High School Mathematics Transition Course

Theme 2: Functions Part 1 **Context:** 6	**Number of Class Periods:** 4 (45 minutes)
Lesson Title: Follow the Bouncing Ball	
Summary: Using experimental design and mathematical modeling, students will collect, organize, and analyze data to construct a model to describe the relationship between initial drop height and height of first bounce. Productive struggle is inherently necessary for students to create meaningful connections and conceptual understanding.	**Standards of Mathematical Practices:** ● MP.1 ● MP.2 ● MP.3 ● MP.4 ● MP.5 ● MP.6

Goals and Objectives:

Students will:

- Determine a functional relationship between rebound height and initial drop height of a ball;
- Determine control, independent, and dependent variable;
- Design and perform experiments;
    - Measure and collect data;
    - Organize and represent data in table and graph; and
    - Determine the pattern and establish and test a regression model.

Teachers will:

- Implement the noticings and wonderings routine.
- Implement "Convince Me Routine" from Theme 0: Day 11.
- Focus on questioning, answering questions with questions, using advancing questions.

Concepts from Previous Mathematical Experiences to be Applied:

- Regression Modeling
- Construct Scatter Plots and Graphs
- Dimensional Analysis
- Measurement
- Multiple Representations of Linear Functions

Instructional Procedures:

Part One:

- Establish groups of 3-4 students.
- Provide student handout and materials.
- Show the video of a commercial from the 1960's or bounce a ball:
    - NOTE: It is recommended to use the 1960s commercial to show students first.
- Ask students what they notice and what they wonder.
- Show the students the Convince Me diagram and review the Convince Me routine. Review with students that they should always be asking "why?" Remind them to act like defenders, providing rationale for their decisions, and prosecutors, challenging assumptions and asking for more information. Although, remind them that all discourse should be centered around ideas and questions should be asked respectfully.
- Allow students to work in groups to begin the activity.
- Expect this activity to take 2-4 days. 1-2 days is spent orienting students to the challenge and discussions, collecting, organizing, and representing data. 1-2 day to evaluate, reassess data, recollect data, and create and test functions.
- **Teacher Note:**
  The experiment that students should design is to drop the ball from varying heights and measure the first rebound. For example, drop it from 2 m, record the rebound; drop it from 3 m, record the rebound; repeat for a variety of heights. The data table they create will compare the different heights with the first rebound heights. Specific directions are not in the student handout because students

need to create this experiment on their own in groups. Allow students to struggle by designing the "wrong" experiment and even carrying out the wrong experiment. There is time built in this lesson for struggle. Use the Convince Me routine and questioning to coach the students to the correct experiment.

- Consider providing a variety of balls of varying weights, composition, and diameter. Ask students what they notice and what they wonder.
- A few guiding questions for facilitation checkpoints are:
    - Do the groups have a variety of variables identified? Have the students identified the types of variables? Ask for reasoning behind any proposed functional relationship.
    - How did you decide on the number of data points? How did you decide on which points to test? How did you decide on your scale of measurement? How did you decide how to measure the height of the bounce? Emphasize that all individuals will be held accountable to explain and utilize the group's work.
    - Verify that the group has put forth a good faith effort to answer all questions to this point before handing out the individual assignment. Ask the groups justify their answers using both the algebraic and graphical forms of their functions.
- Once the groups have completed the group task, bring them back for a whole class discussion. A few guiding questions for the classroom discussion are:
    - What are some possible sources of error?
    - How precise were your measurements?
    - How do the different models compare?
    - What does the slope mean? How does this help us to compare models? Could we have a slope of one or more? Why or Why Not?
    - What is a reasonable domain and range for the functional relationship you determined? (What are your smallest or greatest drop heights) Explain your reasoning.
    - Follow up questions as necessary.
- Provide each student with the Individual Take Home assignment "Drop This".
- **NOTE:** either before or after handing out the Take Home Assignment you must assign individual practical heights for questions 2 and 4.

Follow-up:

- The following day, have students get into their groups and discuss their answers to the Take Home assignment, problems 2 and 4.
- Collect the assignment and perform the test drops to test the students' models.

