# Mathematics 2 Critical Areas of Focus

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. These descriptions have been used to develop critical areas for each of the courses in both the Traditional and Integrated pathways. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

## DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.
- What are the concepts?
- What are the procedures and skills?
- What are the key mathematical practices?
- What are the relationships students are to make?
- What further information is needed? For example, what does prove mean?
- What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards <u>within</u> each course's Critical Areas of Focus.
- What are the relationships among the conceptual categories, domains, clusters and standards?
- Why is each relationship important?
- What are the differences?
- How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections <u>across</u> the Critical Areas of Focus within a course. This information will help create a sequence of units for the course. For example, the learning in Critical Area of Focus #2 is needed prior to Critical Area of Focus #5.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.
- What understandings does this learning build upon?
- What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.
- What is taught now but not in Ohio's Learning Standards?
- What content is essentially the same? Identify the differences.
- What will be new content for this grade?

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# Mathematics 2 Critical Areas of Focus

## CRITICAL AREA OF FOCUS #1

### Applications of Probability

Building on probability concepts that began in grade 7, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions related to real-world situations.

Statistics and Probability – Conditional Probability and the Rules of Probability

**Understand independence and conditional probability and use them to interpret data.**

**S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). ★

**S.CP.2** Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★

**S.CP.3** Understand the conditional probability of A given B as $^{P(A \text{ and } B)}/_{P(B)}$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ★

**S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ★

**S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. ★

**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

**S.CP.6** Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. ★

**S.CP.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model. ★

**(+) S.CP.8** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model. ★ (+) S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems. ★

**(+) S.CP.9** Use permutations and combinations to compute probabilities of compound events and solve problems. ★

# Mathematics 2 Critical Areas of Focus

## CRITICAL AREA OF FOCUS #2

### Expressions and Equations

Students apply the properties of operations with real numbers, the relationships between the operations, along with the properties of exponents to operations with polynomials. Also, students focus on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

### Algebra – Arithmetic with Polynomials and Rational Expressions

**Perform arithmetic operations on polynomials.**

**A.APR.1** Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **a.** Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)

### Algebra – Reasoning with Equations and Inequalities

**Solve equations and inequalities in one variable.**

**A.REI.4** Solve quadratic equations in one variable.

- **a.** Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions.
- **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.
- (+) **c.** Derive the quadratic formula using the method of completing the square.

**Solve systems of equations.**

**A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

**A.REI.11** Explain why the $x$-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

### Algebra – Seeing Structure in Expressions

**Interpret the structure of expressions.**

**A.SSE.1**. Interpret expressions that represent a quantity in terms of its context. ★

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

# Mathematics 2 Critical Areas of Focus

## CRITICAL AREA OF FOCUS #2, CONTINUED

### Expressions and Equations

#### Algebra – Seeing Structure in Expressions

**Write expressions in equivalent forms to solve problems.**

**A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, $8^t$ can be written as $2^{3t}$.*

#### Algebra – Creating Equations

**Create equations that describe numbers or relationships.**

**A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★

- **b.** Focus on applying simple quadratic expressions. (A1, M2)

**A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

- **b.** Focus on applying simple quadratic expressions. (A1, M2)

**A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★

- **c.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r.* (M2)

# Mathematics 2 Critical Areas of Focus

## Critical Area of Focus #3

### Quadratic Functions and Modeling

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to gather information about quadratic and exponential functions by interpreting various forms of expressions representing the functions. For example, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. Students relate their prior experience with transformations to that of building new functions from existing ones and recognize the effect of the transformations on the graphs. Formal work with complex numbers and more specialized functions—absolute value, step, and piecewise-defined, will occur in Algebra 2/Mathematics 3.

