Statistics and Probability Critical Areas of Focus

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.

- What are the concepts?
- What are the procedures and skills?
- What are the key mathematical practices?
- What are the relationships students are to make?
- What further information is needed? For example, what does prove mean?
- What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards *within* each course's Critical Areas of Focus.

- What are the relationships among the conceptual categories, domains, clusters and standards?
- Why is each relationship important?
- What are the differences?
- How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections *across* the Critical Areas of Focus within a course. This information will help create a sequence of units for the course.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.

- What understandings does this learning build upon?
- What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.

- What is taught now but not in Ohio's Learning Standards?
- What content is essentially the same? Identify the differences.
- What will be new content for this grade?

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CRITICAL AREA OF FOCUS #1

Communication and Analysis

Within this critical area students develop conclusions based on quantitative information and critical thinking. They recognize, make, and evaluate underlying assumptions in estimation, modeling, and data analysis. Students then organize and present thoughts and processes using mathematical and statistical evidence. They communicate clear and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation. Students demonstrate numerical reasoning orally and in writing coherent statements and paragraphs.

In the context of real-world applications, students make and investigate mathematical conjectures. They are able to defend their conjectures and respectfully question conjectures made by their classmates. This leads to the development of mathematical arguments and informal proofs, which are ways of expressing particular kinds of reasoning and justification. Explanations (oral and written) include mathematical arguments and rationales, not just procedural descriptions or summaries. Listening to others' explanations gives students opportunities to develop their own understandings. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language. Additionally, conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. **This critical area of focus cross cuts all the other critical areas of focus.**

This critical area cross cuts all the rest of the standards, so all standards in this document also fall under this crucial area!!!

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CRITICAL AREA OF FOCUS #2

Exploring One-Variable Data

Beginning in middle school, students developed an understanding of statistical problem solving at level B of the GAISE Model, displaying numerical data, using measures of center and variability, and applying those concepts in the context of real-world applications. In this course, students move to level C of the GAISE Model through the practices of gathering, displaying, summarizing, examining, and interpreting data to discover patterns and deviations from patterns. Students develop more formal methods of evaluating one-variable data and comparing two or more distributions. Students will use technology to generate and compare plots, and perform repetitive calculations.

Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation[G], interquartile range[G], and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

Note: Standards S.ID.10-13 are included in this course to be considered for next standards revision.

Reason quantitatively about numerical data including covariation to solve real-world problems.

S.ID.*10* *Understand discrete and continuous distributions and distinguish between them. Compare to uniform continuous, uniform discrete, binomial, and normal distributions.★*

a. *Calculate probabilities for various distributions.*

b. *Use one kind of distribution to approximate the other.*

S.ID.*11* *Visually compare a data distribution to the standard normal distribution. Recognize that normal distributions can be used to represent some population distributions. Understand that a normal distribution is determined by its mean and standard deviation.*

S.ID.*12* *Calculate and use probability from a normal distribution. ★*

a. *Determine proportions and percentiles from a normal distribution. Understand that the 50th percentile is a measure-of-center, the median.*

b. *Compare measures of relative position in data sets: z-scores and percentiles.*

S.ID.*13* *Use the standard normal distribution to approximate binomial distributions.★*

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #2, CONTINUED

Exploring One-Variable Data

Algebra – Arithmetic with Polynomials and Rational Expressions

Use polynomial identities to solve problems.

(+) **A.APR.5** *Use models to solve and communicate about contextual financial questions such as budgets, credit card debt, installment savings, amortization schedules, mortgage and other loan scenarios.*★

Algebra – Seeing Structure in Expressions

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1)

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #3

Exploring Two-Variable Data

Beginning in middle school, students create scatterplots to recognize linear trends in data. In this course they will explore linear and non-linear data and use regression techniques to describe and interpret the relationships between quantities. Students use graphical representations and knowledge of the context to make judgements about the appropriateness of regression models. They will use algebra skills and technology to transform and plot these functions to improve the fit. Students will use r^2 and residuals to analyze the goodness of fit.

Students analyzing empirical situations using regression to understand them better is at the heart of this unit. The modeling cycle should be referenced when functions and geometric transformations are applied. For example, students may identify a non-linear function to model a situation, adjust parameters to improve the model, and then compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit.

Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3)

b. Informally assess the fit of a function by discussing residuals. (A2, M3)

c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★

S.ID.9 Distinguish between correlation and causation.★

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #3, CONTINUED

Modeling with Algebra and Functions

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3)

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

(+) F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

Functions – Trigonometric Functions

Model periodic phenomena with trigonometric functions.

(+) F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #4

Collecting Data: Sampling Techniques and Experimental Design

Building on students' prior exposure to some of these concepts, student understanding will move to GAISE level C. Students will understand what constitutes good practice in designing a sample survey, an experiment, and an observational study. This includes the role of random selection in sample surveys and random assignment in experiments as they relate to the appropriateness of conclusions. For example, students will demonstrate an understanding of the effect of sample size on the variability of estimates and the implications of random assignment for cause-and-effect interpretations.

Further, students will understand the issues of bias and confounding variables in observational studies and their implications for interpretation. Students will also understand that in some circumstances the data collected or considered may not generalize to the desired population.

Statistics and Probability – Making Inferences and Justifying Conclusions

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★

Note: This critical area cross cuts other critical areas such as Critical Area #5: Probability, Randomness, and Distributions and Critical Area #6: Inference for One-Sample Categorical and Quantitative Data.

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #5

Probability, Randomness, and Distributions

In this course, students will build on prior learning to deepen fluency with probability, including work with compound probabilities. Additionally, students will understand the impact of probability on statistical reasoning. For example, probability will be applied by answering the question: Could the outcomes observed be due solely to chance? Students will use technology to perform simulations in an effort to more fully understand random processes.

Students will recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. Students will explore expected value by defining random variables and developing a probability distribution model. Students will use technology to calculate expected value (mean) and standard deviation and use the results to evaluate outcomes of decisions.

Statistics and Probability –Conditional Probability and the Rules of Probability

Understand independence and conditional probability, and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★

S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

S.CP.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #5, CONTINUED

Probability, Randomness, and Distributions

Statistics and Probability –Conditional Probability and the Rules of Probability

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★

S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.★

S.CP.8 Apply the general Multiplication Rule in a uniform probability model[G], P(A and B) = P(A)·P(B|A) = P(B)·P(A|B), and interpret the answer in terms of the model.★

S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.★

Statistics and Probability – Using Probability to Make Decisions

Calculate expected values, and use them to solve problems.

S.MD.1 Define a random variable[G] for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution[G] using the same graphical displays as for data distributions.★

S.MD.2 Calculate the expected value[G] of a random variable; interpret it as the mean of the probability distribution.★

S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★

S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*★

Use probability to evaluate outcomes of decisions.

S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★

a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

S.MD.6 Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #5, CONTINUED

Probability, Randomness, and Distributions

Statistics and Probability – Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions.

S.MD.7 Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

Statistics and Probability Critical Areas of Focus

CRITICAL AREA OF FOCUS #6

Inference for One-Sample Categorical and Quantitative Data

A foundational understanding of statistical inference is critical for understanding the practice of statistics. In this course students develop that foundation by creating margins of error and comparing treatments through simulation. These experiences of inference will include the discussion the reasonableness of generalizations. In particular they will estimate the mean or proportion of a population, given a sample from that population.

Statistics and Probability – Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.★

S.IC.6 Evaluate reports based on data.★

Number and Quantity – Quantities

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★