High School Mathematics Transition Course

Theme 0: Problem Solving: Introduction to Mathematical Process **Days:** 13 - 16	**Number of Class Periods:** 4 (45 minutes)
Lesson Title: Days 13-16: Modeling-Driving for Gas	
Summary: ● Students use the Spy, Analyze, Model routine to model the Driving for Gas problem.	**Standards of Mathematical Practices:** ● MP.1 ● MP.2 ● MP.3 ● MP.4 ● MP.6

Goals and Objectives:

- Students will
  - Implement the Spy, Analyze and Model routine.
  - Make sense of problems and persevere in solving them.
  - Construct viable arguments and critique the reasoning of others.
  - Model with Mathematics.
  - Attend to Precision.
- Teachers will
  - Implement the Spy, Analyze, Model routine.
  - Ask advancing questions.

Concepts from Previous Mathematical Experiences to be Applied:

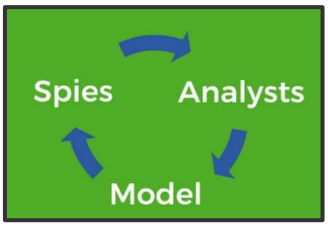
- Multiplication, proportions, rates, distance

Instructional Procedures:

Day 1

- **Part 1** ***(10 minutes)***
  - Give Students Modeling Process Handout. Note: There are several versions in your folder. Choose your favorite. *Note: the other versions are from Ohio's Learning Standards and the GAIMME report. This version is from Robert Kaplinsky's Spies, Analyze, Model graphic*.
  - Explain to students that they are going to do a modeling task. Ask students how many of them like to make their own decisions? Tell them modeling is what people do in real life. It's about decisions—recognizing decisions, making a decision, and revisiting decisions. (Whenever possible, let students "drive" the discussion, helping them especially with the "recognize" part of their own decision-making.)
  - Explain that modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight to real-world phenomena. Relate this to Covid and how decisions were based on models, and when new information came out those models and decisions had to be revisited.

- Show students a modeling graphic to help them understand the process.
  - **Step 1: Spies**–*What information do I need and how do I acquire it?*
  - **Step 2: Analyze**– *Take the information and work with it to figure out how to use it.*
  - **Step 3: Model**–*Use the model and verify that it works.*
  - *Note: For a video about Robert Kaplinsky's Model and the CIA show "Why We Should Reconsider Using Word Problems (And What We Should Be Doing Instead, High School" from 17:40-23:15.*



- Explain the General Modeling Principles
  - It is usually easier to develop useful models by starting with a simplified version of a situation than with one that is closer to reality. The first model is rarely the final model.
  - Pay attention to what you "want." If you need a number, make up a value, but note what you did. That number may become a variable later.
  - Be conscious of decisions/assumptions.
  - Ask, "What if?" What would happen if (pick a number or assumption) changed?
  - Ask, "What question are we trying to answer? How can I 'measure' that?"

- **Part 2 (25-35 minutes)**
  - Give students the Driving for Gas Modeling Problem handout. Students will need access to technology. *Note: In the beginning as students are learning about modeling, problems may need to be scaffold with handouts like this, as the year progresses, take away the scaffold.*
  - Read the problem together. Have students vote on whether the drive across town is worth it. Have students defend their choices. Explain to them, that we have to defend our answers mathematically. Students may respond that they have not been given enough information to answer the question, and that is the point. Most real-world problems require you to seek out information. *(2-5 minutes)*
  - Have students read the problem individually and sketch out their ideas under Step 1: Spies. *(3-5 minutes)*
  - Have students then discuss the problem in their groups sharing ideas and researching information. Ask groups these types of questions: *(5-10 minutes)*
    - Describe the problem that your team has been asked to solve. What information do you need in order to solve this problem?
    - What does a solution to your problem "look like"? Is it possible that your solution will have more than one reasonable answer? Why?
    - What is the specific problem your model is going to solve? How can you complete this sentence "Our model will tell you _____"
  - Discuss Step 1 as a whole group. Have groups share their thought processes about how to approach the problem.
    - To help clarify the question. Ask students "When wouldn't it be worth it to drive across town for gas?"
    - Help students refine their problem if needed. *(5-10 minutes)* Examples could include something like---
    - Determine which costs less---
    - Purchasing gas at Gas Station 1, which is on our planned route, or

        - Traveling out of our way to Gas Station 2 (which sells gas at a cheaper rate) to purchase gas there.
        - Encourage them to draw a diagram.
    - Stating assumptions will be new to students. Explain to students what assumptions are. Discuss why assumptions are needed. Give them a few examples and discuss why they would be helpful. such as— *(3-5 minutes)*
        - Gas costs less at the gas station that is out of our way.
        - The fuel economy of the car remains constant.
        - If we choose to go out of our way, we will consider the added cost of the mileage between the gas station we would have gone to and the further station, and back.
        - We will purchase the same amount of gas, no matter which gas station we choose.
        - We will be driving a _____ car.
- **Part 3 (time remaining)**
    - If time permits, in groups, have students finish working on the Spies section or begin to work on the Analyze section of the modeling process.

