that?" "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

4. Model with mathematics.
In grade 7, students model problem situations symbolically, graphically, in tables, and contextually. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use experiments or simulations to generate data sets and create probability models. Proportional relationships present opportunities for modeling. For example, for modeling purposes, the number of people who live in an apartment building might be taken as proportional to the number of stories in the building. Students should be encouraged to answer questions such as "What are some ways to represent the quantities?" or "How might it help to create a table, chart, or graph?"

5. Use appropriate tools strategically.
Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms. Teachers might ask, "What approach are you considering?" or "Why was it helpful to use ___?"

Standards for Mathematical Practice, continued

6. Attend to precision.
In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities. Teachers might ask, "What mathematical language, definitions, or properties can you use to explain ___?"

7. Look for and make use of structure.
Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2n = 2\ (3 + n)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality; $c = 6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve realworld problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities. Solving an equation such as $8 = 4\ (n - \frac{1}{2})$ is easier if students can see and make use of structure, temporarily viewing $(n - \frac{1}{2})$ as a single entity.

8. Look for and express regularity in repeated reasoning.
In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$ and construct other examples and models that confirm their generalization. Students should be encouraged to answer questions such as "how would we prove that ___?" Or "how is this situation both similar to and different from other situations using these operations?

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

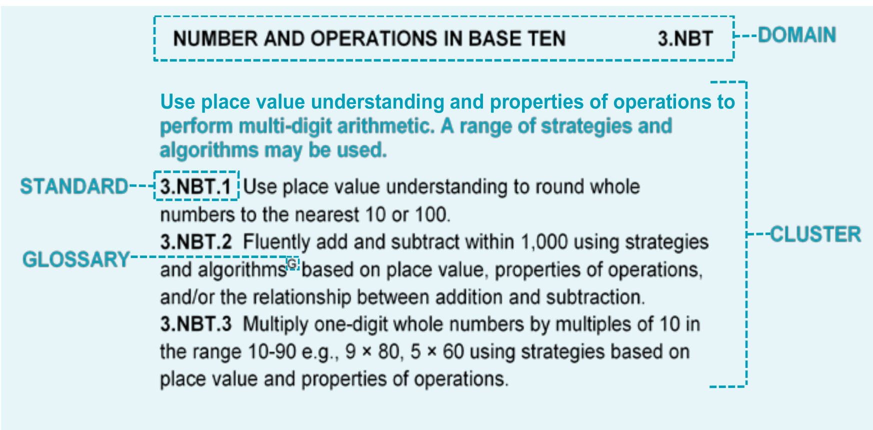
How to Read the Grade Level Standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

[G] shows there is a definition in the glossary for this term.



These standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, does not necessarily mean that teachers must teach topic A before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Therefore, educators, researchers, and mathematicians used their collective experience and professional judgment along with state and international comparisons as a basis to make grade placements for specific topics.

Grade 7

In Grade 7, instructional time should focus on five critical areas:

Critical Area 1: Developing understanding of and applying proportional relationships

Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

Critical Area 2: Developing understanding of operations with rational numbers and working with expressions and linear equations

Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts, e.g., amounts owed or temperatures below zero, students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

Critical Area 3: Solving problems involving scale drawings and informal geometric constructions, angles, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume

Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Critical Area 4: Drawing inferences about populations based on samples

Students build on their previous work with statistical problem solving through the use of the GAISE model framework. They summarize and describe distributions representing one population and informally compare two populations. Students interpret numerical data sets using mean absolute deviation. They begin informal work with sampling to generate data sets: learn about the importance of representative samples for drawing inferences and the impact of bias.

Critical Area 5: Investigating chance

Students build upon previous understandings as they develop concepts of probability. They investigate relevant chance events and develop models to determine and compare probabilities. They analyze the frequencies of the experimental results against their predictions, justifying any discrepancies. Students extend their investigations with compound events representing the possible outcomes in tree diagrams, tables, lists, and ultimately through designing and using simulations.

GRADE 7 OVERVIEW

RATIOS AND PROPORTIONAL RELATIONSHIPS

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

THE NUMBER SYSTEM

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

EXPRESSIONS AND EQUATIONS

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

GEOMETRY

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.

STATISTICS AND PROBABILITY

- Use sampling to draw conclusions about a population.
- Broaden understanding of statistical problem solving.
- Summarize and describe distributions representing one population and draw informal comparisons between two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 7

RATIOS AND PROPORTIONAL RELATIONSHIPS 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks* $^1/_2$ *mile in each* $^1/_4$ *hour, compute the unit rate as the complex fraction*[G] $^{(1/2)}/_{(1/4)}$ *miles per hour, equivalently 2 miles per hour.*

7.RP.2 Recognize and represent proportional relationships between quantities.

- **a.** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **c.** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*
- **d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

THE NUMBER SYSTEM 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- **a.** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
- **b.** Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- **c.** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- **d.** Apply properties of operations as strategies to add and subtract rational numbers.

