Mathematics DOK Levels

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics, a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels, depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond an habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of objects or phenomena and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret," could be classified at different levels depending on the object of the action. For example, interpreting information from a simple graph, or reading information from the graph, also are at Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited only to number skills, but may involve visualization skills and probability skills. Other Level 2 activities include noticing or describing non-trivial patterns, explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and deciding which concepts to apply in order to solve a complex problem.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking, most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if

a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing *and* conducting experiments and projects; developing and proving conjectures, making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

Examples Applied to Objectives and Assessment Items

i. Sample Mathematics Objectives

Use the mathematics DOK levels on the previous pages to determine the DOK levels for the following five sample objectives. When you are finished, turn the page to see whether you agree with the way *we* coded these objectives! Then try using the DOK levels on the 13 sample mathematics items in Part ii.

Objective 1.   Read, write, and compare decimals in scientific notation.

Objective 2.   (Grade 8) Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of results.

Objective 3.   (Grade 8, from the NEAP Mathematics Framework): Design a statistical experiment to study a problem and communicate the outcomes.

Objective 4.   Compute with numbers (that is, add, subtract, multiply, divide).

Objective 5.   Construct two-dimensional patterns for three-dimensional models, such as cylinders and cones.

DOK Levels of the Sample Mathematics Objectives

Objective 1.   This objective is an example of <u>Level 1</u>. The highest demand for students to successfully meet this expectation requires them to use recall and use a routine method to convert a decimal to scientific notation.

Objective 2.   This objective is an example of <u>Level 3</u>. The expectation expressed in this objective is that students will not only solve a two-step linear equation, but will also interpret the solution and verify the results. This will require students to do some reasoning in order to interpret the solution and could be fairly complex, depending on the context. If students were only required to solve linear equations and verify solutions, then the expectation would be <u>Level 2</u>.

Objective 3.   To plan a statistical experiment, a student must define the problem and develop a procedure for solving it. This requires that the student identify the correct statistical model, apply the model to data, and communicate the outcome of the selected model. The student must interpret findings and make reasonable and rationed inferences from obtained data. This represents complex, multistep reasoning and reflects a <u>Level 4</u> task.

Objective 4.   This objective requires students to conduct basic calculations. This is <u>Level 1</u> because it involves routine processing and involves a one-step process.

Objective 5.   This objective is an example of <u>Level 2</u>. Although recognizing and drawing a two-dimensional pattern, or a regular cylinder, is expected to be routine (Level 1), building a three-dimensional model would not be as routine. It would require at least two steps: first, recognizing the shape and, second, drawing a two-dimensional object to reflect the shape in three dimensions.

ii. Sample Mathematics Assessment Items

Now try coding some sample assessment items using the Mathematics DOK Levels. Sample items, for three different grade levels. After you are finished coding these, read our "Answers" on the following page.

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The following five items are from <u>Grade 4</u> mathematics assessments:

1)

The class went on a field trip. The students left school at 9:00 a.m. They returned to school at 1:30 p.m. How long were they gone?

A 8 hr 30 min
B 8 hr
C 4 hr 30 min
D 4 hr

2)

Sam, Terry, and Kim each own some baseball cards that Ted is willing to trade them for. Here is what they are worth:

Sam's cards:			Tetsuo's cards:			Kim's cards:	
Bret Boone	$0.80		Sammy Sosa	$1.30		Randy Johnson	$0.70
Andres Galarraga	$0.40		Greg Maddux	$1.00		Barry Bonds	$2.30
Mark McGuire	$1.50						

Ted will trade his Alex Rodriguez card for $6.75 worth of cards. What is the best trade that Sam, Tetsuo, and Kim can make for Ted's Alex Rodriguez card?

What trade could Sam, Tetsuo, and Kim offer Ted that would be the most fair between Sam, Tetsuo, and Kim?

Explain your thinking and show all your work.

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3)

Bill lives on the side of the street with even-numbered addresses. Which addresses below would be found on Bill's side of the street?

A 1020, 1022, 1024
B 2021, 2023, 2025
**C** 3168, 3169, 3170
D 4167, 4168, 4170

4)

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.

Laura wanted to enter the number 8375 into her calculator. By mistake, she entered the number 8275. Without clearing the calculator, how could she correct her mistake?

Without clearing the calculator, how could she correct her mistake another way?