Day 2

- **Part 1 (20-25 minutes)**
    - Have students work to solve the problem in groups. Depending on where they are in the process (spies or analyze), ask students some of the following questions:
        - Of the factors you have identified as being important to the problem, which values change, and which stay the same?
        - What assumptions do you need to make in order to find a solution? What prompted you to make these assumptions?
        - What would cause you to change an assumption?
        - What are the primary factors that you have identified as being important to the problem? How do you plan to incorporate these values into your model?
        - Where did you find numbers (or data) to use in your model?
        - When researching, did you find more than one value for a factor? How will you determine which value(s) to use in your model?
        - Describe the specific problem your model needs to solve. What are the units associated with the solution(s)? (If appropriate) How does your solution imply the need for quantification?
- **Part 2 (15-20 minutes)**
    - As a whole group. Refer students back to the Model section of their process. Explain that “modeling” is a process, whereas a “model” is a thing (graph, picture, chart, equation, etc.) that we use to communicate a real-world situation. Explain that now that they have the model, they have to use it to answer the original question and verify if it works. Although their model may be specific to one situation, push students to generalize it to more situations. (2-5 minutes)
    - Let students work in groups to reflect on their model. Explain they many need to make a better model (5-10 minutes)

Day 3

- If time permits start with a number talk.
- Show students the presentation rubric.
- Have students work on their presentation.

Day 4

- Have groups present and grade each other using the rubric.
- Discuss any questions that the models produce and how they could be explored.
- Have students use the math practices rubric and discuss which practices they used and which level they are at.

Teacher Focus:

- **Day 1 Prep**
  - Read the GAIMME Report. It's rather long, so read the following pages (28 total) that address the task directly:
  - But What is Mathematical Modeling? (page 8)
  - From Mathematics Setting to Application to Modeling Problems (page 8)
  - At the High School Level (page 9-10)
  - The Modeling Process and background on today's example (pages 12-21)
  - Classroom Facilitator and Coach (pages 99-101)
  - Driving for Gas (pages 196-207) Note: Save for day 2 if you've had too much reading.
  - Read "Mathematical Modeling: Do You Need Better Spies or Analysts" a blog post by Robert Kaplinsky
- **Day 2 Prep**
  - The focus of this day is questioning. Practice answering questions with questions. Review the questions to ask in the instructional procedures section above.
- **Days 3 and 4 During Class**
  - As students work on their presentation ask them some of the following questions:
    - Explain the process your team used to develop a solution.
    - Explain the mathematics used to develop your team's solution.
    - Who are you sharing your results with? Who is the audience for your report?
    - (If appropriate) How did each of your teammates participate in the modeling process? What did you learn from the other members of your team?
    - What are three (to five) things you want anyone reading/hearing your report to understand about your model?
    - With your audience in mind, how can you share your results in a clear and concise way? What are three (to five) things you want anyone reading/hearing your report to understand about your solution? With your audience in mind, how can you share your results in a clear and concise way?
  - As students finish up ask them one or two reflection questions (Some of these may be discussed as a whole class after the presentations):
    - Did you need to revise your model at any point during the activity? If so, why? How did you fix the model?
    - Can you identify a math idea that was key to your ability to develop a model?
    - How did your modeling strategy change throughout the work period?
    - What advice would you give to a classmate (or yourself) prior to developing a mathematical model?
    - Given the chance to do this activity again, what would you do? Would you use the same approach, or would you alter your plan?

<table>
<tr><td colspan="3">■ If given more time, what would you (or your team) do to improve your model or results? What was the most surprising (or unexpected) aspect of this project?</td></tr>
<tr><td colspan="2">Differentiation Strategies:</td><td>Assessment:</td></tr>
<tr><td>Extension Suggestions:</td><td>Materials and Resources:
● Modeling Process Handout-Spies and Analyze version
● Gas Station Modeling Problem Spies and Analysts
● GAIMME Presentation Rubric</td><td>Supporting Documents:
● GAIMME report</td></tr>
<tr><td>Ohio Learning Standards:
● N.Q.1
● N.Q.2
● N.Q.3</td><td>Remediation-Free Standards:</td><td>Notes:</td></tr>
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