THE NUMBER SYSTEM, continued

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- **a.** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- **b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- **c.** Apply properties of operations as strategies to multiply and divide rational numbers.
- **d.** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

EXPRESSIONS AND EQUATIONS 7.EE

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p − 0.15p) is equivalent to (1 − 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example, if a woman making $25 an hour gets a 10% raise, she will make an additional* $1/10$ *of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar* 9 ¾ *inches long in the center of a door that is 27* $1/2$ *inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- **a.** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
- **b.** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where *p*, *q*, and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

GEOMETRY 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals.

- **a.** Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.
- **b.** Represent proportional relationships within and between similar figures.

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions.

- **a.** Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- **b.** Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.

7.G.4 Work with circles.

- **a.** Explore and understand the relationships among the circumference, diameter, area, and radius of a circle.
- **b.** Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

STATISTICS AND PROBABILITY 7.SP

Use sampling to draw conclusions about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population.

- **a.** Differentiate between a sample and a population.
- **b.** Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.

Broaden understanding of statistical problem solving.

7.SP.2 Broaden statistical reasoning by using the GAISE model:

- **a.** Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?"* (GAISE Model, step 1)
- **b.** Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)
- **c.** Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3)
- **d.** Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)

STATISTICS AND PROBABILITY, continued

Summarize and describe distributions representing one population and draw informal comparisons between two populations. (continued)

7.SP.3 Describe and analyze distributions.

a. Summarize quantitative data sets in relation to their context by using mean absolute deviation[G] (MAD), interpreting mean as a balance point.

b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot[G] (line plot), the separation between the two distributions of heights is noticeable.*

7.SP.4 [Deleted standard]

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around $^1/_2$ indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7.SP.7 Develop a probability model[G] and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model[G] by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space[G] for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 − 5 = 9 is a subtraction within 20, and 55 − 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

Algorithm. *See also:* computation algorithm.

Associative property of addition. See Table 3, page 19.

Associative property of multiplication. See Table 3, page 19.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1] *See also:* first quartile and third quartile.

Commutative property. See Table 3, page 19.

Complex fraction. A fraction $^{A}/_{B}$ where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Dot plot. *See also:* line plot.

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fluency. The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

Fluently. *See also:* fluency.

Fraction. A number expressible in the form $^a/_b$ where *a* is a whole number and *b* is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3, page 19.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or −*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 − 6 = 9. *See also:* first quartile, third quartile.

Justify: To provide a convincing argument for the truth of a statement to a particular audience.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

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[3] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

[4] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $^3/_4$ and $^4/_3$ are multiplicative inverses of one another because $^3/_4 \times {}^4/_3 = {}^4/_3 \times {}^3/_4 = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $^5/_{50}$ = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of equality. See Table 4, page 96.

Properties of inequality. See Table 5, page 97.

Properties of operations. See Table 3, page 19.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a nonzero denominator.

Rational number. A number expressible in the form $^a/_b$ or $-^a/_b$ for some fraction $^a/_b$. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Standard Algorithm. See computational algorithm.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Trapezoid. 1. A trapezoid is a quadrilateral with at least one pair of parallel sides. (inclusive definition) 2. A trapezoid is a quadrilateral with exactly one pair of parallel sides. (exclusive definition) *Districts may choose either definition to use for instruction. Ohio's State Tests' items will be written so that either definition will be acceptable.*

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Verify: To check the truth or correctness of a statement in specific cases.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

[5] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Table 1. Common Addition and Subtraction Situations.

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN[1]**
PUT TOGETHER/ TAKE APART[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	DIFFERENCE UNKNOWN	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
COMPARE[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean "makes" or "results in" but always does mean "is the same number as."

[2] Either addend can be unknown, so there are three variations of these problem situations. *Both Addends Unknown* is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the *Bigger Unknown* or *Smaller Unknown* situations, one version directs the correct operation (the version using more for the *Bigger Unknown* and using less for the *Smaller Unknown*). The other versions are more difficult.

Table 2. Common Multiplication and Division Situations[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 X 6 = ?	3 X ? = 18, AND 18 ÷ 3 = ?	? X 6 = 18, AND 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? **Measurement example.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? **Measurement example.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? **Measurement example.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? **Measurement example.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? **Measurement example.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. Properties of Operations.

Here *a, b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system and the complex number sytem.

ASSOCIATIVE PROPERTY OF ADDITION	$(a+b)+c=a+(b+c)$
COMMUTATIVE PROPERTY OF ADDITION	$a+b=b+a$
ADDITIVE IDENTITY PROPERTY OF 0	$a+0=0+a=a$
EXISTENCE OF ADDITIVE INVERSES	For ever a there exists $-a$ so that $a+(-a)=(-a)+a=0$
ASSOCIATIVE PROPERTY OF MULTIPLICATION	$(a\times b)\times c=a\times(b\times c)$
COMMUTATIVE PROPERTY OF MULTIPLICATION	$a\times b=b\times a$
MULTIPLICATIVE IDENTITY PROPERTY OF 1	$a\times 1=1\times a=a$
EXISTENCE OF MULTIPLICATIVE INVERSES	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION	$a\times(b+c)=a\times b+a\times c$

Table 4. Properties of Equality.