5)
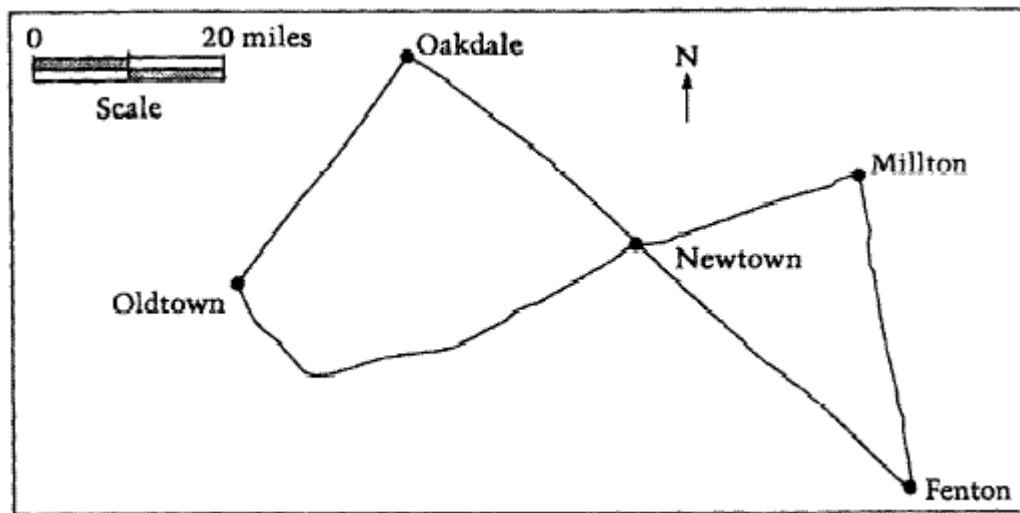
Based on the map above, about how many miles is the shortest route from Oakdale to Fenton?

A) 100
B) 70
C) 40
D) 20

The following five items are from <u>Grade 8</u> assessments:

6)

From any vertex of a 4-sided polygon, 1 diagonal can be drawn.
From any vertex of a 5-sided polygon, 2 diagonals can be drawn.
From any vertex of a 6-sided polygon, 3 diagonals can be drawn.
From any vertex of a 7-sided polygon, 4 diagonals can be drawn.

How many diagonals can be drawn from any vertex of a 20-sided polygon?

7)

A triangle has 0 diagonals, a quadrilateral has 2 diagonals, a pentagon has 5 diagonals, and a hexagon has 9 diagonals. If the pattern continues, how many diagonals will an octagon have?

Sides	3	4	5	6
Diagonals	0	2	5	9

A 11
B 14
C 18
D 20

8)

In which set are the numbers equivalent?

A $\frac{1}{3}, \frac{3}{27}, 33\%$

B 0.090, 90%, 0.90

C 88%, $\frac{88}{100}, \frac{22}{25}$

D 0.66%, $\frac{2}{3}, 66.7\%$

9)

The school newspaper conducted a survey about which ingredient was most preferred as a pizza topping. This graph appeared in the newspaper article.

Favorite Pizza Toppings

What information would best help you determine the number of people surveyed who preferred sausage?

A number of people surveyed and type of survey used
B type of survey used and ages of people surveyed
C percent values shown on chart and number of people surveyed
D ages of people surveyed and percent values shown on chart

10)

Look at the drawing. The numbers alongside each column and row are the total of the values of the symbols within each column and row. What should replace the question mark?

A. 23
B. 25
C. 28
D. 30
E. 32

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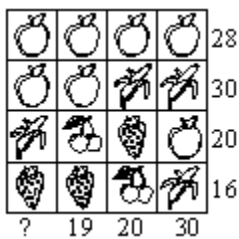
January 2006

The following five items are from <u>Grade 11</u> assessments:

11) Which of the following is NOT true for any value of x?

A $x < x^2 < x^3$
B $x^3 < x < x^2$
C $x^2 < x < x^3$
D $x < x^3 < x^2$
E $x^3 < x^2 < x$

12)
     Players A and B are playing a game. On a table between them is a stack of n pennies. First, Player A removes either one or two pennies from the stack. Then Player B removes either one or two pennies from the stack. They alternate in this way until no pennies remain. The loser is the player who removes the last penny from the stack.
     If they start the game with 5 pennies in the stack, how many pennies should Player A take from the stack on her first turn? Why?
     If the game starts with 7 pennies in the stack, would you rather be Player A or B? Why?
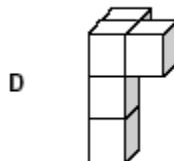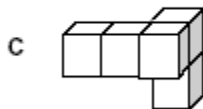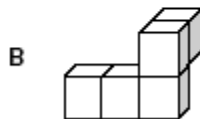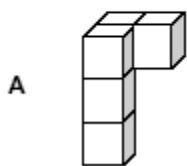   For what values of n, if any, is it best to be player A?
     For what values of n, if any, is it best to be player B?
     Explain and justify your answers.

13)

Which pentacube is **not** congruent to the others?

A
B
C
D

14)

One plan for a state income tax requires those persons with income of $10,000 or less to pay no tax and those persons with income greater than $10,000 to pay a tax of 6 percent only on the part of their income that exceeds $10,000.

A person's <u>effective</u> tax rate is defined as the percent of total income that is paid in tax.