### Functions – Interpreting Functions

**Interpret functions that arise in applications in terms of the context.**

**F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ (A2, M3)

- **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)

**F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★

- **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)

**Analyze functions using different representations.**

**F.IF.7** Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. ★

- **b.** Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2)

**F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)
  - **i.** Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2)
- **b.** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay.* (A2, M3)
  - **i.** Focus on exponential functions evaluated at integer inputs. (A1, M2)

# Mathematics 2 Critical Areas of Focus

## CRITICAL AREA OF FOCUS #3 CONTINUED

### Quadratic Functions and Modeling

#### Functions – Interpreting Functions

**Analyze functions using different representations.**

**F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

- **b.** Focus on linear, quadratic, and exponential functions. (A1, M2)

#### Functions – Building Functions

**Build a function that models a relationship between two quantities.**

**F.BF.1** Write a function that describes a relationship between two quantities. ★

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - **ii.** Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)

**Build new functions from existing functions.**

**F.BF.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)

- **a.** Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)

#### Functions – Linear, Quadratic, and Exponential Models

**Construct and compare linear, quadratic, and exponential models, and solve problems.**

**F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2)

# Mathematics 2 Critical Areas of Focus

## CRITICAL AREA OF FOCUS #4

### Similarity, Right Triangle Trigonometry and Proof

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use it as a familiar foundation for the development of informal and formal proofs, problem solving and applications to similarity in right triangles. This will assist in the further development of right triangle trigonometry, with particular attention to special right triangles, right triangles with one side and one acute angle given and the Pythagorean Theorem. Building on their work associated with finding distances, students use a rectangular coordinate system to verify geometric relationships.

.Geometry – Similarity, Right Triangles, and Trigonometry

**Understand similarity in terms of similarity transformations.**

**G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor:

- **a.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
- **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

**G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**Prove and apply theorems both formally and informally involving similarity using a variety of methods.**

**G.SRT.4** Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

**G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

**Define trigonometric ratios, and solve problems involving right triangles.**

**G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

**G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.

**G.SRT.8** Solve problems involving right triangles. ★

- **a.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)

Geometry – Expressing Geometric Properties with Equations

**Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.**

**G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

# Mathematics 2 Critical Area of Focus

## CRITICAL AREA OF FOCUS #5

### Circles With and Without Coordinates

As an application of similarity, students prove that all circles are similar and derive the formula for the area of a circular sector. Students use the distance formula to verify geometric relationships and write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane and apply techniques for solving quadratic equations which relates back to work done with systems of equations.

Geometry – Expressing Geometric Properties with Equations

**Translate between the geometric description and the equation for a conic section.**

**G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

**Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.**

**G.GPE.4** Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2)

Geometry – Circles

**Understand and apply theorems about circles.**

**G.C.1** Prove that all circles are similar using transformational arguments.

**Find arc lengths and areas of sectors of circles.**

**G.C.5** Find arc lengths and areas of sectors of circles.

- **a.** Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems.
- **b.** Derive the formula for the area of a sector, and use it to solve problems.

# Mathematics 2 Critical Areas of Focus

## Critical Area of Focus #6

### Extending to Three Dimensions

Students develop informal arguments justifying common formulas for circumference, area, and volume of two-dimensional and three dimensional shapes. Students develop the understanding of how changes in dimensions result in similar and non-similar shapes and how scaling changes lengths, areas and volumes. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. Students apply previously gained knowledge about 2 dimensional and 3 dimensional geometric shapes and their properties to real-life situations where geometry is used in modeling context.

### Geometry – Geometric Measurement and Dimension

**Explain volume formulas and use them to solve problems.**

**G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

**G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

**Visualize the relation between two-dimensional and three-dimensional objects.**

**G.GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

**Understand the relationships between lengths, areas, and volumes.**

**G.GMD.5** Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.

**G.GMD.6** When figures are similar, understand and apply the fact that when a figure is scaled by a factor of $k$, the effect on lengths, areas, and volumes is that they are multiplied by $k$, $k^2$, and $k^3$, respectively.

### Geometry – Modeling with Geometry

**Apply geometric concepts in modeling situations.**

**G.MG.1** Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. ★

**G.MG.2** Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot. ★

**G.MG.3** Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. ★