Here *a, b* and *c* stand for arbitrary numbers in the rational, real or complex number sytems.

REFLEXIVE PROPERTY OF EQUALITY	$a=a$
SYMMETRIC PROPERTY OF EQUALITY	If $a=b$, then $b=a$.
TRANSITIVE PROPERTY OF EQUALITY	If $a=b$ and $b=c$, then $a=c$.
ADDITION PROPERTY OF EQUALITY	If $a=b$, then $a+c=b+c$.
SUBTRACTION PROPERTY OF EQUALITY	If $a=b$, then $a-c=b-c$.
MULTIPLICATION PROPERTY OF EQUALITY	If $a=b$, then $a\times c=b\times c$.
DIVISION PROPERTY OF EQUALITY	If $a=b$ and $c\neq 0$, then $a\div c=b\div c$.
SUBSTITUTION PROPERTY OF EQUALITY	If $a=a$, then b may be substituted for a in any expression containing a.

Table 5. Properties of Inequality.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number sytems.

Exactly one of the following is true: $a < b, a = b, a > b$.
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Acknowledgements

ADVISORY COMMITTEE MEMBERS

Aaron Altose
The Ohio Mathematics Association of Two-Year Colleges

Jeremy Beardmore
Ohio Educational Service Center Association

Jessica Burchett
Ohio Teachers of English to Speakers of Other Languages

Jeanne Cerniglia
Ohio Education Association

Margie Coleman
Cochair

Jason Feldner
Ohio Association for Career and Technical Education

Brad Findell
Ohio Higher Education

Gregory D. Foley
Ohio Mathematics and Science Coalition

Margaret (Peggy) Kasten
Cochair

Courtney Koestler
Ohio Mathematics Education Leadership Council

Scott Mitter
Ohio Math and Science Supervisors

Tabatha Nadolny
Ohio Federation of Teachers

Eydie Schilling
Ohio Association for Supervision and Curriculum Development

Kim Yoak
Ohio Council of Teachers of Mathematics

WORKING GROUP MEMBERS

Darry Andrews
Higher Education, Ohio State University, C

Bridgette Beeler
Teacher, Perrysburg Exempted Local, NW

Melissa Bennett
Teacher, Minford Local, SE

Dawn Bittner
Teacher, Cincinnati Public Schools, SW

Katherine Bunsey
Teacher, Lakewood City, NE

Hoyun Cho
Higher Education, Capital University, C

Viki Cooper
Curriculum Specialist/Coordinator, Pickerington Local, C

Ali Fleming
Teacher, Bexley City, C

Linda Gillum
Teacher, Springboro City Schools, SW

Gary Herman
Curriculum Specialist/Coordinator, Putnam County ESC, NW

William Husen
Higher Education, Ohio State University, C

Kristen Kelly
Curriculum Specialist/Coordinator, Cleveland Metropolitan School District, NE

Endora Kight Neal
Curriculum Specialist/Coordinator, Cleveland Metropolitan School District, NE

Julie Kujawa
Teacher, Oregon City, NW

Sharilyn Leonard
Teacher, Oak Hill Union Local Schools, SE

Michael Lipnos
Curriculum Specialist/Coordinator, Aurora City, NE

Dawn Machacek
Teacher, Toledo Public Schools, NW

Janet McGuire
Teacher, Gallia County Schools, SE

Jill Madonia
Curriculum Specialist/Coordinator, Akron Public Schools, NE

Cindy McKinstry
Teacher, East Palestine City, NE

Cindy Miller
Curriculum Specialist/Coordinator, Maysville Local, SE

Anita O'Mellan
Higher Education, Youngstown State University, NE

Sherryl Proctor
Teacher, Vantage Career Center, NW

Diane Reisdorff
Teacher, Westlake City, NE

Susan Rice
Teacher, Mount Vernon City, C

Tess Rivero
Teacher, Bellbrook-Sugarcreek Schools, SW

Benjamin Shaw
Curriculum Specialist/Coordinator, Mahoning County ESC, NE

Julia Shew
Higher Education, Columbus State Community College, C

Tiffany Sibert
Teacher, Lima Shawnee Local, NW

Jennifer Statzer
Principal, Greenville City, SW

Karma Vince
Teacher, Sylvania City, NW

Jennifer Walls
Teacher, Akron Public Schools, NE

Gaynell Wamer
Teacher, Toledo City, NW

Victoria Warner
Teacher, Greenville City, SW

Mary Webb
Teacher, North College Hill, SW

Barb Weidus
Curriculum Specialist/Coordinator, New Richmond Exempted Village, SW

Sandra Wilder
Teacher, Akron Public Schools, NE

Tong Yu
Teacher,Cincinnati Public Schools, SW