Part Two:

- Give students the Student Handout Part 2.
- Students will use their model from the 1st day to predict and fill in the Prediction line of the table.
- When gathering the "Actual" data, students are encouraged to run the experiment 2-3 times and take the average for each rebound height. These data points can be used at the "Actual" measurements. (This is consistent with what they are taught to do for data collection in science class.)
- Students will need access to a graphing calculator.
- As students work through this handout, they will be encouraged to fit a linear regression to an exponential set of data. This is to help them discern when to use linear regressions and when to use nonlinear regressions.
- In #8, the NEXT-NOW language is used. This is a more accessible way of teaching how to write recursive formulas. See the example shown on the student handout. In general, NEXT-NOW formulas follow the pattern of:
    - NEXT = NOW *math operation* (common difference/ratio)

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4 (45 minutes)

Lesson Title: Follow the Bouncing Ball

Teacher Focus:
Before the Lesson
- Review the Convince Me and Noticings and Wonderings routine.
- As you review the lesson list 3-8 questions that you think students might have.
- Write out your questions that you would use in response to each student question that would assess student thinking.
- Write out your questions that you would use in response to each student question that would advance student thinking.
- Write out your questions that you would use in response to each student question that would support the student in making mathematical connections.
- Reflect on the questions that you wrote. Do they help you achieve your learning goals for this context. If not, revise them as necessary.

During the Lesson
- Focus on asking questions that advance student thinking and support students in making mathematical connections.
- Don't forget about your four main questions that you learned on Day 5.
  - "Why do you say ___________________?"
  - "What do you mean by_______________?"
  - "Tell me more about_________________."
  - "How do you see ___________________?"

Differentiation Strategies:
- Some students need to be directed to use drop height as an independent variable.
- Provide students with an empty table or chart. Consider labeling the table or not.
- Provide students with a blank graph.
- Provide students with additional guidance as necessary.
- Have individual groups analyze a variety of balls.

Assessment:
- Evaluate the performance task of this activity.

Extension Suggestions:
- Predict how many bounces would be required for the height to reach zero.
- Does a high correlation mean that this is a good model?
- Are all golf balls going to have the same bounce coefficient? What would cause the difference?
- Discuss to see if students can find an exponential function for repeated rebound heights when given an initial drop height.

Materials and Resources:
- Drop This Individual Take Home Assignment
- Follow the Bouncing Ball Student Handout Part 1
- Follow the Bouncing Ball Student Handout Part 2
- Balls
- Meter/Yard Sticks/Tape Measure
- Graphing Utility
- Convince Me diagram.

Supporting Documents:
Below are some commercials and videos of interest.
1960s Commercial
1970s Helicopter Commercial
1998 vs 2012 Mold Commercial
Comparison of cheap ball to Wham-O Super Ball

<table>
<tr><td colspan="2">Theme 2: Functions Part 1
Context: 6</td><td>Number of Class Periods:
4 (45 minutes)</td></tr>
<tr><td colspan="3">Lesson Title: Follow the Bouncing Ball</td></tr>
<tr><td>Ohio Learning Standards:
A.CED.2
A.CED.3
A.REI.10
F.IF.4
F.LE.1
F.LE.2
S.ID.6
S.ID.8
S.ID.9</td><td>Remediation-Free Standards:
MP.PS.A
MP.PS.D
MP.AUTT.B
MP.CMI.A
MP.CMI.B
MP.CMC.A
A.EI.D
A.FA.E
A.OAO.C
A.G.A
PS.RUD.A
PS.RUD.C</td><td>Notes:
Special thanks to Rodney Null for this lesson.
Students may choose the independent variable to be bounces and then measure each height. If they do, it will lead to the wrong type of function and not the data we want them to get.
The use of slow-motion video (phone) assists in finding accurate measurements of the height of bounces.
Correlation coefficient, r, is appropriate for this context because the data relationship is linear. Note that correlation coefficient should NOT be used when talking about nonlinear relationships.</td></tr>
<tr><td colspan="3">Suggested Reinforcements:
Pay It Forward task found in your folder.
How Much Money Should Doctor Evil Demand? by Robert Kaplinsky is a task exploring exponential equations.</td></tr>
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