Based on this definition, could any person's effective tax rate be 5 percent? Could it be 6 percent? Explain your answer. Include examples if necessary to justify your conclusions.

15)

$S = a/b + c/d$

If $0 < a < b < c < d$ in the equation above, then the greatest increase in S would result from adding 1 to the value of which variable?

   (A) a
   (B) b
   (C) c
   (D) d
   (E) *There is not enough information to know for certain.*

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<u>DOK Levels for the Mathematics Sample Assessment Items</u>

Grade 4 Items:

1)   <u>Level 2</u>. The choices offered indicate that this item is intended to identify students who would simply subtract 9 minus 1 to get an 8. More than one step is required here. The students must first recognize the difference between a.m. and p.m. and make some decisions about how to make this into a subtraction problem, then do the subtraction.

2)   <u>Level 4</u>. This is a complex open-ended problem requiring students "to make several connections and apply one approach among many." It requires the students to plan and organize, and to weigh solutions based on different kinds of criteria. Students should be allowed at least 20 minutes for this problem, which is an extended period of time for a test item.

3)   <u>Level 1</u>. Students only need to be able to recognize even numbers.

4)   <u>Level 3</u>. "An activity that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3." Since there are multiple possible approaches to this problem, the student must make strategic decisions about how to proceed, which is more cognitively complex than simply applying a set procedure or skill.

5)   <u>Level 1</u>. This measurement item requires no analysis of the map itself, since the route in question is a straight line. If the line was not straight, then this item would require estimation and perhaps even calculation, making it Level 2 or 3.

Grade 8 Items:

6)   <u>Level 1</u>. The first thing to note is that this is not really a geometry item. Rather, it simply requires students to notice an easy, routine pattern. DOK levels are difficult to assign for many pattern-recognition problems, because they depend on how routine the pattern is. This particular pattern is immediately recognizable and requires no processing, but a more complex pattern could make this item Level 2 or even Level 3.

7)   <u>Level 2</u>. This item is included in order to contrast it with the previous one. Pattern recognition is required, but the non-routine nature of this pattern brings this up to a higher DOK level. Some analysis and generalization is required in order to understand and extend this pattern.

8)   <u>Level 2</u>. There are a number of different concepts and procedures that can be used for this problem, rather than just one obvious, simple one. Students must not only be able to *identify* different representations of rational numbers (Level 1), but also to *manipulate*

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and *compare* these representations (Level 2). This means that numerous interdependent and non-trivial steps are involved here. However, this does not require any conjecturing, planning, abstracting, or explaining, so it is not Level 3.

This item demonstrates the importance of knowing whether calculators are allowed on the examinations or not. If a calculator were allowed on this examination, this would clearly be Level 1, instead of Level 2.

9)   Level 2. This is an example of how a problem that is multiple choice can reduce its DOK level. If the multiple choices were removed here and the problem were left open-ended, it would be Level 3. But here the student need only weigh the options against one another, easily discarding "type of survey used" and "ages of people surveyed" as bogeys. So they can easily determine that C is immediately better than A or D, without even having to think analytically or creatively about why percent values shown or number of people surveyed would be important information to know.

10)   Level 3. This item can be approached through a number of viable strategies: pattern recognition, guess-and-check, algebra, etc. This freedom means that the student must make choices and assumptions. Furthermore, no matter what strategy she employs, she must keep track of a complex logical chain. The multiple choices provided do not make this task any less complex.

Grade 11 Items:

11)   Level 3. This is another example of an item that is at Level 3 without being open-ended. This item requires generalization, reasoning, and hypothesis testing, involving some creativity in choosing examples that test the hypotheses.

12)   Level 4. This problem requires students to form game strategies, create data, notice number patterns, and justify how and why those patterns arise. It involves inductive, deductive, and strategic reasoning over an extended period of time, perhaps 30 minutes. This may even be a problem best done in pairs or groups within a testing environment.

13)   Level 2. This item is not Level 1 because it is not routine, nor does it focus on a memorized definition or procedure. In fact, it involves numerous steps, because it requires students to compare several different pairs of shapes before arriving at the correct answer. For these reasons, many spatial reasoning items are Level 2.

Note that this may be a coded as a source-of-challenge item, because choice C seems to be drawn in a misleading way.

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14)   Level 3. This item gives the student a new definition and asks her to reason using it. In order to ascertain whether the student really understands the asymptotic behavior that makes a 6% effective rate impossible, this item *must* be open-ended. This is why most Level 3 items are open-response items, because the complexity of thinking they require the students to display could not be displayed using multiple choices.

15)   Level 3. If a multiple-choice item is Level 3, often it is because the multiple choices do not constrain or guide the possible solutions. The choices here allow for *all* possible responses to this item, including the response that the problem cannot be solved. This gives such an item the character of an open-ended item, even though it is not one.